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Research Article

Robust Stability Analysis for Uncertain Switched Discrete-Time Systems

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This paper is concerned with the robust stability for a class of switched discrete-time systems with state parameter uncertainty. Firstly, a new matrix inequality considering uncertainties is introduced and proved. By means of it, a novel sufficient condition for robust stability of a class of uncertain switched discrete-time systems is presented. Furthermore, based on the result obtained, the switching law is designed and has been performed well, and some sufficient conditions of robust stability have been derived for the uncertain switched discrete-time systems using the Lyapunov stability theorem, block matrix method and inequality technology. Finally, some examples are exploited to illustrate the effectiveness of the proposed schemes.

1. Introduction

A switched system is a hybrid dynamical system consisting of a finite number of subsystems and a logical rule that manages switching between these subsystems. Switched systems have drawn a great deal of attention in recent years, see [1–24] and references therein. The motivation for studying switched systems comes partly from the fact that switched systems and switched multi-controller systems have numerous applications in control of mechanical systems, process control, automotive industry, power systems, aircraft and traffic control, and many other fields. An important qualitative property of switched system is stability [1–3]. The challenge of analyzing the stability of switched system lies partly in the fact that, even if the individual systems are stable, the switched system might be unstable. Using a common quadratic Lyapunov function on all subsystems, the quadratic Lyapunov stability facilitates the analysis and synthesis of switched systems. However, the obtained results within this framework have been recognized to be conservative. In [10], various algorithms both for stability and performance analysis of discretetime piece-wise affine systems were presented. Different classes of Lyapunov functions were considered, and how

to compute them through linear matrix inequalities was also shown. Moreover, the tradeoff between the degree of conservativeness and computational requirements was discussed. The problem of stability analysis and control synthesis of switched systems in the discrete-time domain was addressed in [11]. The approach followed in [11] looked at the existence of a switched quadratic Lyapunov function to check asymptotic stability of the switched system under consideration. Two different linear matrix inequality-based conditions allow to check the existence of such a Lyapunov function. These two conditions have been proved to be equivalent for stability analysis.

There are many methodologies and approaches developed in the switched systems theory: approaches of looking for an appropriate switching strategy to stabilize the system [4], dwell-time and average dwell-time approaches for stability analysis and stabilization problems [5], approaches of studying stability and control problems under a specific class of switching laws [1] or under arbitrary switching sequences [6–9]. Reference [19] investigated the quadratic stability and linear state feedback and output feedback stabilization of switched delayed linear dynamic systems with, in general, a finite number of non-commensurate constant internal point delays. The results were obtained based on Lyapunov's stability analysis via appropriate Krasovskii-Lyapunov's functionals, and the related stability study was performed to obtain both delay-independent and delay-dependent results. The problem of fault estimation for a class of switched nonlinear systems of neutral type was considered in [20]. Sufficient delay-dependent existence conditions of the H_∞ fault estimator were given in terms of certain matrix inequalities based on the average dwell-time approach. The problem of robust reliable control for a class of uncertain switched neutral systems under asynchronous switching was investigated in [21]. A state feedback controller was proposed to guarantee exponential stability and reliability for switched neutral systems, and the dwell-time approach was utilized for the stability analysis and controller design. The exponential stability for a class of nonlinear hybrid time-delay systems was addressed in [24]. The delay-dependent stability conditions were presented in terms of the solution of algebraic Riccati equations, which allows computing simultaneously the two bounds that characterize the stability rate of the solution.

On another research front line, it has been recognized that parameter uncertainties, which often occur in many physical processes, are main sources of instability and poor performance. Therefore, much attention has been devoted to the study of various systems with uncertainties, and a great number of useful results have been reported in the literature on the issues of robust stability, robust H_∞ control, robust H_∞ filtering, and so on, by considering different classes of parameter uncertainties [12–14].

Recently, some stability condition and stabilization approaches have been proposed for the switched discrete-time system [15–18]. In [15], the quadratic stabilization of discrete-time switched linear systems was studied, and quadratic stabilization of switched systems with norm bounded time varying uncertainties was investigated. In [16], the stability property for the switched systems which were composed of a continuous-time LTI subsystem and a discretetime LTI subsystem was studied. There existed a switched quadratic Lyapunov function to check asymptotic stability of the switched discrete-time system in [17].

The objective of this paper is to present novel approaches for the asymptotical stability of switched discrete-time system with parametric uncertainties. The parameter uncertainties are time-varying but norm-bounded. Firstly, a new inequality is given. Using the new result, a new sufficient condition for robust stability of a class of uncertain switched discrete-time systems is proposed. Furthermore, using the block matrix method, inequality technology, and the Lyapunov stability theorem, some sufficient conditions for robust stability have been

presented for the uncertain switched discrete-time systems, and the switched law design has been performed. Comparing with [22, 23], the uncertainty in system was not considered in [22, 23], but we consider the uncertainty in systems and the design switching law is simple and easy for application.

The rest of this paper is organized as follows. The problem is formulated in Section 2. Section 3 deals with robust stability criteria for a class of discrete-time switched system with uncertainty. Numerical examples are provided to illustrate the theoretical results in Section 4, and the conclusions are drawn in Section 5.

Notation 1. The notation used in this paper is fairly standard. The superscript "T" stands for matrix transposition; R^n denotes the n-dimensional Euclidean space. $\mathrm{diag}\{\cdots\}$ stands for a block-diagonal matrix. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations. A symmetric matrix $P > 0$ (≥ 0) means P is positive (semipositive) definite. $\|\cdot\|$ stands for the Euclidean vector norm of the vector. $\lambda(A)$ stands for the eigenvalues of matrix A. $\|A\|$ denotes the norm of matrix A, that is, $\|A\| = \mathrm{Max}[\lambda(A^T A)]^{1/2}$. I and 0 represent, respectively, identity matrix and zero matrix.

2. Systems Description and Problem Statement

Consider a class of uncertain switched discrete-time systems given by

$$x(k+1) = \left(A_{\sigma(x(k))} + \Delta A_{\sigma(x(k))}\right)x(k), \quad x(0) = x_0, \tag{2.1}$$

where $x(k) \in R^n$ is the state, $A_{\sigma(x(k))} \in R^n$, x_0 is the initial state, $\sigma(x(k)) : R^n \to \{1, 2, \ldots, N\}, N \geq 2$, is a piecewise constant scalar function, called a switch signal, and N is the number of the individual systems, that is, the matrix $A_{\sigma(x(k))}$ switched between matrices $A_1, A_2, \ldots, A_N$ belonging to the set $A = \{A_1, A_2, \ldots, A_N\}$. $\Delta A_{\sigma(x(k))}$ denotes the parameter uncertainty and is assumed to be in the following form:

$$\Delta A_{\sigma(x(k))} = F_{\sigma(x(k))}(k) E_{\sigma(x(k))}, \tag{2.2}$$

where $E_{\sigma(x(k))}$ is real constant matrices of appropriate dimensions and $F_{\sigma(x(k))}$ is unknown matrix, satisfying $F^T_{\sigma(x(k))} F_{\sigma(x(k))} \leq I$. The switched discrete-time system (2.1) can be described as follows:

$$x(k+1) = (A_i + \Delta A_i)x(k), \quad i = 1, 2, \ldots, N, \tag{2.3}$$

where $\Delta A_i = F_i(k) E_i$ with $F_i^T F_i \leq I$.

For the stability of switched discrete-time system (2.3), some helpful lemmas are given in the following.

Lemma 2.1. *For any matrices $A_1, A_2, \ldots, A_N$ with the same dimensions and $\Delta A_i, i = 1, 2, \ldots, N$, which are given by* (2.3)*, the following inequality holds for any positive constant c:*

$$\begin{aligned}
&\left(\sum_{i=1}^{N}(A_i + \Delta A_i)\right)^T \left(\sum_{i=1}^{N}(A_i + \Delta A_i)\right) \\
&\quad \le (1+c)\left\{\sum_{i=1}^{N-1}\left(1+c^{-1}\right)^{i-1}\overline{A}_i^T\overline{A}_i\right\} + \left(1+c^{-1}\right)^{N-1}\overline{A}_N\overline{A}_N^T \\
&\quad \le (1+c)^2\left\{\sum_{i=1}^{N-1}\left(1+c^{-1}\right)^{i-1}A_i^T A_i\right\} + \left(1+c^{-1}\right)^{N-1}(1+c)A_N^T A_N \\
&\qquad + (1+c)\left\{\sum_{i=1}^{N-1}\left(1+c^{-1}\right)^{i}E_i^T E_i\right\} + \left(1+c^{-1}\right)^{N}E_N^T E_N,
\end{aligned} \tag{2.4}$$

where $\overline{A}_i = A_i + \Delta A_i,\ i \in \{1, 2, \ldots, N\}$.

