Research Article

A Parametric Investigation of Geometric Variation on Fluid Dynamic Instabilities in Axial Compression Systems

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The ability to predict the nature of instabilities is highly important from the compressor design point of view since their consequences could result in widely varying difficulties with the fluid dynamic performance of the systems. Even though the behavior of surge and rotating stall is reported in many literatures, it is noticed that an in-depth analysis is not undertaken. Hence in view of the importance for a deeper understanding, the present paper is aimed at tracking the chaos of these instabilities in a more detailed manner. Primarily the influence of geometric parameters on the nature of surge and rotating stall is investigated. The effect of each of the major geometric parameters such as compressor effective length, annulus area, and plenum volume is discussed. The physical reason for the onset of instabilities is also explained in each case, and the well-accepted Moore-Greitzer model has been used for the present study. The combined effect of physical parameters is determined through the Greitzer *B* parameter. The results shown in this paper clearly elucidate the dominating effect of the geometric parameters on the development of flow instabilities like rotating stall and surge and hence can serve as a design guideline to avoid such instabilities.

1. Introduction

The operability of compression systems is limited at low mass flow rates by fluid dynamic instabilities leading to rotating stall or surge. The compressor surge and rotating stall are primary design constraints which effectively reduce the engine performance and consume a major fraction of engine development program. These unsteady aerodynamic instabilities can lead to large penalties in performance because they are difficult to predict during the design stage. By proper modeling and control of these instabilities, compressor performance can be improved.

Betchov and Criminale [1] have defined stability as the quality of being immune to small disturbances. The analysis of the stability of simple compression systems has been carried out by Emmons et al. [2], Taylor [3], Stenning [4], and many others. The mechanism for the generation of surge and rotating stall phenomena was highlighted in axial flow fans by using Euler equation in [5, 6]. The general result that emerges is that the system will be dynamically unstable near the peak of the pressure rise/mass flow characteristic at

some slight positively sloped operating point [7]. Greitzer [8] developed a nonlinear model to predict the transient response of a compression system subsequent to a perturbation from the steady operating conditions. Greitzer asserted that there is an important nondimensional parameter B that determines which mode of compressor instability-rotating stall or surge will be encountered at the stall line depending on whether it is below or above a critical value. A basic conclusion of this model is that at the same value of *B*, the compression system will exhibit the same transient behavior, independent of whether this value is obtained with a larger plenum volume and low speed or vice versa. Experiments of Pearson [9], Huppert and Benser [10], and Huppert [11] found out that a compressor may exhibit surge at "high" speed but not at "low" speed, where only rotating stall is encountered at the stall limit. Also the volume of the exit plenum into which the compressor discharges has been found to be important; that is, at a given speed a compressor can surge with a large plenum but not with a small one [8]. Another important result of experiments is that the character of surge oscillation varies appreciably for lower and higher *B* values [8]. Also as the number of compressor stage increases, *B* needed to encounter surge decreases. As per Greitzer two distinct surge types can be seen, one known as classic surge where the flow will be pulsating without any reversal and the other in which there is significant flow reversal during part of the surge cycle that is identified to be deep surge.

A heuristic model [12] had been proposed in order to understand the nature of the type of oscillation occurring during deep surge where there is considerable reverse flow during part of the cycle. Based on the work of Moore [13], Moore and Greitzer developed an approximate theory [14, 15] capable of predicting the poststall transients in multistage axial compression systems. This model explains the coupling between rotating stall-like and surge-like motions and gives the nature of the operating point motion during a transient stall phenomenon. In the case of classic surge the blade passes in and out of stalled flow regime which induces vibration in blades as discussed by Horlock [16] and Pinsley et al. [17]. If the frequency of rotating stall happens to match the natural frequency of compressor blades, the ultimate result will be destruction of blade rows. De Jager [18] found that surge and rotating stall restrict the operating efficiency and pressure rise capability of the compressor as they can lead to heating of the turbine blades and to an increase in exit temperature of the compressor.

In the present study by using the Moore-Greitzer theoretical model [14, 15], the effect of change in geometric parameters on axial compression systems is examined to describe the nature of poststall transients that could be expected as the instantaneous operating point of the compressor crosses the peak point in the compressor performance curve.

2. Modeling and Solution Procedure

Several dynamic models for the unstable operation of the compression systems have been proposed but the model of Moore and Greitzer [14, 15] stands out as unique in the sense that the rotating stall amplitude is included as a state and not manifested as a pressure drop which is the case in other models [19-21]. The lower order model of Moore and Greitzer captures the poststall transients of a low speed axial compressor-plenum-throttle system. The lower order refers to simplicity of the model which describes the compression system behavior in three states in spite of the complex fluid dynamic phenomena that it models. Figure 1 [22] shows the schematic of the model compression system used in this study. The assumptions [14] that are involved in this model are that inviscid and irrotational flow having no radial variations enters the compressor, a large hub to tip radius ratio so that a 2D description seems to be reasonable, incompressible flow on account of having low mach number, compressible plenum gas with uniform static pressure (but unsteady), low pressure rises compared to ambient conditions, constant rotor speed, and short throttle duct.

