Hindawi
International Transactions on Electrical Energy Systems
Volume 2022, Article ID 2857358, 18 pages
https://doi.org/10.1155/2022/2857358

Research Article

Fast Flexible Direct Power Flow for Unbalanced and Balanced Distribution Systems

Debasis Kumar Patel, Swapna Mansani, and Arup Kumar Goswami

Electrical Engineering, NIT Silchar, Silchar 788010, Assam, India

Correspondence should be addressed to Debasis Kumar Patel; debasispatel89@gmail.com

Received 31 January 2022; Revised 21 October 2022; Accepted 26 October 2022; Published 29 November 2022

Academic Editor: Paulo Moisés Almeida Costa

The study proposes a fast flexible direct power flow solution for radial distribution systems and a fast flexible direct weakly meshed power flow solution for weakly meshed distribution systems. The algorithm is based on the direct forward sweep power flow solution, which is an updated version of the backward/forward sweep solution. The fast flexible direct power flow uses a unique conversion matrix (CM) to rapidly determine the power flow solution. The inverted conversion matrix and its slide-modified matrix are used to solve the power flow problem in a single forward sweep step, which is a novel feature of this work. To ensure the invertibility of the conversion matrix, it is constructed to have a small condition number and a determinant of minus one, and all of its eigenvalues must be equal to that of minus one. Additionally, by modifying the conversion matrix to accommodate the loop branch using the break-point idea, a new weakly meshed conversion matrix (WMCM) is generated with the same following modification as for the radial network and employed in the fast flexible direct weakly meshed power flow (FFDWMPF) solution for the weakly meshed distribution network. The usage of a single matrix in the power flow solution and advanced direct techniques decreases the number of iterations and CPU execution time when MATLAB programming is executed. Furthermore, the proposed method is flexible enough to incorporate any changes in the radial or weakly meshed distribution system just by incorporating the changes in the CM and WMCM for any radial or weakly meshed system. Moreover, the robustness of FFDPF and FFDWMPF is evaluated under various loading scenarios on balanced radial and weakly meshed distribution networks. Finally, to validate the proposed algorithm, the proposed strategy is applied to numerous balanced and unbalanced distribution systems.

1. Introduction

Analyzing the power flow of the distribution system is crucial to understanding its behavior. The power flow study provides the steady-state value for the bus voltage and angle on an unspecified bus. This bus voltage and angle data assist in determining the total system parameters in the power distribution network, such as power flow (real and reactive) and total line loss, which are critical for system stability analysis. Furthermore, the modern smart grid faces the challenge of collecting instant bus parameters (that is, the magnitude and angle of bus voltage) through sensors [1]. To fully meet the operational and planning requirements of modern distribution systems, a robust, fast, and flexible power flow technique is required.

Numerous solutions to the power flow problem have been proposed since the mid-nineteenth century. Initially, Dusten [2] introduced the power flow using a digital approach. Ward and Hale [3] successfully implemented this work. Following that, most power flow techniques solve power flow problems using the Ybus admittance matrix and the Gauss–Seidel iterative method. Due to the difficulties associated with convergence, these methods have been replaced in power flow solutions for the power system's primary transmission line by Newton–Raphson (NR) [4], decouple [5], and fast decouple [6] techniques. These updated methods are highly efficient for primary transmission lines, but ineffective for distribution system power flow solutions. Since the distribution system has distinct characteristics from the transmission line of the power system, the characteristics of the distribution system are as follows:

(i) The distributed line has a high r/x ratio
(ii) Transmission lines are highly looped, whereas distribution lines are radial and weakly meshed
(iii) Because the loads on the phases are not equal, the distribution system is naturally unbalanced, whereas the transmission system is naturally balanced

As a result, numerous solution techniques for the power flow (PF) in the distribution system have been proposed by researchers. Researchers have suggested using modified NR [7, 8] and fast decouple [9] methods to solve PF solutions of the distribution system in the first research stage. Moreover, recently, a modified version of the NR with the concept of trust region has been used to solve PF for an unbalanced distribution system with a shorter CPU execution time [10]. In addition to that, Liu et al. [11] introduced the NR method to solve the sequential PF solution for the AC/DC microgrid for an unbalanced radial distribution system. However, these methods require more time-consuming derivative calculations of the Jacobian matrix. In some cases, the optimization-based power management system is used for the PF solution for the hybrid AC/DC microgrid [12]. However, the solution to the optimization problem for power management systems requires a difficult calculation, and the accuracy of the result is not satisfactory. Therefore, most of the researchers are sticking to the rule-based PF solution. In rule-based PF solution, the backward/forward method is used most popularly for PF solution of distribution systems.

The popular backward/forward method has evolved in subsequent stages and continues to evolve to the present day. The PF problem is solved using the straightforward KCL-KVL technique in this approach. The backward sweep is used to calculate branch currents, while the forward sweep is used to calculate bus voltages. To begin with, Shirmohammadi et al. [13] successfully implemented the complete BFSM for radial and weakly meshed radial systems using the compensation technique. The weakly meshed system is converted to a radial system by breaking the loop branch, and the injected current at the break point is calculated using the compensation technique. Additionally, the system is solved using the BFSM as a radial system. Additionally, an enhanced version of this method is used to solve an unbalanced feeder in real time [14]. The method described in [13] is repeated and slightly modified in [15, 16]. In these studies, simple algebraic equations (that is, KCL and KVL) are used to solve the BFSM for the solution of PF of an unbalanced distribution system [15, 16]. In [17], the PF solution for the entire practical unbalanced distribution system is developed using an algebraic equation and the admittance matrix. Additional modifications to [13] are made in [18, 19] for the loop system. Wu and Zhang and Ju et al. [18, 19] solve the PF solution using loop-based mathematical equations. Augugliaro et al. [20] introduce a PF technique for weakly and radially distributed networks. This study uses some algebraic equations to calculate the bus voltage through a backward sweep. Again, Dilek et al. [21] replicate another effective PF technique for the heavily weakly meshed system, in which the new stepping load method is used to address the BFSM's convergence problem at higher loading rates. Similarly, in [22], the generalised BFSM is used to solve the problem of a highly unbalanced radial and weakly meshed distribution system. In [22–24], the authors follow the conventional BFSM to solve the PF analysis for the unbalanced distribution system in the presence of distributed generation (DG). In [23], the author proposed the power summation method in the analysis of the BFS PF for an unbalanced radial distribution system with the incorporation of DG. Similarly, in [24], the authors solve the PF for the unbalanced radial distribution with the addition of optimal incorporation of DG with its uncertainty characteristics. In addition to that, the BFSM and NR methods are used together for the PF solution for the grid and the islanded condition of unbalanced loop systems in [25].

In all the studies mentioned, they are well equipped to find the result of the PF solution with greater accuracy. However, these techniques have some common flaws. All BFSM-based PF solutions require careful observation to determine the load path or unique branch numbering to perform a backward/forward sweep. Furthermore, to obtain the PF solution, the weakly meshed system requires compensation techniques and complex loop-based equations to be solved, which is a difficult task. In addition to that, in conventional BFSM, any bus voltage at the receiving end of a distribution system is calculated as a function of its sending end bus voltage. This chain work method significantly slows down the convergence speed of PF solutions.

This limitation of traditional BFSM has been overcome by BFS techniques based on matrix formation. The BFS technique based on matrix formation is described in [26–38]. Using the substation voltage (that is, the root node) and the branch voltage drop, the bus voltage is calculated (that is, the product of the branch current and the branch impedance). The most widely cited matrix-based DLF technique is described in [25], and this method is implemented for the PF solution of an unbalanced distributed system with DG integration in [26]. Also, the modified form of the DLF method is used for the PF solutions for the AC island microgrid in the radial distribution system in [27]. The DLF method performs a backward-to-forward sweep on two matrices to obtain the PF solution for both radial and weakly meshed distribution systems. The matrix structure eliminates the tedious task of numbering branches. However, matrix formation requires determining the number of load paths between the source and the load end, which is a difficult task when dealing with an extensive network or when reconfiguring a network. Additionally, the two matrices require different works to form for each type of distribution network (radial and weakly meshed). When two different matrices are involved, the calculation step is increased. Although the iteration is optimal, there is a slight increase in CPU time. In [29], a 3-phase PF technique is introduced. This methodology uses the K matrix and the branch impedance matrix to perform the backward and forward sweep to obtain the PF solution. The formation of the K matrix, on the contrary, requires an additional two matrix formations using graph theory, which is a difficult task. Furthermore, in graph theory, a unique branch numbering scheme is used to obtain the K matrix. This method [29] is extended again to the weakly meshed network using the compensation technique. However, the compensation technique itself is a complicated and time-consuming process. In [29], a fast and flexible radial PF technique is proposed for the 3-phase unbalanced system using the BFSM. In addition to that, a similar incidence matrix method is used in the BFSM to solve the PF solution for the 3-phase unbalanced radial distribution system with high penetration of DG [30]. In these methodologies, only one reconfiguration matrix or one incidence matrix is required to perform the PF. This method uses the most straightforward bus numbering scheme. The bus numbers are in ascending order, and the branch number is less than the receiving end bus number. Finally, the line data must be sorted in ascending order of branch number. However, the BFS calculation requires more step calculations, increasing the convergence speed and time-consuming task. Moreover, this method is not implemented in a weakly meshed system. An advanced PF technique is proposed for the unbalanced multiphase system [32]. This method uses the load current matrix and its transformation to obtain the branch current matrix. This branch current matrix is used in BFSM to obtain the PF solution. However, getting the branch current matrix from the load impedance matrix in a backward sweep is a complex task. This matrix formation is difficult in large complex networks, and the treatment for weakly meshed systems is not presented. Furthermore, some methods use graphical methods to create matrices [33–35], which are then used in the PF solutions of radial and weakly meshed distribution systems. To solve the PF solution for the distribution systems, a single incidence matrix is used for radial and two matrices for weakly meshed [33]. Six significant matrices are used in [34] to solve PF solutions for radial and weakly meshed distribution systems using BFSM. However, the difficult task in these two methods is determining the load path, and the need for the formation of a large number of matrices is greater, which requires more iteration and CPU time. Similarly, in [35], a special topological-order matrix derived from graph theory is used to solve the PF solution with BFSM in less time. On the contrary, this method requires careful observation to form the topological matrix in a series of steps. Montoya et al. [36] propose PF solutions for a three-phase unbalanced distribution system using an upper triangular matrix and the BFSM matrix. However, this method necessarily requires the load path-finding work to form the upper triangular matrix with the multistep calculation from backward-to-forward to perform the PF, which increases the number of iterations required to converge. In the next step, the more direct forward approach is introduced [37, 38]. These direct methods are derived from the BFSM and have a faster converge rate with less CPU time. In [37], a load impedance matrix is proposed, which represents the topological structure of the radial network calculated from the set theory. Furthermore, the load impedance matrix is used only in the direct forward method for the PF solution. This method is implemented for both radial and weakly meshed distribution systems. However, the task of constructing the set of branch impedances for each load path (i.e., the path from the source node load) is a difficult one. Again, the intersection of the branch impedance sets is required to find the branch current, which helps to find the bus voltage in a forward sweep. This is computationally significant. Similarly, GhatakMukherjee [38] presented a direct forward approach with only a forward sweep with the help of a unique path matrix. Despite the fact that the convergence rate and CPU time are excellent, forming the path matrix directly from the network topology is a difficult task for both complex radial and weakly meshed networks.