Proof. For a positive constant c, A and B with the same dimensions, it is an obvious fact that

$$\begin{aligned}
(A+B)^T(A+B) &= A^T A + B^T B + A^T B + B^T A \\
&\le A^T A + B^T B + cA^T A + c^{-1}B^T B \\
&= (1+c)A^T A + \left(1+c^{-1}\right)B^T B.
\end{aligned} \tag{2.5}$$

So, we have

$$\begin{aligned}
(A_i + \Delta A_i)^T(A_i + \Delta A_i) &\le (1+c)A_i^T A_i + \left(1+c^{-1}\right)\Delta A_i^T \Delta A_i \\
&= (1+c)A_i^T A_i + \left(1+c^{-1}\right)(F_i(k)E_i)^T(F_i(k)E_i) \\
&= (1+c)A_i^T A_i + \left(1+c^{-1}\right)E_i^T F_i(k)^T(F_i(k)E_i) \\
&\le (1+c)A_i^T A_i + \left(1+c^{-1}\right)E_i^T E_i.
\end{aligned} \tag{2.6}$$

Using (2.5) and (2.6), we have

$$
\begin{aligned}
&\left(\sum_{i=1}^{N}(A_i+\Delta A_i)\right)^T\left(\sum_{i=1}^{N}(A_i+\Delta A_i)\right)\\
&\quad=\left(\sum_{i=1}^{N}\overline{A}_i\right)^T\left(\sum_{i=1}^{N}\overline{A}_i\right)\\
&\quad=\left(\overline{A}_1+\sum_{i=2}^{N}\overline{A}_i\right)^T\left(\overline{A}_1+\sum_{i=2}^{N}\overline{A}_i\right)\le(1+c)\overline{A}_1\overline{A}_1^T+\left(1+c^{-1}\right)\left(\sum_{i=2}^{N}\overline{A}_i\right)^T\left(\sum_{i=2}^{N}\overline{A}_i\right)\\
&\quad\le(1+c)\overline{A}_1\overline{A}_1^T+\left(1+c^{-1}\right)\left\{(1+c)\overline{A}_2\overline{A}_2^T+\left(1+c^{-1}\right)\left(\sum_{i=3}^{N}\overline{A}_i\right)^T\left(\sum_{i=3}^{N}\overline{A}_i\right)\right\}\\
&\quad\le(1+c)\overline{A}_1\overline{A}_1^T+\left(1+c^{-1}\right)(1+c)\overline{A}_2\overline{A}_2^T+\left(1+c^{-1}\right)^2\\
&\qquad\times\left\{(1+c)\overline{A}_3\overline{A}_3^T+\left(1+c^{-1}\right)\left(\sum_{i=4}^{N}\overline{A}_i\right)^T\left(\sum_{i=4}^{N}\overline{A}_i\right)\right\}\\
&\quad\le(1+c)\overline{A}_1\overline{A}_1^T+\left(1+c^{-1}\right)(1+c)\overline{A}_2\overline{A}_2^T+\left(1+c^{-1}\right)^2(1+c)\overline{A}_3\overline{A}_3^T\\
&\qquad+\cdots+\left(1+c^{-1}\right)^{N-2}(1+c)\overline{A}_{N-1}\overline{A}_{N-1}^T+\left(1+c^{-1}\right)^{N-1}\overline{A}_N\overline{A}_N^T\\
&\quad=(1+c)\left\{\sum_{i=1}^{N-1}\left(1+c^{-1}\right)^{i-1}\overline{A}_i^T\overline{A}_i\right\}+\left(1+c^{-1}\right)^{N-1}\overline{A}_N\overline{A}_N^T\\
&\quad\le(1+c)\left\{\sum_{i=1}^{N-1}\left(1+c^{-1}\right)^{i-1}\left((1+c)A_i^TA_i+\left(1+c^{-1}\right)E_i^TE_i\right)\right\}\\
&\qquad+\left(1+c^{-1}\right)^{N-1}\left((1+c)A_N^TA_N+\left(1+c^{-1}\right)E_N^TE_N\right)\\
&\quad=(1+c)^2\left\{\sum_{i=1}^{N-1}\left(1+c^{-1}\right)^{i-1}A_i^TA_i\right\}+\left(1+c^{-1}\right)^{N-1}(1+c)A_N^TA_N\\
&\qquad+(1+c)\left\{\sum_{i=1}^{N-1}\left(1+c^{-1}\right)^{i}E_i^TE_i\right\}+\left(1+c^{-1}\right)^{N}E_N^TE_N.
\end{aligned}
\tag{2.7}
$$

This completes the proof of Lemma 2.1. □

Set

$$
\beta_i=\frac{1}{(1+c)\left(1+c^{-1}\right)^{i-1}},\quad i=1,2,\ldots,N-1,\ \beta_N=\frac{1}{\left(1+c^{-1}\right)^{N-1}}. \tag{2.8}
$$

We have

$$\sum_{i=1}^{N}\beta_i = \sum_{i=1}^{N-1}\frac{1}{(1+c)\left(1+c^{-1}\right)^{i-1}} + \frac{1}{\left(1+c^{-1}\right)^{N-1}}$$
$$\frac{1/(1+c)\ \ -1/\left((1+c)\left(1+c^{-1}\right)^{N-1}\right)}{1-1/\left(1+c^{-1}\right)} + \frac{1}{\left(1+c^{-1}\right)^{N-1}} = 1. \tag{2.9}$$

Considering the following system (2.10):

$$x(k+1) = \sum_{i=1}^{N}\beta_i(A_i+\Delta A_i)x(k), \tag{2.10}$$

where $\beta_i, i = 1, 2, \ldots, N$, is given by (2.8), we have the following result.

Lemma 2.2. *If there exist $c > 0$ and a symmetric matrix $P > 0$ such that the following inequality holds:*

$$\sum_{i=1}^{n}\beta_i\left[(1+c)A_i^T P A_i + \left(1+c^{-1}\right)\|P\|E_i^T E_i - P\right] < 0, \tag{2.11}$$

then system (2.10) *is asymptotically stable, and there exists a switching law for the uncertain switched discrete-time system* (2.3) *such that the system* (2.3) *is asymptotically stable.*

Proof. We choose the Lyapunov function candidate as

$$V(k) = x^T(k)Px(k). \tag{2.12}$$

Since P is a symmetric positive-definite matrix, there exists a matrix Q such that $P = Q^T Q$.

We have

$$\begin{aligned}
\Delta V(k)|_{(2.10)} &= V(k+1) - V(k) \\
&= x^T(k+1)Px(k+1) - x^T(k)Px(k) \\
&= \left(\sum_{i=1}^{N}\beta_i(A_i+\Delta A_i)x(k)\right)^T P\left(\sum_{i=1}^{N}\beta_i(A_i+\Delta A_i)x(k)\right) - x^T(k)Px(k) \\
&= x^T(k)\left(\sum_{i=1}^{N}\beta_i(A_i+\Delta A_i)\right)^T P\left(\sum_{i=1}^{N}\beta_i(A_i+\Delta A_i)\right)x(k) - x^T(k)Px(k)
\end{aligned}$$