In the model for pursuing the studies, an ideal form of compressor performance curve is needed in a sense that the performance of the compressor in the absence of any of the disturbances has to be known. In other words a



FIGURE 1: Schematic of the compressor showing nondimensional lengths (Erskine and Hensman [22]).

performance curve, which would hypothetically be measured for a compressor with the rotating stall and surge absent, is required. In [23, 24] it has been argued that the characteristic curve is typically a smooth S-shaped curve, and, hence, a physically realistic choice would be a simple cubic curve as shown in Figure 2. The expression for the curve [14] is as given below:

$$\psi(\phi) = \psi_{c0} + H \left[1 + \frac{3}{2} \left(\frac{\phi}{W} - 1 \right) - \frac{1}{2} \left(\frac{\phi}{W} - 1 \right)^3 \right].$$
(1)

The nondimensional quantities used in (1) are as follows. Pressure coefficient,

$$\Psi = \frac{(p_S - p_T)}{\rho U^2}.$$
 (2)

Flow coefficient,

$$\Phi = \frac{C_x}{U}.$$
 (3)

Nondimensional time,

$$\varepsilon = \frac{Ut}{R}.$$
 (4)

The three ordinary differential equations of the model [14] arise from a Galerkin approximation of the local momentum balance, annulus averaged momentum balance, and mass balance of the plenum. These equations are

$$\frac{d\Psi}{d\varepsilon} = \frac{W/H}{4B^2} \left[\frac{\Phi}{W} - \frac{1}{W} \sqrt{\frac{2\Psi}{K_T}} \right] \frac{H}{l_c},$$

$$\frac{d\Phi}{d\varepsilon} = \left[-\frac{\Psi - \psi_{c0}}{H} + 1 + \frac{3}{2} \left(\frac{\Phi}{W} - 1 \right) \times \left(1 - \frac{1}{2}A^2 \right) - \frac{1}{2} \left(\frac{\Phi}{W} - 1 \right)^3 \right] \frac{H}{l_c},$$

$$\frac{dA}{d\varepsilon} = \frac{A}{2} \left[1 - \left(\frac{\Phi}{W} - 1 \right)^2 - \frac{1}{4}A^2 \right] \frac{3aH}{(1 + ma)W},$$
(5)

where

$$B = \frac{U}{2a_s} \sqrt{\frac{V_p}{A_c L_c}}.$$
 (6)

Here H and W are the semiheight and semiwidth of the cubic axisymmetric characteristic (Figure 2), a is a parameter

that accounts for the inertial effects of the flow through the compressor [14], and *m* is a measure of the duct length l_E , relative to l_I [14] in Figure 1 (m = 2 would refer to a long enough exit duct l_E , and m = 1 would refer to a very short one). *B* is the familiar Greitzer parameter, and l_c is the effective flow passage length for the system in Figure 1 [14]. ψ_{c0} is the nondimensionalised pressure rise through the compressor when there is no flow, and K_T is the constant throttle coefficient.

In the present study Moore-Greitzer model equations (5) have been solved by a MATLAB R2009a code using the inbuilt function "ode 45", which uses a variable step Runge-Kutta method to solve the ordinary differential equations numerically. The initial conditions used for solving the equations are obtained as follows.

Figure 3 shows the cubic axisymmetric characteristic used in the Moore-Greitzer model [14] to perform calculations for the poststall transients. The curve shows a plot of nondimensional quantities of plenum pressure rise versus the compressor mass flow for a constant rotor speed. Throttle curve passing through the peak point of the ideal cubic compressor curve is also shown. The stall line marks the limit of stable operation of the compressor. Stable operating point corresponds to the point of intersection of the cubic compressor curve and the throttle curve. The instability sets in when the operating point reaches the stall line and is represented by point B in Figure 3. Thus the values of nondimensional plenum pressure Ψ and annulus averaged compressor mass flow Φ corresponding to point *B* are chosen as the initial conditions [14, 15] to solve the Moore-Greitzer equations. At these conditions a level of circumferential nonuniformity A is imposed on the compressor flow, and the initial conditions of the system of equations to be solved are (taken from Moore and Greitzer [15]),

$$\Psi(0) = 0.66,$$

 $\Phi(0) = 0.5,$ (7)
 $A(0) = 0.02.$

The value of parameters and constants, appearing in the equations, used are [14]

$$H = 0.18, \qquad W = 0.25,$$

$$a = \frac{1}{3.5}, \qquad m = 1.75,$$

$$B = 1, \qquad l_c = 8,$$

$$\psi_{c0} = 0.3, \qquad K_T = 5.5.$$
(8)

The effect of operating parameter on compressor flow field transients is determined by solving the three sets of coupled Moore-Greitzer equations (5) for different cases as discussed later.

3. Results and Discussion

The physical parameters of the compression system that are included in the model are plenum volume, compressor



FIGURE 2: Cubic axisymmetric characteristic of Moore and Greitzer [14, 15].



FIGURE 3: Compressor performance characteristic.

mean annulus area, and the effective system length of the compressor.

In order to clearly depict the behavior of instabilities the different cases explained later are discussed in detail by including a sufficient number of plots between various quantities, which characterise the compression system behavior.

3.1. Effect of Plenum Volume. Here the effect of change (increase or decrease) on plenum volume by a fixed percentage with respect to B = 1 has been examined. The expression for *B* is given by (6).

The effect of change on plenum volume is manifested as a change in B. It is evident from the above expression that B varies as the square root of plenum volume. The different cases that are examined are shown in Table 1, and the relevant plots are shown in Figures 4 to 7. These plots depict the compression system behavior subsequent to the initial system instability.

