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Research Article

Fuzzy Linguistic Induced Ordered Weighted Averaging Operator and Its Application

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With respect to multiple attribute group decision making (MAGDM) problems, in which the attribute weights take the form of real numbers, and the attribute values take the form of fuzzy linguistic scale variables, a decision analysis approach is proposed. In this paper, we develop a new fuzzy linguistic induce OWA (FLIOWA) operator and analyze the properties of it by utilizing some operational laws of fuzzy linguistic scale variables. A method based on the FLIOWA operators for multiple attribute group decision making is presented. Finally, a numerical example is used to illustrate the applicability and effectiveness of the proposed method.

1. Introduction

In multiple attribute group decision making (MAGDM) analysis with linguistic information, the aggregation operators are required to deal with the aggregation of linguistic information and rankings of alternatives. Since Yager and Filev [1] developed the induced ordered weighted averaging (IOWA) operator, which takes as its argument pairs, called OWA pairs, in which one component is used to induce an ordering over the second components which are exact numerical values and then aggregated, many operators have been developed to aggregate linguistic information in group decision making such as the induced ordered weighted geometric (IOWG) operators that is based on the IOWA operator and the geometric mean [2, 3], the Quasi-IOWA operator based on the quasi-arithmetic means [4], and the induced ordered weighted harmonic averaging (IOWHA) operator based the IOWA operator and the harmonic mean [5], the induced trapezoidal fuzzy ordered weighted harmonic averaging (ITFOWHA) operator that is based on the IOWHA operator with the trapezoidal fuzzy information [6] using the idea of [7], we propose a fuzzy linguistic induce order weight averaging (FLIOWA) operator based on the extended ordered weighted averaging (EWOA) operator [8–10], which aims to resolve the input arguments taking the form of fuzzy linguistic scale information for the EOWA operator.

The rest of this paper is organized as follows. In Section 2, we briefly describe some basic concepts about the fuzzy number. In Section 3 we present the FLIOWA operator and its properties. A method based on the FLIOWA operators for multiple attribute group decision making (MAGDM) is presented in Section 4. In Section 5, we develop a numerical example of the new approach. Finally, in Section 6 we summarize the main conclusions of the paper.

2. Preliminaries

For the convenience of analysis, we give several definitions in this section. These definitions are used throughout the paper.

Definition 2.1. A triangular fuzzy number $\tilde{a}$ can be defined by a triplet $\tilde{a} = (a_1, a_2, a_3)$, $a_1, a_2, a_3 \in \Re$. The membership function [11, 12] $\mu_{\tilde{a}}(x)$ is defined as

$$\mu_{\tilde{a}}(x) = \begin{cases} 0, & x < a_1, \\ \dfrac{x - a_1}{a_2 - a_1}, & a_1 \le x \le a_2, \\ \dfrac{a_3 - x}{a_3 - a_2}, & a_2 \le x \le a_3, \\ 0, & x > a_3, \end{cases} \tag{2.1}$$

where $a_1 \le a_2 \le a_3$, if $a_2 - a_1 = a_3 - a_2$, $\tilde{a}$ is a symmetrical triangular fuzzy number. If $a_1 = a_2 = a_3$, then $\tilde{a}$ is a real number, that is, $\tilde{a} = a_1 = a_2 = a_3$. If $a_1 > 0$, we call $\tilde{a}$ as a positive triangular fuzzy number.

Definition 2.2 (see [11]). Let $\tilde{a} = (a_1, a_2, a_3)$ and $\tilde{b} = (b_1, b_2, b_3)$ be any two positive triangular fuzzy numbers, $\lambda \in \Re$, the basic operational laws related to triangular fuzzy numbers are as follows:

$$\begin{aligned} &(1)\ \tilde{a} \oplus \tilde{b} = (a_1 + b_1, a_2 + b_2, a_3 + b_3), \\ &(2)\ \tilde{a} \otimes \tilde{b} = (a_1 \times b_1, a_2 \times b_2, a_3 \times b_3), \\ &(3)\ \lambda \odot \tilde{a} = (\lambda a_1, \lambda a_2, \lambda a_3), \\ &(4)\ \frac{1}{\tilde{a}} = \left(\frac{1}{a_3}, \frac{1}{a_2}, \frac{1}{a_1}\right). \end{aligned} \tag{2.2}$$

Definition 2.3. Let $\tilde{a} = (a_1, a_2, a_3)$ be the positive triangular fuzzy number, the graded mean value representation of a triangular fuzzy number $\tilde{a}$ is defined as [13]

$$P(\tilde{a}) = \frac{a_1 + 4a_2 + a_3}{6}. \tag{2.3}$$

Definition 2.4. Let $\tilde{a} = (a_1, a_2, a_3)$ and $\tilde{b} = (b_1, b_2, b_3)$ be any two positive triangular fuzzy numbers, if $P(\tilde{a}) \leq P(\tilde{b})$, then $\tilde{a} \leq \tilde{b}$.