$$
\begin{aligned}
&= x^T(k)\left(\sum_{i=1}^{N}\beta_i Q(A_i+\Delta A_i)\right)^T\left(\sum_{i=1}^{N}\beta_i Q(A_i+\Delta A_i)\right)x(k) - x^T(k)Px(k)\\
&\le x^T(k)\left\{(1+c)\sum_{i=1}^{N-1}\left(1+c^{-1}\right)^{i-1}\left(\beta_i Q(A_i+\Delta A_i)\right)^T\left(\beta_i Q(A_i+\Delta A_i)\right)+\left(1+c^{-1}\right)^{N-1}\right.\\
&\qquad\left.\times\left(\beta_N Q(A_N+\Delta A_N)\right)^T\left(\beta_N Q(A_N+\Delta A_N)\right)\right\}x(k) - x^T(k)Px(k)\\
&\le x^T(k)\left\{(1+c)\sum_{i=1}^{N-1}\left(1+c^{-1}\right)^{i-1}\left[(1+c){\beta_i}^2 A_i^T P A_i+\left(1+c^{-1}\right){\beta_i}^2\Delta A_i^T P\Delta A_i\right]+\left(1+c^{-1}\right)^{N-1}\right.\\
&\qquad\left.\times\left[(1+c)\beta_N^2 A_N^T P A_N+\left(1+c^{-1}\right)\beta_N^2\Delta A_N^T P\Delta A_N\right]\right\}x(k) - x^T(k)Px(k)\\
&\le x^T(k)\left\{(1+c)\sum_{i=1}^{N-1}\beta_i A_i^T P A_i+\sum_{i=1}^{N-1}\left(1+c^{-1}\right)\beta_i\Delta A_i^T P\Delta A_i+(1+c)\beta_N A_N^T P A_{Ni}\right.\\
&\qquad\left.+\left(1+c^{-1}\right)\beta_N\Delta A_N^T P\Delta A\right\}x(k) - x^T(k)Px(k)\\
&\le x^T(k)\left\{(1+c)\sum_{i=1}^{N-1}\beta_i {A_i}^T P A_i+\sum_{i=1}^{N-1}\left(1+c^{-1}\right)\beta_i\|P\|E_i^T E_i+(1+c)\beta_N A_N^T P A_N\right.\\
&\qquad\left.+\left(1+c^{-1}\right)\beta_N\|P\|E_N^T E_N\right\}x(k) - x^T(k)Px(k)\\
&= x^T(k)\left\{(1+c)\sum_{i=1}^{N}\beta_i A_i^T P A_i+\sum_{i=1}^{N}\left(1+c^{-1}\right)\beta_i\|P\|E_i^T E_i - P\right\}x(k)\\
&= x^T(k)\left\{\sum_{i=1}^{N}\beta_i\left[(1+c)A_i^T P A_i+\left(1+c^{-1}\right)\|P\|E_i^T E_i - P\right]\right\}x(k) < 0, \quad x(k)\ne 0.
\end{aligned}
\tag{2.13}
$$

Equation (2.11) implies that $\Delta V(k)|_{(2.10)} < 0, x(k)\ne 0$. Hence the system (2.10) is asymptotically stable.

For system (2.3), we have

$$
\begin{aligned}
\Delta V(k)|_{(2.3)} &= x^T(k+1)Px(k+1) - x^T(k)Px(k)\\
&= x^T(k)(A_i+\Delta A_i)^T P(A_i+\Delta A_i)x^T(k) - x^T(k)Px(k)\\
&= x^T(k)\left\{[Q(A_i+\Delta A_i)]^T[Q(A_i+\Delta A_i)] - P\right\}x^T(k)
\end{aligned}
$$

$$\begin{aligned}
&\leq x^T(k)\left[(1+c)A_i^T P A_i + \left(1+c^{-1}\right)\Delta A_i^T P \Delta A_i - P\right]x^T(k) \\
&\leq x^T(k)\left[(1+c)A_i^T P A_i + \left(1+c^{-1}\right)\|P\|E_i^T E_i - P\right]x^T(k).
\end{aligned} \tag{2.14}$$

From (2.11), we know that, for any k, at least there exists an $i \in \{1,2,\ldots,N\}$such that

$$\Delta V(k)|_{(2.3)} \leq x^T(k)\left[(1+c)A_i^T P A_i + \left(1+c^{-1}\right)\|P\|E_i^T E_i - P\right]x^T(k) < 0, \quad x(k) \neq 0. \tag{2.15}$$

The switching signal is given as follows.

(1) If the following inequality holds:

$$x^T(k)\left[(1+c)A_i^T P A_i + \left(1+c^{-1}\right)\|P\|E_i{}^T E_i - P\right]x^T(k) < 0, \tag{2.16}$$

then

$$\sigma(x(k)) = i. \tag{2.17}$$

(2) If $x(k) = 0$, then

$$\sigma(x(k)) = \gamma, \quad \gamma \in \{1,2,\ldots,N\}. \tag{2.18}$$

Thus, the switched discrete-time system (2.3) is asymptotically stable. This completes the proof of Lemma 2.2. □

3. Stability Analysis of Uncertain Switched Discrete-Time System

Consider the asymptotical stability of the following system:

$$x(k+1) = (A_i + \Delta A_i)x(k), \quad i = 1,2, \tag{3.1}$$

where

$$A_i = \begin{bmatrix} A_{i11} & A_{i12} \\ A_{i21} & A_{i22} \end{bmatrix} \in R^{n\times n}, \quad A_{i11} \in R^{n_1\times n_1},\ A_{i12} \in R^{n_2\times n_2}. \tag{3.2}$$

Remark 3.1. The motivation of considering (3.1) is that when the diagonal blocks satisfy Lyapunov matrix inequalities (3.9), system (3.1) has some good property.

According to Lemma 2.2, the study of asymptotical stability for (3.1) can be transformed into the study for (3.3):

$$x(k+1) = \left[\beta(A_1 + \Delta A_1) + (1-\beta)(A_2 + \Delta A_2)\right]x(k), \tag{3.3}$$

where $\beta = 1/(1+c)$.

We choose the Lyapunov function candidate as

$$V(k) = x^T(k)Px(k), \tag{3.4}$$

where $x = [x_1^T, x_2^T]^T, x_1 \in R^{n_1}, x_2 \in R^{n_2}$, and $P = \operatorname{diag}\{P_1, P_2\}$, $P_1 \in R^{n_1\times n_1}, P_2 \in R^{n_2\times n_2}$, are real symmetric positive-definite matrices.

Computing the product, we have

$$\begin{aligned} A_i^T P A_i &= \begin{bmatrix} A_{i11}^T & A_{i21}^T \\ A_{i12}^T & A_{i22}^T \end{bmatrix} \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix} \begin{bmatrix} A_{i11} & A_{i12} \\ A_{i21} & A_{i22} \end{bmatrix} \\ &= \begin{bmatrix} A_{i11}^T P_1 A_{i11} + A_{i21}^T P_2 A_{i21} & A_{i11}^T P_1 A_{i12} + A_{i21}^T P_2 A_{i22} \\ A_{i12}^T P_1 A_{i11} + A_{i22}^T P_2 A_{i21} & A_{i12}^T P_1 A_{i12} + A_{i22}^T P_2 A_{i22} \end{bmatrix}. \end{aligned} \tag{3.5}$$

So,

$$\begin{aligned} x^T A_i^T P A_i x &= \begin{bmatrix} x_1^T & x_2^T \end{bmatrix} \begin{bmatrix} A_{i11}^T P_1 A_{i11} + A_{i21}^T P_2 A_{i21} & A_{i11}^T P_1 A_{i12} + A_{i21}^T P_2 A_{i22} \\ A_{i12}^T P_1 A_{i11} + A_{i22}^T P_2 A_{i21} & A_{i12}^T P_1 A_{i12} + A_{i22}^T P_2 A_{i22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \\ &= \left[x_1^T\left(A_{i11}^T P_1 A_{i11} + A_{i21}^T P_2 A_{i21}\right) + x_2^T\left(A_{i12}^T P_1 A_{i11} + A_{i22}^T P_2 A_{i21}\right)\right]x_1 \\ &\quad + \left[x_1^T\left(A_{i11}^T P_1 A_{i12} + A_{i21}^T P_2 A_{i22}\right) + x_2^T\left(A_{i12}^T P_1 A_{i12} + A_{i22}^T P_2 A_{i22}\right)\right]x_2 \\ &= x_1^T\left(A_{i11}^T P_1 A_{i11} + A_{i21}^T P_2 A_{i21}\right)x_1 + x_2^T\left(A_{i12}^T P_1 A_{i11} + A_{i22}^T P_2 A_{i21}\right)x_1 \\ &\quad + x_1^T\left(A_{i11}^T P_1 A_{i12} + A_{i21}^T P_2 A_{i22}\right)x_2 + x_2^T\left(A_{i12}^T P_1 A_{i12} + A_{i22}^T P_2 A_{i22}\right)x_2. \end{aligned} \tag{3.6}$$

