Composite Shaft Optimization Using Simulated Annealing, Part II: Strength and Stresses

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Part I of this paper deals with natural frequency optimization. In Part II, shaft weight is minimized with the constraints imposed on the natural frequency obtained from Part I, shaft buckling torque and strength criteria. The shafts are subjected to unbalance excitation and transmitted torque. Tsai-Wu (Quadratic Failure Criteria) is adopted in the formulation. Optimization is further extended to reduce stresses along the shaft axial length in an attempt to obtain uniform stresses. Similar to that of Part I, shaft of uniform/variable (UV), variable/uniform (VU), and variable/variable (VV) fiber layup and wall thickness are analyzed.

Keywords: Composite shafts; Optimization; Stresses; Strength; Simulated annealing; Unbalance excitation

In the present paper the natural frequency analysis presented in Part I has been extended to evaluate stresses induced in the shaft due to unbalance excitation and torque transmission. The literature on dynamics of composite shafts deals primarily with critical speeds and unbalance response which provides only part of the information needed by the designer. There is not much work available in the literature which deals with stresses in composite shafts. Singh and Gupta (1996) have determined modal stresses in the uniform wall thickness composite shaft cross-section by a layer-wise beam theory. Gupta (1997) identified the important considerations in the dynamic design of composite rotors. These include torque transmission, stresses, and proper placement of critical speeds. Stresses induced in composite shaft for different cases have been evaluated and studied in detail. Gubran et al. (2000) have studied the stresses induced in composite shafts due to

unbalance excitation and transmitted torque. Uniform and axially variable wall thickness shafts have been considered. Their analysis has shown the possibilities of improvement in the dynamic performance and reduction of maximum stresses induced in the shaft by extending the design of uniform wall thickness shaft to axially variable wall thickness shaft, keeping the mass of the shaft same as that of the uniform wall thickness shaft. In the present paper, Simulated Annealing Algorithm (SA) described in Part I is used as a mathematical tool for optimizing different ply angles, ply thicknesses and their stacking sequence along the shaft length for achieving superior dynamic performance and to reduce maximum stresses induced in the shaft.

ANALYSIS

Rayleigh–Ritz procedure described in Part I of this paper is extended for evaluating stresses induced in the shaft due to unbalance excitation. The stresses induced in the shaft due to torque transmission are evaluated and superimposed on stresses due to unbalance excitation.

Unbalance Response Formulation

The unbalance of any mass m_r situated at a distance r from the geometric centre of the shaft and at an angular location Φ_r (as shown in Figure 1) is given by $u_r = m_r r \angle \Phi_r$. The kinetic energy of all unbalance masses is obtained by summing up the kinetic energy of the individual masses. For a synchronous circular whirl, the whirl speed Ω is equal to the rotational speed of the shaft ω . The total kinetic energy can be expressed as

$$T_u = \omega e^{i\omega t} \sum_{r=1}^n [u_r e^{i\phi_r} \{\dot{v}(x_r) - i\dot{w}(x_r)\}] \quad [1]$$

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FIGURE 1 Unbalance excitation.

Adding the terms of Eq. [1] to the kinetic energy expression [5] in Part I and setting the Lagrangian stationary with respect to the solution coefficients leads to a nonhomogeneous complex quadratic equations given below:

$$[-\omega^2[M] + i\omega[D] + [K]]\{X\} = [F] \quad [2]$$

The unbalance response is obtained from the solution of the simultaneous algebraic equations which give the unknown coefficients.

Displacement Field

The coefficients given by vector $\{X\}$ obtained from the solution of algebraic Eq. [2] can be used for evaluating the

displacement field. The vector $\{X\}$ is

$$\{X\} = [W_1, W_2, \dots, W_k, V_1, V_2, \dots, V_k, A_1, A_2, \dots, A_k, B_1, B_2, \dots, B_k, \Psi_1, \Psi_2, \dots, \Psi_k]^T \quad [3]$$

Substituting the coefficients from vector $\{X\}$ in Eq. [6] of Part I gives beam displacement field $\bar{w}$, $\bar{v}$, $\bar{\alpha}$, $\bar{\beta}$, and $\bar{\varphi}$ in Cartesian coordinates. Referring to Figure 2, the displacements in cylindrical coordinates can be written as

$$\begin{aligned} \bar{U}(x, \theta, z) &= -(r+z)(\bar{\alpha}(x) \cos \theta + \bar{\beta}(x) \sin \theta) \\ \bar{V}(x, \theta, z) &= \bar{v}(x) \cos \theta - \bar{w}(x) \sin \theta \\ \bar{W}(x, \theta, z) &= \bar{w}(x) \cos \theta + \bar{v}(x) \sin \theta \end{aligned} \quad [4]$$

