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General Influence Coefficient Algorithm in Balancing of Rotating Machinery

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The General Influence Coefficient Algorithm (GICA), developed in this article, is a new calculation method for Influence Coefficients (ICs) with a general formula. Compared to the traditional calculation method, GICA can solve the ICs' calculation task when the group of trial weights are installed on the rotor each time, the trial weights are retained on the rotor systems, or there is redundant trial balancing data, when even part of the ICs is known. GICA is also a powerful tool for refining the ICs from redundant balancing data or historical balancing data and serves as a general algorithm. With the general matrix formula, GICA is ready to be applied in a computer-aided balancing system as the key part of calculation software. Examples in industry are also presented to demonstrate the aplication of this new algorithm.

Keywords Rotating machinery, Balancing, Influence coefficient, Computer aided, Optimization, Rotor

INTRODUCTION

High-speed balancing is a requirement for many types of rotating machinery such as pumps, fans, steam turbine generators, centrifugal compressors, and so on. Especially in the power generation industry, steam turbine generators require periodic field balancing in order to control vibrations. Field balancing is a challenging task because it must be finished with a minimum number of balancing trial runs as soon as possible to reduce the expenditure. There is a great need for effective balancing especially field balancing.

The Influence Coefficient method and modal balancing method are two major balancing methods that are used extensively today (Goodman, 1964; Zori et al., 1982). The essence of the IC method is to apply m time's trial masses for m rotor balance planes existing in the rotor machinery to obtain all ICs between the rotor and the bearings (Rieger, 1988; Darlow, 1987; Tecza et al., 1991; Holdt, 1988). The principle of the modal balancing method is to balance rotors by their modal shapes. The shortcomings of the modal method and its variations, as well as the newly developed methods, lies not in the theoretical method, but in required pre-knowledge of the rotor system or the inconvenience for the field balancing of large scale of rotating machinery (Parkinson et al., 1980; Austrow, 1994; Ling and Cao, 1996; Hassan, 1995; Kang et al., 1996; Xu and Qu, 2001).

The IC method requires no more assumption other than the linearity of the rotor system and the measuring system, which makes it well suited to field balancing. It is an empirical method which characterizes a rotor system in terms of its response to single trial weights. A multi-speed, multi-plane matrix of ICs generated from this response is used to compute a correction mass. The IC is readily computer-automated and can address many classes of rotating machinery. As the balancing of rotating machinery is under their normal operating condition and incorporated with many auxilliary machines, the procedure is time-consuming and expensive, especially in the balancing of steam turbine generator units. There is a great need for minimization of the trial runs while the balancing quality can be maintained. Although there are a lot of variations of the IC method developed to optimize both the balance weight and the vibration level, the accuracy of the IC is the most inportant prerequisite. The GICA gives the opportunity to refine the ICs with good accuracy by making full use of the balancing data.

NEW CALCULATION METHOD OF IC

In order to minimize the number of trial runs, much work has been concentrated on the balancing calculation. Many methods belong to the category of the IC method; therefore, the accuracy of the ICs should be maintained. Furthermore, the way to deal with a lot of historical balancing data for a specific machine or similar type of machine is still an urgent problem. According to

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the conventional IC method, a single trial mass must be added to each plane and the IC for a specific plane can be derived with the data acquired after a trial run. In practice, instead of adding single trial masses one by one, groups of trial weights are always adopted in order to minimize the number of trial runs, some of which may be retained on the rotor system due to its good contribution to the attenuation of the vibrations. Furthermore, when the groups of trial weights are applied, the part of the influence coefficients of the specific planes may be known due to previous balancing tests. Under the conditions mentioned above, the traditional IC calculation Method seems to be inadequate to solve all these problems. Generally, the main features of GICA include:

- When groups of trial weights are applied, the IC for each plane can be extracted from the balancing data with the necessary number of trial runs.
- When balancing a machine, for which the historical balancing data is available, the groups of trial weights can be applied on the rotor, and the IC for the unknown planes can be extracted with the necessary number of trial runs.
- When the number of trial runs is greater than the necessary number for determining the unknown ICs, the algorithm can make full use of the redundant balancing information to refine the ICs for each plane.
- It can deal with the multi-plane and multi-point balancing task.
- The IC can be calculated even under the condition that the trial weights be retained on the rotor system due to good contribution to vibration reduction.