Using the properties of matrix norm, we have

$$\begin{aligned} &x^T A_i^T P A_i x \\ &\quad \le x_1^T A_{i11}^T P_1 A_{i11} x_1 + \|x_1\|^2 \|A_{i21}\|^2 \|P_2\| + 2\|x_1\|\|x_2\|(\|A_{i11}\|\|P_1\|\|A_{i12}\| + \|A_{i21}\|\|P_2\|\|A_{i22}\|) \\ &\qquad + \|x_2\|^2 \|P_1\|\|A_{i12}\|^2 + x_2^T A_{i22}^T P_2 A_{i22} x_2 \end{aligned}$$

$$
\begin{aligned}
&\le x_1^T A_{i11}^T P_1 A_{i11} x_1 + x_2^T A_{i22}^T P_2 A_{i22} x_2 + \left(\|x_1\|^2 + \|x_2\|^2\right) \\
&\quad \times (\|A_{i11}\|\|P_1\|\|A_{i12}\| + \|A_{i21}\|\|P_2\|\|A_{i22}\|) + \|x_1\|^2\|A_{i21}\|^2\|P_2\| + \|x_2\|^2\|P_1\|\|A_{i12}\|^2 \\
&= x_1^T A_{i11}^T P_1 A_{i11} x_1 + \|x_1\|^2\left(\|A_{i11}\|\|P_1\|\|A_{i12}\| + \|A_{i21}\|\|P_2\|\|A_{i22}\| + \|A_{i21}\|^2\|P_2\|\right) \\
&\quad + x_2^T A_{i22}^T P_2 A_{i22} x_2 + \|x_2\|^2\left(\|A_{i11}\|\|P_1\|\|A_{i12}\| + \|A_{i21}\|\|P_2\|\|A_{i22}\| + \|P_1\|\|A_{i12}\|^2\right).
\end{aligned} \tag{3.7}
$$

It will be convenient throughout this section to use the following notations:

$$
\begin{aligned}
e_1 &= \|A_{111}\|\|P_1\|\|A_{112}\| + \|A_{121}\|\|P_2\|\|A_{122}\| + \|A_{121}\|^2\|P_2\|, \\
e_2 &= \|A_{211}\|\|P_1\|\|A_{212}\| + \|A_{221}\|\|P_2\|\|A_{222}\| + \|A_{221}\|^2\|P_2\|, \\
d_1 &= \|A_{111}\|\|P_1\|\|A_{112}\| + \|A_{121}\|\|P_2\|\|A_{122}\| + \|P_1\|\|A_{112}\|^2, \\
d_2 &= \|A_{211}\|\|P_1\|\|A_{212}\| + \|A_{221}\|\|P_2\|\|A_{222}\| + \|P_1\|\|A_{212}\|^2.
\end{aligned} \tag{3.8}
$$

Theorem 3.2. *There exists a switched law such that the discrete-time system* (3.1) *is asymptotically stable, if there exist symmetric positive-definite matrices* $P_1, P_2 > 0$ *and positive constants* $c, \lambda_{ij}, i, j = 1, 2$, *such that the following inequalities hold:*

$$
A_{ijj}^T P_j A_{ijj} - P_j \le -\lambda_{ij} I, \quad i, j = 1, 2, \tag{3.9}
$$

$$
\begin{aligned}
e_1 + ce_2 - \lambda_{11} - c\lambda_{21} + c\lambda_{\max}(P_1) + c^{-1}\|P\|\left\|E_1^T E_1\right\| + \|P\|\left\|E_2^T E_2\right\| < 0, \\
d_1 + cd_2 - \lambda_{12} - c\lambda_{22} + c\lambda_{\max}(P_2) + c^{-1}\|P\|\left\|E_1^T E_1\right\| + \|P\|\left\|E_2^T E_2\right\| < 0.
\end{aligned} \tag{3.10}
$$

Proof. By Lemma 2.1, the difference of the Lyapunov function (3.4) is as following

$$
\begin{aligned}
\Delta V(k) &\le x^T(k)\Big\{\beta\Big[(1+c)A_1^T P A_1 + \left(1 + c^{-1}\right)\|P\|E_1^T E_1\Big] \\
&\qquad + (1-\beta)\Big[(1+c)A_2^T P A_2 + \left(1 + c^{-1}\right)\|P\|E_2^T E_2\Big] - P\Big\}x(k) \\
&= x^T(k)\Big\{\Big[\beta(1+c){A_1}^T P A_1 + \beta\left(1 + c^{-1}\right)\|P\|E_1^T E_1\Big] \\
&\qquad + \Big[(1-\beta)(1+c)A_2^T P A_2 + (1-\beta)\left(1 + c^{-1}\right)\|P\|E_2^T E_2\Big] - P\Big\}x(k) \\
&= x^T(k)\Big\{\Big[A_1^T P A_1 + c^{-1}\|P\|E_1^T E_1\Big] + \Big[cA_2^T P A_2 + \|P\|E_2^T E_2\Big] - P\Big\}x(k).
\end{aligned} \tag{3.11}
$$

Using (3.7), it follows

$$
\begin{aligned}
\Delta V(k) \le{}& x_1^T A_{111}^T P_1 A_{111} x_1 + x_2^T A_{122}^T P_2 A_{122} x_2 \\
&+ \|x_1\|^2 \left(\|A_{111}\| \|P_1\| \|A_{112}\| + \|A_{121}\| \|P_2\| \|A_{122}\| + \|A_{121}\|^2 \|P_2\| \right) \\
&+ \|x_2\|^2 \left(\|A_{111}\| \|P_1\| \|A_{112}\| + \|A_{121}\| \|P_2\| \|A_{122}\| + \|P_1\| \|A_{112}\|^2 \right) \\
&+ c \Big\{ x_1^T A_{211}^T P_1 A_{211} x_1 + x_2^T A_{222}^T P_2 A_{222} x_2 \\
&\quad + \|x_1\|^2 \left(\|A_{211}\| \|P_1\| \|A_{212}\| + \|A_{221}\| \|P_2\| \|A_{222}\| + \|A_{221}\|^2 \|P_2\| \right) \\
&\quad + \|x_2\|^2 \left(\|A_{211}\| \|P_1\| \|A_{212}\| + \|A_{221}\| \|P_2\| \|A_{222}\| + \|P_1\| \|A_{212}\|^2 \right) \Big\} \\
&+ \|x\|^2 \left(c^{-1} \|P\| \left\| E_1^T E_1 \right\| + \|P\| \left\| E_2^T E_2 \right\| \right) - \left(x_1^T P_1 x_1 + x_2^T P_2 x_2 \right) \\
\le{}& -\lambda_{11} \|x_1\|^2 - \lambda_{12} \|x_2\|^2 + c x_1^T (-\lambda_{21} I + P_1) x_1 + c x_2^T (-\lambda_{22} I + P_2) x_2 \\
&+ \|x_1\|^2 \Big[\left(\|A_{111}\| \|P_1\| \|A_{112}\| + \|A_{121}\| \|P_2\| \|A_{122}\| + \|A_{121}\|^2 \|P_2\| \right) \\
&\qquad + c \left(\|A_{211}\| \|P_1\| \|A_{212}\| + \|A_{221}\| \|P_2\| \|A_{222}\| + \|A_{221}\|^2 \|P_2\| \right) \Big] \\
&+ \|x_2\|^2 \Big[\left(\|A_{111}\| \|P_1\| \|A_{112}\| + \|A_{121}\| \|P_2\| \|A_{122}\| + \|P_1\| \|A_{112}\|^2 \right) \\
&\qquad + c \left(\|A_{211}\| \|P_1\| \|A_{212}\| + \|A_{221}\| \|P_2\| \|A_{222}\| + \|P_1\| \|A_{212}\|^2 \right) \Big] \\
&+ \|x\|^2 \left(c^{-1} \|P\| \left\| E_1^T E_1 \right\| + \|P\| \left\| E_2^T E_2 \right\| \right) \\
\le{}& \|x_1\|^2 \Big[\left(\|A_{111}\| \|P_1\| \|A_{112}\| + \|A_{121}\| \|P_2\| \|A_{122}\| + \|A_{121}\|^2 \|P_2\| \right) \\
&\qquad + c \left(\|A_{211}\| \|P_1\| \|A_{212}\| + \|A_{221}\| \|P_2\| \|A_{222}\| + \|A_{221}\|^2 \|P_2\| \right) \\
&\qquad - \lambda_{11} - c\lambda_{21} + c\lambda_{\max}(P_1) + c^{-1} \|P\| \left\| E_1^T E_1 \right\| + \|P\| \left\| E_2^T E_2 \right\| \Big] \\
&+ \|x_2\|^2 \Big[\left(\|A_{111}\| \|P_1\| \|A_{112}\| + \|A_{121}\| \|P_2\| \|A_{122}\| + \|P_1\| \|A_{112}\|^2 \right) \\
&\qquad + c \left(\|A_{211}\| \|P_1\| \|A_{212}\| + \|A_{221}\| \|P_2\| \|A_{222}\| + \|P_1\| \|A_{212}\|^2 \right) \\
&\qquad - \lambda_{12} - c\lambda_{22} + c\lambda_{\max}(P_2) + c^{-1} \|P\| \left\| E_1^T E_1 \right\| + \|P\| \left\| E_2^T E_2 \right\| \Big] \\
\le{}& \|x_1\|^2 \Big[e_1 + c e_2 - \lambda_{11} - c\lambda_{21} + c\lambda_{\max}(P_1) + c^{-1} \|P\| \left\| E_1^T E_1 \right\| + \|P\| \left\| E_2^T E_2 \right\| \Big] \\
&+ \|x_2\|^2 \Big[d_1 + c d_2 - \lambda_{12} - c\lambda_{22} + c\lambda_{\max}(P_2) + c^{-1} \|P\| \left\| E_1^T E_1 \right\| + \|P\| \left\| E_2^T E_2 \right\| \Big].
\end{aligned}
\tag{3.12}
$$