As a result of the abovementioned literature review, it is clear that there is room for research in developing a fast, flexible, and direct-forward-sweep PF method for radial and weakly meshed distribution systems as the modern power system requires more rapid and flexible PF solutions to accommodate significant work such as innovative grid development, network expansion, and reconfiguration.

This study proposes a fast, flexible direct approach PF (FFDPF) solution for radial and weakly meshed distribution networks to address the needs mentioned above. This FFDPF technique is based on a single novel CM, which enables the direct forward sweep method to be used in the FFDPF analysis. A simple bus numbering scheme is required to create the CM. The CM captures the topological structure of the distribution network, allowing easy change of the CM for network expansion and reconfiguration. CM inversion is also required for PF calculation in the FFDPF solution. In a single step, the inversed CM and its transpose are multiplied by the diagonal impedance matrix to obtain the voltage drop matrix for each bus connected to the substation bus. This voltage drop is subtracted from the substation voltage to get the bus voltage in a single forward step. The only forward sweep step cannot be classified as a BFS method, and this proposed method is what this work claims to be novel. This method employs minimal step and matrix computations. Therefore, it converges faster and consumes less CPU time during execution.

Another novel work is implementing a fast flexible direct weakly meshed power flow (FFDWMPF) solution for a weakly meshed distribution system with slide changes in the conversion matrix. Additionally, the proposed method demonstrated a high level of robustness under higher loading conditions due to the matrix calculation in the algorithm. To validate the algorithms of both FFDPF and FFDWMPF, these algorithms are applied to a variety of IEEE standard radial and weakly meshed systems (that is, balanced 33-bus and 69-bus and unbalanced 10-bus and 25-bus radial and weakly meshed distribution systems).

The remaining sections of the study are divided into the following sections: Section 2 discusses three-phase line modeling; Section 3 discusses power flow formulation; Section 4 discusses power flow formulation for weakly meshed systems, Section 5 discusses results and discussion, and Section 6 discusses conclusions.

2. Three-Phase Line Modeling

The three-phase line model extends single-phase transmission into a three-phase line section. Figure 1 illustrates a branch section of a three-phase line. Carson's rule can be used to calculate the parameters of any line connecting two buses. After that, Korn's reduction technique is applied to eliminate the neutral impedance and phase-to-neutral impedance, and the modified line impedance matrix of dimension 3×3 is obtained [26], which is represented as

$$Z_{abc} = \begin{bmatrix} Z_{aa\_n} & Z_{ab\_n} & Z_{ac\_n} \\ Z_{ba\_n} & Z_{bb\_n} & Z_{bc\_n} \\ Z_{ca\_n} & Z_{cb\_n} & Z_{cc\_n} \end{bmatrix}, \quad (1)$$

where Z_{aa_n}, Z_{bb_n}, and Z_{cc_n} are represent the self-impedance and Z_{ab_n}, Z_{bc_n}, and Z_{ca_n} are the mutual impedances in phase *a*, *b,* and *c*. Now, the bus phase voltages, line current, and line impedance of the line section can be represented in the equation below:

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} - \begin{bmatrix} Z_{aa\_n} & Z_{ab\_n} & Z_{ac\_n} \\ Z_{ba\_n} & Z_{bb\_n} & Z_{bc\_n} \\ Z_{ca\_n} & Z_{cb\_n} & Z_{cc\_n} \end{bmatrix} \begin{bmatrix} I_{Aa} \\ I_{Bb} \\ I_{Cc} \end{bmatrix}, \quad (2)$$

where V_A, V_B, V_C and V_a, V_b, V_c are the sending and receiving end bus voltage of the 3-phase line sections for each phase. Furthermore, I_{Aa}, I_{Ba}, and I_{Ca} are the line section current for phase *A*, *B,* and *C,* respectively. In the case of a missing phase, the row and column elements of the corresponding phase in the impedance matrix must be filled with zeros. For example, if phase *C* is absent, the line section impedance matrix is denoted as the following equation:

$$Z_{abc} = \begin{bmatrix} Z_{aa\_n} & Z_{ab\_n} & 0 \\ Z_{ba\_n} & Z_{bb\_n} & 0 \\ 0 & 0 & 0 \end{bmatrix}. \quad (3)$$

3. Formulation of Power Flow

The proposed FFDPF methodology is based on the simple formulation of the CM. It represents the complete connection between the sending and receiving end buses of the distribution system under consideration. The CM is formed after applying the simple bus numbering rule in any distribution network, which is a simple work compared with the complex numbering, load pathfinding, and the number of buses ahead finding, which helps in reducing the observation work and provides better flexibility. The next step is the inversion of the CM matrix to obtain the corresponding two slide modification matrices and multiply them by the diagonal branch impedance matrix. This multiplier product matrix is again multiplied by the load current matrix to obtain the voltage drop matrix for the different buses from the substation bus. The voltage drop matrix is directly subtracted from the substation-rated voltage to obtain the corresponding updated bus voltages. Furthermore, the load current matrix is represented by the diagonalised complex conjugate of complex load power and bus voltages to reduce the BFSM to a direct forward method. So, all this helps to obtain the voltage drop matrix for the different buses from the substation bus in a single step. This single step reduces the forward and backward steps to a single forward step, which helps to fasten the convergence rate and save CPU time. Apart from that, the CM provides the solution for the weakly meshed system with sliding modification to include the meshed branch of the network. The proposed method is first implemented on a balanced system as a single-phase for better understanding. Later, this work is implemented for the 3-phase systems. The derivation of the proposed work using the CM matrix and the procedure of the PF is presented in the following sections.

3.1. Flexibility and Simplicity in Bus Numbering and Line Section. The radial and weakly meshed networks are configured uniquely, with the distribution substation serving as the source bus. The substation serves as the root node (source bus), from which all power is distributed to all load ends through the lateral and sublateral feeders of the distribution network. According to most reviews of the literature, a complex bus numbering rule is required to perform power flow, which presents difficulties for network reconfiguration, mesh formation, and network extension for distribution system planning. As a result, this proposed load flow method for radial and weakly meshed distribution systems adheres to the simple bus numbering rule. The proposed FFDPF method for radial networks adheres to the simple bus numbering rule, which states that the receiving bus number should be greater than the sending end bus, which results in an overrepresentation of bus numbers in ascending order in the lateral and sublateral feeders. The following step renumbers the branch numbers by one less than the receiving end. Now, as specified in [30], we arrange the branch numbers in ascending order and use them as input line data for programming. This straightforward bus numbering scheme is illustrated in Figure 2 for the six-bus radial distribution network.

3.2. Formulation of Configuration Matrix and Its Role in PF. Only the CM matrix is required for the FFDPF solution. The CM would be in charge of implementing any necessary changes to the existing structure or the addition of new networks. FFDPF requires only the results of the CM; no additional matrices are required for the direct forward sweep method. CM is a square matrix with the dimension of $NB \times NB$. Algorithm 1 is used to construct the CM for a balanced distribution system.

Algorithm 1 can be summarised in a mathematical representation of the entry of elements into CM as presented in the following equation:

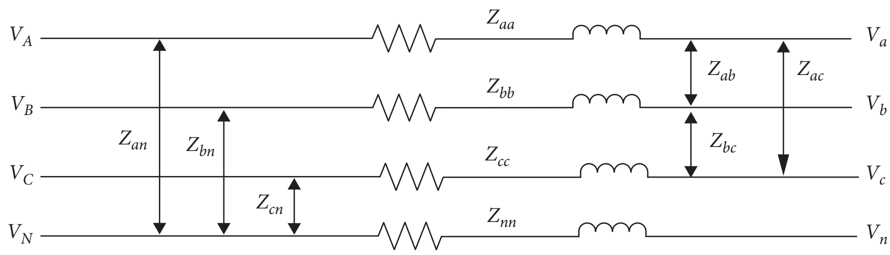

Figure 1: Three-phase branch section model.

