Stabilization Design of Three-Phase LCL-Filtered Grid-Connected Inverter Using IDA-PBC Controller

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1. Introduction

In renewable energy generation system, the grid-connected inverter has become one of the core parts to interface grid [1]. Its output current harmonic distortion is an important technical indicator; thus, LCL filter with low-pass characteristics is usually connected to inject high quality current. LCL filter has better filtering effect, smaller system volume, and lower loss at the same cost compared with L filter [2]. However, as a result of inherent resonance of high-order filter, the circuit is prone to oscillating or even unstable outputs. [3] Therefore, extra requirements are put forward for the controller design for LCL-filtered inverter system. Additionally, 1-beat or 1.5-beat delay could be brought by discrete control and pulse width modulation [4]. The control delay will typically reduce the phase margin as well as complicate the design of the controller.

Apart from the stability of the inverter system itself, another concern of the controller design is the interactive stability with the grid [5]. During actual operation, grid impedance is varying, resulting in stimulated resonances for the stable inverter system during independent operation and even undesirable tripping. Recently, the concept of passivity theory was applied to access the stability of grid-connected inverters in frequency domain, and many research works have been proposed to extend nonnegative real part of inverter output admittance [5–10]. It indicates that the resonance will not be stimulated if the interactive point falls in the passive region. Most control methods mainly consider passive regions up to the Nyquist frequency. However, passivity analysis and instability of inverter admittance above the Nyquist frequency should also be considered [8].

Currently, most closed-loop control strategies rely on the classic linear control theory, which can be used to accurately design the controller parameters and evaluate the influence of the delay. These control methods are however difficult to fully eliminate interactive resonances under complex weak grid, and the traditional proportional integral control does...
not have sufficient disturbances rejection capability. Compared with the classical linear control theory, the modern nonlinear control theory has better robustness and performance, such as sliding mode control [11, 12], predictive control [13, 14], and passivity-based control (PBC) [15–26]. PBC has proven useful in the design of robust controllers for railway systems [19, 20], renewable generation systems [21], etc. In terms of the PBC design process, energy formation and damping injection are considered to be of obvious physical significance [17], and two branches are proposed by the Euler–Lagrange (EL) model and the port-controller Hamiltonian (PCH) model, called EL-PBC and IDA-PBC, respectively. [22–26] The modeling process of the EL-PBC method is simpler, but IDA-PBC offers more flexibility in the type of damping interconnection and the modeling process. Moreover, for the IDA-PBC, fast convergence speed of the error energy function can be determined by calculating the derivative of the closed-loop energy function and the injected damping [23]. For EL-PBC, when the error is large, the convergence speed is fast, and injected damping plays the main role. In contrast, the convergence speed becomes slower and the role of injected damping becomes weak when the error is small. Towards this end, IDA-PBC has been adopted in this study so as to achieve better performance under complex grid conditions.

However, it is often overlooked that time delay can affect IDA-PBC [25]. As to the design of damping matrix of IDA-PBC, few research studies analyze the specific design guideline and the try-and-error method is usually adopted. Therefore, for the purpose of assessing the stability of weak grid, the detailed design of digital controlled IDA-PBC is necessary. A brief summary of the main contributions is as follows.

(1) On the premise of ensuring Lyapunov stability, to simplify the design process for PCH model-based PBC, a more flexible interconnection matrix design method is used.

(2) This study demonstrates how to select the injected damping considering the effect of control delay using frequency-domain passivity theory based on linear control design.

(3) With the proposed controller parameters’ design method, the nonnegative real part of the LCL-filtered inverter output admittance can be achieved within switching frequency. If the passivity of subsystem is ensured, then the stability of the whole interlinked system in parallel form is guaranteed regardless of grid impedance.

Moreover, in Section 2, a brief description of the LCL-filtered inverter system is given, followed by the design of IDA-PBC methodology in Section 3. Instead of the original antisymmetric matrix, the interconnection matrix is simplified by using the upper triangular matrix with zero corner elements. In order to precisely design the injected damping gains, in Section 4, frequency-domain passivity theory is applied with the derived impedance model of IDA-PBC. In Section 5 and Section 6, system performance is demonstrated by simulations and experimental results. Furthermore, the study found that IDA-PBC also performed well in unbalanced grid. Section 7 concludes the study.