From (3.10), we get that

$$\Delta V(k) < 0, \quad x(k) \neq 0. \tag{3.13}$$

According to Lemma 2.2, the discrete-time system (3.1) is asymptotically stable. Proof of Theorem 3.2 is completed. □

Consider the system (2.3) with

$$A_i = \begin{bmatrix} A_{i11} & A_{i12} \\ A_{i21} & A_{i22} \end{bmatrix} \in R^{n\times n}, \quad A_{i11} \in R^{n_1\times n_1},\ A_{i12} \in R^{n_2\times n_2}. \tag{3.14}$$

According to Lemma 2.2, we have the following theorem for the asymptotically stability of (2.3).

Theorem 3.3. *There exists a switched law such that the discrete-time system* (2.3) *is asymptotically stable, if there exist symmetric positive-definite matrices* $P_1, P_2 > 0, P_1 \in R^{n_1\times n_1}, P_2 \in R^{n_2\times n_2}$, *and positive constants* $c, \lambda_{ij}, i, j = 1, 2$, *satisfying the follow inequalities:*

$$A_{ijj}^T P_j A_{ijj} - \frac{1}{1+c} P_j \leq -\lambda_{ij} I, \quad i = 1, 2, \ldots N,\ j = 1, 2, \tag{3.15}$$

$$\sum_{i=1}^{N} \beta_i \left(\|A_{i11}\| \|P_1\| \|A_{i12}\| + \|A_{i21}\| \|P_2\| \|A_{i22}\| + \|A_{i21}\|^2 \|P_2\| - \lambda_{i1} + c^{-1} \|P\| \left\| E_i^T E_i \right\| \right) < 0,$$

$$\sum_{i=1}^{N} \beta_i \left(\|A_{i11}\| \|P_1\| \|A_{i12}\| + \|A_{i21}\| \|P_2\| \|A_{i22}\| + \|P_1\| \|A_{i12}\|^2 - \lambda_{i2} + c^{-1} \|P\| \left\| E_i^T E_i \right\| \right) < 0, \tag{3.16}$$

where $P = \operatorname{diag}\{P_1, P_2\}$, *and*

$$\beta_i = \frac{1}{(1+c)\left(1+c^{-1}\right)^{i-1}}, \quad i = 1, 2, \ldots, N-1,\ \beta_N = \frac{1}{\left(1+c^{-1}\right)^{N-1}}. \tag{3.17}$$

Proof. We choose the Lyapunov function candidate as

$$V(k) = x^T(k) P x(k), \tag{3.18}$$

where $x = [x_1^T, x_2^T]^T, x_1 \in R^{n_1}, x_2 \in R^{n_2}$ and $P = \operatorname{diag}\{P_1, P_2\}$ are real symmetric positive-definite matrices.

Using Lemma 2.1 and (3.15), we get

$$\begin{aligned}
\Delta V(k)|_{(2.10)} &\le x^T(k)\left\{\sum_{i=1}^{N}\beta_i\left[(1+c)A_i^T P A_i + \left(1+c^{-1}\right)\|P\|E_i^T E_i - P\right]\right\}x(k) \\
&\le \sum_{i=1}^{N}\beta_i(1+c)\Big[x_1^T A_{i11}^T P_1 A_{i11} x_1 + x_2^T A_{i22}^T P_2 A_{i22} x_2 \\
&\qquad + \|x_1\|^2\left(\|A_{i11}\|\|P_1\|\|A_{i12}\| + \|A_{i21}\|\|P_2\|\|A_{i22}\| + \|A_{i21}\|^2\|P_2\|\right) \\
&\qquad + \|x_2\|^2\left(\|A_{i11}\|\|P_1\|\|A_{i12}\| + \|A_{i21}\|\|P_2\|\|A_{i22}\| + \|P_1\|\|A_{i12}\|^2\right)\Big] \\
&\quad + \sum_{i=1}^{N}\beta_i\left[\|x\|^2\left(1+c^{-1}\right)\|P\|\left\|E_i{}^T E_i\right\| - x^T(k)Px(k)\right] \\
&\le \sum_{i=1}^{N}\beta_i(1+c)\left[x_1^T\left(A_{i11}^T P_1 A_{i11} - \frac{1}{(1+c)}P_1\right)x_1 + x_2^T\left(A_{i22}^T P_2 A_{i22} - \frac{1}{(1+c)}P_2\right)x_2\right. \\
&\qquad + \|x_1\|^2\left(\|A_{i11}\|\|P_1\|\|A_{i12}\| + \|A_{i21}\|\|P_2\|\|A_{i22}\| + \|A_{i21}\|^2\|P_2\|\right) \\
&\qquad + \|x_2\|^2\left(\|A_{i11}\|\|P_1\|\|A_{i12}\| + \|A_{i21}\|\|P_2\|\|A_{i22}\| + \|P_1\|\|A_{i12}\|^2\right) \\
&\qquad \left. + \|x\|^2\frac{1+c^{-1}}{1+c}\|P\|\left\|E_i^T E_i\right\|\right] \\
&\le \sum_{i=1}^{N}\beta_i(1+c)\left[\|x_1\|^2\left(\|A_{i11}\|\|P_1\|\|A_{i12}\| + \|A_{i21}\|\|P_2\|\|A_{i22}\| + \|A_{i21}\|^2\|P_2\| - \lambda_{i1}\right.\right. \\
&\qquad \left. + \frac{1+c^{-1}}{1+c}\|P\|\left\|E_i^T E_i\right\|\right) \\
&\qquad + \|x_2\|^2\Big(\|A_{i11}\|\|P_1\|\|A_{i12}\| + \|A_{i21}\|\|P_2\|\|A_{i22}\| \\
&\qquad \left. + \|P_1\|\|A_{i12}\|^2 - \lambda_{i2} + c^{-1}\|P\|\left\|E_i{}^T E_i\right\|\Big)\right] \\
&= \|x_1\|^2\sum_{i=1}^{N}\beta_i(1+c)\Big(\|A_{i11}\|\|P_1\|\|A_{i12}\| + \|A_{i21}\|\|P_2\|\|A_{i22}\| + \|A_{i21}\|^2\|P_2\| \\
&\qquad - \lambda_{i1} + c^{-1}\|P\|\left\|E_i^T E_i\right\|\Big) \\
&\quad + \|x_2\|^2\sum_{i=1}^{N}\beta_i(1+c)\Big(\|A_{i11}\|\|P_1\|\|A_{i12}\| + \|A_{i21}\|\|P_2\|\|A_{i22}\| + \|P_1\|\|A_{i12}\|^2 \\
&\qquad - \lambda_{i2} + c^{-1}\|P\|\left\|E_i^T E_i\right\|\Big).
\end{aligned} \tag{3.19}$$

From (3.16), we have $\Delta V(k) < 0,\ x(k) \neq 0$.