Strain-Displacement Relations

The displacement field can be written in a more general form as

$$\begin{aligned} \bar{U}(x) &= \bar{U}_0(x, \theta, z) + z\bar{\psi}(x, \theta, z), \\ \bar{W}(\theta) &= \bar{V}_0(x, \theta, z) + z\bar{\phi}(x, \theta, z), \\ \bar{V}(z) &= \bar{W}_0(x, \theta, z) \end{aligned} \quad [5]$$

where $\bar{U}_0$, $\bar{V}_0$, $\bar{W}_0$ are the displacements in x , θ , and z directions at the mid surface (thickness $z=0$), and $\bar{\psi}$, $\bar{\phi}$ are the rotations of the normal to the middle surface in $x-z$ and $x-\theta$ planes respectively.

The strain displacement relations can be written as

$$\begin{aligned} \epsilon_{xx} &= U_{,xx} \quad \epsilon_{\theta\theta} = (W + V_{,\theta})/(r+z) \\ \gamma_{x\theta} &= V_{,x} + U_{,\theta}/(r+z) \end{aligned} \quad [6]$$

where, $\Psi = \alpha \cos \theta + \beta \sin \theta$ and Φ is assumed to be zero.

Stress-Strain Relations

The stresses at any k th lamina are calculated by the stress-strain relation as

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{\theta\theta} \\ \tau_{x\theta} \end{Bmatrix}^k = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{21} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{61} & \bar{Q}_{62} & \bar{Q}_{66} \end{bmatrix}^k \begin{Bmatrix} \epsilon_{xx} \\ \epsilon_{\theta\theta} \\ \gamma_{x\theta} \end{Bmatrix}^k \quad [7]$$

Torque Formulation

For symmetric laminate, the steady torque applied to the shaft gives rise to shear strain $\gamma_{x\theta}$. This can be calculated by

$$\gamma_{x\theta} = \frac{T}{2\pi r_m^2 t G_{xy}} \quad [8]$$

FIGURE 2 Shaft cross-section during bending.

Equation [8] is used for evaluating the stresses induced in the shaft due to torque transmission. For linear analysis, the stresses induced in the shaft due to unbalance excitation are superimposed on those due to torque transmission to obtain total stresses.

Failure Strength Analysis (Tsai-Wu Failure Criterion)

Shaft Optimization for minimum weight with constraints imposed on the natural frequency and buckling torque has been extended to include shaft stresses and strength as a part of the constraint. Different loading conditions namely, unbalance excitation only or unbalance excitation, transmitted torque and axial force either singly or in conjunction with each other are used. Tsai-Wu criterion, which takes into account interactions of stresses, is used in the analysis applied at ply level. Tsai-Wu failure criteria can be written as

$$F_1\sigma_1 + F_2\sigma_2 + F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + F_{66}\tau_{12}^2 - \sqrt{F_{11}F_{22}}\sigma_1\sigma_2 = 1 \quad [9]$$

where

$$F_1 = \left(\frac{1}{\sigma_1^T} + \frac{1}{\sigma_1^C} \right), \quad F_{11} = -\frac{1}{\sigma_1^T \sigma_1^C}$$

$$F_2 = \left(\frac{1}{\sigma_2^T} + \frac{1}{\sigma_2^C} \right), \quad F_{22} = -\frac{1}{\sigma_2^T \sigma_2^C}$$

$$F_{66} = \left(\frac{1}{\tau_{12}^F} \right)^2$$

For safe design, Tsai-Wu criteria can be written as

$$TW \leq 1 \quad [10]$$

where TW represents the L.H.S. of Eq. [9]. Table I gives the values of the failure stresses and values of $F_1, \dots, F_6$ for graphite reinforced material, Hyer (1997).