Figures 4(a) and 4(b) show the temporal variation of annulus averaged flow coefficient and plenum pressure rise coefficient. For higher values of B, a behavior could be observed in which high-frequency rotating stall exists during

Sl no.	Plenum volume	В
1	V_p	1
2	$0.2V_p$	0.4472
3	$0.3V_p$	0.5477
4	$0.4V_p$	0.6324
5	$0.6V_p$	0.7745
6	$1.2V_p$	1.0954
7	$1.3V_p$	1.1401
8	$1.4V_p$	1.1832

a part of surge cycle. As could be seen, with decrease in plenum volume (a decrease in B) the surge modulations in axial flow and plenum pressure slowly vanish and the amplitude of pulsation gets damped. For B = 0.4472 where the plenum volume is the lowest, the compression system settles down to a rotating stall operation. In this mode, the annulus averaged flow and plenum pressure remain steady with time after an initial drop. Figure 4(c) shows the temporal variation of rotating stall amplitude. The existence of nonzero rotating stall amplitude signifies the time instants where there is circumferential flow distortion inside the compressor. The peak values correspond to the time instants where the annulus averaged compressor mass flow is minimum. This condition favors the development of rotating stall because the compressor blades will get stalled when the low velocity flow approaches the blade at large incidence. This type of flow pulsation where high-frequency (compared to surge frequency) rotating stall exists during a part of surge cycle is known as classic surge [8]. Similar observations are also reported by the experiments of Greitzer [12, 25] and Bammert and Mobarak [26]. In other words rotating stall is coupled to surge during this mode of behavior. As the plenum volume decreases, the time range of zero rotating stall amplitude becomes shortened. Finally for a sufficiently low plenum volume, the stall cell amplitude reaches a steady state indicating the development of permanent rotating stall pattern inside the compressor as represented by B = 0.4472case. Figures 5(a) and 5(b) show the variation of flow coefficient and plenum pressure coefficient with stall cell amplitude. The sense of operating point motion is in the clockwise direction. These plots show the value of axial flow and plenum pressure coefficients at which the stall cell amplitude attains its maximum and minimum values. The operation at fully developed rotating stall subsequent to an initial perturbation from steady-state conditions is identified by a single point in these plots. It is interesting to note in Figure 5(b) that for higher values of *B*, during classic surge, the compressor continues to deliver higher pressure even under conditions of peak stall for some period of time. Figure 5(c) shows the variation of plenum pressure with the compressor axial flow plotted in the form of a limit cycle oscillation. The operating point moves in a counterclockwise sense. The starting point of limit cycle (Φ, Ψ) as stipulated by the initial conditions is (0.5, 0.66). The plot depicts the behavior that during a classic surge, the compressor operating point traces a path where there will be flow

pulsation about the starting point as the mean position. Unlike the axial flow, the plenum pressure value does not rise up much above the starting value during a limit cycle. This is due to the fact that the plenum pressure rise cannot increase beyond a limit even under conditions of flow instability because it is the compressor output. The fully developed rotating stall is indicated by steady-state operation at a reduced value of axial flow and plenum pressure as identified by a point in the compressor performance characteristic.

Thus it is clear that with decrease in plenum size, rotating stall will be favored subsequent to initial system instability. The physical reason is that for a smaller plenum, the stored pressure will not be sufficient enough to create a flow pulsation or to reverse the flow through the compressor.

The effects of increase in plenum size on poststall transients are shown in Figures 6 and 7. The type of variation in annulus averaged flow and plenum pressure rise as depicted by Figures 6(a) and 6(b) shows that the system has a tendency to surge with increase in plenum volume. A clear transition in poststall behavior could be seen from B = 1 to 1.1832. The compression system changes its unsteady behavior from classic surge (B = 1 case) to constant amplitude classic surge (B = 1.0954 case) to varying amplitude deep surge (B =1.1401 case) to constant amplitude deep surge (B = 1.1832case). It is interesting to note in the last case that the amplitude of oscillation remains steady for all the cycles once the surge gets fully established. The deep surge qualitatively differs from classic surge in that there will be considerable reverse flow during a part of surge cycle. Figure 6(c) shows the plot for temporal variation of stall cell amplitude which indicates that the rotating stall starts diminishing at larger plenum volumes. The possible reason could be that as the pressure forces inside the plenum increase with the plenum size, the surge oscillations become rapid which precludes the formation of any rotating stall pattern in the compressor. The stall cell inception becomes impossible when there is a reverse flow through the compressor. This fact is also supported by the experimental results shown in [12]. The local flow coefficient [15] at any point inside the compressor blade row is given by an equation of the form

$$\phi(\varepsilon) = \Phi(\varepsilon) + WA(\varepsilon)\sin(\theta - f_0\varepsilon). \tag{9}$$

Here θ is the angular coordinate around the compressor, f_0 is the nondimensional propagation speed of the rotating stall-like disturbances [15], and *A* is the amplitude of rotating stall-like disturbances, which is time (ε) dependent.

The harmonic sine function alternates its sign periodically, and the stall cell amplitude becomes negative for some cases (e.g., B = 1.1401 case) for a very short period of time. So the second quantity on the right hand side of (9) may be positive or negative periodically, and the local flow coefficient at a particular time instant will be more or less than its corresponding annulus averaged value. For B = 1.1401, the negative peak amplitude of stall acts as the precursor for surge to occur.