Switching Law. The switching law is given by (2.17) and (2.18). In the light of Lemma 2.2, the discrete-time system (2.3) is asymptotically stable. Proof of Theorem 3.3 is completed. □

Remark 3.4. Conditions (3.15) and (3.16) in Theorem 3.3 are nonlinear with respect to unknown $c > 0$. If c is a given positive constant, then (3.15) is a linear matrix inequality. In order to find the solution for (3.15) and (3.16), $c > 0$ is given in advance, then we solve (3.15) and (3.16).

Consider the system (2.3) with

$$A_i = \begin{bmatrix} A_{i11} & A_{i12} & A_{i13} \\ A_{i21} & A_{i22} & A_{i23} \\ A_{i31} & A_{i32} & A_{i33} \end{bmatrix} \in R^{n\times n}, \quad A_{i11} \in R^{n_1\times n_1},\ A_{i22} \in R^{n_2\times n_2},\ A_{i33} \in R^{n_3\times n_3}. \tag{3.20}$$

According to Lemma 2.2, we have the following theorem for the asymptotical stability of (2.3).

Theorem 3.5. *There exists a switched law such that the discrete-time system* (2.3) *is asymptotically stable, if there exist positive constants* $c, \eta_{ij}, i, j = 1, 2,$ *and symmetric positive-definite matrices* $P_1, P_2, P_3 > 0, P_1 \in R^{n_1\times n_1}, P_2 \in R^{n_2\times n_2}, P_3 \in R^{n_3\times n_3}$ *satisfying the following inequalities:*

$$A_{ijj}^T P_j A_{ijj} - \frac{1}{1+c} P_j \le -\eta_{ij} I, \quad i = 1, 2, \ldots N,\ j = 1, 2, 3, \tag{3.21}$$

$$\begin{gathered}
\sum_{j=2}^{3} \|A_{ij1}\|^2 \|P_j\| + \sum_{j=1}^{3} \|A_{ij1}\| \|A_{ij2}\| \|P_j\| + \sum_{j=1}^{3} \|A_{ij1}\| \|A_{ij3}\| \|P_j\| + c^{-1} \|P\| \left\| E_i^T E_i \right\| - \eta_{i1} < 0, \\
\sum_{\substack{j=1 \\ j\neq 2}}^{3} \|A_{ij2}\|^2 \|P_j\| + \sum_{j=1}^{3} \|A_{ij1}\| \|A_{ij2}\| \|P_j\| + \sum_{j=1}^{3} \|A_{ij3}\| \|A_{ij2}\| \|P_j\| + c^{-1} \|P\| \left\| E_i^T E_i \right\| - \eta_{i2} < 0, \\
\sum_{j=1}^{2} \|A_{ij3}\|^2 \|P_j\| + \sum_{j=1}^{3} \|A_{ij1}\| \|A_{ij3}\| \|P_j\| + \sum_{j=1}^{3} \|A_{ij3}\| \|A_{ij2}\| \|P_j\| + c^{-1} \|P\| \left\| E_i^T E_i \right\| - \eta_{i3} < 0,
\end{gathered} \tag{3.22}$$

where $P = \operatorname{diag}\{P_1, P_2, P_3\}$, *and*

$$\beta_i = \frac{1}{(1+c)\left(1+c^{-1}\right)^{i-1}}, \quad i = 1, 2, \ldots, N-1,\ \beta_N = \frac{1}{\left(1+c^{-1}\right)^{N-1}}. \tag{3.23}$$

Proof. We choose the Lyapunov function candidate as

$$V(k) = x^T(k) P x(k), \tag{3.24}$$

where $x = [x_1^T, x_2^T, x_3^T]^T$ and $P = \operatorname{diag}\{P_1, P_2, P_3\}$ are real symmetric positive-definite matrices.

Computing the product, we have

$$A_i^T P A_i = \begin{bmatrix} A_{i11}^T & A_{i21}^T & A_{i31}^T \\ A_{i12}^T & A_{i22}^T & A_{i32}^T \\ A_{i13}^T & A_{i23}^T & A_{i33}^T \end{bmatrix} \begin{bmatrix} P_1 & 0 & 0 \\ 0 & P_2 & 0 \\ 0 & 0 & P_3 \end{bmatrix} \begin{bmatrix} A_{i11} & A_{i12} & A_{i13} \\ A_{i21} & A_{i22} & A_{i23} \\ A_{i31} & A_{i32} & A_{i33} \end{bmatrix}$$

$$= \begin{bmatrix} \sum_{j=1}^{3} A_{ij1}^T P_j A_{ij1} & \sum_{j=1}^{3} A_{ij1}^T P_j A_{ij2} & \sum_{j=1}^{3} A_{ij1}^T P_j A_{ij3} \\ \sum_{j=1}^{3} A_{ij2}^T P_j A_{ij1} & \sum_{j=1}^{3} A_{ij2}^T P_j A_{ij2} & \sum_{j=1}^{3} A_{ij2}^T P_j A_{ij3} \\ \sum_{j=1}^{3} A_{ij3}^T P_j A_{ij1} & \sum_{j=1}^{3} A_{ij3}^T P_j A_{ij2} & \sum_{j=1}^{3} A_{ij3}^T P_j A_{ij3} \end{bmatrix}. \tag{3.25}$$

So, we get

$$x^T A_i^T P A_i x = \begin{bmatrix} x_1^T & x_2^T & x_3^T \end{bmatrix} \begin{bmatrix} \sum_{j=1}^{3} A_{ij1}^T P_j A_{ij1} & \sum_{j=1}^{3} A_{ij1}^T P_j A_{ij2} & \sum_{j=1}^{3} A_{ij1}^T P_j A_{ij3} \\ \sum_{j=1}^{3} A_{ij2}^T P_j A_{ij1} & \sum_{j=1}^{3} A_{ij2}^T P_j A_{ij2} & \sum_{j=1}^{3} A_{ij2}^T P_j A_{ij3} \\ \sum_{j=1}^{3} A_{ij3}^T P_j A_{ij1} & \sum_{j=1}^{3} A_{ij3}^T P_j A_{ij2} & \sum_{j=1}^{3} A_{ij3}^T P_j A_{ij3} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$= \left(x_1^T \sum_{j=1}^{3} A_{ij1}^T P_j A_{ij1} + x_2^T \sum_{j=1}^{3} A_{ij2}^T P_j A_{ij1} + x_3^T \sum_{j=1}^{3} A_{ij3}^T P_j A_{ij1} \right) x_1$$

$$+ \left(x_1^T \sum_{j=1}^{3} A_{ij1}^T P_j A_{ij2} + x_2^T \sum_{j=1}^{3} A_{ij2}^T P_j A_{ij2} + x_3^T \sum_{j=1}^{3} A_{ij3}^T P_j A_{ij2} \right) x_2$$

$$+ \left(x_1^T \sum_{j=1}^{3} A_{ij1}^T P_j A_{ij3} + x_2^T \sum_{j=1}^{3} A_{ij2}^T P_j A_{ij3} + x_3^T \sum_{j=1}^{3} A_{ij3}^T P_j A_{ij3} \right) x_3$$

$$\leq {x_1}^T A_{i11}^T P_1 A_{i11} x_1 + \|x_1\|^2 \sum_{j=2}^{3} \|A_{ij1}\|^2 \|P_j\| + 2\|x_1\|\|x_2\| \sum_{j=1}^{3} \|A_{ij1}\| \|A_{ij2}\| \|P_j\|$$

$$+ 2\|x_1\|\|x_3\| \sum_{j=1}^{3} \|A_{ij1}\| \|A_{ij3}\| \|P_j\| + x_2^T A_{i22}^T P_2 A_{i22} x_2 + \|x_2\|^2 \sum_{\substack{j=1 \\ j \neq 2}}^{3} \|A_{ij2}\|^2 \|P_j\|$$

$$+ 2\|x_3\|\|x_2\| \sum_{j=1}^{3} \|A_{ij3}\| \|A_{ij2}\| \|P_j\| + \|x_3\|^2 \sum_{\substack{j=1 \\ j \neq 3}}^{3} \|A_{ij3}\|^2 \|P_j\| + x_3^T A_{i33}^T P_3 A_{i33} x_3$$