Suppose that $S = \{s_1, s_2, \ldots, s_T\}$ is the preestablished finite and totally ordered discrete linguistic term set with odd cards, where s_i denotes the ith linguistic term of S. It can be seen that T is the cardinality of S. An alternative possibility for reducing the complexity of defining a grammar consists of directly supplying the term set by considering all terms as primary ones and distributed on a scale on which a total order is defined [14–16]. For example, a set of nine terms S could be given as follows:

$$\begin{aligned} S = \{ & s_1 = \text{none (N)},\ s_2 = \text{extremely low (EL)},\ s_3 = \text{very low (VL)}, \\ & s_4 = \text{low (L)},\ s_5 = \text{medium (M)},\ s_6 = \text{High (H)},\ s_7 = \text{very high (VH)}, \\ & s_8 = \text{extremely high (EH)},\ s_9 = \text{perfect (P)}\}, \end{aligned} \tag{2.4}$$

in which $s_i < s_j$ if and only if $i < j$. Usually, in these cases, it is often required that the linguistic term set satisfies the following additional characteristics.

(1) There is a negation operator, for example, $\text{Neg}(s_i) = s_j$, $j = T - 1$ ($T + 1$ is the cardinality).

(2) Maximization and minimization operator: $\text{Max}(s_i, s_j) = s_i$ if $s_i \geq s_j$, $\text{Min}\ (s_i, s_j) = s_i$ if $s_i \leq s_j$.

The fuzzy linguistic scale $\tilde{S}$ is used by the decision making to evaluate the performance of decisions, and these and fuzzy linguistic terms are denoted by triangular fuzzy numbers as follows:

$$\begin{aligned} \tilde{S} = \{ & s_{\tilde{9}} = s_{(0.8,0.9,1)},\ s_{\tilde{8}} = s_{(0.7,0.8,0.9)},\ s_{\tilde{7}} = s_{(0.6,0.7,0.8)}, \\ & s_{\tilde{6}} = s_{(0.5,0.6,0.7)},\ s_{\tilde{5}} = s_{(0.4,0.5,0.6)},\ s_{\tilde{4}} = s_{(0.3,0.4,0.5)}, \\ & s_{\tilde{3}} = s_{(0.2,0.3,0.4)},\ s_{\tilde{2}} = s_{(0.1,0.2,0.3)},\ s_{\tilde{1}} = s_{(0,0.1,0.2)}\}. \end{aligned} \tag{2.5}$$

Definition 2.5. Let $\tilde{a} = (a_1, a_2, a_3)$ and $\tilde{b} = (b_1, b_2, b_3)$ be any two positive triangular fuzzy numbers, $s_{\tilde{a}}, s_{\tilde{b}} \in \tilde{S}$, if $\tilde{a} \leq \tilde{b}$, then $s_{\tilde{a}} \leq s_{\tilde{b}}$.

According to Definition 2.5, there is obviously

$$s_{\tilde{9}} > s_{\tilde{8}} > s_{\tilde{7}} > s_{\tilde{6}} > s_{\tilde{5}} > s_{\tilde{4}} > s_{\tilde{3}} > s_{\tilde{2}} > s_{\tilde{1}}. \tag{2.6}$$

Definition 2.6 (see [9]). Let $s_{\tilde{\alpha}}, s_{\tilde{\beta}} \in \tilde{S}$, $y, z \in [0,1]$, then:

$$\begin{aligned}
&(1)\ s_{\tilde{\alpha}} \oplus s_{\tilde{\beta}} = s_{\tilde{\alpha}+\tilde{\beta}} = s_{\tilde{\beta}} \oplus s_{\tilde{\alpha}},\\
&(2)\ y \odot s_{\tilde{\alpha}} = s_{y\tilde{\alpha}},\\
&(3)\ y \odot \left(s_{\tilde{\alpha}} \oplus s_{\tilde{\beta}}\right) = y \odot s_{\tilde{\alpha}} \otimes y \odot s_{\tilde{\beta}} = s_{y\tilde{\alpha}+y\tilde{\beta}},\\
&(4)\ (y+z) \odot s_{\tilde{\alpha}} = y \odot s_{\tilde{\alpha}} \oplus z \odot s_{\tilde{\alpha}} = s_{(y+z)\tilde{\alpha}}.
\end{aligned} \tag{2.7}$$

3. Fuzzy Linguistic Induced Ordered Weighted Averaging (FLIOWA) Operator

Inspired by Mitchell and Estrakh's work [17], Yager and Filev introduced in [18] a more general type of OWA operator, which they named the induced ordered weighted averaging (IOWA) operator.