The proposed algorithm answers the questions of how to separate the IC for each plane and how to make full use of the redundant trial balancing information including present and historical data. Of course, if the single trial weight is applied, the algorithm for calculating ICs reduces to the traditional IC calculation method. Furthermore, the algorithm has a uniform formula for different calculation task.

THE DERIVATION OF GICA

For the balancing procedure, m balancing planes (axial location) are used for the addition of a correction mass. A total of n vibration measurements are taken to determine the response of the rotor. In general, these n measurements consist of data from sensors for each of the balancing speeds at which the vibration should be controlled.

It is essential that we take a sufficient number of trial runs in order to determine the ICs for each unknown correction plane. Generally, in order to determine the unknown ICs, the number of trial runs must be no less than the number of the unknown planes. It is admitted that the groups of trial weights can be added to the rotor; of course, it covers the case with a single trial weight at each trial run.

In order to make the derived formula applied in a more general condition, we suppose that there are u planes whose ICs are unknown among m planes, here $u \leq m$. Furthermore, there are h trial runs and h must be no less than u in order to fully determine the unknown ICs completely. Obviously, the number of trial runs is no less than the necessary number, i.e., h must be no less than u. The information of the trial runs may be redundant under specific conditions. For example, the number of runs including the trial runs and the check run(s) will be greater than the number of the unknown planes; the final correction mass added to the rotor can also be regarded as a "trial run." With the redundant information of the trial runs, we can refine the IC with the method developed in this article. This refining method has more advantages than the vector average method, which has been applied in engineering practice. In this article, the general formula of the IC method is derived using a matrix algorithm.

Matrix Construction

As vibration is a vector in the complex domain, vibration has its magnitude and angle. Thus, the entire matrix and its element are built in the complex domain. Before derivation, matrix construction should be made as follows:

1. Vibration influence matrix
   V_j is defined as the vector of vibration measurements just after adding the j-th set of trial weights; V_j^0 is the vector of vibration measurements just before adding the j-th set of trial weights. V_j and V_j^0 have the same format.

$$V_j = \begin{Bmatrix} v_{1j} \\ v_{2j} \\ \vdots \\ v_{nj} \end{Bmatrix} \quad V_j^0 = \begin{Bmatrix} v_{1j}^0 \\ v_{2j}^0 \\ \vdots \\ v_{nj}^0 \end{Bmatrix}$$

where $v_{ij} \in C$, $1 \leq i \leq h$, $1 \leq j \leq h$.

Note that V_j^0 is the vector of vibration measurements just before adding the j-th set of trial weights. This is the vector of vibration measurements, and it can be adjusted according to the status of the set of trial weights because the set of the trial weights may be removed or retained during the balancing procedure. Generally, V_j^0 can be expressed

$$V_j^0 = \begin{cases} V_0 & (j = 1) \\ V_r & (j \neq 1) \end{cases}$$

Here, r is the sequence number, i.e., the number of the last set of trial weights which is retained on the rotor system before the j-th set of trial weights is installed.

ΔV_j is defined as the vibration influence vector formed by the n different readings before and after adding j-th set of trial weights to the rotor. So ΔV_j can be expressed as

$$\Delta V_j = V_j - V_j^0$$

In its matrix form,

$$\Delta V_j = \begin{Bmatrix} \Delta v_{1j} \\ \Delta v_{2j} \\ \vdots \\ \Delta v_{nj} \end{Bmatrix}$$

Then the vibration influence matrix $[\Delta V]_{n\times h}$ is formed by ΔV_j, which can be expressed as follows:

$$[\Delta V]_{n\times h} = [\Delta V_1, \Delta V_2, \ldots \Delta V_h]_{n\times h}$$

