

Retraction

Retracted: Theoretical Correction of Viscosity Coefficient Measurement by Falling Ball Method

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This article has been retracted by Hindawi, as publisher, following an investigation undertaken by the publisher [1]. This investigation has uncovered evidence of systematic manipulation of the publication and peer-review process. We cannot, therefore, vouch for the reliability or integrity of this article.

Please note that this notice is intended solely to alert readers that the peer-review process of this article has been compromised.

Wiley and Hindawi regret that the usual quality checks did not identify these issues before publication and have since put additional measures in place to safeguard research integrity.

We wish to credit our Research Integrity and Research Publishing teams and anonymous and named external researchers and research integrity experts for contributing to this investigation.

The corresponding author, as the representative of all authors, has been given the opportunity to register their agreement or disagreement to this retraction. We have kept a record of any response received.

References

- [1] R. Wang, Q. Zhang, S. Xu, L. Cui, and H. Jiang, "Theoretical Correction of Viscosity Coefficient Measurement by Falling Ball Method," *International Transactions on Electrical Energy Systems*, vol. 2022, Article ID 3897120, 12 pages, 2022.

Research Article

Theoretical Correction of Viscosity Coefficient Measurement by Falling Ball Method

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The measurement of viscosity coefficient of liquids is vital in plentiful fields. There are numerous ways to measure the viscosity coefficient such as capillary and rotary rheometry in which falling ball method is mostly widely used. The uncertainty of the terminal velocity and the limit of the viscosity coefficient scale are the principled disadvantages of the method. In our work, the relationship between the displacement and time is obtained by integral calculation. What's more, we considered the radius of the ball, the radius of the graduated cylinder, and the height of the graduated cylinder by theoretical correction of the stokes expression. The new correction expression broadens the measured viscosity coefficient scale and also improves the precision dramatically. Experimental verification is also well processed which fits well with our theory. The deviation between the viscosity coefficient of castor oil measured by the new correction method and the standard value is only 0.045. The relationship between displacement and time is consistent with the theoretical prediction, and the room mean square error of the two is 0.065. Based on them, this method can be widely used in scientific researches and teaching practices.

1. Introduction

In the process of fluid flow, due to the different flow rates of each flow layer, there is an equal and opposite friction force between two adjacent flow layers. This pair of friction forces is called internal friction force or viscous force. This property of fluid is called viscosity. The viscosity coefficient is a key parameter for dynamics calculation via flow mechanics theory and the measurement of it is required in normal productions and researches [1]. The viscosity coefficient of a liquid is an important physical quantity that characterizes the strength of the viscous force of the liquid. The liquid viscosity coefficient is an important parameter commonly used in fluid mechanics, material science, and engineering technology to describe the internal friction characteristics of a liquid. The important parameter mainly reflects the degree of obstruction to the liquid movement when there is relative movement inside the liquid. The measurement of its viscosity coefficient has a huge demand in production and

research and is widely used in industrial production, medicine and health, mechanical engineering equipment research and development, and other fields. The methods which have achieved significant progress are capillary and rotary rheometry or falling ball method. The falling ball method is a kind of fundamental method in measuring the viscosity coefficient with simple devices, easy operation, and great application prospects. In the production process, the big experiment error and the small measuring range limit the application requirements. People desire to measure the viscosity coefficient of liquids easily with wide viscous range in industry (e.g., tetrahydrofuran) [2, 3]. The measuring devices in the laboratories are often unable to meet the needs. In addition, the falling ball method has other limitations: the measuring devices are required to approximate the infinite extension space (large liquids level areas and the pellets must fall along the central axis of the liquids), with small radius of the pellets, high liquids level height, slow speed of the pellets, and large viscosity coefficient of the

liquids. Furthermore, the turbulence of the liquids is not considered. And the falling ball method is only suitable for measuring the viscosity coefficient of translucent liquids, such as castor oil, glycerin, etc., but because of the obvious physical phenomenon, intuitive principle, and many experimental operations and training contents, this method is still widely used. College physics experiments in science and engineering universities and basic physics experiments in lower grades of physics-related majors. The falling ball method experiment requires the ball to fall along the central axis of the container. The traditional viscosity coefficient experiment is realized by the experimenter's release by experience, and it is difficult to ensure that the small ball falls just along the central axis of the container; the falling ball method experiment requires the uniform velocity of the ball to be measured. Through a certain distance, the traditional viscosity coefficient experiment is only realized by the experimenter's stopwatch. Due to the parallax and reaction time of the artificial stopwatch, it is difficult to accurately measure the falling time of the ball during the experiment.

Many scholars have done a lot of research to solve the above problems of the falling ball method and improved the traditional viscosity coefficient test. Some scholars use photogate and digital millisecond meter instead of artificial stopwatch and add a plexiglass cover on the top of the container, and a pull rod with a magnet is placed above the cover. Accuracy: Liu et al. [4] systematically explored the influencing factors of the measurement results in the experiment of measuring the viscosity coefficient of castor oil by the falling ball method and analyzed the effect of the drop of the ball from different positions, the diameter of the ball, and the experimental temperature on the measurement of the viscosity coefficient of castor oil. Impact: By drawing and analyzing the error curve, an improvement scheme of the experiment is proposed to improve the experimental accuracy and reduce the experimental error. Sevryugin and Skirda [5] proposed an empirical expression for the viscosity coefficient of molecular liquids based on the existing general concepts of viscous flow structure and diffusion theory. This expression determines the relationship between the viscosity coefficient, self-diffusion coefficient, surface tension coefficient, and steric hindrance parameters of liquid molecules. The validity of the proposed expression has been demonstrated by its applicability to some liquids with different physicochemical properties. Liu [6] used the least squares method to obtain the conclusion that the correction coefficient is not always a constant value based on the experimental data obtained by testing and also used MATLAB to simulate the displacement of the ball when the falling ball reaches a uniform speed, this allows the experimenter to more accurately choose the starting point of the timing and the size of the ball, and mastering the measurement of the ultimate speed, thereby reducing the error; it is found that whether the ball falls from the center of the measuring cylinder has a greater impact on the measurement results, and the same attention should be paid to control. The development of fluid mechanics researches and emergence of the correction of Stokes' formula in recent years provide feasible ways to solve the above problems [7]. Shen [8]

started from the analysis of the establishment conditions of Stokes' law and conducted a theoretical discussion on the selection of experimental conditions and the correction of results for the determination of the viscosity coefficient of liquids by the falling ball method. De-wen and Zhao [9] performed a linear fit to the experimental data through experiments and software, gave the correction coefficients of Stokes' formula under laboratory conditions, and compared the calculated results with the accepted values. The calculation result of the revised formula has a small error and a small degree of data dispersion. In addition, the falling ball method requires that the viscosity coefficient of the liquid can be measured only when the small ball falls in the liquid to reach a uniform speed, but in some cases, the small ball cannot reach a uniform speed because the viscosity coefficient of the liquid is too small or the height of the liquid is too low. In the state of motion, the falling ball method fails to measure the viscosity coefficient of liquids. In domestic and foreign research, this problem has not been solved so far.