Figures 7(a) and 7(b) depict the variation of rotating stall disturbance with axial flow and plenum pressure. The plot indicates that the rotating stall continues to develop



FIGURE 4: Temporal variation of nondimensional flow properties (a), (b), and (c) with decrease in size of plenum: (a) axial flow coefficient, (b) plenum pressure rise coefficient, and (c) disturbance amplitude of rotating stall.



FIGURE 5: Effect of decrease in plenum size: (a) variation of axial flow coefficient with disturbance amplitude of rotating stall. (b) Variation of plenum pressure rise coefficient with disturbance amplitude of rotating stall. (c) Limit cycle oscillation: axial flow coefficient versus plenum pressure rise coefficient.



FIGURE 6: Temporal variation of nondimensional flow properties (a), (b), and (c) with increase in size of plenum: (a) axial flow coefficient, (b) plenum pressure rise coefficient, and (c) disturbance amplitude of rotating stall.



FIGURE 7: Effect of increase in plenum size: (a) variation of axial flow coefficient with disturbance amplitude of rotating stall. (b) Variation of plenum pressure rise coefficient with disturbance amplitude of rotating stall. (c) Limit cycle oscillation: axial flow coefficient versus plenum pressure rise coefficient.

and decay out periodically as the mass flow through the compressor and plenum pressure changes. The points of maximum and zero stall cell amplitudes are seen in these figures. Clear hysteresis behavior could also be seen. This is because when mass flow decreases below the mean value, the development of rotating stall is favored, and when it starts increasing back towards the mean value, during a surge cycle, the stall still persists for some more range of mass flows due to the highly nonlinear behavior of the phenomenon under consideration [27]. When deep surge occurs, the operating point will trace a to-and-fro motion in a horizontal line corresponding to zero stall amplitude (B = 1.1832 case). In the limit cycle plots of Figure 7(c) one could see a large amount of hysteresis when there is a transition in behavior from classic surge to deep surge. This is attributed to the varying amplitude of surge oscillations. There is a severe drop in plenum pressure during a part of surge cycle for larger plenum sizes, and the cycle shifts more towards the negative range of mass flow rates. With reference to Figures 6(a) and 7(c) deep surge behavior at higher B values (e.g., B = 1.1832) can be identified to exhibit different characteristics during a cycle: (1) a sudden huge drop in compressor flow (to negative flows), (2) plenum pressure dropping down slowly in the reverse flow regime, (3) a sudden increase in mass flow towards the stable flow regime at almost constant plenum pressure, and (4) rise in plenum pressure with decreasing flow. The results from these plots agree very closely with Greitzer's experimental observations [12]. From Figures 6(c) and 7(c) it is clear that the peak stall cell amplitude corresponds to the time instants where the annulus averaged flow and plenum pressure are at their lower range of values.

It has become evident that with increase in plenum size, the compression system has a tendency to surge than to operate in rotating stall mode. The physical reason is that a larger plenum has a larger pressure storage capability which is sufficient enough to reverse the flow as the compressor blade row stalls.

3.2. Effect of Compressor System Length. An important fact worthy of mention is that the non-dimensional term (l_c) represents the ratio of compressor system length (L_c) to its mean radius (R). The effective system length of the compressor [14] implies the effective flow-passage length through the compressor and its ducts. In other words in addition to the geometric length it also accounts for the tortuous flow path through the compressor. Thus the effective (inertial) path length for the system is longer than the axial length. It is to be noted that the present analysis assumes [14] that the pipe length (l_E) after the compressor duct is not smaller than the length (l_I) before the compressor duct. Thus to account for the relative size of the exit duct with the inlet one, the parameter *m* is introduced in the analysis as discussed earlier. Following [14], the effective system length (l_c) is given by an equation of the form

$$l_c = l_I + \frac{1}{a} + l_E,$$
 (10)

where *a* is a parameter that accounts for the inertial effects of the flow through the compressor.

TABLE 2: Different compressor effective lengths with respect to B = 1 and $l_c = 8$ case.

Sl no.	Compressor effective length	В	l_c
1	L _c	1	8
2	$3l_c$	0.5773	24
3	$6l_c$	0.4082	48
4	$8l_c$	0.3535	64
5	$0.6l_{c}$	1.2909	4.8

Here the effect of change (increase or decrease) in system length by a fixed percentage with respect to B = 1 has been examined. As the effective length L_c changes, the parameter l_c will change and B varies as the inverse square root of L_c (6).

The different cases that are examined are given in Table 2, and the relevant plots are shown in Figures 8 and 9 where the compression system behavior subsequent to initial system instability is shown.