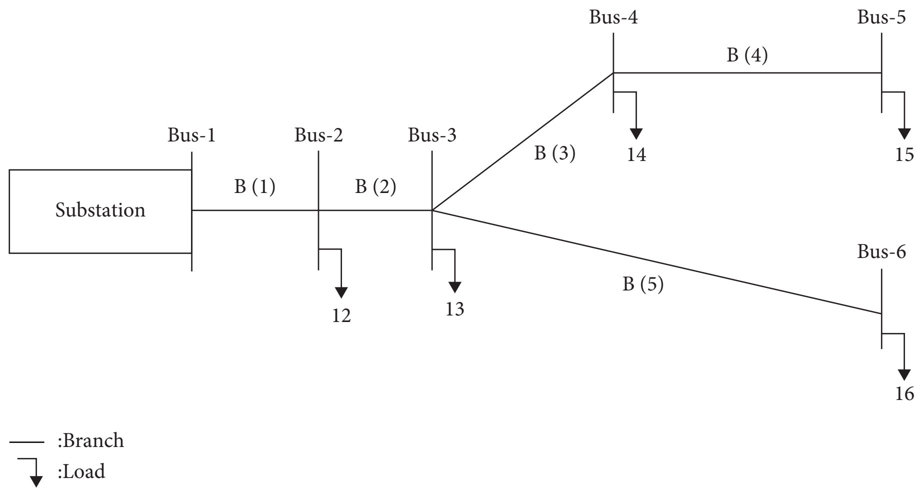

Figure 2: Six-bus balanced radial distribution system.

$$\mathrm{CM}_{\mathrm{row}}(i) = \begin{cases} -1, & i, \quad \text{if } i \text{ is either} \begin{cases} a - \text{sending bus } i, \\ b - \text{dead} - \text{end bus } i, \end{cases} \\ 1, & j, k, l, \ldots, \quad \begin{array}{l} \text{if } j, k, l, \ldots \text{ are receving end buses} \\ \text{physically connected to bus } i, \end{array} \\ 0, & \text{otherwise.} \end{cases} \tag{4}$$

Algorithm 1 should be followed for the 3-phase unbalanced system with an increment in the dimension of the CM by just multiplying three by the dimension of the balanced CM. If any bus is missing for any phase or is a dead-end bus, the only entry in its corresponding row is zero's diagonal entry similar to equation (3). For example, the resultant CM is as follows when the CM formation algorithm is applied to the six-bus radial distribution system (Figure 2):

$$\mathrm{CM} = \begin{bmatrix} -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 1 \\ 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}. \tag{5}$$

3.2.1. Significance of CM. The CM is a nonsingular (i.e., nonzero determinant matrix), and the condition number (CN) is small (i.e., 8.5048 for the CM of a 6-bus system). The condition number is understood as, if $Ax = b$, then the maximum ratio of the relative error in x to the relative error in b. The condition number is small for CM, which signifies the condition of a well-conditioned matrix. The CM matrix consists of only three elements minus 1, plus 1, and zero. Therefore, if there is an increase in the loop number in the system, there is only an extra addition of the column number and a row number in the WMCM. In addition to that, there is a change of minus 1 and 1 in the newly added column of the WMCM matrix corresponding to the row of the adjacent bus. As a result of the new addition of rows and columns, the matrix dimension of WMCM is increased. This change in the matrix does not change much in the condition number (CN) or the matrix's nonsingularity (i.e., the eigenvalue and the determinant are always minus 1 for CM). As a result, the

CM matrix's inversion is always possible, and a unique solution is possible even for the more extensive system.

3.3. Path Matrix (PM). After finding the CM, the path matrix is formed by inverting the CM and removing the first column and first row of the inverse configuration matrix. The resulting path matrix is depicted in the following equation:

$$\mathrm{PM} = \begin{bmatrix} -1 & -1 & -1 & -1 & -1 \\ 0 & -1 & -1 & -1 & -1 \\ 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix}. \tag{6}$$

3.4. Bus Voltage Calculation in Forward Sweep. Now, to calculate the bus voltage, the bus voltage drop is subtracted from substation bus in the single forward sweep step, which is represented as follows:

$$\begin{gathered} V = \left[V_{\text{rated}}\right] - [\Delta V], \\ V = V_{\text{rated}} - [[\text{Transpose}(\text{PM}) \times \text{diag}Z] \times [\text{PM}]] \times \left[I_L\right], \\ \begin{bmatrix} V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix} = V_{\text{rated}} - \begin{bmatrix} Z_{12} & Z_{12} & Z_{12} & Z_{12} & Z_{12} \\ Z_{12} & Z_{12}+Z_{23} & Z_{12}+Z_{23} & Z_{12}+Z_{23} & Z_{12}+Z_{23} \\ Z_{12} & Z_{12}+Z_{23} & Z_{12}+Z_{23}+Z_{34} & Z_{12}+Z_{23}+Z_{34} & Z_{12}+Z_{23} \\ Z_{12} & Z_{12}+Z_{23} & Z_{12}+Z_{23}+Z_{34} & Z_{12}+Z_{23}+Z_{34}+Z_{45} & Z_{12}+Z_{23} \\ Z_{12} & Z_{12}+Z_{23} & Z_{12}+Z_{23} & Z_{12}+Z_{23} & Z_{12}+Z_{23}+Z_{36} \end{bmatrix} \begin{bmatrix} I_{L2} \\ I_{L3} \\ I_{L4} \\ I_{L5} \\ I_{L6} \end{bmatrix}. \end{gathered} \tag{7}$$

The voltage drop matrix is a positive matrix because the multiplication of the two negative matrices (that is, Transpose (PM) and PM) is resulted as the positive matrix. Also, due to the positive diagonalise impedance matrix, the complete voltage drop is positive. In the next step, the load current can be represented in terms of the load complex power and voltage. The voltage equation is shown in the following equation:

$$V = V_{\text{rated}} - [[[\text{Transpose}(\text{PM}) \times \text{diag}Z] \times [\text{PM}]] \times \text{diag}(S)^*]\left[\frac{1}{V^*}\right], \tag{8}$$

$$\begin{bmatrix} V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix} = V_{\text{rated}} - \begin{bmatrix} (Z_{12})S_2^* & (Z_{12})S_3^* & (Z_{12})S_4^* & (Z_{12})S_5^* & (Z_{12})S_6^* \\ (Z_{12})S_2^* & (Z_{12}+Z_{23})S_3^* & (Z_{12}+Z_{23})S_4^* & (Z_{12}+Z_{23})S_5^* & (Z_{12}+Z_{23})S_6^* \\ (Z_{12})S_2^* & (Z_{12}+Z_{23})S_3^* & (Z_{12}+Z_{23}+Z_{34})S_4^* & (Z_{12}+Z_{23}+Z_{34})S_5^* & (Z_{12}+Z_{23})S_6^* \\ (Z_{12})S_2^* & (Z_{12}+Z_{23})S_3^* & (Z_{12}+Z_{23}+Z_{34})S_4^* & (Z_{12}+Z_{23}+Z_{34}+Z_{45})S_5^* & (Z_{12}+Z_{23})S_6^* \\ (Z_{12})S_2^* & (Z_{12}+Z_{23})S_3^* & (Z_{12}+Z_{23})S_4^* & (Z_{12}+Z_{23})S_5^* & (Z_{12}+Z_{23}+Z_{36})S_6^* \end{bmatrix} \begin{bmatrix} \dfrac{1}{V_2^*} \\ \dfrac{1}{V_3^*} \\ \dfrac{1}{V_4^*} \\ \dfrac{1}{V_5^*} \\ \dfrac{1}{V_6^*} \end{bmatrix}, \tag{9}$$

(1) Make a square matrix with the required dimensions and fill in each element with zero.
(2) Substitute the '−1'entries for all diagonal elements representing all sending end and dead-end buses.
(3) In each row, if the column index corresponds to an existing receiving bus, change the entry to 1.

ALGORITHM 1: Formation of CM.

where diag$Z$ is the diagonal matrix of the branch impedance, "I_L" is the load current matrix, and diag(S^*) is the diagonal matrix of the complex conjugate of the complex load power, which are represented by the following equations, respectively:

$$\text{diag}Z = \begin{bmatrix} Z_{12} & 0 & 0 & 0 & 0 \\ 0 & Z_{23} & 0 & 0 & 0 \\ 0 & 0 & Z_{34} & 0 & 0 \\ 0 & 0 & 0 & Z_{45} & 0 \\ 0 & 0 & 0 & 0 & Z_{36} \end{bmatrix}, \tag{10}$$

$$I_L = \begin{bmatrix} I_{L1} \\ I_{L2} \\ I_{L3} \\ I_{L4} \\ I_{L5} \end{bmatrix}, \tag{11}$$

$$\text{diag}(S^*) = \begin{bmatrix} S_2^* & 0 & 0 & 0 & 0 \\ 0 & S_3^* & 0 & 0 & 0 \\ 0 & 0 & S_4^* & 0 & 0 \\ 0 & 0 & 0 & S_5^* & 0 \\ 0 & 0 & 0 & 0 & S_6^* \end{bmatrix}. \tag{12}$$

At last, the solution of the PF can be obtained by solving the following equation iteratively:

$$V^{k+1} = V_{\text{rated}} - [\text{Factor\_matrix}]\left[\frac{1}{V^k}\right]. \tag{13}$$

3.4.1. Convergence Criteria of FFDPF. There should be a convergence criterion for the FFDPF algorithm. The convergence criteria of FFDPF is the difference between the two successive iterative complex voltages, and its absolute value must be less than the tolerance value of "ε." The convergence criteria are stated below in equations (14) and (15). The selection of the epsilon value is important in determining the convergence rate of the algorithm. Because epsilon is the difference between the two consecutive voltage values of the two consecutive iterations, the epsilon decides up to how many decimal points of accuracy is required. On the basis of the need for accuracy, the epsilon value will be decided. For instance, if up to three decimal points of accuracy are required, the epsilon value will be 0.0001:

$$\Delta V_{\text{bus}}^{k+1} = V^{k+1} - V^k, \tag{14}$$

$$\max\left(\Delta V_{\text{bus}}^{k+1}\right) \leq \varepsilon, \tag{15}$$

where $\Delta V_{\text{bus}}^{k+1}$ = voltage difference between the two consecutive iterations, $k = k^{\text{th}}$ iteration, and ε = convergence tolerance value.

3.5. The Algorithm for the Radial Distribution Network for FFDPF. Algorithm 2 summarizes the proposed FFDPF method that can be used to solve the power flow solutions for the radial distribution system.

3.5.1. Required Modification in Algorithm 2 for 3-Phase Unbalanced System. The proposed Algorithm 2 can be easily implemented into the 3-phase unbalance system by changing the dimension by $3 \times (\text{NB} \times \text{NB})$. In the dimension of the next step, the PM dimension is turned into $3 \times (\text{NBR} \times \text{NBR})$. On the contrary, the line impedance matrix has a size according to the number of phases present in each line branch section, similar to equations (1) and (3). The diagonal impedance matrix is the diagonalization of all branch impedance matrices and its dimension $3 \times (\text{NBR} \times \text{NBR})$. Similarly, the diagonalization of the complex power matrix is the diagonalization of each load power at each bus (that is, $[S_a\ 0\ 0;\ 0\ S_b\ 0;\ 0\ 0\ S_c]$), which has the dimension of $3 \times (\text{NB} - 1 \times \text{NB} - 1)$. The corresponding Factor_matrix is obtained with a size of $3 \times (\text{NBR} \times \text{NBR})$. The updated voltage vector matrix in equation (13) is of dimension $3 \times (\text{NB} - 1 \times 1)$, which starts from the second bus number. The bus vector matrix is represented as $[V_{2a}, V_{2b}, V_{2c}, \ldots V_{NB\_a}, V_{NB\_b}, V_{NB\_c}]^t$, and the V_{rated} matrix is defined as $[V_{2\text{rated}\_a}, V_{2\text{rated}\_b}, V_{2\text{rated}\_c}, \ldots V_{\text{NB\_rated}\_a}, V_{\text{NB\_rated}\_b}, V_{\text{NB\_rated}\_c}]^t$.