Based on this, in the new correction method, the velocity, acceleration, and the relationship between the velocity and displacement of the pellets over time are obtained by integrating during the whole process of the ball falling. In our experiments, Newton's method was used in calculating the viscosity coefficient by measuring the falling displacement and time of the pellets [10]. Therefore, in the process of using the falling ball method, the viscosity coefficient of the liquid to be measured can be calculated only by considering the falling time and displacement of the small ball. In order to test the reliability of theory, two experiments were carried out in this study: One was to measure the viscosity coefficient of liquids by using the new correction method at different temperatures and compare them with the standard values to verify the correctness of the correction method. In another experiment, we compared the relationship between the falling displacement and time of the pellets at 23°C based on the standard values with relationship between the falling displacement and time calculated by measured values according to the new correction method. This method broadens the scope of application of the falling sphere method and is expected to further broaden the scope of application of the falling sphere method, provide theoretical guidance for new viscosity coefficient measuring instruments, or inspire university physics experimental teaching and become a new method for measuring liquid viscosity coefficient [11].

1.1. Theoretical Basis and Falling Ball Method

1.1.1. Stokes' Formula. Stokes' formula not only has been verified by experiments, but also can be derived by logical reasoning through existing theories. There are many derivation methods for Stokes formula, among which the more commonly used channel is the solution of flow around a sphere as the Navier-Stokes equation. As a partial differential equation, Navier-Stokes equation is difficult to solve mathematically, and it is still the focus and difficulty in mathematical physics. The Stokes formula simplifies the

Navier-Stokes equation according to the specific situation and then obtains the solution of small Reynolds number. The basic theory used in the falling ball method is Stokes' formula: A small ball with a radius is used to translate in a fluid marked by a viscosity coefficient. The velocity of pellets is not very large; thus the fluid resistance is $f = 6\pi r\eta v$ [12].

The following conditions are important conditions for simplifying and solving the Navier-Stokes equation, which directly affect whether the Stokes formula is established or not. Stokes' formula is valid only if the following conditions are met: i. The infinite extension space: for small pellets, the liquid level is infinitely vast in all directions.

This is a relative concept. Since the sphere is much smaller than the liquid, the liquid boundary can be approximately ignored when considering the motion of the sphere. When the ball descends along the central axis of the liquid, it is necessary to ensure that the radius of the ball is less than 1% of the liquid surface radius and the liquid surface height at the same time. Therefore, before the ball drops by 100 radius units, strictly speaking, the traditional Stokes formula does not hold. Fortunately, in order to ensure that the ball reaches the end speed, the traditional ball drop method starts at a later time, which is much larger than 100 times the radius of the ball drop, which ensures the measurement effect. ii. There is no turbulence in the liquids; that is to say; the Reynolds number should satisfy $Re < 0.1$.

In the formula: $Re = 2\rho'vR/\eta$. ρ' is the density of the liquids and R is the radius of the liquids' surface. Reynolds number is a dimensionless quantity that can characterize the state of liquids flow [13].

Obviously, the Reynolds number is only related to the physical properties of the liquid to be measured and the velocity of the ball. This article does not discuss the effect of liquid density. To keep the liquid laminar, the Reynolds number should be as small as possible. Therefore, the liquid with a larger viscosity coefficient and a small ball with a smaller radius are more suitable for the traditional falling ball method. During the falling process of the ball, the velocity v increases, and when the Reynolds number $Re > 0.1$, turbulence will be induced. Therefore, when turbulence is not triggered, the maximum velocity of the pellets is

$$v_1 = 0.05 \frac{\eta}{\rho' R} \quad (1)$$

If the closing speed is less than v_1 , the traditional ball drop method will not fail. Therefore, it is very important to select the appropriate ball radius according to the physical properties of the liquid to be tested for the use of the traditional falling ball method. The radius of the ball cannot be shortened infinitely, so the traditional falling ball method is ineffective for the measurement of many liquids.

1.1.2. Millikan Correction Factor. In experiment, the measuring devices are unable to get the infinite extension space, and the fluid resistance of the pellets is always larger than we expect. At this point, the Millikan correction factor needs to be applied. The Millikan correction factor is $\beta = (1 + 2.4(r/R))(1 + 3.3(r/R))$. It is not difficult to find

that the two factors of the Millikan correction factor correspond to the influence of the radius of the liquid surface and the height is not much larger than the radius of the sphere, respectively. And in the infinite extension space, the Millikan correction factor is equal to 1, and the conclusion of Stokes formula is self-consistent; then the fluid resistance can be corrected as

$$f = 6\pi r\eta v\beta = 6\pi r\eta v \left(1 + 2.4 \frac{r}{R}\right) \left(1 + 3.3 \frac{r}{H}\right). \quad (2)$$

When the Millikan correction factor is applied in practical measurements, the liquids' height is not taken into account due to the difference of magnitudes of the two factors of the Millikan correction factor. For example, in the experiments described later, the radius of the pellet is $r = 1\text{mm}$, while the liquid radius is $R = 10.5\text{mm}$, and the liquid height is $H = 30\text{cm}$. At this time, $1 + 2.4(r/R) = 43/35$, and $1 + 3.3(r/R) = 1011/1000$. Obviously, the former formula plays a decisive role in the revision, and it is appropriate to ignore the latter.