Problem Description and Formulation

The shaft weight is minimized with the constraints imposed on different ply strengths, shaft buckling torque and shaft

natural frequency. Different ply fiber angles, their stacking sequence, and their thicknesses are kept as design variables. Stresses induced in the shaft due to unbalance excitation, transmitted torque and axial loading have been evaluated. Different shaft design alternatives described earlier (in Part I) have been considered. Dynamic stresses are strongly dependent on shaft operating speed and shaft natural frequency. In order to have a basis for comparison of stresses induced on different shaft designs, their fundamental natural frequency is kept same as that of the uniform fibre layup and uniform wall thickness shaft (UU). Operating speed for different shaft design alternatives is also kept the same as that of the uniform fibre layup and uniform wall thickness (UU) shaft which is 90% of its first natural frequency.

In order to evolve an iterative procedure of design and optimization, the optimization subroutines are linked with the main subroutine which calculate shaft natural frequencies, buckling torque and shaft stresses. The detailed procedure of shaft design and optimization is described in Figure 3. The formulation of the problem here is the same as that in step 2, (shaft weight minimization) described in Part I, except that the constraint of failure strength criterion (Tsai-Wu failure criterion) described in Eq. [10] is added. However, for the case of shaft optimization for minimum weight and reduction of maximum stresses (uniform stress case) along the shaft length, two objective functions are considered with constraints imposed on shaft buckling torque and shaft natural frequency.

RESULTS AND DISCUSSION

The shaft of 1 m length studied earlier (in Part I) is considered. A disc of 7 kg mass is placed at its mid-span. Shaft material properties are kept the same as that given in Table I (of Part I). Two separate cases have been considered here. In the first, Tsai-Wu failure criterion applied to different plies at different locations along the shaft axial length is added to the constraints of step 2 (shaft weight minimization described in Part I). In the second, reduction of maximum stresses along the shaft length is added to the objective function of shaft weight minimization; however the constraints are kept the same as that in step 2 of Part I.

In all cases the initial UU case (un-optimized) results for stresses induced in the uniform fiber layup and uniform wall thickness design, which is optimized in step 2 of Part I, is taken as the initial design and basis for comparison with different design alternatives.

The optimization subroutines have been linked to the main subroutines for evaluation of natural frequency and stresses. All subroutines together constitute a closed loop system for shaft design and optimization.

TABLE I Failure stresses for graphite/epoxy

$\sigma_1^T = 1500$ Mpa	$F_1 = 0.1333$ (1/Gpa)
$\sigma_1^C = -1250$ Mpa	$F_2 = 15.0$ (1/Gpa)
$\sigma_2^T = 50$ Mpa	$F_{11} = 0.533$ (1/Gpa) ²
$\sigma_2^C = -200$ Mpa	$F_{22} = 100$ (1/Gpa) ²
$\tau_{12}^F = 100$ Mpa	$F_{66} = 100$ (1/Gpa) ²

FIGURE 3 Flowchart for shaft design and optimization.

Case 1. Shaft Optimization Including Tsai-Wu Failure Criterion

Shaft optimization have been studied under different loading conditions of unbalance excitation of 0.01 kgcm, transmitted torque and axial loading. In all loading conditions, the operating speed for different design alternatives is kept as 90% of the first natural frequency for the initial uniform (UU) design. However, different loads have been considered for transmitted torque and axial loading.

(i) *Unbalance Excitation, $F=T=0$* For this case the constraint imposed on the failure criterion is not active. This is clear from the total sum of the left hand side of Tsai-Wu failure criteria which is found to be much less than unity. For this, the problem is reduced to minimization of shaft weight under the constraints of buckling

torque of 1910 Nm and natural frequency of (57 Hz). Results presented in Table II show the reduction of the shaft weight as obtained by using different design alternatives (i.e., UU, UV, VU and VV). Optimized results for VV case show a reduction in the shaft weight by 11% (i.e., from 0.239 kg to 0.213 kg) compared to that of the initial design. Results for this case also show that the thickness of the shaft at the mid-section is more compared to the thicknesses of the first and second sections. It is about 0.307 mm for the first section, 0.368 mm for the second section and 0.902 mm for the mid-section. Similarly the ply angles at the mid section (14.054° and -16.912°) are such that it has higher stiffness. Results for other designs (UV and VU) are almost similar to that of the initial uniform design. This is expected for the present load case in which the strength constraint is not operative.