2. System Description and PCH Modeling

2.1. Description of the System. Figure 1 depicts an inverter with a power supply voltage $U_{dc}$ and is filtered by a LCL filter including inverter-side inductor $L_1$, filter capacitor $C$, and grid-side inductor $L_2$. The parasitic resistances of $L_1$ and $L_2$ are represented by $R_1$ and $R_2$, respectively. The voltage $v_{pcc}$ is used for the synchronization of the grid current. Additionally, the inverter-side current $i_1$, the capacitor voltage $u_C$, and the grid-side current $i_2$ should also be measured to control the injected current as well as suppress the resonances. It should be noted that the state observer or Kalman filter can be used to reduce the number of required sensors by estimating state variables [9, 11]. The equivalent grid reactance of $L_g$ and $C_g$ is connected to the grid at the point of common coupling (PCC), which can take a wide range of values. Therefore, the controller must be robust enough to withstand grid disturbance. Table 1 summarizes basic system parameters for analysis.

\[
\begin{align*}
L_1 \frac{di_{1k}}{dt} &+ R_1 i_{1k} + u_{Ck} = u_{ik}, \\
C \frac{du_{Ck}}{dt} + i_{2k} - i_{1k} &= 0, \\
L_2 \frac{di_{2k}}{dt} &+ R_2 i_{2k} - u_{Ck} = -v_{pck}.
\end{align*}
\]

In accordance with Figure 1, the equations for a three-phase LCL filtered inverter system can be derived from Kirchhoff’s law in a stationary abc frame as follows: where $k$ stands for the cases in abc coordinates. Through the rotation transformation, from (1), the mathematical model of the grid-connected inverter in d-q coordinates appears as

\[
\begin{align*}
L_1 \frac{di_{1d}}{dt} &= u_{id} - R_1 i_{1d} + \omega L_1 i_{1q} - u_{Cd}, \\
L_1 \frac{di_{1q}}{dt} &= u_{iq} - R_1 i_{1q} - \omega L_1 i_{1d} - u_{Cq}, \\
C \frac{du_{Cd}}{dt} &= i_{1d} - i_{2d} + \omega C u_{Cd}, \\
C \frac{du_{Cq}}{dt} &= i_{1q} - i_{2q} - \omega C u_{Cd}, \\
L_2 \frac{di_{2d}}{dt} &= u_{Cd} - R_2 i_{2d} + \omega L_2 i_{2q} - v_{pccd}, \\
L_2 \frac{di_{2q}}{dt} &= u_{Cq} - R_2 i_{2q} - \omega L_2 i_{2d} - v_{pccq}.
\end{align*}
\]
2. PCH Modeling. In order to construct a robust controller for the above system, a PCH model with dissipation can be used [24]. It is possible to express the system’s PCH model as follows:

\[ \dot{x} = [J(x) - R(x)] \frac{\partial H(x)}{\partial x} + g(x)u, \]  

where \( J(x) \) represents the system interconnection properties and normally satisfy formula \( J(x) = -\frac{\partial H(x)}{\partial x} \), \( R(x) \) indicates the system dissipation characteristics, and \( H(x) \) is the Hamiltonian function; \( u \) indicates the exchange of energy between the outside world and the system. The coefficient matrix \( g(x) \) stands for the interconnections. The following is a definition of system variables:

\[
x = [x_1 \ x_2 \ x_3 \ x_4 \ x_5 \ x_6]^T
\]

\[
= \begin{bmatrix}
L_{11} & L_{12} & L_{13} & L_{14} & L_{15} & L_{16} \\
L_{21} & L_{22} & L_{23} & L_{24} & L_{25} & L_{26} \\
L_{31} & L_{32} & L_{33} & L_{34} & L_{35} & L_{36} \\
L_{41} & L_{42} & L_{43} & L_{44} & L_{45} & L_{46} \\
L_{51} & L_{52} & L_{53} & L_{54} & L_{55} & L_{56} \\
L_{61} & L_{62} & L_{63} & L_{64} & L_{65} & L_{66}
\end{bmatrix}.
\]

The following is the expression for each of the vectors and matrices for the LCL-filter inverter according to (2):

\[
J = \begin{bmatrix}
0 & \omega L_1 & -1 & 0 & 0 & 0 \\
-\omega L_1 & 0 & 0 & -1 & 0 & 0 \\
1 & 0 & 0 & \omega C & -1 & 0 \\
0 & 1 & -\omega C & 0 & 0 & -1 \\
0 & 0 & 1 & 0 & 0 & \omega L_2 \\
0 & 0 & 0 & 0 & 1 & -\omega L_2
\end{bmatrix},
\]

\[
g(x) = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix},
\]

\[
u = [u_d, u_q, v_{pccd}, v_{pqcd}]^T,
\]

\[
R = \text{diag}[R_1, R_1, 0, 0, R_2, R_2].
\]

The stored energy in the LCL filter can be expressed as

\[ H(x) = \frac{x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 + x_6^2}{2L_1} + \frac{2x_1x_2 + 2x_1x_4 + 2x_1x_5 + 2x_3x_4 + 2x_3x_5 + 2x_6^2}{2L_2}. \]  

3. IDA-PBC Controller Design

The primary goal of the controller is to find the Hamiltonian energy storage function \( H_d(x) \), interconnection matrix \( J_d(x) \), and damping matrix \( R_d(x) \), which satisfy the following equation with a balance point at \( x^* \):

\[ [I_d(x) - R_d(x)]K(x) = -[J_d(x) - R_d(x)] \frac{\partial H(x)}{\partial x} + g(x)u, \]

where

\[ K(x) = \text{diag}[\partial H_d(x)/\partial x], \]

\[ I_d(x) = J(x) + J_d(x), \]

\[ R_d(x) = R(x) + R_d(x), \]

and \( H_d(x) \) is the undetermined function.