1.1.3. Osson-Golls Formula. When the Reynolds number satisfies $Re > 0.1$, turbulence will occur in the liquids, and the nonlinear term of the inertial force has great impact which cannot be ignored [14]. According to the Osson-Golls formula, the fluid resistance is $f = 6\pi r\eta v(1 + (3/16)Re - (19/1080)Re^2 + \dots)$. When using the Osson-Golls formula for correction, the correction series should be determined according to the size of the Reynolds number [15] (the degree of Reynolds number which appears in the polynomial). For example, when $Re \in (0.1, 1)$, a first-order correction is required. In particular, Stokes' formula is zero-order correction form of the Osson-Golls formula.

However, it must be mentioned that the modification of the Stokes formula by the Osson-Golls formula is not unlimited. When the third-order correction is required, the inertial force is often greater than the viscous resistance, and the Osson-Golls formula will also be invalid at this time, and the Navier-Stokes equation must be returned to solve it again. The practicality of the method proposed in this paper lies in the fact that the liquids extended by the first-order correction and the second-order correction can be measured by the new method.

1.1.4. Falling Ball Method and Its Correction. When the pellets fall in the liquids, they are mainly affected by three forces: gravity, buoyancy, and fluid resistance. Among them, gravity is $G = mg = \rho Vg = (4/3)\pi\rho gr^3$. In the formula, m is the quality of the pellets, g is the acceleration of gravity, ρ is the density of the pellets, and V is the volume of the pellets. The direction of gravity is straight down. The buoyancy force is $F_b = \rho'Vg = (4/3)\pi\rho'gr^3$; the force direction is straight up. The two forces are constant forces (but the fluid resistance is constantly changing as the ball falls), and they do not change over time, so its resultant force is

$$C = G - F_b = \frac{4}{3}\pi(\rho - \rho')gr^3. \quad (3)$$

When the pellets fall in the liquids, they do an accelerated movement with gradually decreasing acceleration. When the velocity of pellets increases to v_0 , that is, terminal velocity, the forces on the ball reach equilibrium and record $G - F_b = f$ with $V(\rho - \rho')g = 6\pi r\eta v_0$, so the viscosity coefficient is $\eta = (2(\rho - \rho')gr^2/9v_0)$ [16]. In the formula $v_0 = s/t_s$, in which t_s is the time it takes for the pellets to fall uniformly in the liquid for a distance of s [17].

In fact, if the pellets can reach the terminal speed, we can use the falling ball method [11] only to modify the fluid resistance to obtain the viscosity coefficient of the liquid [18]. Considering both the effects of the measuring devices and turbulence of the liquid, then the fluid resistance is

$$\begin{aligned} f &= 6\pi r\eta v \left(1 + 2.4\frac{r}{R}\right) \left(1 + 3.3\frac{r}{H}\right) \\ &\quad \left(1 + \frac{3}{16}R_e - \frac{19}{1080}R_e^2 + \dots\right) \\ &= 6\pi r\eta v \left(1 + 2.4\frac{r}{R}\right) \left(1 + 3.3\frac{r}{H}\right) \end{aligned} \quad (4)$$

$$\left[1 + \frac{3}{16}\left(\frac{2\rho'vR}{\eta}\right) - \frac{19}{1080}\left(\frac{2\rho'vR}{\eta}\right)^2 + \dots\right].$$

If the velocity is stable, and $R_e \in (0.1, 1)$, the fluid resistance needs to be made first-order correction. For example,

$$\begin{cases} A = 6\pi r\beta = 6\pi r \left(1 + 2.4\frac{r}{R}\right) \left(1 + 3.3\frac{r}{H}\right), \\ B = \frac{3}{8}\rho'R. \end{cases} \quad (5)$$

According to the force equilibrium, we know that $ABv_0^2 + A\eta v_0 - C = 0$, and discriminant is $\Delta = A^2\eta^2 + 4ABC \geq 0$. Because $v \geq 0$, according to the general solution of a quadratic equation of one variable, we can get

$$v_0 = \frac{-A\eta + \sqrt{A^2\eta^2 + 4ABC}}{2AB}, \quad (6)$$

$$\text{and } \eta = \frac{C - ABv_0^2}{Av_0}.$$

2. Theoretical and Experimental Methods

2.1. Full-Time Falling Ball Method. During the experiment, because the ball is too large to meet the requirements of the falling ball method, or the viscosity of the liquid is too small, the gravity of the ball is always greater than the reasonable buoyancy and viscous resistance of the ball during the falling process. The ball is always in a state of accelerated motion. In the whole experimental device, there is no ending speed. It

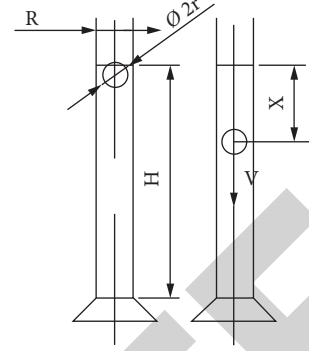


FIGURE 1: Schematic diagram of the experiment.

can be considered to analyze the whole process of the drop of the ball and try to find the conditions that can be directly obtained during the falling process of the ball, such as the falling time and displacement. Then, the viscosity coefficient is obtained.

2.1.1. Experimental Situation. The pellets are placed in the center of the liquids level and are just submerged by the liquids. At the $t = 0$ moment, the pellets are released from their resting state and the timing begins at the same time. The radius of the pellets is r , and the height of the liquids level is H and the radius is R . Record $t = t$ moment, the velocity of pellets is v , and the displacement of pellets is x . The schematic diagram of the experiment is shown in Figure 1,

2.2. Derivation and Solution. Because the Reynolds number will change during the whole falling process of pellets, the correction series of fluid resistance also needs to be determined according to specific experiments. There is no specific universal form so this section will give abstract algorithms.

According to the momentum theorem, within the time dt , the increment of the momentum of the pellets is $dp = I$. I is the impulse of the external force, that is, $mdv = (C - f)dt$. By separating the variables, we can obtain

$$t = \int_0^t dt = \int_0^v u(v)dv = U(v) - U(0) \triangleq W(v). \quad (7)$$

And

$$u(v) = \frac{m}{C - f}, \quad (8)$$

$u(v)$ is a piecewise function which is the reason why the formula form of the full-time falling ball method changes and it is impossible to give a specific universal formula. But it is not difficult to find out that $u(v)$ is always a continuous and bounded function within the integral interval in the context of the actual situation; therefore, it is Riemann integral. So we can find a function $U(v)$ which is the primitive function of $u(v)$. In the experiment, it can be obtained by using MATLAB's integration command.