The temporal variation of axial flow and plenum pressure coefficient are shown in Figures 8(a) and 8(b). The plots indicate that the shorter compressors system length is more prone to classic surge $(B = 1, l_c = 8 \text{ and } B = 1, l_c = 24)$, and as the compressor effective length increases, the surge modulation in the flow decreases and the system has a tendency to settle down in rotating stall mode. This nature of compression system behavior is also mentioned in [8, 25]. A slight increase in oscillation amplitude is observed for plenum pressure as shown in Figure 8(b) for the case B = 1and $l_c = 24$ as compared to B = 1 and $l_c = 8$ case. This is due to the highly nonlinear behavior of the phenomenon under consideration. Figure 8(c) depicts the unsteady variation of stall cell amplitude. The periodic development of stall cells is evident when classic surge exists inside the compressor. The plot shows that after an initial transient, the stall cell amplitude reaches a steady-state value and permanent rotating stall pattern stabilizes inside the compressor of larger effective length. Figures 8(d) and 8(e) show the axial flow and plenum pressure variation with stall cell amplitude. One could see the mass flow and pressure values at which there is no stall and where stalling is severe. As earlier, hysteresis is evident in the case of classic surge. Figure 8(f) shows the limit cycle plot between the axial flow and plenum pressure. For the case of B = 0.5773 and $l_c = 24$, the amplitude of plenum pressure oscillation goes appreciably above the initial starting value during each cycle for a short period of time. This behavior is not seen in other cases, which again emphasizes the strongly non-linear features of the compressor flow instabilities. When a permanent rotating stall is established inside a compressor of larger length, the operation is identified by a single point in the compressor performance curve, after an initial transient.

Thus as the length of compressor flow increases, the system has a tendency to operate in rotating stall than surge. The physical reason is that for a longer flow path inside the compressor the inertial forces will be strong enough to resist any pulsation or flow reversal that may happen due to the plenum pressure force, so that the instability will collapse into a rotating stall.



FIGURE 8: Effect of increase in compressor effective length: variation of nondimensional flow properties (a), (b), and (c) with time. (a) Axial flow coefficient, (b) plenum pressure rise coefficient, and (c) disturbance amplitude of rotating stall. (d) Variation of axial flow coefficient with disturbance amplitude of rotating stall. (e) Variation of plenum pressure rise coefficient with disturbance amplitude of rotating stall. (f) Limit cycle oscillation: axial flow coefficient versus plenum pressure rise coefficient.



FIGURE 9: Effect of decrease in compressor effective length: variation of nondimensional flow properties (a), (b), and (c) with time. (a) Axial flow coefficient, (b) plenum pressure rise coefficient, and (c) disturbance amplitude of rotating stall. (d) Variation of axial flow coefficient with disturbance amplitude of rotating stall. (e) Variation of plenum pressure rise coefficient with disturbance amplitude of rotating stall. (f) Limit cycle oscillation: axial flow coefficient versus plenum pressure rise coefficient.

Figure 9(a) shows the variation of axial flow and plenum pressure with time when the compressor effective length is shortened. It is seen that as the length of the compressor decreases, both the frequency and amplitude of oscillation increase, and for a sufficiently shorter compressor the flow reverses during a part of surge cycle leading to deep surge. It is seen that the deep surge amplitude remains constant with time throughout the cycle. Figure 9(c) shows the variation of stall cell amplitude with time for the two different cases that are examined here. The initial peak for the two cases is due to the effect of initial imposed stall cell amplitude. When the deep surge occurs, the stall cell amplitude goes to zero indicating the nonexistence of rotating stall during deep surge. Figures 9(d) and 9(e) show the variation of compressor flow and plenum pressure with stall cell amplitude. A large hysteresis could be seen in the plenum pressure variation with stall amplitude compared to the flow variation. Thus if one traces the cycle in a clockwise sense, it means that the compressor continues to deliver larger pressure output during a part of cycle even in the presence of rotating stall (indicated by the top flat portion for the case of B = 1 and $l_c = 8$). For shorter compressor length case, the operating point traces a to-and-fro motion in a horizontal line corresponding to zero stall amplitude. The limit cycle oscillation in Figure 9(f) shows that when deep surge occurs, hysteresis increases dramatically and the curve shifts more towards the negative flow regime. Also the deep surge behavior seems to be extremely repeatable along the same flow path. Again the four distinct features in a single deep surge cycle could be seen as discussed earlier.

It can thus be concluded that as the effective system length of the compressor decreases, the system has a tendency to surge than to operate in rotating stall subsequent to an initial instability. The physical reason behind this fact is that as the characteristic flow length through the compressor decreases, the inertial forces become weak to resist the plenum pressure force thereby leading to flow pulsation or reversal.

3.3. Effect of Compressor Annulus Area. The effect of compressor annulus area on transient behavior of flow is studied by varying the compressor mean radius, R. Here the effect of change (increase and decrease) in mean radius by a fixed percentage with respect to B = 1 and $l_c = 8$ has been examined.

From (6) it can be seen that the mean blade speed, U, varies directly with R, and compressor mean annulus area varies directly with square of R which in effect makes B independent of R. So the effect of change in R is manifested as a change in l_c alone. The different cases that are examined are given in Table 3, and relevant plots are shown in Figures 10 and 11.

The temporal variation of axial flow and plenum pressure, when compressor mean annulus area reduces, is shown in Figures 10(a) and 10(b). With decrease in area the compression system shows a tendency to surge. Unlike the kind of oscillation encountered in the previous cases, the character of classic surge oscillation is different as the annulus area reduces. The variation of mass flow and plenum

TABLE 3: Different compressor mean annulus areas with respect to B = 1 and $l_c = 8$ case.