In the form with its elements,

$$[\Delta V]_{n\times h} = \begin{bmatrix} \Delta v_{11} & \Delta v_{12} & \cdots & \Delta v_{1h} \\ \Delta v_{21} & \Delta v_{22} & \cdots & \Delta v_{2h} \\ \vdots & \vdots & & \vdots \\ \Delta v_{n1} & \Delta v_{n2} & \cdots & \Delta v_{nh} \end{bmatrix}_{n\times h} \quad [1]$$

2. Trial weight matrix

   P_j is defined as the trial weight vector formed by weights installed on m planes at the j-th trial run:

$$P_j = \begin{Bmatrix} p_{1j} \\ p_{2j} \\ \vdots \\ p_{mj} \end{Bmatrix}$$

Where $p_{ij} \in C$, $1 \le i \le m$, $1 \le j \le h$. Note that the expression above is a general form. P_j may contain '0' elements, meaning that no trial weight is installed on some individual planes in the set of the trial weights. The trial weight matrix formed by the trial weights vector after h trial runs can be defined as $[P]_{m\times h}$:

$$[P]_{m\times h} = [P_1, P_2, \cdots P_j]_{m\times h}, \quad 1 \le j \le h$$

In its matrix form with elements,

$$[P]_{n\times h} = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1h} \\ p_{21} & p_{22} & \cdots & p_{2h} \\ \vdots & \vdots & & \vdots \\ p_{n1} & p_{n2} & \cdots & p_{nh} \end{bmatrix}_{n\times h} \quad [2]$$

3. The IC matrix relating n measurements and m planes can be expressed as:

$$[K]_{n\times m} = \begin{bmatrix} k_{11} & k_{12} & \cdots & k_{1m} \\ k_{21} & k_{22} & \cdots & k_{2m} \\ \vdots & \vdots & & \vdots \\ k_{n1} & k_{n2} & \cdots & k_{nm} \end{bmatrix}_{n\times m}$$

$$k_{ij} \in C, \quad 1 \le i \le n, \quad 1 \le j \le m \quad [3]$$

Here, k_{ij} is the IC element, relating the IC on the i-th vibration measurement due to the trial weight installed on j-th plane. Note that there are unknown columns in $[K]_{n\times m}$, namely, $u(u \le h)$ columns are unknown and need to be determined.

Derivation Procedure

In practical field balancing, the supposition, in which there exists a linear relationship between the trial weights and the influence of the adding weights, is widely adopted. Obviously, this is the pre-condition of all IC methods and their relevant improved method or their variations. Consistent with the IC approaches to balancing, a linear relationship between excitation and response is assumed. This basic relationship is given by:

$$[K]_{n\times m}[P]_{m\times h} = [\Delta V]_{n\times h} \quad [4]$$

The above equation can be expressed in the following form with their elements,

$$\begin{bmatrix} k_{11} & k_{12} & \cdots & k_{1m} \\ k_{21} & k_{22} & \cdots & k_{2m} \\ \vdots & \vdots & & \vdots \\ k_{n1} & k_{n2} & \cdots & k_{nm} \end{bmatrix}_{n\times m} \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1h} \\ p_{21} & p_{22} & \cdots & p_{2h} \\ \vdots & \vdots & & \vdots \\ p_{m1} & p_{m2} & \cdots & p_{mh} \end{bmatrix}_{m\times h} = \begin{bmatrix} \Delta v_{11} & \Delta v_{12} & \cdots & \Delta v_{1h} \\ \Delta v_{21} & \Delta v_{22} & \cdots & \Delta v_{2h} \\ \vdots & \vdots & & \vdots \\ \Delta v_{n1} & \Delta v_{n2} & \cdots & \Delta v_{nh} \end{bmatrix}_{n\times h} \quad [5]$$

Note that there exist unknown columns in matrix $[K]_{n\times m}$. There are u planes which are to be determined among m planes, here $u \le m$. In order to solve Eq. [5] the IC matrix $[K]_{n\times m}$ can be separated into two matrices; thus, the matrix $[K]_{n\times m}$ can be divided into two parts. The left columns in $[K]_{n\times u}$ are linked with the planes with unknown ICs, and the right columns in