Moreover, in order to ensure \( x^* \) is locally stable equilibrium point, the following conditions should also be satisfied [15].

1. In traditional design, strict structure preservation is provided

\[ \left\{ \begin{array}{l}
I_d(x) = J(x) + J_d(x) = -[J(x) + J_d(x)]^T, \\
R_d(x) = R(x) + R_d(x) = [R(x) + R_d(x)]^T.
\end{array} \right. \]

2. \( K(x) \) is the integrated gradient of a scalar function:

\[ \frac{\partial K(x)}{\partial x} = \left( \frac{\partial K(x)}{\partial x} \right)^T. \]

3. Expect the balance \( x^* \); the desired dynamics is achieved if \( K(x) \) is satisfied:

\[ K(x^*) = -\frac{\partial H(x)}{\partial x}|_{x=x^*}, \]

\[ H_d(x) = H(x) + H_d(x). \]

4. At the balance point, \( K(x) \) should satisfy the following function based on the Lyapunov stability:

\[ \frac{\partial K(x)}{\partial x}|_{x=x^*} > -\frac{\partial^2 H(x)}{\partial x^2}|_{x=x^*}. \]

For EL-PBC, \( J(x) \) is fixed and the damping is injected with constructed energy function. For IDA-PBC, it is more flexible to design \( J_d(x) \) and \( R_d(x) \) corresponding to desired energy function. At the stable equilibrium point \( x^* \), the closed-loop system based on PCH is described as [22].
\[
\dot{x} = [J_d(x) - R_d(x)] \frac{\partial H_d(x)}{\partial x}, \tag{14}
\]

At the desired \(x^*\), the tracking error of selected Hamiltonian function \(H_d(x)\) should be minimum. The derivation of the closed-loop Hamiltonian energy function \(H_d(x)\) is depicted as

\[
\dot{H}_d(x) = \left[\frac{\partial H_d(x)}{\partial x}\right]^T \dot{x}
= \left[\frac{\partial H_d(x)}{\partial x}\right]^T [R_d(x) - J_d(x)] \frac{\partial H_d(x)}{\partial x}.	ag{15}\]

Based on Lyapunov’s second criterion, the derivative of \(H_d(x)\) of the closed-loop energy function must be negative to attain an asymptotically stable equilibrium point. Conventional, \(J_d\) is designed as positive definite symmetric matrix based on structure preservation in (9). A more flexible method is proposed to design \(J_d\) in this study. The derivative of the energy function can be less than zero if \([R_d - J_d]\) is a positive definite matrix. So, if injected damping \(R_a\) is positive, \(J_d\) can be designed as a lower triangular matrix or an upper triangular matrix with zero diagonal elements to simplify the design procedure, and it is also capable of ensuring that \(\dot{H}_d(x) < 0\). So, \(J_a\), \(J_d\), and \(R_a\) are defined as

\[
J_a = \begin{bmatrix}
0 & -\omega L_1 & 1 & 0 & 0 & 0 \\
\omega L_1 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & -\omega C & 1 & 0 & 0 \\
0 & 0 & \omega C & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & -\omega L_2 & 0 \\
0 & 0 & 0 & 0 & \omega L_2 & 0 \\
\end{bmatrix},
\]

\[
J_d = R_a = \text{diag} \{ r_1, r_1, r_2, r_2, r_3, r_3 \}.	ag{16}\]

The elements of \(R_a\) should be positive. Through the control states \(x_5 \rightarrow x_5^*\) and \(x_6 \rightarrow x_6^*\), the states \(x_1, x_2, x_3, x_4, x_5, x_6\) can gradually reach the equilibrium points, which are defined as follows:

\[
x^* = \begin{bmatrix} L_1 i_{1d}^* & L_1 i_{1q}^* & C \dot{i}_{q_d}^* & C \dot{i}_{q_d}^* & L_2 i_{2d}^* & L_2 i_{2q}^* \end{bmatrix}^T. \tag{17}\]

The derivative of \(H_d(x)\) can be defined as \(K(s)\) and corresponding components of \(K(s)\) can be viewed as follows:

\[
K(x) = \begin{bmatrix} k_1(x), k_2(x), k_3(x), k_4(x), k_5(x), k_6(x) \end{bmatrix}^T, \tag{18}\]

\[
\begin{align*}
k_1(x) &= \frac{x_1^*}{L_1} \\
k_2(x) &= \frac{x_2^*}{L_1} \\
k_3(x) &= \frac{x_3^*}{C} \\
k_4(x) &= \frac{x_4^*}{C} \\
k_5(x) &= \frac{x_5^*}{L_2} \\
k_6(x) &= \frac{x_6^*}{L_2}.
\end{align*}
\]

When \(J_a\), \(R_a\), and \(K(x)\) are substituted to (1), reference values for the state variables of the system can be found in (20) which determined the passive controller \(u\). As a result of (20), as shown in Figure 2, the cascaded three loops controller can be depicted. It is worth mention that “11e,” “12e,” “Ce,” “R1e,” and “R2e” are the initial filter parameters given in the controller, and the subscript “e” is used to differentiate with actual parameters of the physical filter.