4. Formulation of the Power Flow for Weakly Meshed Network

Open tie switches can sometimes cause loops in the distribution network in crowded load areas. Weakly meshed distribution systems are those with such feeders. The presence of a tie switch improves the feeder voltage profile, but loop currents make the system analysis more complicated. The following section discusses the changes that must be made to the CM to create the WMCM and the extension of the proposed method to handle weakly meshed systems. Thus, discussing the formulation of the PF problem for the weakly meshed network is essential. To handle a weakly meshed system, some modifications to the CM are required. The following section focuses on this aspect as a continuation of the proposed approach.

4.1. Weakly Meshed Conversion Matrix (WMCM) and Its Role in PF. The first step in bus numbering is to adhere to a simple bus numbering rule that is analogous to the radial

Step 1: Apply the simple bus numbering rule to find the CM.
Step 2: Find the PM, transpose the PM, and diagonalise the branch impedance matrix.
Step 3: Calculate the factor matrix by multiplying the transpose of the PM and the diagonal impedance matrix, and then multiply by the PM. At last, multiply the product with diagonal conjugate complex load power to obtain the Factor_matrix.
Step 4: Start the iterative process by assuming that all bus voltages are equal to the substation-rated voltage.
Step 5: Update the bus voltage.
Step 6: Multiply the factor matrix and $[1/V^*]$ matrix to get the voltage drop matrix of different buses from the source node and calculate the bus voltage by subtracting the substation-rated voltage.
Step 7: Determine the absolute difference of bus voltage with the current iteration value and with pervious iteration value for all bus voltages, and pick the maximum difference.
(i) If the difference is $\leq \varepsilon$
Solution achieved
Stop and terminate the FFDPF procedure.
Get the final bus voltage and branch current.
(ii) If not, use the result of this iteration (the bus voltage) to start the new one by returning to Step 5.

Algorithm 2: Formation of FFDPF.

distribution network [30]. However, the branch that creates the mesh in the weakly meshed network is numbered randomly, as there is no need for proper numbering, i.e., left-to-right numbering, after the nonmeshed branches are numbered (i.e., the branch does not participate in mesh formation). The relationship between the NBR and NB is indicated by the formula $\mathrm{NBR} = (\mathrm{NB} - 1) + \mathrm{NL}$.

Steps to obtain the WMCM matrix are shown in Algorithm 3.

4.1.1. Break-Point Concept. Most importantly, in the break-point concept, an imaginary bus created by the loop branch behaves like a receiving end bus for the two buses in which the loop branch is connected [26]. However, the imaginary receiving bus is denoted with a plus one and a minus one for the two adjacent buses, respectively. The mathematical representation of the entry of elements into WMCM is presented in equation (16), which is a summarised way of forming Algorithm 3:

$$\mathrm{WMCM}_{\mathrm{row}}(i) = \begin{cases} -1 \quad i \quad \text{if } i \text{ is either} \begin{cases} a - \text{sending bus } i \\ b - \text{dead} - \text{end bus } i \end{cases} \\ 1, \quad j,k,l,\ldots, \quad \begin{matrix} \text{if } j,k,l,\ldots \text{ are receiving end buses} \\ \text{physically connected to bus } i, \end{matrix} \\ \pm 1 \text{ and } \mp 1, \quad m, \quad \begin{matrix} \text{if } m \text{ is the imaginary bus acting like} \\ \text{receiving bus for buses } p \text{ and } q\,(m) \text{ is connected,} \\ \text{in between the bus } p \text{ and } (q), \end{matrix} \\ 0, \quad \text{otherwise.} \end{cases} \tag{16}$$

After applying Algorithm 3 to the weakly meshed network shown in Figure 3, the WMCM is as follows:

$$\mathrm{WMCM} = \begin{bmatrix} -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & +1 \\ 0 & 0 & 0 & 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}. \tag{17}$$

Again, the WMPM can be derived from the WMCM matrix just by an inversion of the WMCM matrix and eliminating the first row and first column as shown in the following equation:

$$\mathrm{WMPM} = \begin{bmatrix} -1 & -1 & -1 & -1 & -1 & 0 \\ 0 & -1 & -1 & -1 & -1 & 0 \\ 0 & 0 & -1 & -1 & 0 & -1 \\ 0 & 0 & 0 & -1 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}. \tag{18}$$

(1) Build a square matrix of the required dimensions and fill each element with zero.
(2) Modify all diagonal elements with −1 entries, representing all sending ends and dead-end buses.
(3) Now, the receiving end bus represents 1, corresponding to the sending end bus.
(4) For the loop's column, change the loop element with a plus one and a minus one, or vice-versa for the two adjacent buses. In between, the loop branch is connected.

Algorithm 3: Formation of WMCM.

4.2. Bus Voltage Calculation in Forward Sweep. The next step is to multiply the three matrices such as transpose of the WMPM, diagonalise path impedance matrix, and WMPM, to get the total branch path impedance matrix, which can be again multiplied with the load current matrix to get the total voltage drop matrix of each bus voltage from the substation voltage, which can be mathematically represented as the following equation:

$$\begin{bmatrix} \Delta V \\ 0 \end{bmatrix} = [[\text{Transpose}(\text{WMPM}) \times \text{diag}Z] \times [\text{WMPM}]] \begin{bmatrix} I_L \\ 0 \end{bmatrix}, \tag{19}$$

$$\begin{bmatrix} \Delta V_1 \\ \Delta V_2 \\ \Delta V_3 \\ \Delta V_4 \\ \Delta V_5 \\ 0 \end{bmatrix} = \begin{bmatrix} Z_{12} & Z_{12} & Z_{12} & Z_{12} & Z_{12} & 0 \\ Z_{12} & Z_{12}+Z_{23} & Z_{12}+Z_{23} & Z_{12}+Z_{23} & Z_{12}+Z_{23} & 0 \\ Z_{12} & Z_{12}+Z_{23} & Z_{12}+Z_{23}+Z_{34} & Z_{12}+Z_{23}+Z_{34} & Z_{12}+Z_{23} & Z_{34} \\ Z_{12} & Z_{12}+Z_{23} & Z_{12}+Z_{23}+Z_{34} & Z_{12}+Z_{23}+Z_{34}+Z_{45} & Z_{12}+Z_{23} & Z_{34} \\ Z_{12} & Z_{12}+Z_{23} & Z_{12}+Z_{23} & Z_{12}+Z_{23} & Z_{12}+Z_{23}+Z_{56} & -Z_{56} \\ 0 & 0 & Z_{34} & Z_{34}+Z_{45} & -Z_{56} & Z_{34}+Z_{45}+Z_{56}+Z_{36} \end{bmatrix} \begin{bmatrix} I_{L2} \\ I_{L3} \\ I_{L4} \\ I_{L5} \\ I_{L6} \\ 0 \end{bmatrix}. \tag{20}$$

The change in bus voltage is represented as

$$\left[\frac{V^{k+1}}{0}\right] = \left[V^k_{\text{rated}}\right] - [\text{Transpose}(\text{WMPM}) \times \text{diag}Z] \times [\text{WMPM}] \left[\frac{I_L}{0}\right]. \tag{21}$$

Here, the sum of the branch impedance of weakly meshed (BIWM) matrix from the substation to bus is denoted as

$$\text{BIWM} = [\text{Transpose}(\text{WMPM}) \times \text{diag}Z] \times [\text{WMPM}] = \begin{bmatrix} P & Q \\ R & S \end{bmatrix}. \tag{22}$$

By substituting the BIWM representation, equation (22) is represented as

$$\left[\frac{V^{k+1}}{0}\right] = \left[V^k_{\text{rated}}\right] - \begin{bmatrix} P & Q \\ R & S \end{bmatrix} \left[\frac{I_L}{0}\right]. \tag{23}$$

As the zeroth row is present in the equation due to the weakly meshed branch, to bring the matrix to the required dimension, Korn's reduction technique must be used to remove the zeroth row (i.e., the number of buses in the system) [37]. As a result, Korn's reduction is applied to the factor matrix. After that, equation (23) can be represented as

$$\left[V^{k+1}\right] = \left[V^k_{\text{rated}}\right] - \left[\left([P] - \left([Q][S]^{-1}[R]\right)\right)[I_L]\right]. \tag{24}$$

Now, equation (24) can be represented with only the complex conjugate of the load complex power and bus voltage instead of load current as shown in equation (25). The resultant single forward step equation (26) is solved iteratively to obtain the PF solution [37]:

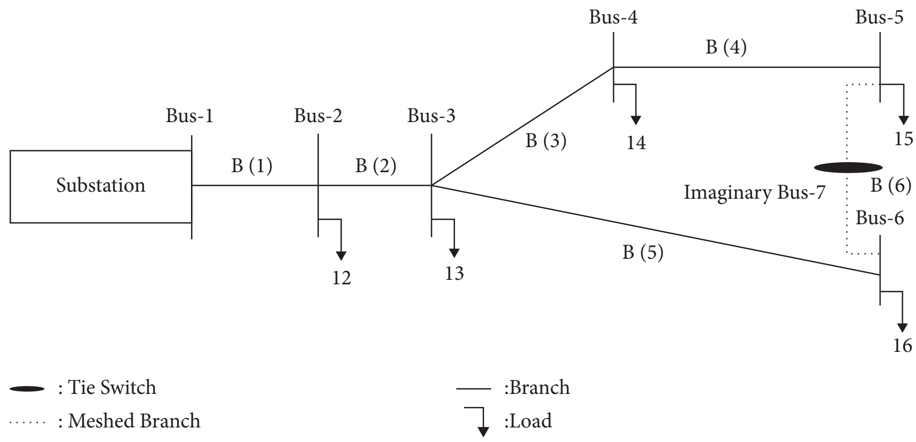

Figure 3: Six-bus balanced weakly meshed distribution system.

$$\left[V^{k+1}\right] = \left[V^{k}_{\text{rated}}\right] - \left[\left([P] - \left([Q][S]^{-1}[R]\right)\right) \times \operatorname{diag}\left(S^{*}\right)\right] \times \left[\frac{1}{V^{*}}\right], \tag{25}$$

$$\left[V^{k+1}\right] = \left[V^{k}_{\text{rated}}\right] - \left[\text{Factor\_Matrix}_{\text{weakly}}\right] \times \left[\frac{1}{V^{*}}\right], \tag{26}$$

where the dimensions of the matrix P, Q, R, and S matrix are (NB-1) × (NB − 1), (NB − 1 × NL), (NL × NB − 1), and (NL × NL), respectively. Equation (26) solves iteratively to get the PF solution of weakly meshed system. The same convergence criterion discussed for FFDPF is also applied for the weakly meshed system. For implementing FFDWMPF algorithm in a 3-phase system, the dimension of WMCM, WMPMFactor_Matrix$_{\text{weakly}}$, V^{k}_{rated}, diag$Z$, and diag(S^{*}) is similar to the 3-phase dimensions of radial distribution system as discussed in radial PF section. Whereas the dimensions of the matrix P, Q, R, and S are 3 × ((NB − 1) × (NB − 1)), 3 × (NB − 1 × NL), 3 × ((NL × NB − 1), and 3 × (NL × NL), respectively. The complete algorithm of FFDWMPF is shown in Section 4.3.