Sl no.	Compressor mean radius	В	l_c
1	R	1	8
2	2 <i>R</i>	1	4
3	4R	1	2
4	R/2	1	16
5	<i>R</i> /4	1	32

pressure has become gradual (e.g., B = 1, $l_c = 32$ case). The mass flow variation follows an approximately triangular wave pattern, and plenum pressure almost shows a sinusoidal variation. The deep surge behavior could not be seen even after reducing the compressor annulus area considerably. Figure 10(c) shows the temporal variation of stall cell amplitude. It is evident that the peak amplitude is retained for some longer time range with decrease in annulus area, unlike the kind of variation seen in other parametric cases. This is attributed to the gradual variation of mass flow, facilitating the existence of rotating stall over a substantial period of time.

Figures 10(d) and 10(e) show the variation of flow and pressure rise with stall amplitude. A mirror image pattern of variation is seen about A = 0 line for the last two cases due to the existence of negative stall amplitude peaks. As observed in previous cases hysteresis behavior still prevails as seen in these plots. Figure 10(f) depicts the variation of compressor axial flow with plenum pressure which is the limit cycle oscillation. It is evident that the amplitude of mass flow and plenum pressure fluctuations in all the three cases is nearly the same. This is essentially because as seen in Figures 10(a) and 10(b) not the amplitude but the character of classic surge oscillations changes with decrease in flow through area of the compressor.

Thus with decrease in annulus area of the compressor, the system has a tendency to surge than to operate in rotating stall. The physical reason is that a smaller area compressor connected to a plenum will be encountering a greater pressure differential than a larger one connected to the plenum of same size.

The effect of increase in annulus area on compressor flow field transients is represented in Figure 11. It could be seen from the unsteady variation of compressor axial flow and plenum pressure, as shown in Figures 11(a) and 11(b), that as the flow through area of the compressor increases, the surge modulations in flow and plenum pressure decrease and the compressor has a tendency to get settled in rotating stall operation. From the temporal variation of stall cell amplitude shown in Figure 11(c), as flow area increases, one could see high-frequency fluctuations near to the base of stall amplitude peaks indicating the tendency to enter into a permanent stall condition. The rotating stall becomes fully developed in the third case.

Figures 11(d) and 11(e) show the axial flow and plenum pressure variation with stall cell amplitude. It could be noted from these plots that for the third case, before settling into a steady-state value of mass flow and plenum pressure, there are significant fluctuations during the transient.



FIGURE 10: Effect of decrease in mean annulus area of compressor: variation of nondimensional flow properties (a), (b), and (c) with time. (a) Axial flow coefficient, (b) plenum pressure rise coefficient, and (c) disturbance amplitude of rotating stall. (d) Variation of axial flow coefficient with disturbance amplitude of rotating stall. (e) Variation of plenum pressure rise coefficient with disturbance amplitude of rotating stall. (f) Limit cycle oscillation: axial flow coefficient versus plenum pressure rise coefficient.



FIGURE 11: Effect of increase in mean annulus area of compressor: variation of nondimensional flow properties (a), (b), and (c) with time. (a) Axial flow coefficient, (b) plenum pressure rise coefficient, and (c) disturbance amplitude of rotating stall. (d) Variation of axial flow coefficient with disturbance amplitude of rotating stall. (e) Variation of plenum pressure rise coefficient with disturbance amplitude of rotating stall. (f) Limit cycle oscillation: axial flow coefficient versus plenum pressure rise coefficient.

TABLE 4: Different *B* values with respect to B = 1 case.

Sl no.	В
1	1
2	0.6
3	0.55
4	1.13
5	1.14
6	1.15
7	1.3
8	2
9	3

The hysteresis behaviour seen in these plots is also evident from the results of Moore and Greitzer [15]. The limit cycle plots for all the three cases are shown in Figure 11(f) which shows the variation of axial flow with plenum pressure rise. The amplitude of pressure drop that occurs from the initial starting value when operating in steady state (in terms of annulus averaged compressor mass flow) rotating stall could be clearly seen.

It is thus seen that with increase in compressor annulus area the system has a tendency to operate in rotating stall because of the inadequacy of pressure differential to initiate surge with a compressor of larger annulus area.

In previous analysis an increase in compressor system length (means an increase in l_c) favors operation in rotating stall whereas in the present analysis a decrease in compressor mean radius (an increase in l_c) favors surge. Even if l_c increases, in both cases the instabilities are different. This proves the necessity for analysing the system parameters individually rather than merely focussing on a representative non-dimensional quantity.

3.4. Effect of Greitzer B Parameter. In order to examine the combined effect of plenum volume and compressor system length, the B parameter is subjected to variation to examine its effect on the system transients. The different cases that are examined are given in Table 4, and Figures 12–15 show the relevant plots.

Figures 12(a) and 12(b) show the effect of decrease in *B* on flow field transients by means of temporal variation in axial flow and plenum pressure. When *B* decreases, both the frequency and amplitude of surge oscillation decrease, and at a sufficiently low value of *B* (e.g., B = 0.55), rotating stall will be completely established. The only exception to this behavior is seen for B = 0.6 where the plenum pressure oscillation slightly increases in amplitude. This is again due to the highly non-linear behavior of transients under investigation. The regimes of flow behavior before the occurrence of rotating stall depict that of classic surge.