Continue to integrate; then we can get

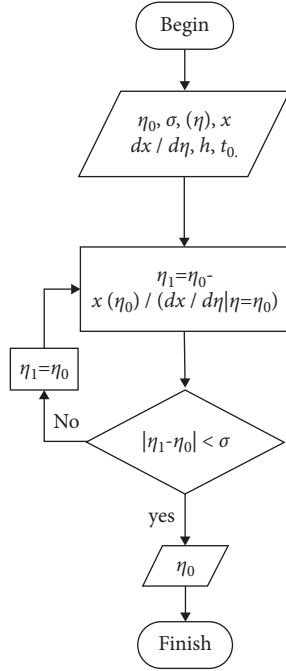


FIGURE 2: Block diagram of Newton's iterative method.

$$x = \int_0^t v(t) dt = X(t) - X(0). \quad (9)$$

$v(t) = W(t)^{-1}$ (because $W(t)$ is monotonically increasing within the interval, its inverse function exists). $X(t)$ is primitive function of $v(t)$. Plug

$$\begin{cases} x = h, \\ t = t_0. \end{cases} \quad (10)$$

Into the formula; then get the transcendental equation about η . It is difficult to find its analytical solution, so the new method uses Newton's method to find its approximate numerical solution.

To do this, it is necessary to differentiate η in the formula $X(\eta)$, so as to get the following formula:

$$X'(\eta) = \frac{dX}{d\eta}. \quad (11)$$

Then write the program and execute and calculate. The algorithm is shown in Figure 2.

η_0 is the estimated viscosity coefficient, σ is the maximum error we can tolerate, and h and t_0 are measured data in experiments.

$f(\eta, v)$ is a M-function about formula (3). According to Newton's second law, Section 2.2 is essentially solving a second-order differential equation which is missing x and t (formula (11)) [19]. Therefore, the relationship between the displacement and the time of the pellets can also be obtained by using the general solution of ordinary differential equations, or use ODEsolvers, numerical analysis, and other methods to directly obtain numerical solutions.

$$m \frac{d^2 x}{dt^2} = 6\pi r \eta v \left(1 + 2.4 \frac{r}{R}\right) \left(1 + 3.3 \frac{r}{H}\right)$$

$$\left[\frac{dx}{dt} + \frac{3}{8} \frac{\rho' R}{\eta} \left(\frac{dx}{dt}\right)^2 - \frac{19}{270} \left(\frac{\rho' R}{\eta}\right)^2 \left(\frac{dx}{dt}\right)^3 + \dots \right] - \frac{4}{3} \pi (\rho - \rho') g r^3. \quad (12)$$

3. Example

The above article only gives an abstract algorithm of the full-time falling ball method. In order to facilitate the readers' understanding and use, the later will deduce and solve two specific situations.

3.1. There Is No Turbulence. If the viscosity of the liquids to be measured is large and the terminal velocity of the pellets is generally not very fast, it will not cause turbulence. At this time, there is no need to use the Ossion-Golls formula to correct the formula, so $f = A\eta v$; $u_1(v) = m/C - A\eta v$; we can get the following formula by integrating:

$$t = \int_0^v \frac{m}{C - A\eta v} dv = -\frac{m}{A\eta} \ln \left| C - A\eta v \right| \Big|_0^v \quad (13)$$

$$= -\frac{m}{A\eta} \ln \left| \frac{C - A\eta v}{C} \right|.$$

Because $f \leq C$:

$$t = -\frac{m}{A\eta} \ln \left(1 - \frac{A}{C} \eta v \right). \quad (14)$$

The inverse function of formula (12) is

$$v = \frac{C}{A} \frac{1 - e^{-(A\eta/m)t}}{\eta}. \quad (15)$$

Then continue to integrate to get the relationship between x and t :

$$x = \int_0^t v dt = \frac{C}{A\eta} \left(t - \int_0^t e^{-(A\eta/m)t} dt \right)$$

$$= \frac{C}{A\eta} t + \frac{mC}{A^2 \eta^2} e^{-(A\eta/m)t} \Big|_0^t = \frac{C}{A\eta} t + \frac{mC}{A^2 \eta^2} (e^{-(A\eta/m)t} - 1). \quad (16)$$

To use Newton's method, it is necessary to differentiate η in formula (15). Then get the following formula:

$$\frac{dx}{d\eta} = -\frac{C}{A} \frac{t}{\eta^2} - 2 \frac{mC}{A^2 \eta^3} (e^{-(A\eta/m)t} - 1) - \frac{C}{A\eta} e^{-(A\eta/m)t}. \quad (17)$$

And write the following program.

3.2. A Situation That Requires Only First-Order Correction. According to formula (1), if $v_1 < v_0$, the pellets will eventually cause turbulence. Plug it into formula (12), so as to get

$$t_1 = -\frac{m}{A\eta} \ln\left(1 - \frac{A}{C} \eta v_1\right). \quad (18)$$

$$\begin{aligned} u_2(v) &= \frac{m}{C - A\eta v(1 + B(v/\eta))} = \frac{-m}{ABv^2 + A\eta v - C} \\ &= \frac{-m}{2\sqrt{(A\eta^2/4B) + C}} \left(\frac{1}{\sqrt{AB}v + \sqrt{A/B}\eta/2 - \sqrt{(A\eta^2/4B) + C}} - \frac{1}{\sqrt{AB}v + \sqrt{A/B}\eta/2 + \sqrt{(A\eta^2/4B) + C}} \right) v \in (v_1, v_2). \end{aligned} \quad (19)$$

Check the integral table; get formula as follows:

$$\int \frac{dx}{ax^2 + bx + c} = \frac{1}{\sqrt{b^2 - 4ac}} \ln \left| \frac{2ax + b - \sqrt{b^2 - 4ac}}{2ax + b + \sqrt{b^2 - 4ac}} \right| + \text{const}^2 > 4ac. \quad (20)$$