TABLE II Optimized results for a shaft of 1 m length with disk subjected to unbalance excitation only

Shaft design	Shaft configuration		Freq. (Hz)	Buckling torque (Nm)	Shaft weight (kg)
	Ply angles (Deg.)	Thickness (mm)			
Initial UU	(10.643/−10.666) ₂	(0.0902/0.163) ₂	57	1910	0.2400
opt. UU	(13.026/−14.029) ₂	(0.1482/0.1051) ₂	57	1910	0.2399
opt. UV	(13.374/−18.281) ₂	(0.0551/0.151) ₂ (0.192/0.1008) ₂ (0.158/0.1045) ₂	58	1910	0.2388
opt. VU	(24.490/−26.712) ₂ (16.282/−7.293) ₂ (16.960/−6.982) ₂	(0.1430/0.1048) ₂	58	1910	0.2350
opt. VV	(11.171/−26.098) ₂ (33.189/−16.961) ₂ (14.054/−16.912) ₂	(0.0614/0.0923) ₂ (0.0203/0.1635) ₂ (0.437/0.01395) ₂	58	1910	0.2135

(ii) *Unbalance Excitation*, $T = 500 \text{ Nm}$ and $F = 100 \text{ N}$
 Analysis of this case showed that the initial design could not withstand these loads. This was clear from the fact that parameter TW became more than unity. For this case, the constraint imposed on failure criterion becomes active and

parameter TW for the optimized results of UU, UV, VU, and VV is made equal or less than unity. Optimized results for different designs are presented in Table III which give a solution that satisfies both constraints of buckling torque and natural frequency. Results also show the superiority of

TABLE III Optimized results for a shaft of 1 m length with disk subjected to unbalance excitation, transmitted torque of 500 Nm, and axial force of 100 N

Shaft design	Shaft configuration		Freq. (Hz)	Buckling torque (Nm)	Shaft weight (kg)
	Ply angles (Degrees)	Thickness (mm)			
Initial UU	(10.643/−10.666) ₂	(0.0902/0.1631) ₂	57	1910	0.2400
opt. UU	(12.511/−3.552) ₂	(0.177/0.076) ₂	58	1935	0.2399
opt. UV	(10.297/−9.414) ₂	(0.162/0.0335) ₂ (0.112/0.078) ₂ (0.0518/0.367) ₂	58	1910	0.2263
opt. VU	(10.279/−9.282) ₂ (10.256/−9.506) ₂ (10.437/−9.537) ₂	(0.250/0.010) ₂	58	2006	0.2464
opt. VV	(10.242/−8.586) ₂ (10.283/−8.364) ₂ (10.265/−9.253) ₂	(0.1566/0.010) ₂ (0.1881/0.0201) ₂ (0.389/0.037) ₂	57.4	1895	0.2229

TABLE IV Optimized results for a shaft of 1 m length with disk at mid-span subjected to unbalance excitation only

Shaft design	Shaft configuration		Freq. (Hz)	Buckling torque (Nm)	Shaft weight (kg)
	Ply angles (Degrees)	Thickness (mm)			
Initial UU	(10.666/−10.643) ₂	(0.0902/0.163) ₂	57.27	1910	0.2399
opt. UU	(6.021/−13.346) ₂	(0.132/0.124) ₂	57.41	1910	0.2421
opt. UV	(0/−16.092) ₂	(0.0251/0.271) ₂ (0.063/0.0845) ₂ (0.268/0.0623) ₂	56.93	1910	0.2307
opt. VU	(3.135/−17.872) ₂ (6.582/−0.394) ₂ (6.449/−4.323) ₂	(0.022/0.231) ₂	60	1910	0.2400
opt. VV	(14.558/−30.234) ₂ (8.279/−26.95) ₂ (1.749/−12.603) ₂	(0.152/0.100) ₂ (0.1400/0.0644) ₂ (0.228/0.1252) ₂	57.20	1910	0.2390

design VV over the other designs. It shows that UV design is having maximum thickness at the mid-section of the shaft. It is observed that about 6% (i.e., from 0.239 kg to 0.226 kg) reduction in weight is achieved from UU to UV case as compared to 7% (i.e., from 0.239 kg to 0.222 kg)

between UU and VV case. This clearly shows that for this case substantial improvement is achieved by variable wall thickness and relatively lesser effect obtained by variable fibre layup. For this load of maximum transmitted torque (500 Nm), shear stress is maximum. This is clear from the

FIGURE 4 Stresses distribution (unbalance excitation only): (a) Longitudinal stress (b) Hoop stress, and (c) Shear stress.

major contribution of the term $F_{66}\tau_{12}^2$ to the total sum of the left hand side (TW) of the Tsai-Wu failure criteria. Generally for metallic shafts, shear stress induced in the shaft as a result of transmitted torque will remain constant through out the entire shaft length. However for composite shafts this is not the case because of varying degree of shear-normal coupling present along the shaft axis.