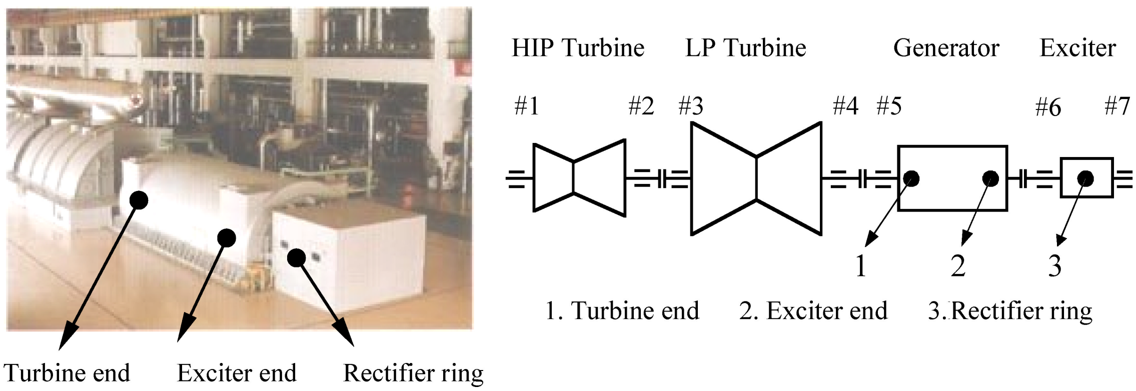

FIGURE 1
Correction planes for a 300 MW steam turbine generator unit.

$[K]_{n\times(m-u)}$ are linked with planes with known ICs.

$$\left[\begin{array}{cccc|cc} k_{11} & k_{12} & \cdots & k_{1u} & \cdots & k_{1m} \\ k_{21} & k_{22} & \cdots & k_{2u} & \cdots & k_{2m} \\ \vdots & \vdots & & \vdots & & \vdots \\ k_{n1} & k_{n2} & \cdots & k_{nu} & \cdots & k_{nm} \end{array}\right]_{n\times m} \left[\begin{array}{cccc} p_{11} & p_{12} & \cdots & p_{1h} \\ p_{21} & p_{22} & \cdots & p_{2h} \\ \vdots & \vdots & & \vdots \\ p_{u1} & p_{u2} & \cdots & p_{uh} \\ \hline \vdots & \vdots & & \vdots \\ p_{m1} & p_{m2} & \cdots & p_{mh} \end{array}\right]_{m\times h}$$

$$= \begin{bmatrix} \Delta v_{11} & \Delta v_{12} & \cdots & \Delta v_{1h} \\ \Delta v_{21} & \Delta v_{22} & \cdots & \Delta v_{2h} \\ \vdots & \vdots & & \vdots \\ \Delta v_{n1} & \Delta v_{n2} & \cdots & \Delta v_{nh} \end{bmatrix}_{n\times h} \qquad [6]$$

That is, $[K]_{n\times m}$ is divided into $[A]_{n\times u}$ and $[B]_{n\times(m-u)}$. Accordingly, trial weighting matrix $[P]_{m\times h}$ is also divided into two parts: the upper rows are linked with the trial weights which are installed on the unknown planes, and the lower rows are linked with the trial weights which are installed on the planes where ICs are known. $[P]_{m\times h}$ is divided into $[C]_{u\times h}$ and $[D]_{(m-u)\times h}$. By substituting the matrix $[A]_{n\times u}$, $[B]_{n\times(m-u)}$, $[C]_{u\times h}$, and $[D]_{(m-u)\times h}$ into Eq. (6), in abbreviated form, it can be rewritten as,

$$[A_{n\times u}|B_{n\times(m-u)}]\left[\frac{C_{u\times h}}{D_{(m-u)\times h}}\right] = [\Delta V_1, \Delta V_2, \ldots, \Delta V_h]_{n\times h} \qquad [7]$$