A difference in pattern of variation is seen with the temporal variation of stall cell amplitude as shown in Figure 12(c). When *B* decreases, the stall peak shows a tendency to rise and finally attains a steady amplitude at B = 0.55 representing the development of permanent rotating stall. Figures 12(d) and 12(e) show the axial flow and plenum

pressure variation with stall cell amplitude. A linear rise in stall cell amplitude with decrease in axial flow during a part of surge cycle is clearly evident in Figure 12(d). The top flat portion in Figure 12(e) during classic surge again shows the delivery of high pressure (for some period of time) under conditions of maximum stall as discussed earlier. Figure 12(f) represents the axial flow variation with plenum pressure rise in the form of a limit cycle oscillation for the three cases. It is clear that as *B* decreases, the system exhibits tendency for the rotating stall operation. *B* could physically be interpreted as the ratio of pressure force to the inertial force [12, 25]. A decrease in *B* signifies the dominance of inertial force over the pressure force which makes the system operate in rotating stall rather than surge under conditions of instability.

Figures 13(a) and 13(b) show the temporal variation of axial flow and plenum pressure for three values of *B* (i.e., 1, 1.13, and 1.14). Here in these plots a transition in behavior could be observed. The flow and plenum pressure behavior changes from varying amplitude classic surge to constant low amplitude surge to varying amplitude deep surge. One notable observation is that even when there is no flow reversal, rotating stall is not developed for B = 1.13 case. This might be because of the lack of sufficient decrease in mass flow needed to initiate the development of rotating stall. This type of surge cannot be categorised as classic surge, since there is no stall-like disturbances involved. As *B* increases, the downward amplitude of oscillation increases with the upward amplitude remaining more or less the same.

Figure 13(c) shows the time variation of stall cell amplitude for the three cases of *B*. The first peak for all the three cases is due to the effect of imposed initial condition on stall cell amplitude. The periodic development of rotating stall is evident for B = 1 case, which is the classic surge behavior. The sharp negative peak for B = 1.14 acts as the precursor for deep surge to occur. Figures 13(d) and 13(e) show the flow and plenum pressure variation with stall cell amplitude. The hysteresis behavior in plenum pressure variation with stall cell amplitude dramatically increases with B as deep surge is approached. In the limit cycle oscillation between axial flow and plenum pressure, as shown in Figure 13(f), one could see a large amount of hysteresis involved in the deep surge case of B = 1.14. This is due to the occurrence of varying amplitude oscillation at this value of B. Similar behavior is reported by the experiments of Greitzer [12].

Figures 14(a) and 14(b) show the temporal plots for axial flow and plenum pressure corresponding to *B* values of 1.15 and 1.3. The case of B = 1.14 is also showed for the sake of comparison. With further increase in *B* it is seen that the behavior changes from varying amplitude to constant amplitude deep surge. It is evident from Figure 14(c) for temporal variation of stall cell amplitude that no rotating stall development takes place during deep surge. When B =1.14, a slight tendency for the stall to get initiated is seen at different time instants but the rapid mass flow oscillation prevents its formation as emphasized earlier.

Figures 14(d) and 14(e) show the variation of axial flow and plenum pressure with stall cell amplitude. The negative stall amplitude values for B = 1.14 case are due to existence



FIGURE 12: Effect of decrease in *B* parameter: variation of nondimensional flow properties (a), (b), and (c) with time. (a) Axial flow coefficient. (b) Plenum pressure rise coefficient, and (c) disturbance amplitude of rotating stall. (d) Variation of axial flow coefficient with disturbance amplitude of rotating stall. (e) Variation of plenum pressure rise coefficient with disturbance amplitude of rotating stall. (f) Limit cycle oscillation: axial flow coefficient versus plenum pressure rise coefficient.



FIGURE 13: Effect of increase in *B* parameter from 1 to 1.14: variation of nondimensional flow properties (a), (b), and (c) with time. (a) Axial flow coefficient, (b) plenum pressure rise coefficient, and (c) disturbance amplitude of rotating stall. (d) Variation of axial flow coefficient with disturbance amplitude of rotating stall. (e) Variation of plenum pressure rise coefficient with disturbance amplitude of rotating stall. (f) Limit cycle oscillation: axial flow coefficient versus plenum pressure rise coefficient.



FIGURE 14: Effect of increase in *B* parameter from 1.14 to 1.3: variation of nondimensional flow properties (a), (b), and (c) with time. (a) Axial flow coefficient. (b) Plenum pressure rise coefficient, and (c) disturbance amplitude of rotating stall. (d) Variation of axial flow coefficient with disturbance amplitude of rotating stall. (e) Variation of plenum pressure rise coefficient with disturbance amplitude of rotating stall. (f) Limit cycle oscillation: axial flow coefficient versus plenum pressure rise coefficient.



FIGURE 15: Effect of increase in *B* parameter from 1.3 to 3: variation of nondimensional flow properties (a), (b), and (c) with time. (a) Axial flow coefficient, (b) plenum pressure rise coefficient, and (c) disturbance amplitude of rotating stall. (d) Variation of axial flow coefficient with disturbance amplitude of rotating stall. (e) Variation of plenum pressure rise coefficient with disturbance amplitude of rotating stall. (f) Limit cycle oscillation: axial flow coefficient versus plenum pressure rise coefficient.

of a negative peak as seen in Figure 14(c). The variation between compressor flow and plenum pressure in the form of a limit cycle is shown in Figure 14(f). It depicts clearly the hysteresis behavior of surge in the transition regime (for B = 1.14) and the highly repeating behavior of oscillation along the same path at higher *B* values (e.g., B = 1.3). Also the cycle shifts more towards the negative flow region at higher *B*.