From formula (6), we can get the following equation:

$$t = t_1 + \int_{v_1}^v u_2(v) dv = t_1 + \frac{-m}{\sqrt{\Delta}} \ln \left| \frac{2ABv + A\eta - \sqrt{\Delta}}{2ABv + A\eta + \sqrt{\Delta}} \right| \Big|_{v_1}^v, \quad (21)$$

where

$$D = \ln \left| \frac{2ABv_1 + A\eta - \sqrt{\Delta}}{2ABv_1 + A\eta + \sqrt{\Delta}} \right| - \frac{\sqrt{\Delta}}{A\eta} \ln \left(1 - \frac{A}{C} \eta v_1\right). \quad (22)$$

So $D - (\sqrt{\Delta}/m)t = \ln|2ABv + A\eta - \sqrt{\Delta}/2ABv + A\eta + \sqrt{\Delta}|$. Consider the absolute value of RHS: it is known that the function should be in the decline interval according to the physical meaning, and the graph is shown in Figure 3, so

$$D - \frac{\sqrt{\Delta}}{m} t = \ln \left(\frac{2\sqrt{\Delta}}{2ABv + A\eta + \sqrt{\Delta}} - 1 \right). \quad (23)$$

Rearrange formula (21), so as to get

$$v = \frac{\left(2\sqrt{\Delta}/1 + e^{D - (\sqrt{\Delta}/m)t}\right) - A\eta - \sqrt{\Delta}}{2AB}. \quad (24)$$

And

$$v_0 = \lim_{t \rightarrow \infty} v = \frac{-A\eta + \sqrt{A^2\eta^2 + 4ABC}}{2AB}. \quad (25)$$

Corresponding to formula (5), this shows that the full-time falling ball method is self-consistent. Plot a graph according to formula (22). The relationship between v and t of the pellets is shown in Figure 4.

When $t > t_1$, the Ossion–Golls formula needs to be used to correct the fluid resistance (in fact, the critical velocity and time of each order of correction can be calculated by imitating formulas (1) and (16)).

When $v < v_2 = \eta/2\rho'R$, we can use first-order correction. So there is

As can be seen from Figure 4, the pellets quickly reach the terminal speed, which shows that the traditional falling ball method is applicable in many cases, and the function $v(t)$ is also piecewise. According to formula (15), when turbulence occurs, the displacement of the pellets is

$$x_1 = \frac{C}{A\eta} t_1 + \frac{mC}{A^2\eta^2} \left(e^{-(A\eta/m)t_1} - 1 \right). \quad (26)$$

At this point, formula (8) is

$$\begin{aligned} x &= x_1 + \frac{\sqrt{\Delta}}{AB} \int_{t_1}^t \frac{e^{(\sqrt{\Delta}/m)t}}{e^{(\sqrt{\Delta}/m)t} + e^D} dt - \frac{\eta}{2B} t - \frac{\sqrt{\Delta}}{2AB} t \\ &= x_1 + \frac{m}{AB} \ln \left| e^{(\sqrt{\Delta}/m)t} + e^D \right| \Big|_{t_1}^t - \frac{\eta}{2B} t - \frac{\sqrt{\Delta}}{2AB} t \\ &= x_1 + \frac{m}{AB} \ln \left(\frac{e^{(\sqrt{\Delta}/m)t} + e^D}{e^{(\sqrt{\Delta}/m)t_1} + e^D} \right) - \frac{\eta}{2B} t - \frac{\sqrt{\Delta}}{2AB} t. \end{aligned} \quad (27)$$

The function here is very complex; in addition to using Newton's method to calculate this function, MATLAB's fsolve command can also solve this. It is not described here.

4. Results and Discussion

4.1. Practice Verification. In order to verify the reliability of the theory, two sets of experiments were processed to evaluate the accuracy and stability of the corrected full-time falling ball methods. In order to ensure the accuracy of the test results, during the test, a relatively constant ambient humidity was maintained, and the viscosity coefficient test instrument was used to keep the liquid temperature constant, and the method of taking the average of multiple measurements was used to ensure the rationality of the test data. The viscosity coefficient of castor oil is very sensitive to changes in temperature. The fluidity of castor oil is good, almost no oxidative rancidity occurs in the air, and the storage stability is good. It is a typical non-drying liquid oil

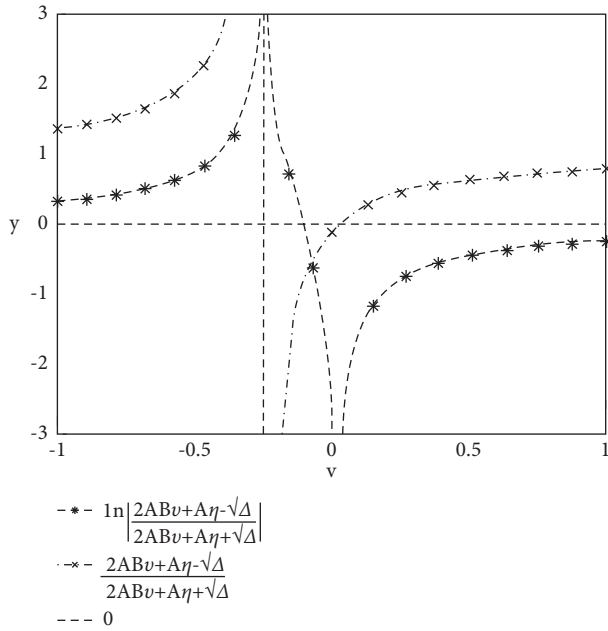


FIGURE 3: Function graphic.

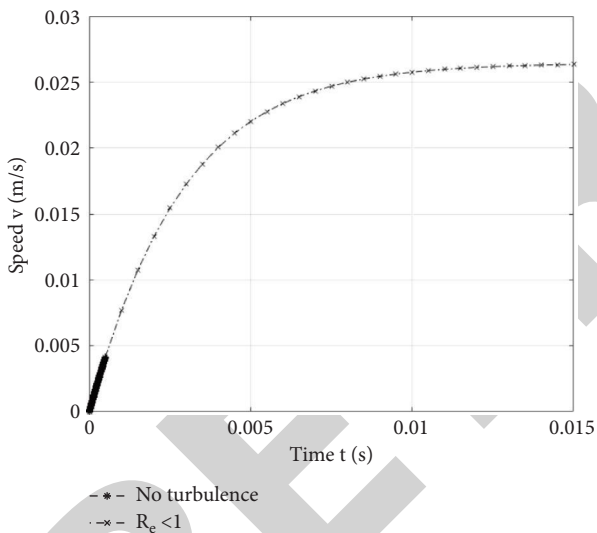


FIGURE 4: Velocity-time relationship of the pellets.

with color, flexibility, pigment dispersion, wetting, lubricity, low-temperature properties, electrical properties, and biological properties. Therefore, castor oil is mostly used as the liquid to be tested in university physics experiments, and the change of the viscosity coefficient of castor oil with temperature is measured by the falling ball method. The experimental material in this study is castor oil, which is made in the laboratory and is a commonly used experimental liquid in the teaching of university physics experiments and has standard viscosity coefficients to compare. The density of the castor oil is $\rho' = 0.96 \times 10^3 \text{ kg/m}^3$. The density of the pellet used in the test is calculated by measuring its volume and mass, and the density is $\rho = 15.1176 \times 10^3 \text{ kg/m}^3$.