Then Eq. (7) can be expressed as

$$\begin{aligned} A_{n\times u}C_{u\times h} + B_{n\times(m-u)}D_{(m-u)\times h} &= \Delta V_{n\times h} \\ A_{n\times u}C_{u\times h} &= \Delta V_{n\times h} - B_{n\times(m-u)}D_{(m-u)\times h} \end{aligned} \qquad [8]$$

Under general conditions that $u \neq h$, and $C_{u\times h}$ is not a square matrix with the same dimension u and h, Eq. (8) is inconsistent when the number of trial runs is great than the number of unknown planes. According to matrix theory, we can get an optimized solution. As far as the physics mechanism is concerned, the redundant information of the trial runs gives us the opportunity to approach the real ICs. This optimization method can avoid the deviation due to reading error, operation difference of the machine.

In Eq. (8), $A_{n\times u}$ is to be determined. Because h is no less than u, $C_{u\times h}$ is not always a Squares matrix but it has a Moore-Penrose pseudo inverse matrix provided that the columns of $C_{u\times h}$ are independent. In practical balancing, we can get independent trial weight sets, so $C_{u\times h}$ has a Moore-Penrose pseudo inverse matrix. With the $M - P$ inverse matrix, we can get the

TABLE 1
Balancing Data and Shaft Vibration (Peak-Peak): Amplitude/Phase Angle

	Trial weights/check runs	#5	#6	#7
1	Magnitude of initial vibration before balancing	148/6	115/337	151/341
2	Adding 1st trial weights, turbine end: 0.320 Kg at 113° (retained) exciter end: 0.417 Kg at 315° rectifier ring: 0.113 Kg at 320°	192/353	62/166	53/184
3	Adding 2nd trial weights, turbine end: 0.514 Kg at 80° exciter end: 0.420 Kg at 270° rectifier ring: 0.113 Kg at 320°	131/339	64/38	71/41

optimized solution of Eq. [8]. Thus, the unknown ICs matrix $C_{u\times h}$ can be expressed as

$$A_{n\times u} = \left(\Delta V_{n\times h} - B_{n\times(m-u)} D_{(m-u)\times h}\right) C^{+}_{u\times h} \qquad [9]$$

Note that $C^{+}_{u\times h}$ is the Moore-Penrose pseudo inverse matrix of $C_{u\times h}$. According to the matrix theory, the Moore-Penrose pseudo inverse matrix of $C_{u\times h}$ is $C^{*}_{u\times h}(C_{u\times h}C^{*}_{u\times h})^{-1}$, provided that $C_{u\times h}$ is of full rank, where $C^{*}_{u\times h}$ is the conjugated transpose of $C_{u\times h}$. When applying the expression of $C^{+}_{u\times h}$ into Eq. [9], we get the final expression for the general ICs calculation expression,

$$A_{n\times u} = \left(\Delta V_{n\times h} - B_{n\times(m-u)} D_{(m-u)\times h}\right) C^{*}_{u\times h}(C_{u\times h}C^{*}_{u\times h})^{-1} \qquad [10]$$

Equation [10] is the general solution for the ICs. It can be concluded that Eq. [10] can deal with the entire ICs calculation task.

If the redundant trial runs exist, i.e., h is greater than u, the solution of Eq. [10] is an optimized one according to matrix theory. The optimized method gives us an opportunity to make full use of all the information obtained in field balancing and refines the ICs. When adding a single trial weight, Eq. [10] yields the conventional ICs method.

PRACTICAL APPLICATION IN INDUSTRY

Equation [10] is very useful in field balancing. It has a uniform formula for diverse ICs calculation tasks and the ability to refine the ICs with all the information including present and historical balancing data. The calculation software based on GICA is developed with an interactive interface, which guides the user to follow the balancing steps. Due to the general formula of GICA, the algorithm is easy to program. In order to demonstrate the application of IC calculations, two examples applied in industry are given.