\[
\begin{align*}
&u_{1d}^* = u_{cd} - \omega L_1 i_{1d} + r_1 (i_{1d} - i_{1d}) + R_1 i_{1d}, \\
&u_{1q}^* = u_{cq} + \omega L_1 i_{1q} + r_1 (i_{1q} - i_{1q}) + R_1 i_{1q}, \\
&i_{1d} = \frac{2i}{C}, \\
&i_{1q} = \frac{2i}{C}, \\
&u_{cd} = v_{ped} - \omega L_2 i_{2d} + r_3 (i_{2d} - i_{2d}) + R_2 i_{2d}, \\
&u_{cq} = v_{ped} + \omega L_2 i_{2q} + r_3 (i_{2q} - i_{2q}) + R_2 i_{2q}.
\end{align*}
\]

By using (11) and (12), it is possible to calculate the amount of energy injected into the system by the controller:

\[
H_a(x) = \frac{x_1^*}{L_1} x_1 - \frac{x_2^*}{L_1} x_2 - \frac{x_3^*}{C} x_3 - \frac{x_4^*}{C} x_4 - \frac{x_5^*}{L_2} x_5 - \frac{x_6^*}{L_2} x_6. \tag{19}\]

Accordingly, the closed-loop Hamiltonian energy function for this system is as follows:

\[
H_d(x) = \frac{x_1^*}{L_1} x_1 - \frac{x_2^*}{L_1} x_2 - \frac{x_3^*}{C} x_3 - \frac{x_4^*}{C} x_4 - \frac{x_5^*}{L_2} x_5 - \frac{x_6^*}{L_2} x_6 + \frac{x_1^2 + x_2^2}{2L_1} + \frac{x_3^2 + x_4^2}{2C} + \frac{x_5^2 + x_6^2}{2L_2}. \tag{20}\]

\[
\frac{\partial^2 H_d(x)}{\partial x^2} \bigg|_{x=x^*} = \text{diag} \begin{bmatrix} \frac{1}{L_1} & \frac{1}{L_1} & \frac{1}{C} & \frac{1}{C} & \frac{1}{L_2} & \frac{1}{L_2} \end{bmatrix} > 0. \tag{21}\]

Hence, \(H_d(x)\) has the minimum value at \(x^*\). Furthermore,
is dissipative, this also implies that it is passive. In order to select the values of the injected damping coefficients, Section 4 will analyze the passivity of the closed-loop output admittance using the impedance model.

4. Impedance Model and Stability Oriented Design

4.1. Impedance Modeling. With the traditional design of the IDA-PBC controller outlined, the digital control is ignored. The system is modeled with equivalent Laplace transform for the controller equations to select the appropriate parameters of injected damping gains considering the effect of discretization delay. Based on (20), it is possible to describe the IDA-PBC controller as a three-stage controller for LCL-filtered inverters in the following matrices:

\[
\begin{bmatrix}
    u^*_d \\
    u^*_q
\end{bmatrix} =
\begin{bmatrix}
    E & 0 \\
    0 & E
\end{bmatrix}
\begin{bmatrix}
    i_{1d} \\
    i_{1q}
\end{bmatrix} +
\begin{bmatrix}
    r_1 & \omega L_{1e} \\
    -\omega L_{1e} & r_1
\end{bmatrix}
\begin{bmatrix}
    i_{1d} \\
    i_{1q}
\end{bmatrix} +
\begin{bmatrix}
    u_{Cd} \\
    u_{Cq}
\end{bmatrix},
\]

(26)

\[
\begin{bmatrix}
    i_{2d}^* \\
    i_{2q}^*
\end{bmatrix} =
\begin{bmatrix}
    F & 0 \\
    0 & F
\end{bmatrix}
\begin{bmatrix}
    u_{Cd}^* \\
    u_{Cq}^*
\end{bmatrix} +
\begin{bmatrix}
    r_2 & \omega C_e \\
    -\omega C_e & r_2
\end{bmatrix}
\begin{bmatrix}
    u_{Cd} \\
    u_{Cq}
\end{bmatrix} +
\begin{bmatrix}
    i_{2d} \\
    i_{2q}
\end{bmatrix},
\]

(27)

\[
\begin{bmatrix}
    u_{Cd}^* \\
    u_{Cq}^*
\end{bmatrix} =
\begin{bmatrix}
    G & 0 \\
    0 & G
\end{bmatrix}
\begin{bmatrix}
    i_{2d}^* \\
    i_{2q}^*
\end{bmatrix} -
\begin{bmatrix}
    r_3 & \omega L_{2e} \\
    -\omega L_{2e} & r_3
\end{bmatrix}
\begin{bmatrix}
    i_{2d} \\
    i_{2q}
\end{bmatrix} +
\begin{bmatrix}
    v_{pccd} \\
    v_{pccq}
\end{bmatrix},
\]

(28)

where \( E = R_{1e} + r_1 \), \( F = r_2 \), and \( G = R_{2e} + r_3 \).