4.3. The Algorithm for the Weakly Meshed Distribution Network for FFDPF. The FFDWMPF algorithm can be summarised as follows to solve the power flow solution for weakly meshed distribution system, which is shown in Algorithm 4.

5. Results and Discussion

The complete proposed Algorithms 2 and 4 are written in MATLAB code version R2016a and run on a computer with an Intel Core i3 2.4 GHz processor and 4 GB RAM. The algorithms are used in a variety of distribution power networks, both balanced and unbalanced, and in its radial and weakly meshed network categories. Both the proposed radial and weakly meshed PF methods yield the power flow results presented later in the latter section. The performance of the proposed approach is compared to that of an existing power flow technique, whose studies are mentioned in this study's literature.

5.1. Balanced 3-Phase Network. The proposed Algorithm 2 is applied to two IEEE standard three-phase balanced radial distribution networks, namely, the 33- and 69-bus balanced systems. The single line diagrams for both systems are shown in Figures 4 and 5, respectively, where the meshed branch is omitted for the radial distribution network. The simple bus numbering rule is used to number the buses and branches of the balanced distribution network. In a simple bus numbering rule, the buses in a lateral and sublateral feeder have been numbered ascendingly, and the branch is smaller than the receiving end bus. The results obtained are not tabulated, and the branch number is not depicted in Figure 5. This conserves space and avoids the clumsiness in Figure 5. However, as illustrated in Figures 6(a) and 6(b), the results are graphed for both 33-bus and 69-bus systems. To validate the results of the proposed method, the obtained results are compared to the result published in [39] for both systems. The maximum voltage magnitude difference for a 33-bus system was determined to be 1.54641×10^{-5} p.u, while for a 69-bus radial distribution system, it was determined to be 0.0007009 p.u.

Furthermore, the proposed Algorithm 2 is compared with four other methods in [1, 26, 40] for comparison of iteration number. Moreover, for the sake of clarity, the CPU execution time of the proposed Algorithm 2 is compared to the most frequently cited DLF method [26] implemented on the same computer system. The 33-bus and 69-bus systems are compared in the same way. Tables 1 and 2 illustrate this number of iterations and CPU time comparisons for both 33-bus and 69-bus systems. The convergence tolerance value is defined as 0.0001 p.u for Algorithm 2 and other methods compared. The proposed Algorithm 2 has an optimal iteration, which is consistent with previous research. However, the CPU time required for execution is less than the most

Step 1: Determine the WMCM using the simple bus numbering rule and the break-point concept.
Step 2: Identify the WMPM and its transposition. At the same time, the branch impedance matrix is diagonalised.
Step 3: Multiply the WMPM by the diagonal impedance matrix, and then multiply by the transpose of the WMPM to obtain the BIWM matrix.
Step 4: Apply Korn's reduction to the BIWM matrix's zeroth row until all zeroth rows are eliminated and then multiply the diagonalise complex conjugate load power to reduced BIWM matrix to obtain the $\text{Factor\_Matrix}_{\text{weakly}}$.
Step 5: Begin the iterative process by assuming that all bus voltages are equal to the rated voltage of the substation.
Step 6: Update the bus voltage.
Step 7: Multiply the factor matrix and the $[1/V^*]$ matrix to obtain the voltage drop matrix for each bus connected to the source node. Subtract the bus voltage drop matrix from the substation-rated voltage to get the bus voltage.
Step 8: Determine the absolute difference of bus voltage with current iteration value with pervious iteration value for all bus voltage and pick the maximum difference.
(i) If the difference is $\leq \varepsilon$
Solution achieved
Stop and terminate the FFDWMPF procedure.
Get the final bus voltage and branch current.
(ii) If not, use the result of this iteration (the bus voltage) to start the new one by returning to Step 6.

Algorithm 4: Formation of FFDWMPF.

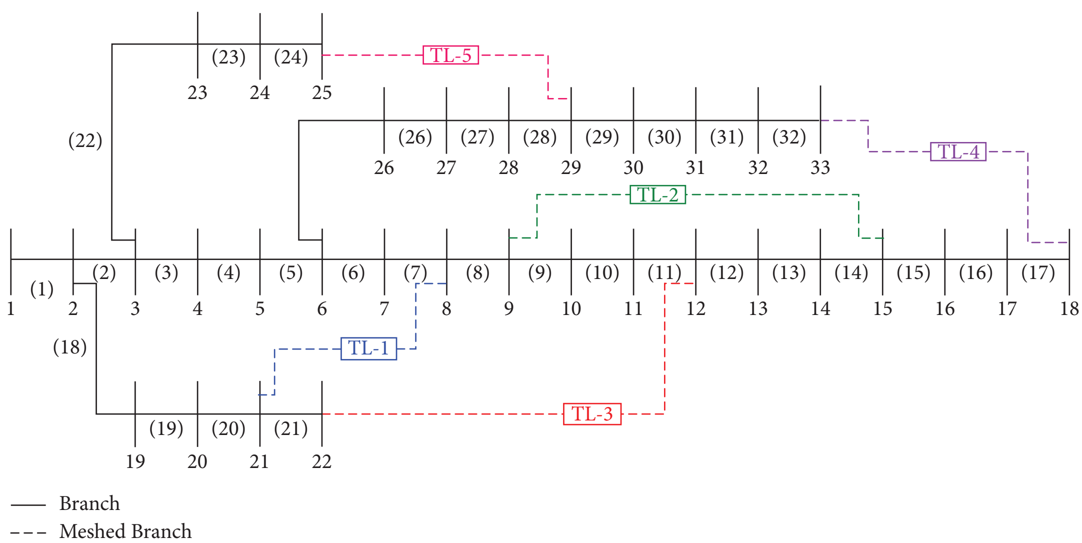

Figure 4: Thirty three-bus weakly meshed distribution system.

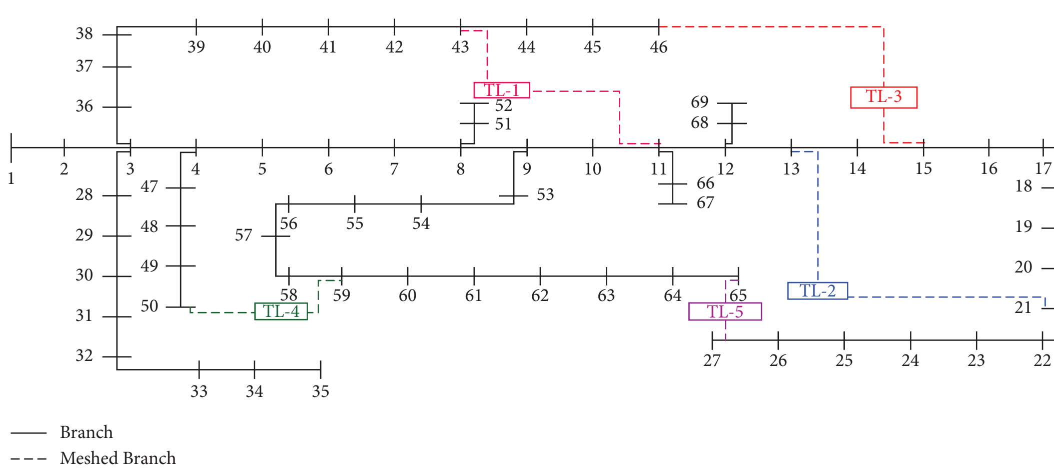

Figure 5: Sixty nine-bus weakly meshed distribution system.

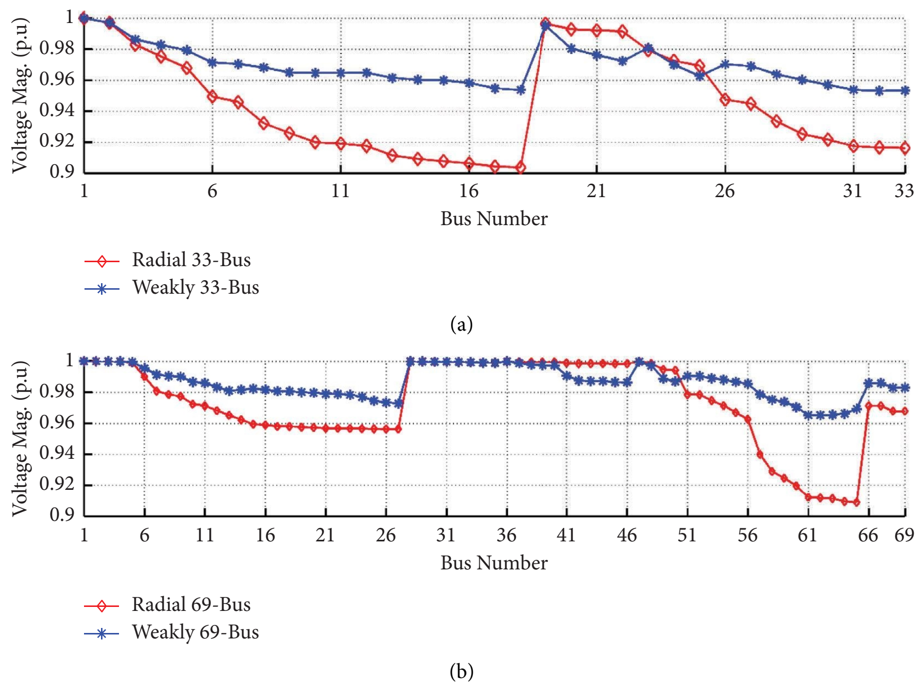

Figure 6: The magnitude of the radial and weakly meshed system voltages: (a) 33-bus and (b) 69-bus.