$$\leq x_1^T A_{i11}^T P_1 A_{i11} x_1$$

$$
\begin{aligned}
&+\|x_1\|^2\left[\sum_{j=2}^{3}\|A_{ij1}\|^2\|P_j\|+\sum_{j=1}^{3}\|A_{ij1}\|\|A_{ij2}\|\|P_j\|+\sum_{j=1}^{3}\|A_{ij1}\|\|A_{ij3}\|\|P_j\|\right]\\
&+x_2^T A_{i22}^T P_2 A_{i22} x_2\\
&+\|x_2\|^2\left[\sum_{\substack{j=1\\ j\neq 2}}^{3}\|A_{ij2}\|^2\|P_j\|+\sum_{j=1}^{3}\|A_{ij1}\|\|A_{ij2}\|\|P_j\|+\sum_{j=1}^{3}\|A_{ij3}\|\|A_{ij2}\|\|P_j\|\right]\\
&+x_3^T A_{i33}^T P_3 A_{i33} x_3\\
&+\|x_3\|^2\left[\sum_{j=1}^{2}\|A_{ij3}\|^2\|P_j\|+\sum_{j=1}^{3}\|A_{ij1}\|\|A_{ij3}\|\|P_j\|+\sum_{j=1}^{3}\|A_{ij3}\|\|A_{ij2}\|\|P_j\|\right].
\end{aligned}
\tag{3.26}
$$

Using the properties of matrix norm, (3.24), and Lemma 2.2, we have

$$
\begin{aligned}
\Delta V(k)|_{(2.10)} &\leq x^T(k)\left\{\sum_{i=1}^{N}\beta_i\left[(1+c){A_i}^T P A_i+\left(1+c^{-1}\right)\|P\|{E_i}^T E_i-P\right]\right\}x(k)\\
&\leq \sum_{i=1}^{N}\beta_i(1+c)\left\{{x_1}^T A_{i11}^T P_1 A_{i11} x_1+\|x_1\|^2\right.\\
&\qquad\times\left[\sum_{j=2}^{3}\|A_{ij1}\|^2\|P_j\|+\sum_{j=1}^{3}\|A_{ij1}\|\|A_{ij2}\|\|P_j\|\right.\\
&\qquad\left.+\sum_{j=1}^{3}\|A_{ij1}\|\|A_{ij3}\|\|P_j\|\right]\\
&\qquad+{x_2}^T A_{i22}^T P_2 A_{i22} x_2+\|x_2\|^2\\
&\qquad\times\left[\sum_{\substack{j=1\\ j\neq 2}}^{3}\|A_{ij2}\|^2\|P_j\|+\sum_{j=1}^{3}\|A_{ij1}\|\|A_{ij2}\|\|P_j\|+\sum_{j=1}^{3}\|A_{ij3}\|\|A_{ij2}\|\|P_j\|\right]\\
&\qquad+{x_3}^T A_{i33}^T P_3 A_{i33} x_3+\|x_3\|^2
\end{aligned}
$$

$$\times \left[\sum_{j=1}^{3} \|A_{ij3}\|^2 \|P_j\| + \sum_{j=1}^{3} \|A_{ij1}\| \|A_{ij3}\| \|P_j\| + \sum_{j=1}^{3} \|A_{ij3}\| \|A_{ij2}\| \|P_j\| \right] \Bigg\}$$

$$+ x^T(k) \sum_{i=1}^{N} \beta_i \left[\left(1 + c^{-1}\right) \|P\| E_i^T E_i - P \right] x(k)$$

$$\leq \sum_{i=1}^{N} \beta_i (1 + c) \Bigg\{ \|x_1\|^2 \left[\sum_{j=2}^{3} \|A_{ij1}\|^2 \|P_j\| + \sum_{j=1}^{3} \|A_{ij1}\| \|A_{ij2}\| \|P_j\| + \sum_{j=1}^{3} \|A_{ij1}\| \|A_{ij3}\| \|P_j\| + c^{-1} \|P\| \left\| E_i^T E_i \right\| - \eta_{i1} \right]$$

$$+ \|x_2\|^2 \left[\sum_{\substack{j=1 \\ j \neq 2}}^{3} \|A_{ij2}\|^2 \|P_j\| + \sum_{j=1}^{3} \|A_{ij1}\| \|A_{ij2}\| \|P_j\| + \sum_{j=1}^{3} \|A_{ij3}\| \|A_{ij2}\| \|P_j\| + c^{-1} \|P\| \left\| E_i^T E_i \right\| - \eta_{i2} \right]$$

$$+ \|x_3\|^2 \left[\sum_{j=1}^{2} \|A_{ij3}\|^2 \|P_j\| + \sum_{j=1}^{3} \|A_{ij1}\| \|A_{ij3}\| \|P_j\| + \sum_{j=1}^{3} \|A_{ij3}\| \|A_{ij2}\| \|P_j\| + c^{-1} \|P\| \left\| E_i^T E_i \right\| - \eta_{i3} \right] \Bigg\}. \tag{3.27}$$

From (3.22), we have $\Delta V(k) < 0, x(k) \neq 0$.

Switching Law. The switching law is given by (2.17) and (2.18). In the light of Lemma 2.2, the discrete-time system (2.3) is asymptotically stable. Proof of Theorem 3.5 is completed. □

Remark 3.6. According to the system matrix that is divided into block matrices of the different dimension, Theorems 3.3 and 3.5 are obtained, respectively. When the system matrix is divided into 2×2 block matrix, Theorem 3.3 can be used. When the system matrix is divided into 3×3 block matrix, Theorem 3.5 can be used.

4. Numerical Examples

Example 4.1. Consider the switched discrete-time system composed of two individual systems given as follows.

$$\begin{aligned}
&\text{Mode 1:}\quad x(k+1) = (A_1 + \Delta A_1)x(k), \quad A_1 = \begin{bmatrix} 0.4 & 0.1 \\ 0 & 1 \end{bmatrix}, \ \Delta A_1 = F_1(k)E_1, \ E_1 = [0, 0.1], \\
&\text{Mode 2:}\quad x(k+1) = (A_2 + \Delta A_2)x(k), \quad A_2 = \begin{bmatrix} -0.2 & 0.1 \\ 1 & 0.3 \end{bmatrix}, \ \Delta A_2 = F_2(k)E_2, E_2 = [0, 0.2].
\end{aligned} \tag{4.1}$$

Set $P_1 = P_2 = 0.5, c = 1$. It is easily obtained that

$$\begin{aligned}
&e_1 = 0.02, \qquad e_2 = 0.66, \qquad d_1 = 0.025, \qquad d_2 = 0.165, \\
&\lambda_{11} = 0.84, \qquad \lambda_{12} = 0.75, \qquad \lambda_{21} = 0.96, \qquad \lambda_{22} = 0.91.
\end{aligned} \tag{4.2}$$

It is validated that

$$\begin{aligned}
&A_{ijj}^T P_j A_{ijj} - P_j \le -\lambda_{ij} I, \quad i, j = 1, 2, \\
&e_1 + ce_2 - \lambda_{11} - c\lambda_{21} + c\lambda_{\max}(P_1) + c^{-1}\|P\|\left\|{E_1}^T E_1\right\| + \|P\|\left\|{E_2}^T E_2\right\| < 0, \\
&d_1 + cd_2 - \lambda_{12} - c\lambda_{22} + c\lambda_{\max}(P_2) + c^{-1}\|P\|\left\|{E_1}^T E_1\right\| + \|P\|\left\|{E_2}^T E_2\right\| < 0.
\end{aligned} \tag{4.3}$$

According to Theorem 3.2, the switched discrete-time system (4.1) is asymptotically stable by the following switching law.