Its geometric features have been described above. The one instrument is performed by PID temperature control

experimental instruments made in Century Zhongke Technology Company. PID temperature control is actually a closed-loop automatic control technology. The closed-loop automatic control technology is based on the concept of feedback to reduce uncertainty. The feedback has three elements, measurement, comparison, and execution. The key to measurement is the actual value of the controlled variable, compared with the expected value, and this deviation is used to correct the response of the system and perform regulatory control. In engineering practice, the most widely used regulator control laws are proportional, integral, and differential control, which is PID control. Another one is variable temperature viscosity coefficient experimental instrument made in Century Zhongke Technology Company, which has the following characteristics:

1. This instrument is equipped with a heating and cooling system, which can be used to study the relationship between the viscosity coefficient of liquid and the temperature, which expands the scope of knowledge and enriches the experimental content.

2. The laser photoelectric sensor combined with the single-chip computer for timing can overcome the parallax and reaction error of artificial stopwatch timing, and the accuracy of measuring the falling speed of the ball is high. It guides students to master a new method of timing, speed measurement, and counting, which reflects the modernization of experimental teaching.

3. The instrument uses a new type of laser transmitter and receiver and has made improvements in adjusting the laser beam, avoiding the problem of "difficult adjustment" in the previous instruments, making the experiment easy to operate.

4. Design the horizontal and vertical adjustment devices of the chassis and the guide tube for the drop of the ball in the center of the beam to ensure that the ball falls from the center of the liquid measuring cylinder to be measured.

This instrument can be used for basic physics experiments, design research experiments, and demonstration experiments in institutions of higher learning and technical secondary schools.

4.2. Use the Full-Time Falling Ball Method to Measure the Viscosity Coefficient of Castor Oil. Experimental methods: Castor oil was placed in an PID temperature control experimental instrument, and variable temperature viscosity coefficient experimental instrument was used to keep the castor oil constant temperature. The timing started when the pellet was released from rest in the center of the liquid level. When the pellet reached a predetermined midpoint, the timing stopped. The distance and time the ball fell were then recorded. After substituting all data into formula (26), the viscosity coefficient was obtained. Form 13°C to 23°C , the viscosity coefficient of castor oil was measured three times by the above method at intervals of 2°C . Before the start of the test, prepare experimental instruments, small balls, and the liquid castor oil to be tested, wipe the surface of the small balls dry, and use this experimental device to accurately release the small balls from the center of the liquid measuring cylinder to be tested. During the test, the displacement

and time of the drop of the ball were measured. After each measurement, the ball was taken out, and the castor oil on its surface was wiped dry and then remeasured at the same temperature. A total of three measurements were made at each temperature. The temperature of the castor oil was increased by using the heating and cooling system of the variable temperature viscosity coefficient tester, and the test was carried out every 2°C and repeated three times. According to the measured displacement and time, the viscosity coefficient of castor oil at the test temperature can be calculated by using the new theoretical formula calculated by the Newton iteration method. The obtained three viscosity coefficient measurements at each temperature are averaged as the final result of the viscosity coefficient of castor oil at that temperature, which can be used to study the variation of liquid viscosity coefficient with temperature. Calculate the average of the three measurements and then plot them on a paper with a standard curve; this is shown in Table 1 and Figure 5.

4.2.1. Results and Discussion. The standard curve reflects the standard value of the viscosity coefficient of castor oil at different temperatures. It decreases with increasing temperature. The measured data points are the viscosity coefficients of castor oil measured using the new correction formula. It can be seen from Figure 5 that both the standard viscosity coefficient and the measured viscosity coefficient decrease with the increase of temperature, and both show the same trend of change. The measurement result of the full-time falling ball method is very close to the standard value. By calculating, room mean square error was $S = 0.045$, and $S = \sqrt{\sum_{i=1}^6 (\hat{\eta}_i - \eta_i)^2 / 6}$. The deviation between the standard viscosity coefficient and the measured viscosity coefficient is 0.045, and the accuracy is high.

In the formula, $\hat{\eta}_i$ were measured value, and η_i were standard values. Room mean square error was less than 1%, indicating that new correction method had a high degree of accuracy and could measure the viscosity coefficients of liquids well.

4.3. Prediction and Test of Relationship between Displacement and Time of Pellets. Experimental methods: Select a temperature, use the revised formula to calculate the relationship between the falling displacement and time of the ball in the liquid to be measured at this temperature, and at this temperature, the relationship between the falling displacement and time of the ball is actually obtained through experiments. The rationality and accuracy of the revised formula are obtained by comparing the two.

According to the test environment conditions, the selected test temperature is 23°C. At 23°C, check the table for the standard values of the viscosity coefficients of castor oil, plug them into formula (25), and get the elapsed time when the ball falls with different displacements, and draw the relationship between displacement and time. Follow the experiment method of 4.1; during the falling process of the pellet, record the time that the pellet passed by every five centimeters (repeat the experiment three times). After one

```

clc, clear;
C = input ("C = ");
m = input ("m = ");
syms eta;
syms v;
u = m/(C-f(eta, v));
W = int (u, v);
V = finverse (W, v);
x = int (V, v);
v = input ("t_0 = ");
h = input ("h = ");
fsolve ("x-h")

```

ALGORITHM 1: Viscosity Coefficient Solver.