Part of the ICs of the Trial Planes is Known

Shown in Figure 1 is a large-scale steam turbine generator unit. The vibration of #5–#7 bearings is too high and this problem must be solved as soon as possible. After a full condition monitoring and diagnostics it was established that the main cause of vibration is unbalance. The unit has three parts where balance weights can be installed—both sides of the generator rotor and the middle part of the exciter rotor (rectifier ring); these planes are the effective ones which can influence the vibrations of #5–#7 bearings. In our previous work on this plant, we have obtained some historic ICs for the rectifier ring which have good coherence (see Table 2), thus the ICs of each side of the generator is unknown. With our experience, the trial weights in both generator rotor and exciter rotor at a time are effective. Therefore, we select these three planes, both sides of the generator rotor, and the middle part of the exciter rotor (rectifier ring).

TABLE 2
Known ICs of the Rotating Rectifier ICs: Amplitude/Angle

Known ICs	#5	#6	#7
The ICs of rotating rectifier to different point	91/167	605/201	640/218

TABLE 3
Calculated Result and ICs: Amplitude/Angle

Calculated result	#5	#6	#7
The ICs of turbine end to different point	97/86	313/227	223/229
The ICs of exciter end to different point	209/20	499/204	485/206

Table 1 gives all the vibration measurements and the relevant balancing data. Note that the trial weights installed are in groups instead of a single trial weight. After two trial runs, the measuring data can be determined because there are only two planes whose ICs are unknown. In this case, the first set of trial weights is retained on the rotor due to good effects in reducing the #6, #7 vibration magnitudes.

For the traditional IC method, the IC of each plane cannot be solved due to the groups of trial weight. However, with the software based on GICA, we get separate ICs for both sides of the generator rotor. The calculated results are shown in Table 3.

Determination of ICs when Redundant Trial Weighting Information is Available

The data shown in Table 4, is from the field balancing case of a centrifugal fan in a power plant. According to Table 4, there is only one correction plane and three trial runs including the check run. Therefore, the number of the 'trial' runs is greater than the

TABLE 4
Balancing Data and Shaft Vibration (Peak-Peak): Amplitude/Phase Angle

Trial runs/check runs	Vertical	Horizontal	Axial
(1) Magnitude of initial vibration before balancing	130/164	176/95	102/342
(2) Adding 1st trial weights, 0.326 Kg at 60° (removed)	154/137	200/86	102/325
(3) Adding 2nd trial weights, 0.579 Kg at 300° (retained)	38/102	60/40	34/290
(4) Adding 3rd trial weights, 0.191 Kg at 250°	10/134	16/88	8/353

TABLE 5
Calculated Result and ICs: Amplitude/Phase Angle

Calculation method	Vertical	Horizontal	Axial
Method A: Only with information (1) and (2)	215/19	116/339	92/183
Method B: Only with information (2) and (3)	215/27	283/341	132/219
Method C: Only with information (3) and (4)	157/21	265/316	163/206
Method D: Refining ICs with information (1), (2), (3), and (4)	210/25	243/339	122/212

number of unknown planes. In this case, there is redundant trial weighting information. With the traditional IC method, we can only calculate the ICs from the information of each single trial run, shown as Method A, B, and C in Table 5.

With the newly developed GICA, we can refine the ICs combining all the testing information. After refining, the ICs are more authentic and ready for later use. The result is shown as Method D.

It can be seen from the above that the ICs are decentralized with only one trial weighting information (see to Table 5). However, if we apply GICA, the defect can be solved by its optimization ability. This method can combine all the trial weighting information and approach the real ICs to some extent. Therefore GICA gives us an effective tool to refine the ICs.

CONCLUSIONS

With the matrix method, a new ICs calculation method, GICA, is derived using the matrix method and giving examples from industry applications. The general formula can deal with various kind of IC calculation task in the balancing of rotating machinery. The trial weight is not limited as a separate trial weight; it only requires that the number of the trial runs is no less than the number of the unknown planes. GICA can also be applied to the case where part of the ICs is known before balancing and groups of trial weights are applied. When the number of the trial runs is greater than the number of the unknown planes, the solution is an optimized result and can work as a refining tool for ICs. The formula derived in this article is ready to be applied into the core part of an IC procedure for a computer-aided balancing system.

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