Switching Law. $\sigma(x(k)) = i$, if the following inequality holds:

$$x^T(k)\left[2A_i^T P A_i + 2\|P\|E_i^T E_i - P\right]x^T(k) < 0. \tag{4.4}$$

If $x(k) = 0$, then

$$\sigma(x(k)) = \gamma, \qquad \gamma \in \{1, 2\}. \tag{4.5}$$

Example 4.2. Consider the switched discrete-time system composed of three individual systems given as follows.

$$\begin{aligned}
&\text{Mode 1 :}\quad x(k+1) = (A_1 + \Delta A_1)x(k), \quad \Delta A_1 = F_1(k)E_1, \ E_1 = [0, 0, 0.1]. \\
&\text{Mode 2:}\quad x(k+1) = (A_2 + \Delta A_2)x(k), \quad \Delta A_2 = F_2(k)E_2, \ E_2 = [0, 0, 0.2]. \\
&\text{Mode 3:}\quad x(k+1) = (A_3 + \Delta A_3)x(k), \quad \Delta A_3 = F_3(k)E_3, \ E_3 = [0, 0.1, 0],
\end{aligned} \tag{4.6}$$

where

$$A_1 = \begin{bmatrix} 0.2 & 0 & 0.1 \\ 0.1 & 0.3 & 0.4 \\ 0 & 0 & 0.7 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} -0.2 & 0.1 & 0 \\ 0 & 0.3 & 0.1 \\ 0.1 & 0 & 0.1 \end{bmatrix}, \qquad A_3 = \begin{bmatrix} -0.1 & 0.1 & 0 \\ 0 & 0.4 & 0.3 \\ 0 & 0.1 & 0.2 \end{bmatrix}. \tag{4.7}$$

Taking $c = 1, P_1 = 0.35, P_2 = 0.45, P_3 = 0.7$, it is easily obtained that

$$\begin{gathered}\eta_{11} = 0.8691, \quad \eta_{12} = 0.8190, \quad \eta_{13} = 0.4191, \quad \eta_{21} = 0.8691, \quad \eta_{22} = 0.8191, \\ \eta_{23} = 0.8991, \quad \eta_{31} = 0.8991, \quad \eta_{32} = 0.7491, \quad \eta_{33} = 0.8691.\end{gathered} \tag{4.8}$$

It is validated that (3.21) and (3.22) hold. According to Theorem 3.5, the switched discrete-time system (4.6) is asymptotically stable by the following switching law.

Switching Law. $\sigma(x(k)) = i$, if the following inequality holds:

$$x^T(k)\left[2A_i^T P A_i + 2\|P\| E_i^T E_i - P\right] x^T(k) < 0. \tag{4.9}$$

If $x(k) = 0$, then

$$\sigma(x(k)) = \gamma, \quad \gamma \in \{1, 2, 3\}. \tag{4.10}$$

5. Conclusion

The robust stability is investigated for a class of switched discrete-time system with state parameter uncertainty. The switched law design method is proposed. By a simple switching law, some sufficient conditions for robust stability have been derived for the uncertain switched discrete-time systems and are presented in terms of inequalities. The present results are straightforward. Two examples are given to show the effectiveness of our approach.

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References

[1] D. Liberzon and A. S. Morse, "Basic problems in stability and design of switched systems," *IEEE Control Systems Magazine*, vol. 19, no. 5, pp. 59–70, 1999.

[2] Y. Dong, J. Liu, S. Mei, and M. Li, "Stabilization for switched nonlinear time-delay systems," *Nonlinear Analysis: Hybrid Systems*, vol. 5, no. 1, pp. 78–88, 2011.

[3] R. Wang, X. Liu, and Z. Guan, "Robustness and stability analysis for a class of nonlinear switched systems with impulse effects," *Dynamic Systems and Applications*, vol. 14, no. 2, pp. 233–248, 2004.

[4] P. Peleties and R. Decarlo, "Asymptotic stability of m-switched systems using Lyapunov-like functions," in *Proceedings of the 1991 American Control Conference*, pp. 1679–1684, Boston, Mass, USA, June 1991.

[5] H. Ishii and B. A. Francis, "Stabilizing a linear system by switching control with dwell time," *IEEE Transactions on Automatic Control*, vol. 47, no. 12, pp. 1962–1973, 2002.

[6] M. S. Branicky, "Multiple Lyapunov functions and other analysis tools for switched and hybrid systems," *IEEE Transactions on Automatic Control*, vol. 43, no. 4, pp. 475–482, 1998, Hybrid control system.

[7] E. Skafidas, R. J. Evans, A. V. Savkin, and I. R. Petersen, "Stability results for switched controller systems," *Automatica*, vol. 35, no. 4, pp. 553–564, 1999.

[8] F. Jianyin and L. Kai, "Stabilization of nonlinear systems under arbitrary switching law," in *Proceedings of the 30th Chinese Control Conference*, pp. 641–643, Beijing, China, 2010.

[9] Y. Dong, J. Fan, and S. Mei, "Quadratic stabilization of switched nonlinear systems," *Science in China F*, vol. 52, no. 6, pp. 999–1006, 2009.
[10] G. Ferrari-Trecate, F. A. Cuzzola, D. Mignone, and M. Morari, "Analysis of discrete-time piecewise affine and hybrid systems," *Automatica*, vol. 38, no. 12, pp. 2139–2146, 2002.
[11] J. Daafouz, P. Riedinger, and C. Iung, "Stability analysis and control synthesis for switched systems: a switched Lyapunov function approach," *IEEE Transactions on Automatic Control*, vol. 47, no. 11, pp. 1883–1887, 2002.
[12] V. N. Phat, "Robust stability and stabilizability of uncertain linear hybrid systems with state delays," *IEEE Transactions on Circuits and Systems II*, vol. 52, no. 2, pp. 94–98, 2005.
[13] Y. G. Sun, L. Wang, and G. Xie, "Delay-dependent robust stability and H_∞ control for uncertain discrete-time switched systems with mode-dependent time delays," *Applied Mathematics and Computation*, vol. 187, no. 2, pp. 1228–1237, 2007.
[14] L. Zhang, P. Shi, C. Wang, and H. Gao, "Robust H_∞ filtering for switched linear discrete-time systems with polytopic uncertainties," *International Journal of Adaptive Control and Signal Processing*, vol. 20, no. 6, pp. 291–304, 2006.
[15] Z. Ji and L. Wang, "Quadratic stabilization of uncertain discrete-time switched linear systems," in *Proceeding of the IEEE International Conference on Systems, Man and Cybernetics (SMC '04)*, pp. 1492–1497, October 2004.
[16] G. Zhai, H. Lin, A. N. Michel, and K. Yasuda, "Stability analysis for switched systems with continuous-time and discrete-time subsystems," in *Proceedings of the American Control Conference (AAC '04)*, pp. 4555–4560, July 2004.
[17] Y. G. Sun and L. Wang, "Stabilization of planar discrete-time switched systems: switched Lyapunov functional approach," *Nonlinear Analysis: Hybrid Systems*, vol. 2, no. 4, pp. 1062–1068, 2008.
[18] J. S. Chiou and C. M. Cheng, "Stabilization analysis for a class of switched discrete-time systems," in *Proceeding of the IEEE International Conference on Systems, Man and Cybernetics (SMC '06)*, pp. 4535–4540, Taipei, China, October 2006.
[19] M. da la Sen, "Quadratic stability and stabilization of switched dynamic systems with uncommensurate internal point delays," *Applied Mathematics and Computation*, vol. 185, no. 1, pp. 508–526, 2007.
[20] X. Z. Liu and S. A. Yuan, "On designing H_∞ fault estimator for switched nonlinear systems of neutral type," *Communications in Nonlinear Science and Numerical Simulation*, vol. 16, no. 1, pp. 4379–4389, 2011.
[21] Z. R. Xiang, Y.-N. Sun, and Q. W. Chen, "Robust reliable stabilization of uncertain switched neutral systems with delayed switching," *Applied Mathematics and Computation*, vol. 217, no. 23, pp. 9835–9844, 2011.
[22] W. Zhang, A. Abate, J. Hu, and M. P. Vitus, "Exponential stabilization of discrete-time switched linear systems," *Automatica*, vol. 45, no. 11, pp. 2526–2536, 2009.
[23] Y. G. Sun, L. Wang, and G. Xie, "Delay-dependent robust stability and stabilization for discrete-time switched systems with mode-dependent time-varying delays," *Applied Mathematics and Computation*, vol. 180, no. 2, pp. 428–435, 2006.
[24] V. N. Phat, T. Botmart, and P. Niamsup, "Switching design for exponential stability of a class of nonlinear hybrid time-delay systems," *Nonlinear Analysis: Hybrid Systems*, vol. 3, no. 1, pp. 1–10, 2009.