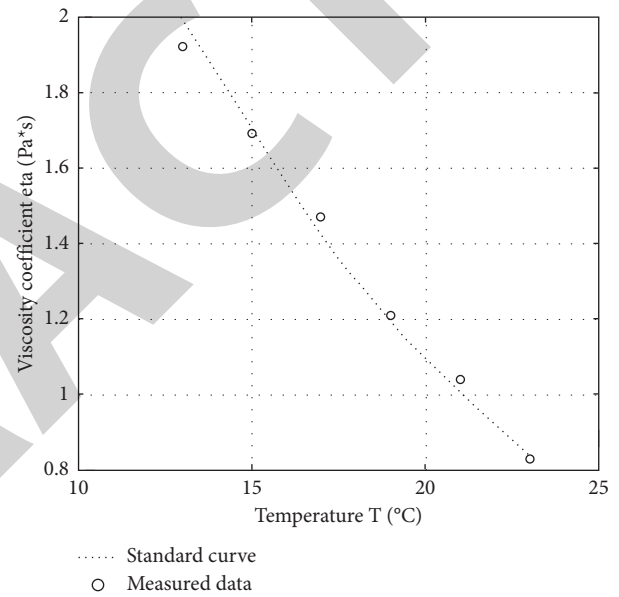


FIGURE 5: The viscosity coefficient of castor oil changes with temperature.

test is completed, the ball is taken out, and the castor oil on its surface is wiped dry, and then a new repeated test is carried out, and the average value of the three experiments is taken as the actual time elapsed by the ball when the ball passes through different displacements, and the average values of the three measurements are regarded as the final measured values. Then the average values of time are plotted on the above graph; the program for drawing is as in Table 2.

```

clc, clear;
eta = input ("eta");
A = 0.02341276413;
B = 3.78;
C = 0.000581765;
m = 0.000065;
v1 = (0.05 * eta)/(960 * 0.0105);
t1 = (-m/(A * eta)) * log(1-(A/C) * eta * v1);
delta = (A^2) * (eta^2) + 4 * A * B * C;

```

```

clc, clear;
%Input the actual parameters. It can generate physical situation
A = input ("A = ");
C = input ("C = ");
m = input ("m = ");
t = input ("t = ");
h = input ("h = ");
eta0 = 0;
% Input estimated viscosity coefficient
eta1 = input ("Please input the expected eta in present temperature = ");
%Input the maximum error we can tolerate
delta = input ("Please input the delta that you can accept = ");
%Enter the loop, output when the condition is met
while abs (eta0-eta1) > delta
eta0 = eta1;
f = C * t/A * eta + (m * C * (exp(-A * eta * t/m)-1))/A^2 * eta^2;
%f1 is the derivative of f
f1 = -C * t/A * eta^2 - 2 * m * C * (exp(-A * eta * t/m)-1)/A^2 * eta^3 - C * exp(-A * eta * t/m)/A * eta;
eta1 = eta0-f/f1;
end
fprintf ("Eta of the measured liquid is %.3fn," eta1)

```

ALGORITHM 2: Drop Depth Prediction Program.

```

gama = log (abs((2 * A * B *, v1 + A * eta -sqrt(delta))/
(2 * A * B *, v1 + A * eta + sqrt(delta)))-(sqrt(delta)/
(A * eta)) * log(1-(A/C) * eta * v1);
g = @(t) (C * t)/(A * eta) + ((m * C)/(A^2 * eta^2)) * (exp(-
A * eta * t/m)-1);
fplot (g,[0 t1], "k-- * ");
hold on;
x = [0 0.61 0.76 1.02 1.78 2.14];
y = [0 1e-2 2e-2 3e-2 4e-2 5e-2];
scatter (x, y, "k"); f = @(t) (log((exp((sqrt(delta) * t)/
m) + exp(gama))/(exp((sqrt(delta) * t1)/
m) + exp(gama))))/(A * B/m)-(eta * t)/(2 * B)-
(sqrt(delta) * t)/(2 * A * B) + (C * t1)/
(A * eta) + ((m * C)/(A^2 * eta^2)) * (exp(-A * eta * t1/
m)-1);
fplot (f, [t1 1.8], "k-.x")
legend ("No turbulence," "Measured data," "R_e < 1")
xlabel ("Time t/s")
ylabel ("Falling length of the pellet x/m")
grid on

```

Then we can get Figure 6.

4.3.1. Results and Discussion. The sphere falls in the liquid, and the phase without turbulence occurs only for a short period of time. The dotted line in the figure represents the viscosity coefficient of castor oil when the Reynolds number is less than 1, and the measured data points represent the relationship between the drop displacement of the ball and time measured at 23°C. It can be seen from Figure 6 that, with the extension of the falling time of the ball, the displacement of the ball gradually increases. The prediction

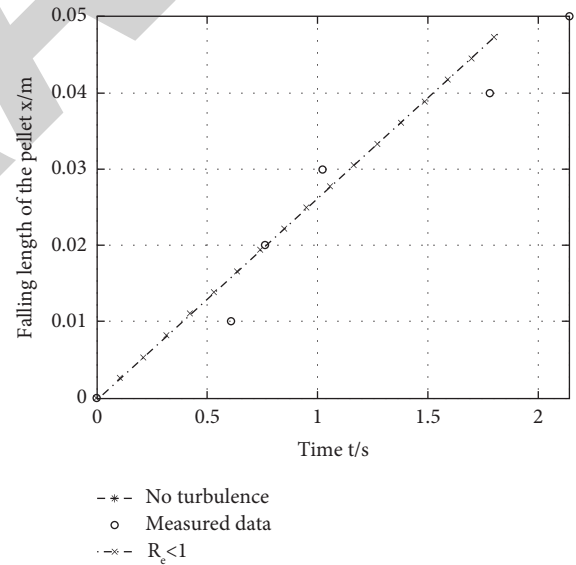


FIGURE 6: Displacement-time relationship of the pellet.

results of the full-time ball drop method are in good agreement with the experiment. By calculating, room mean square error was $S = 0.064$, the result is significant, and the revised formula is more consistent with the traditional ball drop method.

5. Discussion

In the research on the falling ball method and the viscosity coefficient, most scholars did not study the liquid viscosity coefficient itself from the falling ball method, and very few Chinese scholars studied it from the experimental

TABLE 1: Measured values of viscosity coefficient at different temperatures.

	13°C	15°C	17°C	19°C	21°C	23°C
Viscosity coefficient	1.92	1.69	1.47	1.21	1.04	0.83

TABLE 2: The relationship between the falling displacement of the ball and time.