The various plots in Figure 15 represent the influence of B parameter more strongly on the nature of surge oscillations. At higher B values, the deep surge oscillations are of relaxation type with two distinct time scales-one in which the variation of axial flow takes place over a longer period of time and second a rapid flow reversal. From the unsteady plots of axial flow and plenum pressure as given by Figures 15(a) and 15(b) it is also seen that the time period of each cycle increases with B but the amplitude remains the same. In the plot for unsteady plenum pressure, it can be seen that the amplitude of plenum pressure decreases slightly with increase in B. This is because for the same value of compressor and throttle annulus area if plenum volume increases, pressure magnitude decreases, and hence the plenum pressure oscillations decrease. The plenum pressure blowdown at higher B value takes place over a longer time as could be noticed from Figure 15(b).

The unsteady variation of stall amplitude is shown in Figure 15(c). There are no signs of stall development, after an initial transient, during deep surge behavior as stated earlier. The variation of axial flow and plenum pressure with disturbance amplitude of rotating stall is shown in Figures 15(d) and 15(e). The initial existence of stall cell amplitude in Figures 15(c), 15(d), and 15(e) is attributed to the imposed stall disturbance at the beginning which immediately decays out. The hysteresis behavior in Figure 15(e) could be seen to vanish as B increases. The limit cycle oscillation at higher B values as shown in Figure 15(f) is more squareshaped with large amount of hysteresis in each cycle. Also it is characterized by a rapid variation in one quantity with a gradual variation in the other. For B = 2 and 3, it can be inferred that the fluid acceleration in the compressor is quite small during the blowdown part of deep surge cycle. Thus, the pressure difference across the compressor will approximately be the same as the steady-state pressure rise and this portion of the cycle may be considered to represent the steady-state characteristic in this region.

Thus in the present investigation, the Moore-Greitzer model provides a detailed information into the type of transients, for compression systems of different geometry, that one could expect when the operating point crosses the stall limit line. The model prediction appears to be consistent with the experimental results in many aspects and hence can be considered to reveal many important features of surge and rotating stall. It turns out from the present investigation that the geometric parameters alter the behavior of surge and rotating stall significantly, and hence the design of a compression system in gas turbine engine has to be done with utmost care.

Some important aspects that make the model prediction "qualitative" are the assumption of constant rotor speed

and noninclusion of viscosity and inlet flow distortion. In reality viscosity introduces a damping effect on the fluid oscillations, which is not accounted for by the Moore-Greitzer model. Thus the surge oscillation amplitude in reality will be smaller [27] than that predicted by the Moore-Greitzer model for the same physical conditions.

4. Conclusions

- (1) The type of instabilities that could be expected for compressors of relatively different sizes is examined with respect to each of the major geometric parameters. The size of plenum into which the high pressure air gets discharged is shown to have a profound influence in determining system instability behavior.
- (2) There are three possible regimes of instability: pure rotating stall, coupled rotating stall and surge, and pure surge. Limit cycle hysteresis is much more in the transition region of rotating stall and surge.
- (3) The study of the combined effect of system length and plenum volume showed that in the transition region, the transient behavior undergoes change from classic surge to constant low amplitude surge to varying amplitude deep surge and then to constant amplitude deep surge.
- (4) Among the compressors of same radius, a longer one will be more prone to rotating stall than to surge. Among the compressors of same length, one with larger radius is more prone to rotating stall than to surge.
- (5) The deep surge at higher *B* values is identified by a cycle of longer period (relaxation type of oscillation) where plenum pressure blowdown takes place very slowly.
- (6) At higher range of *B* values, the oscillation amplitude of plenum pressure decreases with *B* whereas the mass flow amplitude remains the same.

Nomenclature

- *A*: Amplitude of rotating stall disturbance
- A_c : Mean flow area of the compressor, in m²
- *a*: Reciprocal time lag parameter of the blade passage
- a_s : Sound speed, in m/s
- B: Non-dimensional Greitzer parameter
- C_x : Flow axial velocity inside the compressor, in m/s
- *f*₀: Non-dimensional propagation speed of rotating stall-like disturbances
- *H*: Semiheight of cubic axisymmetric characteristic in Figure 2
- K_T : Constant throttle coefficient
- *L_c*: Effective system length of the compressor duct, in m
- *l_c*: Effective non-dimensional length of the compressor and its ducts

- MG: Moore-Greitzer/Moore-Greitzer solution
- *m*: Compressor duct flow parameter
- p_S : Plenum pressure, in N/m²
- p_T : Atmospheric pressure, in N/m²
- *R*: Mean compressor radius, in m
- *t*: Time, in sec
- U: Mean compressor blade speed, in m/s
- V_p : Plenum volume, in m³
- *W*: Semiwidth of cubic axisymmetric characteristic in Figure 2
- ψ_{c0} : Shut-off value of axisymmetric characteristic
- ϕ : Local axial flow coefficient
- Ψ : Plenum pressure rise coefficient
- ρ : Density, in kg/m³
- Φ: Annulus averaged axial flow coefficient
- *ε*: Non-dimensional time
- θ : Angular coordinate around the compressor.

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