Table 1: Comparison of iterations of the balanced 33-bus and 69-bus system.

Methods	Balanced Iteration	
	33-bus system	69-bus system
Backward/forward sweep [40]	4	4
Current injection [1]	—	5
Improved backward/forward sweep [1]	4	4
DLF [26]	4	4
Proposed method FFDPF (Algorithm 2)	4	4

Table 2: Comparison of the CPU execution time of the balanced 33-bus and 69-bus system.

Methods	Balanced CPU time in sec	
	33-bus system	69-bus system
DLF [26]	0.05	0.066
Proposed method FFDPF (Algorithm 2)	0.02496	0.045

frequently used DLF method [26]. Due to its matrix calculation, the novel direct forward approach method is one of the fastest methods in terms of CPU execution time when compared to other methods. The following conclusions are inferred from the results of the proposed method.

Firstly, the proposed Algorithm 2 has fewer steps because it solves the problem in a single forward sweep, which helps to reduce the number of iterations. Secondly, MATLAB programming takes less CPU time to run because only one CM matrix is used to solve the entire method; other methods, on the contrary, we use multiple matrix formations to run their methods. This matrix calculation saves CPU time by reducing the number of programming loops in MATLAB. Lastly, flexibility can be provided in structuring the network topology by using simple bus numbering rules and uses in the formation of CM. Therefore, the proposed algorithm can be easily used for a more extensive network extension and reconfiguration system by simply changing the CM. This facility is a significant advantage over the previously mentioned methods in the literature.

5.2. Unbalanced Three-Phase Network. Similarly, in the case of an unbalanced system, the extended version of the proposed Algorithm 2 is used for two unbalanced radial distribution systems. The first is a radial distribution system with ten buses [41], and the second is a radial distribution system with twenty-five buses [41]. The bus numbering of the 10-bus system has been retained from the source article, but the bus numbers have been rearranged according to a simple bus numbering rule for the 25-bus system as illustrated in Figure 7. The effects of shunt capacitance are ignored in these unbalanced systems. Additionally, to validate the results, they are compared to previously published data. In comparison with the result of [41], the p.u. maximum voltage difference in phase A, phase B, and phase C is 4.36×10^{-5}, 2.7403×10^{-5}, and 6.09525×10^{-5}, respectively. Likewise, when the results are compared to the result of [42] for a 25-bus system, the p.u maximum voltage difference is 0.000662, 0.002365, and 0.002989 for phase A, phase B, and

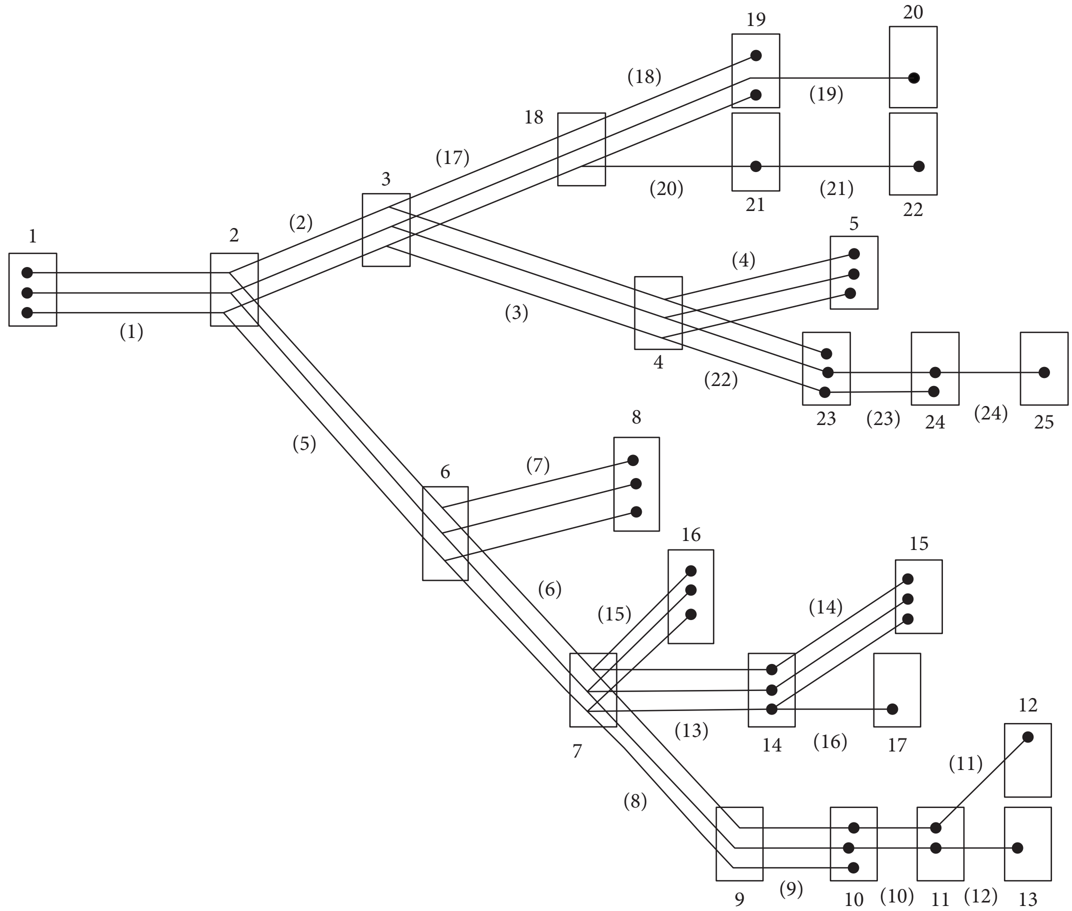

Figure 7: Unbalanced 25-bus radial distribution system.

phase C, respectively. The results are compared with the voltage on the bus that is carrying the load. Additionally, the line graph is illustrated only for the proposed method's results to conserve space. The results are depicted in Figure 8.

As shown in Table 3, the proposed Algorithm 2 has good convergence characteristics compared to the traditional BFS quasi-coupled method, while the number of iterations is identical to the most widely used DLF PF technique. However, as shown in Table 4, the CPU execution time is lower compared to all other methods. It is concluded that while CPU time is not critical in distribution system PF analysis, modern microgrid automation requires faster PF analysis operation. So, in this case, the proposed FFDPF method could be highly beneficial. Furthermore, the performance characteristics are favorable in the case of a practical unbalanced system. The proposed FFDPF method can be used for network reconfiguration and network extension without the need for many matrices or time-consuming, complex path identification, as most methods do in the literature.

Table 3: Comparison of iterations of the balanced 10-bus and 25-bus system.

Methods	Unbalanced Iteration	
	10-bus	25-Bus
Backward/forward sweep [23]	5	4
Quasi-coupled method [41]	5	4
DLF [26]	4	3
Proposed method FFDPF (Algorithm 2)	4	3

Table 4: Comparison of the CPU execution time of the balanced 10-bus and 25-bus system.

Methods	Unbalanced CPU time in sec	
	10-bus	25-bus (sec)
DLF [26]	0.08424	0.0538
Proposed method FFDPF (Algorithm 2)	0.060840	0.04836

5.3. Weakly Meshed Network. The proposed Algorithm 4 for the FFDWMPF method is tested on two weakly meshed distribution systems. The first weakly meshed system has a 33-bus and five meshed areas created by the five tie lines as shown in Figure 4. The second system is a 69-bus system with five meshes created by the five tie lines shown in Figure 5. The 33-bus and 69-bus system data are taken from [16, 43], respectively. Figure 6 shows the voltage solutions obtained using the proposed method. For 33-bus weakly meshed systems, the maximum difference between the voltage obtained by the proposed methodology and that obtained by the DLF [20] is

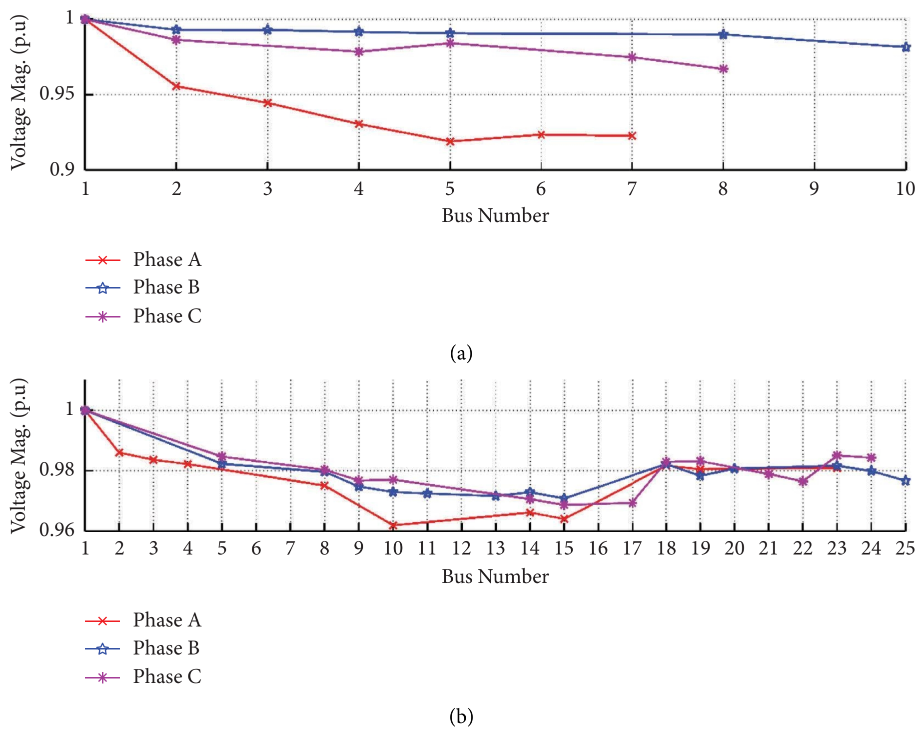

Figure 8: Voltage magnitude of (a) 10-bus and (b) 25-bus unbalanced distribution system.