Falling length (m)	0	0.01	0.02	0.03	0.04	0.05
Viscosity coefficient (s)	0	0.61	0.76	1.02	1.78	2.14

instruments and experimental methods. Therefore, most of the difficult problems remain unsolved in the use of the falling ball method to measure the viscosity coefficient of liquids.

Sato et al. [20] developed a rolling ball viscometer for simultaneous measurement of viscosity, density, and bubble point pressure in CO-expanded liquids. The unit is installed in a high pressure vessel and can be used at pressures up to 30 MPa and temperatures up to 200°C. The calibration of the viscometer was carried out by using water at a temperature of 40°C and 80°C at an inclination angle of 10° to 30° and at pressures up to 20 MPa and confirmed using the toluene viscosity; the viscosity of toluene was found. The measurements were consistent with literature data, with an average relative deviation within 0.66%. Calvignac et al. [21] showed development of an improved falling ball viscometer for high pressure measurement of supercritical CO₂. The device is based on the principle of a falling ball viscometer and is implemented in an autoclave equipped with a visualization window. The ingenuity here is that the ball falls from a tube that is open at both ends and has a diameter slightly larger than that of the ball, which simplifies modeling and numerical simulations. Numerical methods have been used for viscosity determination. Chukwunke [22] showed analysis of the dynamics of free-falling bodies in viscous fluids: a computational fluid dynamics approach. The dynamics of solid objects falling in viscous fluids are studied. The forces governing motion and the effects of fluid viscosity and object density were investigated using analytical models. Use CFD to evaluate wall effects due to the presence of finite fluid boundaries. By further correlating the CFD results, the function approximation model required to predict the wall effect constraints was developed. The analytical solution gives a good approximation of the forces governing motion and shows that the effect of fluid viscosity is crucial in resisting the motion of solid particles in a viscous fluid. Camas-Anzueto [23] reported the viscosity characteristics of biodiesel produced from *Jatropha curcas*. Viscosity measurements were performed using a modified falling ball viscometer as well as optical techniques. Viscosity is measured in the range of 28 to 70°C, which is the key to determining the quality of biodiesel. We found that the viscosity measurement resolution of the falling ball optical viscometer was ± 0.039 mPa·s with a relative error of 1.47933%. A comparison of the measurement process with a commercial viscometer proves that the biodiesel produced in

Chiapas is of good quality. Steigerwald [24] focused on the numerical simulation of the ammonothermal growth of III-V bulk single crystals, where accurate viscosity and density data are urgently needed. In this work, changes in viscosity with temperature and pressure were tracked in a developed ball viscometer. There, the fall time is detected by acquiring the ball's acoustic signal using a high-temperature noise accelerometer. Viscosity results for pure ammonia under ammonothermal conditions have shown good accuracy. The device is designed to measure density and viscosity by substituting rolling ball materials in future experiments. This is important because the mineralizer necessary for the ammonothermal process changes the solubility of GaN in ammonia, so the density of the flowing fluid is not constant. A system was developed by Wang [25], based on a falling ball viscometer interfaced with a personal computer, that greatly facilitates viscosity measurements under low shear conditions. Three optical sensors indicate the rate at which the steel ball is falling through the microcapillary, and a potentiometer detects the angle of the tube. The generated data is passed to a spreadsheet from a custom program with a graphical user interface. The spreadsheet then calculates and stores the viscosity value of the sample using the data passed to it from the viscometer and a look-up table that stores slope and intercept values calculated from precision viscosity standards. We used a viscometer to determine the kinetics of actin polymerization and to measure the viscosity of F-actin-aldolase gels. Compared to the traditional "eyeball and stopwatch" approach, the system offers significantly higher repeatability and speed in data acquisition, and the data reduction is almost instantaneous.

Starting from the ball drop method itself, this paper finds that there are many limitations in the use of the ball drop method. Some problems have been solved by some scholars by studying experimental devices and methods. However, studies have found that, for the falling ball method, the velocity needs to reach a uniform motion state in order to measure the viscosity coefficient of the liquid. The actual situation sometimes cannot meet this condition, such as when the liquid level is too low or the viscosity coefficient of the liquid is too small, resulting in a constant acceleration state when falling in the experimental device, unable to reach its terminal velocity, resulting in the limitation of the falling ball method in these research processes. This problem has not been resolved for a long time.

Based on this, this paper improves the falling ball method. By examining the whole falling distance of the small ball, the relationship between the falling total length and time is obtained by integrating, and then the liquid viscosity coefficient is calculated. This paper considers the correction of Stokes formula, the restrictions on the experimental device and the viscosity coefficient of the liquid to be measured are relaxed, and the application range of the falling ball method is broadened.

6. Conclusions and Outlook

The new correction method improves the precision dramatically with reliable measurement results. The instruments are simple and easy to operate. The deviation of the viscosity coefficient of castor oil measured by the full-time

falling ball method at different temperatures from the standard value is only 0.045, and the result is reliable. The p value is less than 0.95, and the result is significant. It is a new development of the falling ball method to measure the viscosity coefficient of liquids. On the one hand, it is expected to become a new idea for measuring the viscosity coefficient of liquids with wide viscosity range. On the other hand, it can provide a guidance for the teaching of physics experiments in universities.

The method considers the correction of Stokes' formula and broadens the application scope of the falling ball method. In a sense, it is theoretical correction of viscosity coefficient measurement by falling ball method. While the Reynolds number increases further, the correction of Stokes' formula is difficult due to the greater influence of inertial forces. At this point, we can consider using the velocity field equation directly [26, 27]. Nevertheless, this presents a greater challenge to mathematical solutions that requires further exploration [28]. It can be considered from two aspects. On the one hand, the Navier-Stokes equation can be solved directly, but this requires progress in the mathematical solution of partial differential equations; simulate the falling process of the sphere. Fortunately, there are more reliable simulation software programs such as COMSOL for selection.

Data Availability

The datasets generated during and/or analyzed during the current study are not publicly available due to sensitivity and data use agreement.

Disclosure

The authors confirm that the content of the manuscript has not been published or submitted for publication elsewhere.

Conflicts of Interest

The authors declare that there are no conflicts of interest.

Authors' Contributions

Qingyu Zhang is co-first author. All authors have seen the manuscript and approved to submit to the journal.

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