In Figure 2, the delay in the system is added after the deduced reference control law, resulting in the following expression [8]:

\[
G_d(s) = \frac{\sin(\omega T/2)}{\omega T/2}e^{-1.5Ts},
\]

(29)

where \( T \) represents the sampling period. The delay \( G_d(s) \) is written as \( G_d \) to simplify the expression in the following.

Then, the control signal is sent to the LCL filter-based inverter depicted in Figure 2:

\[
\begin{bmatrix}
    u_d \\
    u_q
\end{bmatrix} = G_d \begin{bmatrix}
    u^*_d \\
    u^*_q
\end{bmatrix},
\]

(30)

The LCL-filtered inverter system model is expressed as follows:
\[
\begin{bmatrix}
i_{2d} \\
i_{2q}
\end{bmatrix} =
\begin{bmatrix}
0 & \omega L_2 \\
-\omega L_2 & 0
\end{bmatrix}
\begin{bmatrix}
A & 0 \\
0 & A
\end{bmatrix}
\begin{bmatrix}
i_{2d} \\
i_{2q}
\end{bmatrix} +
\begin{bmatrix}
A & 0 \\
0 & A
\end{bmatrix}
\begin{bmatrix}
u_{Cd} \\
u_{Cq}
\end{bmatrix} -
\begin{bmatrix}
A & 0 \\
0 & A
\end{bmatrix}
\begin{bmatrix}
v_{pced} \\
v_{pcco}
\end{bmatrix},
\]
\[11\]

\[
\begin{bmatrix}
u_{Cd} \\
u_{Cq}
\end{bmatrix} =
\begin{bmatrix}
0 & \omega C \\
-\omega C & 0
\end{bmatrix}
\begin{bmatrix}
B & 0 \\
0 & B
\end{bmatrix}
\begin{bmatrix}
u_{Cd} \\
u_{Cq}
\end{bmatrix} +
\begin{bmatrix}
B & 0 \\
0 & B
\end{bmatrix}
\begin{bmatrix}
i_{1d} \\
i_{1q}
\end{bmatrix} -
\begin{bmatrix}
B & 0 \\
0 & B
\end{bmatrix}
\begin{bmatrix}
i_{2d} \\
i_{2q}
\end{bmatrix},
\]
\[12\]

\[
\begin{bmatrix}
i_{1d} \\
i_{1q}
\end{bmatrix} =
\begin{bmatrix}
0 & \omega L_1 \\
-\omega L_1 & 0
\end{bmatrix}
\begin{bmatrix}
D & 0 \\
0 & D
\end{bmatrix}
\begin{bmatrix}
i_{1d} \\
i_{1q}
\end{bmatrix} +
\begin{bmatrix}
D & 0 \\
0 & D
\end{bmatrix}
\begin{bmatrix}	u_d \\
u_q
\end{bmatrix} -
\begin{bmatrix}
D & 0 \\
0 & D
\end{bmatrix}
\begin{bmatrix}
u_{Cd} \\
u_{Cq}
\end{bmatrix},
\]
\[13\]

where \( A = (1/sL_2 + R_2), B = (1/sC), \) and \( D = (1/sL_1 + R_1). \)

Substituting (30) to (33), \( u_d \) and \( u_q \) can be canceling, and it can be derived that

\[
\begin{bmatrix}
1 & -\omega L_1 D \\
\omega L_1 D & 1
\end{bmatrix}
\begin{bmatrix}
i_{1d} \\
i_{1q}
\end{bmatrix} =
\begin{bmatrix}
D \times G_d & 0 \\
0 & D \times G_d
\end{bmatrix}
\begin{bmatrix}
u_d' \\
u_q'
\end{bmatrix} -
\begin{bmatrix}
D \times G_d & 0 \\
0 & D \times G_d
\end{bmatrix}
\begin{bmatrix}
u_{Cd} \\
u_{Cq}
\end{bmatrix},
\]
\[14\]

Then, substituting (33) to (26), \( u_d^* \) and \( u_q^* \) can be canceling, and it can be obtained that

\[
\begin{bmatrix}
1 & -\omega L_1 D \\
\omega L_1 D & 1
\end{bmatrix}
\begin{bmatrix}
i_{1d} \\
i_{1q}
\end{bmatrix} =
\begin{bmatrix}
DE \times G_d & 0 \\
0 & DE \times G_d
\end{bmatrix}
\begin{bmatrix}
i_{1d} \\
i_{1q}
\end{bmatrix} -
\begin{bmatrix}
r_1 D \times G_d & \omega L_1 D \times G_d \\
-\omega L_1 D \times G_d & r_1 D \times G_d
\end{bmatrix}
\begin{bmatrix}
i_{1d} \\
i_{1q}
\end{bmatrix} +
\begin{bmatrix}
D \times G_d - 1 & 0 \\
0 & D \times G_d - 1
\end{bmatrix}
\begin{bmatrix}
u_{Cd} \\
u_{Cq}
\end{bmatrix},
\]
\[15\]