Table 5: Comparison of iterations of the balanced 33-bus weakly meshed system.

No. of meshed in 33-bus system	Proposed method FFDWMP (Algorithm 4) Iteration	DLF [26] Iteration	Backward/forward sweep [37] Iteration
1	4	4	4
2	4	4	5
3	4	4	5
4	4	4	6
5	3	3	8

found to be 0.000706339 p.u. Similarly, the maximum p.u. voltage difference is 0.002210 compared to the implemented DLF [20] for the 69-bus system. Figure 6 shows the voltage magnitude curve. The convergence rate is compared with the two methods, such as the DLF and the conventional BFS method [28], for 33-bus weakly meshed systems. When compared with these other methods, the proposed FFDWMPF method has the same iteration number as the DLF method and fewer than the BFS and loop-based analysis methods [16].

A careful examination reveals that if the two buses are connected through a tie line, the voltage magnitudes of those buses do not differ significantly. The results of an analysis of the effects of the number of tie lines on the convergence speed are shown in Table 5. Table 5 shows that, as the number of tie lines increases, the convergence speed increases because the tie line helps to improve the voltage profile system, and there is not much voltage difference between the buses, which is shown in Tables 5 and 6 for 33-bus and 69-bus systems, respectively. Furthermore, while the number of iterations for the DLF and proposed FFDWMPF methods is the same, the proposed FFDWMPF method can save a significant amount of CPU time, as shown in Tables 7 and 8 for the 33-bus and 69-bus systems, respectively.

This advantage is possible due to the reduction in the calculation step in the direct forward sweep approach of the proposed method and the only need for WMCM formation, which helps reduce the CPU execution time.

Although previous research focused on reducing CPU execution time and number of iteration using evolved direct forward sweep [37, 38] from the conventional BFS, this proposed work is the first to introduce the flexibility in topology structuring of the network by simple one matrix formation (i.e., CM or WMCM), reducing the physical work of finding the load path and adding the direct forward sweep to both radial and weakly meshed PF solutions to the optimum iteration and CPU execution time. Therefore, it has been a widely preferred PF solution, that has an optimum iteration and execution time. The proposed FFDPF and FFDWMPF can be widely used for the PF solution for the distribution system.

Furthermore, even though some literature studies [37, 38] have a little less iteration and CPU execution time than the proposed method, however, the flexibility provided

Table 6: Comparison of iterations of the balanced 69-bus weakly meshed system.

Number of meshed in 69-bus system	Proposed method FFDWMPF (Algorithm 4) Iteration	DLF [26] Iteration	Loop-analysis- based method [18] Iteration	Backward/forward sweep with compensation [18] Iteration
1	4	4	7	7
2	4	4	7	7
3	4	4	7	8
4	3	3	6	9
5	3	3	5	10

Table 7: Comparison of the CPU execution time of the balanced 33-bus weakly meshed system.

Number of meshed in 33-bus system	Proposed method FFDWMPF (Algorithm 4) CPU time in sec	DLF [26] CPU time in sec
1	0.02652	0.05304
2	0.03276	0.06397
3	0.03588	0.06708
4	0.03744	0.06864
5	0.03900	0.0702

Table 8: Comparison of the CPU execution time of the balanced 69-bus weakly meshed system.

Number of meshed in 69-bus system	Proposed method FFDWMPF (Algorithm 4)		DLF [26]	
	Iteration	CPU time in sec.	Iteration	CPU time in sec.
1	4	0.03276	4	0.05616
2	4	0.03744	4	0.05772
3	4	0.0468	4	0.06864
4	3	0.03588	3	0.0468
5	3	0.04056	3	0.0678

in the formation of the WMCM cannot be achieved by the above algorithm mentioned in [37, 38].

Moreover, the proposed method cannot be directly applied to the reverse power flow condition in the presence of more DG [44] or the islanded condition [45]. The combination of FFDPF and NR methods must be used for the reverse power flow condition. FFDPF is used to calculate the voltage at each node, and NR is used to calculate the power mismatch of the DG node buses, which is the future research scope of the proposed work. The proposed method, on the contrary, is easily applicable to the unidirectional power flow condition in the presence of DG. Specifically, the unidirectional power flow condition is a condition in which the DGs are not causing the reverse power flow condition in the distribution network.

5.4. Robustness Test. In some cases, the distribution network must provide the consumer with the highest load demand. The power flow algorithm should provide the correct data about the system condition to the system operators in order to run the system in a highly demanding environment. As a result, the system power flow algorithm should have good convergence characteristics when subjected to high load demands (i.e., a high level of robustness).

At higher load demand conditions, the FFDPF (Algorithm 2) and FFDWMPF (Algorithm 4) algorithms also exhibit good convergence characteristics. Even if the balanced load increments up to three times, the FFDPF method can give the PF results for 33- and 69-balanced radial distribution systems. Similarly, the FFDWMPF can give the power flow results for the balanced load increments up to 6.5 times for the 33-bus and 8.0 times for the 69-bus distribution systems, respectively, which are illustrated in Table 9. The robustness of the weakly meshed system is higher than that of the radial distribution system because of the better voltage stability of the weakly meshed system. The robustness in Algorithms 2 and 4 is due to the involvement of matrix calculation in the algorithm. Therefore, from the above discussion and observation, the proposed FFDPF and FFDWMPF methods are best suited for PF solutions for the distribution system, with the characteristics of speed, flexibility, and robustness.

Table 9: Different loading conditions in the 33-bus and 69-bus weakly meshed system.

Different loading condition	Number of iterations for 33-bus system		Number of iterations for 69-bus system	
	Radial (Algorithm 2)	Weakly (Algorithm 4)	Radial (Algorithm 2)	Weakly (Algorithm 4)
$(P + jQ) \times 0.5$	3	3	3	3
$(P + jQ) \times 1$	4	4	4	4
$(P + jQ) \times 1.5$	5	4	5	4
$(P + jQ) \times 2$	6	4	6	4
$(P + jQ) \times 2.5$	8	5	8	4
$(P + jQ) \times 3.0$	11	5	14	5
$(P + jQ) \times 3.5$	(NC)	6	(NC)	5
$(P + jQ) \times 4$	(NC)	6	(NC)	5
$(P + jQ) \times 4.5$	(NC)	7	(NC)	6
$(P + jQ) \times 5$	(NC)	8	(NC)	6
$(P + jQ) \times 5.5$	(NC)	9	(NC)	7
$(P + jQ) \times 6.0$	(NC)	12	(NC)	7
$(P + jQ) \times 6.5$	(NC)	21	(NC)	8
$(P + jQ) \times 7$	(NC)	(NC)	(NC)	10
$(P + jQ) \times 7.5$	(NC)	(NC)	(NC)	12
$(P + jQ) \times 8$	(NC)	(NC)	(NC)	18
$(P + jQ) \times 8.5$	Not converge (NC)	Not converge (NC)	Not converge (NC)	Not converge (NC)

6. Conclusions

The study proposes a fast, flexible direct power flow method for radial (FFDPF) and weakly meshed power flow (FFDWMPF) for balanced and unbalanced systems. The power flow solutions suggest that the proposed methods are well equipped to solve the power flow for a high R/X distribution system. The crucial aspect of the proposed method is the conversion matrix, which allows one to solve the power flow with only the direct forward sweep method. In addition, it provides vector calculation, which helps to reduce CPU execution time and the number of iterations. In addition, the CM offers the proposed method with good robustness features. Therefore, the proposed method becomes a fast, flexible, and more direct method with good robustness. In addition to that, the CM matrix provides better flexibility in the extension of the network and reconfiguration of the network. Taking this all together, the proposed method offers a novel perspective on how to solve the power flow problem in a balanced and unbalanced distribution system. In the future, the proposed method is best suited for supervisory control and data acquisition. It may also suit the automation and control of the distribution system, as the algorithm gives results quickly. Moreover, due to the better flexibility in the algorithm, the proposed method is fruitful for the reconfiguration of the distribution network. In addition to that, the proposed method can be used to solve the PF solution for the islanded AC-DC microgrid in a balanced and unbalanced distribution system.

Abbreviations

CM: Conversion matrix
WMCM: Weakly meshed conversion matrix
FFDPF: Fast flexible direct power flow (radial power flow)
FFDWMPF: Fast flexible direct weakly meshed power flow
BFSM: Backward/forward sweep method
BFS: Backward/forward sweep
BFM: Backward/forward method
NB: Number of buses
NBR: Number of branches
NL: Number of loops
CN: Condition number
DLF: Direct load flow
PM: Branch path from the substation bus to the different bus matrix
$[\Delta V]$: The voltage drop matrix
diag$Z$: Diagonalization of branch impedance
Transpose (PM): Transpose of the path matrix
I_L: Load current matrix
V_{rated}: Substation-rated voltage
V: Bus voltage matrix
k: K^{th} iteration
ε: Tolerance value
WMPM: Weakly meshed path matrix
Factor_matrix: Factor matrix of radial system
Factor_Matrix$_{\text{weakly}}$: Factor matrix of weakly meshed system
BIWM: Branch impedance matrix of weakly meshed system
PF: Power flow
NC: Not convergence.

Data Availability

(1) The (33-bus radial and weakly system data) data used to support the findings of this study have been deposited in the (Network reconfiguration in distribution systems for loss reduction and load balancing) repository (DOI: 10.1109/61.25627). (2) The (69-bus radial and weakly system data) data used to support the findings of this study have been deposited in the (Optimal capacitor placement on radial distribution systems) repository (DOI: 10.1109/61.19265).

(3) The (10-bus radial Unbalanced Distribution Data) data used to support the findings of this study have been deposited in the (Quasi-coupled three-phase radial load flow) repository (DOI: 10.1109/TPWRS.2003.821624). (4) The (25-bus radial Unbalanced Distribution Data) data used to support the findings of this study have been deposited in the (Direct solution of distribution systems) repository (DOI: 10.1049/ip-c.1991.0010).

Conflicts of Interest

The authors declare that they have no conflicts of interest.

References

[1] S. Ouali and A. Cherkaoui, "An improved backward/forward sweep power flow method based on a new network information organization for radial distribution systems," *Journal of Electrical and Computer Engineering*, vol. 2020, Article ID 5643410, 11 pages, 2020.

[2] L. A. Dunstan, "Digital load flow studies," *Transactions of the American Institute of Electrical Engineers. Part III: Power Apparatus and Systems*, vol. 73, no. 1, pp. 825–832, 1954.