Case 2. Shaft Optimization for Minimization of Maximum Stresses Along the Shaft Length

In the previous section optimization was carried for weight minimization taking into account the constraints on the shaft natural frequency, buckling torque and strength. This work has been extended here to include stresses induced in the shaft along the axial length as a result of unbalance excitation and/or transmitted torque. The stresses induced in the 1 m uniform fibre layup and uniform wall thickness shaft studied in Part I is kept as the basis for comparisons with different design alternatives. In general, the stresses induced in the shaft are strongly dependent on the shaft operating speed and its natural frequency. In this work, in order to insure that the reduction of stresses is not due to the change of gap between the operating speed and shaft natural frequency, the constraint has been imposed on the shaft natural frequency to remain the same as obtained in the optimized result of the UU case obtained in Part I. The stresses have been evaluated at the outer ply angles for different designs of UU, UV, VU, and VV at different locations along the shaft axial length. The optimization problem is formulated as

$$\text{Min: } f_1 = \text{weight} \quad [11]$$

$$f_2 = \text{stresses} \quad [12]$$

Subjected to:

$$f_s = f_{s1} \quad [13]$$

$$T_{\text{Buc}} \geq F.O.S \times T \quad [14]$$

$$r_{\text{inner}} \leq R \quad [15]$$

Equations [11] and [12] are the objective functions which tend to minimize shaft weight and stresses along the axial length. However, Eqs. [13–15] represent the constraints imposed on the shaft natural frequency, buckling torque, and shaft geometry to remain as that of Part I. Design parameters include ply angles, their stacking sequence and the individual thickness of each ply.

In this study, a shaft of 1 m length has been studied for optimized distribution of stresses along the shaft axial length. Each shaft is subjected to unbalance excitation of 0.01 kg cm, or unbalance excitation and transmitted torque of 100 Nm. The operating speed for different design alternatives is kept as 90% of the first natural frequency of the initial uniform design. Optimization has been carried out for different design alternatives (UU, UV, VU, and VV).

(i) *Shaft of 1 m length with 7 kg disk at its mid span subjected to Unbalance Excitation (U) only* For 1 m long shaft subjected to unbalance excitation only, the maximum stresses developed are normal stresses at the midspan of the shaft. Optimized results obtained for this case have been presented in Table IV and Figures 4a, b, and c. For optimized (UU) shaft, a reduction of the order 5% (i.e., from 2.78 Mpa to 2.65 Mpa) in the maximum normal stress along with 1.2% (i.e., from 0.239 kg to 0.242 kg) increase in the shaft weight is shown. Results for the optimized UV, VU, and VV designs show a reduction in the maximum normal stress by about 75–80% (i.e., from 2.78 Mpa to 0.594 Mpa and 0.553 Mpa) for almost the same shaft weight as in the initial uniform design. This percentage reduction in the maximum stress can be explained by the increase in the shaft thickness at the middle-section of the shaft and a reduction in the shaft thickness at the end sections and also because of placement of ply angles of lower order (close to zero) at the middle-section of the

TABLE V Optimized results for a shaft of 1 m length without disk subjected to unbalance excitation and transmitted torque (U + T)

Shaft design	Shaft configuration		Frequency (Hz)	Buckling torque (Nm)	Shaft weight (kg)
	Ply angles (Degrees)	Thickness (mm)			
Initial UU	(10.643/–10.666) ₂	(0.0902/0.163) ₂	439.50	1910	0.2399
opt. UU	(21.901/–10.207) ₂	(0.0095/0.241) ₂	439.19	1903	0.2374
opt. UV	(23.659/–12.540) ₂	(0.0103/0.240) ₂ (0.010/0.26) ₂ (0.010/0.186) ₂	439.98	1910	0.2357
opt. VU	(39.680/–39.445) ₂ (24.802/–18.460) ₂ (7.836/–5.667) ₂	(0.054/0.184) ₂	440.02	1910	0.2251
opt. VV	(40.00/–40.00) ₂ (30.00/–30.00) ₂ (10.00/–10.00) ₂	(0.0397/0.139) ₂ (0.01/0.101) ₂ (0.186/0.286) ₂	440.21	1910	0.1999

shaft. This leads to stiffening of the shaft at the middle and hence decreases shaft deflection and the stresses at the middle section. Results obtained for VU also show that the constraint imposed on the natural frequency is not fully satisfied, i.e., there is an increase in the shaft natural

frequency by about 5% (i.e., from 57.3 Hz to 60 Hz). Keeping the shaft operating speed same as in the case of initial design, the separation margin increased leading to a reduction in the shaft stresses. It is clearly observed from Figure 4b, that identical results are obtained for normal