Based on (34), a description of the inner loop response would be as follows:

\[
\begin{bmatrix}
i_{1d} \\
i_{1q}
\end{bmatrix} =
\begin{bmatrix}
X_i & \psi_i \\
-\psi_i & X_i
\end{bmatrix}
\begin{bmatrix}
i_{1d} \\
i_{1q}
\end{bmatrix} -
\begin{bmatrix}
Y_i & -\varphi_i \\
\varphi_i & Y_i
\end{bmatrix}
\begin{bmatrix}
u_{Cd} \\
u_{Cq}
\end{bmatrix},
\]
\[16\]

where \( \psi_i \) and \( \varphi_i \) are the coupling components \( d-q \) axis and \( X_i \) and \( Y_i \) correspond to the transfer function of closed-loop and output admittance of the inner loop, respectively:

\[
X_i = \frac{DE \times G_d (1 + r_1 D \times G_d)}{(1 + r_1 D \times G_d)^2 + (\omega L_1 D - \omega L_{1q} D \times G_d)^2},
\]
\[17\]

\[
Y_i = \frac{-D \times G_d - 1 (1 + r_1 D \times G_d)}{(1 + r_1 D \times G_d)^2 + (\omega L_1 D - \omega L_{1q} D \times G_d)^2}.
\]
\[18\]

Similarly, combining functions (26)-(33), the equivalent response of the overall controller can be expressed in (39). The derivation process is omitted here:

\[
\begin{bmatrix}
i_{2d} \\
i_{2q}
\end{bmatrix} =
\begin{bmatrix}
G_o & \psi \\
-\psi & G_o
\end{bmatrix}
\begin{bmatrix}
i_{2d} \\
i_{2q}
\end{bmatrix} -
\begin{bmatrix}
Y_o & -\varphi \\
\varphi & Y_o
\end{bmatrix}
\begin{bmatrix}
v_{pced} \\
v_{pcco}
\end{bmatrix},
\]
\[19\]

4.2. Proposed Parameters’ Design Procedure. As can be seen from Figure 2, the IDA-PBC controller offers a three-loop structure for the LCL inverter system that incorporates three gains \((r_1, r_2, r_3)\) that must be determined. Traditional IDA-PBCs can achieve stable equilibrium conditions with energy shaping techniques. However, the design of interconnected damping gains is not guided by any specific design procedure. The following is a step-by-step design procedure for IDA-PBC gains that aims for fast dynamic response and high level of robustness against external disturbances.

4.2.1. Design of the Inner Loop Gain \( r_1 \). As the gain of the inner loop, \( r_1 \) should be designed to achieve a fast response. For convenience, a first-order inertial model is substituted for the delay in the design of inner loop. Thus, the closed-loop transfer function of the inner loop is expressed in function (40):

\[
\begin{align*}
\frac{\zeta_{in}(s)}{\zeta_{ext}(s)} & = \frac{G_o \times G_{in}(s)}{1 + G_o \times G_{in}(s)} \\
\mathcal{Z} & = \frac{G_o \times G_{in}(s)}{1 + G_o \times G_{in}(s)}
\end{align*}
\]
It can be seen from (40) that the inner loop is a second-order system. According to classical control theory, the critically damped case is preferred to reach shortest time for the system. Since the critical damping ratio is equal to 1, then \( r_1 \) is chosen as 2. It is should, however, be noted that PI regulator can be applied instead of the damping \( r_1 \) with proper integral coefficient to remove the steady-state error.

\[
\begin{align*}
\frac{i_2}{i_{2d}^*} &= \frac{R_1 + r_1}{(1.5Ts + 1)(sL_1 + R_1) + r_1}, \\
r_1 &= \frac{L_1}{6\xi^2 T}, \quad \xi = 1.
\end{align*}
\]

4.2.2. Design of the Middle Loop Gain \( r_2 \) and the Outer Loop Gain \( r_3 \). Here, design of \( r_2 \) and \( r_3 \) relies on passivity of the inverter system impedance with control delay to ensure a stable system. Based on passivity theory, within switching frequency, if positive real part of system output impedance can be ensured, the system is passive [5]. For the analysis, mathematics software can be applied to calculate closed-loop transfer function \( G_o(s) \) and output admittance \( Y_o(s) \) for the outer loop. In this process, the delay function is substituted by Euler’s formula. During the steady state condition, \( i_{2d} / i_{2d}^* = 1 \) and \( i_{2d}^* / i_{2d} = 0 \). Based on function (39), ignoring the coupling items, \( G_o(s) \) is expressed as follows:

After deciding on \( r_1 = 2 \), the relationship between frequency, the real part of the inverter system admittance, and \( r_2 \) or \( r_3 \) can be plotted. The value of \( r_2 \) is not independent, but varies with value of \( r_3 \). Assume that \( r_3 \) is ensured and then determine the approximate range of \( r_2 \). Likewise, select the middle value of \( r_2 \) within the stable range, and determine the stable range of \( r_1 \) again. In addition, the system closed-loop

\[
\begin{align*}
\frac{i_2}{i_{2d}^*} &= \frac{R_1 + r_1}{(1.5Ts + 1)(sL_1 + R_1) + r_1}, \\
r_1 &= \frac{L_1}{6\xi^2 T}, \quad \xi = 1.
\end{align*}
\]
poles are drawn with different $r_1$ and $r_2$. This is demonstrated in Figures 3 and 4, where two groups of $r_1$ and $r_2$ are selected for verification.

$$\frac{I_{2d}}{I_{2d}} = \frac{XBFG}{(1 - BY + Br_2 X)A^{-1} + (XBFr_3 + B - XB)} \quad (42)$$

The relationship between $r_2$, inverter output admittance, and frequency is illustrated in Figure 3(a), assuming that $r_3$ is 25. In order to ensure positive real part of $Y_o(s)$, $r_2$ should be less than 0.19, which can also be proved with the closed-loop pole-zero maps shown in Figure 4(a).

Then, assuming that $r_2$ is 0.15, reverse-seek the range of $r_3$, as demonstrated in Figures 3(b) and 4(b). In the second round, $r_1$ is selected as 10, and the stable range of $r_2$ are illustrated in Figures 3(c) and 4(c). Then, the range of $r_3$ can be reached by reverse-seeking, assuming that $r_2$ is 0.1 and $r_1$ is 2, as shown in Figure 3(d). After the iterations, $r_2$ and $r_3$ are selected as 0.15 and 15, consequently. The overall flow diagram of the proposed design procedure for LCL-filtered grid-connected inverter with IDA-PBC is shown in Figure 5.

According to the figures of closed-loop pole-zero maps and output admittance real parts, the calculated ranges of parameters based on the internal stability of the system basically coincide with the ranges calculated according to the passivity of the output admittance. Due to the fact that the energy function of the closed-loop system represents the physical state of energy at the equilibrium point and accounts for external energy input in its entirety, with the selected parameters, the real part of the output admittance can be always positive and passive within switching frequency. Hence, the overall system can remain stable regardless of the variation of the grid impedance.
Figure 5: Flow diagram of the proposed design procedure for LCL-filtered grid-connected inverter with IDA-PBC.

Table 2: Controller parameters.

<table>
<thead>
<tr>
<th>Symbols</th>
<th>Components</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r_1 )</td>
<td>Inner loop damping</td>
<td>2</td>
</tr>
<tr>
<td>( r_2 )</td>
<td>Middle loop damping</td>
<td>0.15</td>
</tr>
<tr>
<td>( r_3 )</td>
<td>Loop damping</td>
<td>15</td>
</tr>
</tbody>
</table>

Figure 6: Simulated results when grid impedance varies with proposed IDA-PBC strategy.
Simulations were conducted with selected system parameter values from Table 1 and controller parameters from Table 2 in order to evaluate the effectiveness of the proposed strategy.

5.1. Under Balanced Grid Voltage. Figure 6 depicts the waveforms results of system when implementing the proposed IDA-PBC strategy under the steady state. The grid impedance is zero for $0 < t < 0.06s$, and $L_g$ is set as 3.6mH for $0.06s < t < 0.12s$, and then, $L_g = 3.6mH$ and $C_g = 4uF$ for $0.12s < t < 0.18s$. The inductor and the capacitor are added in the grid at 0.6s and 0.12s. The system return to the normal state after one period. The injected currents are stable and clearly sinusoidal in three periods. Figure 7 depicts the results of the system when grid impedance varies with traditional IDA-PBC strategy. It can be seen that traditional IDA-PBC strategy can be stable in strong grid and inductive grid, but it is unstable under some capacitive grid impedance cases. The

![Figure 7: Simulated results when grid impedance varies with traditional IDA-PBC strategy.](image)

![Figure 8: Simulated results of transient responses of grid currents under balanced grid when (a) $L_g = 0$ mH and $C_g = 0$ uF, (b) $L_g = 4.8$ mH and $C_g = 0$ uF, and (c) $L_g = 4.8$ mH and $C_g = 3$ uF.](image)
The proposed IDA-PBC strategy is more robust under complex grid impedance condition. The performance of the proposed method during a transient response is also tested under strong grid, inductive grid, and complex grid, as illustrated in Figure 8. In this analysis, the peak value of the reference current is reduced from 12.86 A to 6.28 A at 0.01 s. In the presence of a strong grid, the currents can track the reference value immediately, and even when the grid is weak or capacitive, the response time is also less than 5 ms.

5.2. Under Unbalanced Grid Voltage. Caused by the grid faults or sudden load changes, the voltage dips are probably happen and then bring challenges to the control of the grid. Figure 9 shows the results of injected currents when $v_{ga}$ and $v_{gb}$ drop by 36% and 18%, with $v_{ga} = 70$ (RMS) and $v_{gb} = 90$ (RMS), respectively. It is apparent that grid currents are still sinusoidal and balanced with the tracked reference value under unbalanced grid.