[3] J. B. Ward and H. W. Hale, "Digital computer solution of power-flow problems (includes discussion)," *Transactions of the American Institute of Electrical Engineers. Part III: Power Apparatus and Systems*, vol. 75, no. 3, 1956.

[4] W. Tinney and C. Hart, "Power flow solution by Newton's method," *IEEE Transactions on Power Apparatus and Systems*, vol. PAS–86, no. 11, pp. 1449–1460, 1967.

[5] B. Stott, "Decoupled Newton load flow," *IEEE Transactions on Power Apparatus and Systems*, vol. PAS–91, no. 5, pp. 1955–1959, 1972.

[6] B. Stott and O. Alsac, "Fast decoupled load flow," *IEEE Transactions on Power Apparatus and Systems*, vol. PAS–93, no. 3, pp. 859–869, 1974.

[7] V. M. Da Costa, N. Martins, and J. L. R. Pereira, "Developments in the Newton raphson power flow formulation based on current injections," *IEEE Power Engineering Review*, vol. 14, no. 4, pp. 1320–1326, 1999.

[8] P. A. N. Garcia, J. L. R. Pereira, S. Carneiro, V. M. Da Costa, and N. Martins, "Three-phase power flow calculations using the current injection method," *IEEE Transactions on Power Systems*, vol. 15, no. 2, pp. 508–514, 2000.

[9] W. M. Lin and J. H. Teng, "Three-phase distribution network fast-decoupled power flow solutions," *International Journal of Electrical Power & Energy Systems*, vol. 22, no. 5, pp. 375–380, 2000.

[10] R. Gianto, "Trust-region method for load flow solution of three-phase unbalanced electric power distribution system," *Journal of Electrical and Computer Engineering*, vol. 2022, Article ID 5415300, 17 pages, 2022.

[11] Y. Liu, Z. Li, and M. Fan, "A newton-raphson-based sequential power flow algorithm for hybrid AC/DC microgrids," *IEEE Transactions on Industry Applications*, vol. 58, no. 1, pp. 843–854, 2022.

[12] M. Hosseinzadeh and F. R. Salmasi, "Robust optimal power management system for a hybrid AC/DC micro-grid," *IEEE Transactions on Sustainable Energy*, vol. 6, no. 3, pp. 675–687, 2015.

[13] D. Shirmohammadi, H. W. Hong, A. Semlyen, and G. X. Luo, "A compensation-based power flow method for weakly meshed distribution and transmission networks," *IEEE Transactions on Power Systems*, vol. 3, no. 2, pp. 753–762, 1988.

[14] C. S. Cheng and D. Shirmohammadi, "A three-phase power flow method for real-time distribution system analysis," *IEEE Transactions on Power Systems*, vol. 10, no. 2, pp. 671–679, 1995.

[15] R. Ranjan, B. Venkatesh, A. Chaturvedi, and D. Das, "Power flow solution of three-phase unbalanced radial distribution network," *Electric Power Components and Systems*, vol. 32, no. 4, pp. 421–433, 2004.

[16] S. Ghosh and D. Das, "Method for load-flow solution of radial distribution networks," *IEE Proceedings - Generation, Transmission and Distribution*, vol. 146, no. 6, p. 641, 1999.

[17] M. Bazrafshan and N. Gatsis, "Comprehensive modeling of three-phase distribution systems via the bus admittance matrix," *IEEE Transactions on Power Systems*, vol. 33, no. 2, pp. 2015–2029, 2018.

[18] W. C. Wu and B. M. Zhang, "A three-phase power flow algorithm for distribution system power flow based on loop-analysis method," *International Journal of Electrical Power & Energy Systems*, vol. 30, no. 1, pp. 8–15, 2008.

[19] Y. Ju, W. Wu, B. Zhang, and H. Sun, "Loop-analysis-based continuation power flow algorithm for distribution networks," *IET Generation, Transmission & Distribution*, vol. 8, no. 7, pp. 1284–1292, 2014.

[20] A. Augugliaro, L. Dusonchet, S. Favuzza, M. G. Ippolito, and E. R. Sanseverino, "A backward sweep method for power flow solution in distribution networks," *International Journal of Electrical Power & Energy Systems*, vol. 32, no. 4, pp. 271–280, 2010.

[21] M. Dilek, F. De León, R. Broadwater, and S. Lee, "A robust multiphase power flow for general distribution networks," *IEEE Transactions on Power Systems*, vol. 25, no. 2, pp. 760–768, 2010.

[22] V. C. Strezoski and P. M. Vidović, "Power flow for general mixed distribution networks," *International Transactions on Electrical Energy Systems*, vol. 25, no. 10, pp. 2455–2471, 2015.

[23] S. M. Moghaddas-Tafreshi and E. Mashhour, "Distributed generation modeling for power flow studies and a three-phase unbalanced power flow solution for radial distribution systems considering distributed generation," *Electric Power Systems Research*, vol. 79, no. 4, pp. 680–686, 2009.

[24] K. N. Maya and E. A. Jasmin, "Optimal integration of distributed generation (DG) resources in unbalanced distribution system considering uncertainty modelling," *International Transactions on Electrical Energy Systems*, vol. 27, no. 1, Article ID e248, 2017.

[25] X. Wang, M. Shahidehpour, C. Jiang, W. Tian, Z. Li, and Y. Yao, "Three-phase distribution power flow calculation for loop-based microgrids," *IEEE Transactions on Power Systems*, vol. 33, no. 4, pp. 3955–3967, 2018.

[26] J. H. Teng, "A direct approach for distribution system load flow solutions," *IEEE Transactions on Power Delivery*, vol. 18, no. 3, pp. 882–887, 2003.

[27] J. H. Teng, "Modelling distributed generations in three-phase distribution load flow," *IET Generation, Transmission & Distribution*, vol. 2, no. 3, pp. 330–340, 2008.

[28] F. Hameed, M. Al Hosani, and H. H. Zeineldin, "A modified backward/forward sweep load flow method for islanded radial microgrids," *IEEE Transactions on Smart Grid*, vol. 10, no. 1, pp. 910–918, 2019.

[29] T. H. Chen and N. C. Yang, "Three-phase power-flow by direct ZBR method for unbalanced radial distribution

systems," *IET Generation, Transmission & Distribution*, vol. 3, no. 10, pp. 903–910, 2009.

[30] M. F. AlHajri and M. E. El-Hawary, "Exploiting the radial distribution structure in developing a fast and flexible radial power flow for unbalanced three-phase networks," *IEEE Transactions on Power Delivery*, vol. 25, no. 1, pp. 378–389, 2010.

[31] H. E. Farag, E. F. El-Saadany, R. El Shatshat, and A. Zidan, "A generalized power flow analysis for distribution systems with high penetration of distributed generation," *Electric Power Systems Research*, vol. 81, no. 7, pp. 1499–1506, 2011.

[32] S. Singh and T. Ghose, "Improved radial load flow method," *International Journal of Electrical Power & Energy Systems*, vol. 44, no. 1, pp. 721–727, 2013.

[33] A. Marini, S. S. Mortazavi, L. Piegari, and M. S. Ghazizadeh, "An efficient graph-based power flow algorithm for electrical distribution systems with a comprehensive modeling of distributed generations," *Electric Power Systems Research*, vol. 170, pp. 229–243, 2019.

[34] K. Murari and N. P. Padhy, "Graph-theoretic based approach for the loadflow solution of three-phase distribution network in the presence of distributed generations," *IET Generation, Transmission & Distribution*, vol. 14, no. 9, pp. 1627–1640, 2020.

[35] D. P. Sharma, A. Chaturvedi, R. Saxena, and J. Krishna R, "Faster load flow algorithm for radial distribution network using graph theory," *International Transactions on Electrical Energy Systems*, vol. 29, no. 2, pp. e2705–e2711, 2019.

[36] O. D. Montoya, J. S. Giraldo, L. F. Grisales-Noreña, H. R. Chamorro, and L. Alvarado-Barrios, "Accurate and efficient derivative-free three-phase power flow method for unbalanced distribution networks," *Computation*, vol. 9, no. 6, pp. 61–21, 2021.

[37] U. Ghatak and V. Mukherjee, "A fast and efficient load flow technique for unbalanced distribution system," *International Journal of Electrical Power & Energy Systems*, vol. 84, pp. 99–110, 2017.

[38] U. Ghatak and V. Mukherjee, "An improved load flow technique based on load current injection for modern distribution system," *International Journal of Electrical Power & Energy Systems*, vol. 84, pp. 168–181, 2017.

[39] R. Ranjan and D. Das, "Simple and efficient computer algorithm to solve radial distribution networks," *Electric Power Components and Systems*, vol. 31, no. 1, pp. 95–107, 2003.

[40] S. Kawambwa, R. Mwifunyi, D. Mnyanghwalo, N. Hamisi, E. Kalinga, and N. Mvungi, "An improved backward/forward sweep power flow method based on network tree depth for radial distribution systems," *Journal of Electrical Systems and Information Technology*, vol. 8, no. 1, p. 7, 2021.

[41] E. R. Ramos, A. Exposito, and G. Á. Cordero, "Quasi-coupled three-phase radial load flow," *IEEE Transactions on Power Systems*, vol. 19, no. 2, pp. 776–781, 2004.

[42] S. K. Goswami and S. K. Basu, "Direct solution of distribution systems," *IEE Proceedings C Generation, Transmission and Distribution*, vol. 138, no. 1, pp. 78–88, 1991.

[43] M. E. Baran and F. F. Wu, "Network reconfiguration in distribution systems for loss reduction and load balancing," *IEEE Transactions on Power Delivery*, vol. 4, no. 2, pp. 1401–1407, 1989.

[44] P. T. de Godoy, A. B. de Almeida, A. C. Zambroni de Souza, and B. Venkatesh, "Global power flow for microgrids," in *Proceedings of the 2021 IEEE Power & Energy Society General Meeting (PESGM)*, pp. 1–5, Washington, DC, USA, July 2021.

[45] C. Li, S. K. Chaudhary, M. Savaghebi, J. C. Vasquez, and J. M. Guerrero, "Power flow analysis for low-voltage AC and DC microgrids considering droop control and virtual impedance," *IEEE Transactions on Smart Grid*, vol. 8, no. 6, pp. 2754–2764, 2017.