FIGURE 5 Stresses distribution (unbalance excitation and transmitted torque): (a) Longitudinal stress (b) Hoop stress, and (c) Shear stress.

stresses from UV and VV cases. This implies that no additional benefit is obtained by allowing fibre layup to vary at different sections along the shaft axial length.

(ii) *Shaft of 1 m length without disk subjected to Unbalance Excitation and Transmitted Torque (U+T)*: A shaft of 1 m length without disk is subjected to unbalance excitation and transmitted torque. The shaft is rotating at a speed of 23733 RPM which is 10% below its first critical. At such a high speed, the stresses developed in the shaft are mainly due to unbalance excitation. Torque transmission also gives rise to a shear stress which due to coupling mechanism gives rise to additional normal stresses. Optimized results for different design alternatives are presented in Table V and Figures 5a, b, and c. Results for UU and UV cases show a reduction in the maximum normal stresses at the shaft midspan by about 25% (i.e., from 167 Mpa to 125 Mpa) and an increase of the same at the shaft ends. This makes the stresses along the shaft axial length more uniformly distributed. Results obtained for the same design (i.e., UU, UV) also show an increase in the shear stress throughout the shaft length. This can be explained as follows. In the optimized results for UU and UV, the outer ply angle (at which the stresses have been evaluated) is moved towards higher values (45°); this leads to an increase in the coupling coefficients ($\bar{Q}_{16}$ and $\bar{Q}_{26}$) and a decrease in the longitudinal term ($\bar{Q}_{11}$) which gives a reduction in the normal stress and an increase in the shear stress. Results for VU and VV designs show a reduction in the normal stress by about 28% (from 167 Mpa to 120 Mpa) and 62% (from 167 Mpa to 63 Mpa) respectively, while the shaft natural frequency is kept same as that in the initial design. Results obtained show a reduction in the shaft weight for designs UU, UV, VU and VV. This reduction is maximum for the VV case, which is about 20% (i.e., from 0.239 kg to 0.199 kg). Comparison of normal stresses for different cases clearly show the benefit achieved by varying shaft wall thickness (UV) in reduction of stress and in uniform distribution of maximum normal stress. However for the case (VU), in spite of the reduction obtained in normal stress, there is no uniform distribution of stress along the shaft length. In general, maximum reduction and uniform distribution of stress are obtained by allowing both the shaft fibre layup and wall thickness (VV) to vary along the shaft length.

CONCLUSION

Optimization using Simulated Annealing Algorithm used in Part I is extended to include strength and stresses

induced in different design alternatives. Two case studies have been taken. In first, Tsai-Wu failure criterion is included in the constraints and the shaft has been subjected to different loading conditions. Results show that optimized design of different alternatives gives rise to a safe design and also it gives a reduction in the shaft weight compared to the initial uniform design. In the second case, reduction of maximum stresses along the shaft length is taken as an objective function in addition to weight minimization. The following are the main conclusions drawn,

- (1) Optimized results for all different designs (UV, VU, and VV) show a reduction in the maximum stress, keeping the shaft natural frequency and weight almost the same as the case of the initial design.
- (2) Optimized results for variable fiber layup and variable wall thickness (VV) show superior results in reducing shaft stresses and even shaft weight, keeping shaft natural frequency same as that of initial design.
- (3) For uniform fiber layup and variable wall thickness (UV), optimized results show that the middle portion of the shaft tends to have maximum thickness compared to the shaft end portions.
- (4) Observation of the optimized results presented in different tables show that there is a tendency to cancel out coupling effects at the location of stress evaluation.

NOMENCLATURE

TW	Tsai-Wu failure criteria
$\bar{U}, \bar{V}$	Inplane displacement in the x and θ directions
$\bar{W}$	Transverse displacement along r direction
u_r	The amount of unbalance, kgm

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