5.3. Grid Impedance Variation in a Wide Range. Figure 10 illustrates a simulation of grid currents to demonstrate the robustness of the proposed control algorithm under a wide range of grid impedance variations when $C_g$ is fixed as 7 uF and $L_g$ varies from 0 to 10 mH.

Figure 11 depicts the corresponding results of the grid currents when $L_g$ is set as 3 mH and $C_g$ varies from 0 uF to 15 uF. It is clear that the system can inject high quality and stable currents both in inductive and capacitive grid in a wide range with the selected parameters for the IDA-PBC method.

Figure 9: Simulated results of grid voltages and grid currents when grid impedance varies under unbalanced grid.

Figure 10: Simulated results of grid currents when $C_g = 7$ uF; $L_g$ varies from 0 mH to 10 mH.

Figure 11: Simulated results of grid currents when $L_g = 3$ mH; $C_g$ varies from 0 uF to 15 uF.

Figure 12: Experimental results of grid voltages and currents with $L_g = 3.6$ mH and $C_g = 0$ uF.
6. Experimental Validation

To further validate the effectiveness of the IDA-PBC for LCL-filtered inverter with selected parameters, an experimental setup of 3-KW/110-V/three-phase grid inverter was built to test. The dc-link source to the Danfoss FC302 inverter is provided by Chroma dc power supply. The power grid is simulated by a programmable ac source, and a dSPACE DS1007 platform is utilized to implement the control algorithm. Except the system parameters listed in Table 1, the selected parameters of $r_1$, $r_2$, and $r_3$ of the IDA-PBC controller are 2, 0.15, and 15, respectively. Based on the simulation results, it is clear that the system is capable of performing optimally in a balanced grid. The experimental results of the proposed controller are displayed here under the unbalanced voltage condition. The voltage of phase A is emulated by a 36% dips with $v_{ga} = 70V$ (RMS).

A study of experimental grid currents and voltages under the steady state is presented in Figures 12 and 13. The grid currents can be controlled to be nearly sinusoidal with the proposed method in both inductive and capacitive grids, even though the grid voltages are unbalanced and distorted. To investigate the performance of transient responses with proposed IDA-PBC and selected control parameters, the experimental results of grid currents when the reference current increases under different grid impedances are demonstrated in Figure 14, 15, and 16. In all cases, the injected grid currents are stable and sinusoidal.

The grid current can rapidly track the reference without the settling time under the ideal strong grid. Even with the inductive and capacitive unbalanced grid, the injected currents can still track the reference current and the response time is less than 3 ms.

7. Conclusion

A modified IDA-PBC strategy and design method for calculating damping gains based on an impedance model are presented in this study. Simulations and experimental tests have been used to determine the effectiveness of the proposed controller, from which the following conclusion can be drawn.

1. The stability of IDA-PBC with the assignment of the desired energy function to the closed-loop system is
coinciding to the passivity of the closed-loop system output admittance. It is a good feature to allow this control method to be widely used in weak grid. The optional range of injected damping parameters can be selected based on the principle to satisfy the output impedance is passive.

(2) The design procedure of IDA-PBC considers the effect of control delay and a more flexible interconnection matrix design method is proposed. With the proposed IDA-PBC strategy, the inverter system is passive within the switching frequency, which allows high quality current to be injected to the grid, regardless of large variations in grid impedance and unbalanced grid voltages.

Abbreviations

IDA: Interconnection and damping assignment
PBC: Passivity-based control
PCH: Port-controller Hamiltonian
EL: Euler–Lagrange
EL-PBC: PBC based on the Euler–Lagrange (EL) model
IDA-PBC: PBC based on the port-controller Hamiltonian
model
PCC: Point-of-common coupling
Udc: Dc-link voltage
L1: Inverter-side inductor
R1: The parasitic resistances of L1
L2: Grid-side inductor
R2: The parasitic resistances of L2
C: Filter capacitor
vpcc: The PCC joint voltage
i1: Inverter-side current
i2: Grid-side current
uC: Capacitor voltage
PCC: Point of common coupling
Lg and Cg: Grid reactance
J(x): Interconnection matrix
R(x): Dissipation matrix
H(x): Hamiltonian function
u: The exchange of energy between the outside world and the system
g(x): Coefficient matrix stands for the interconnections
J0(x): Interconnection matrix
R0(x): Damping matrix
H0(x): Hamiltonian energy storage function
x*: Balance point
K(x): The gradient of a scalar function
H0(x): Closed-loop energy function
J0**: Positive definite symmetric matrix based on structure preservation
R0: Active damping matrix
T: The sampling period
Gd(s): Time delay
ψ and φ: The coupling components of d-q axis
Gc0: The system closed-loop transfer function
Yo: The inverter output admittance
r1: The inner loop gain
r2: The middle loop gain
r3: The outer loop gain.

Data Availability

The authors confirm that the data supporting the findings of this study are available within the article.

Conflicts of Interest

The authors have no conflicts of interest to declare.

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References