Definition 3.1. An IOWA operator of dimension n is a function $\Phi_W : (R \times R)^n \rightarrow R$, to which a weighting vector is associated, $W = (w_1, w_2, \ldots, w_n)$, such that $w_i \in [0,1]$ and $\sum_{i=1}^{n} w_i = 1$, and it is defined to aggregate the set of second arguments of a list of n pairs $\{(u_1, p_1), (u_2, p_2), \ldots, (u_n, p_n)\}$ according to the following expression:

$$\Phi_W((u_1, p_1), (u_2, p_2), \ldots, (u_n, p_n)) = \sum_{i=1}^{n} w_i p_{\sigma(i)}, \tag{3.1}$$

where $\sigma : (1, 2, \ldots, n) \rightarrow (1, 2, \ldots, n)$ is a permutation such that $u_{\sigma(i)} \geq u_{\sigma(i+1)}$, for all $i = 1, 2, \ldots, n-1$,: that is, $(u_{\sigma(i)}, p_{\sigma(i)})$ is the pair with $u_{\sigma(i)}$ the ith highest value in the set $\{u_1, u_2, \ldots, u_n\}$. $p_1, p_2, \ldots, p_n$ is induced by the ordering of the values $u_1, u_2, \ldots, u_n$ associated with them. Because of this use of the set of values $u_1, u_2, \ldots, u_n$, Yager and Filev have called them the values of an order inducing variable and $p_1, p_2, \ldots, p_n$ the values of the argument variable [19].

Xu et al. [8–10] developed the extended ordered weighted averaging (EOWA) operator.

Definition 3.2. An extended ordered weighted averaging (EOWA) operator of dimension n is a mapping, $\Phi : S^n \rightarrow S$, that has an associated weighting vector $w = (w_1, w_2, \ldots, w_n)^T$ with the properties $w_i \in [0,1]$ and $\sum_{i=1}^{n} w_i = 1$, such that:

$$\Phi_w(s_{\alpha_1}, s_{\alpha_2}, \ldots, s_{\alpha_n}) = w_1 \odot s_{\beta_1} \oplus \cdots \oplus w_n \odot s_{\beta_n} = s_{\beta}, \tag{3.2}$$

where $\beta = \sum_{i=1}^{n} w_i \beta_i$, $(s_{\beta_1}, s_{\beta_2}, \ldots, s_{\beta_n}) \rightarrow (s_{\alpha_1}, s_{\alpha_2}, \ldots, s_{\alpha_n})$ being a permutation such that $s_{\beta_i} \geq s_{\beta_{i+1}}$, for all $i = 1, 2, \ldots, n-1$, that is, s_{β_i} is the ith highest value in the set $(s_{\alpha_1}, s_{\alpha_2}, \ldots, s_{\alpha_n})$.

Definition 3.3. Let $\tilde{a}_i = (a_{i1}, a_{i2}, a_{i3})$, $(i = 1, 2, \ldots, n)$ be a collection of triangular fuzzy numbers. A fuzzy linguistic induced ordered weighted averaging (FLIOWA) operator of dimension n is a function, FLIOWA : $\widetilde{S}^n \rightarrow \widetilde{S}$, that has associated a set of weights or weighting vector $w = (w_1, w_2, \ldots, w_n)^T$ to it, so that $w_i \in [0,1]$ and $\sum_{i=1}^n w_i = 1$ and is defined to aggregate a list of values $(s_{\widetilde{\alpha}_1}, s_{\widetilde{\alpha}_2}, \ldots, s_{\widetilde{\alpha}_n})$ according to the following expression:

$$\text{FLIOWA}_w(s_{\widetilde{\alpha}_1}, s_{\widetilde{\alpha}_2}, \ldots, s_{\widetilde{\alpha}_n}) = w_1 \odot s_{\widetilde{\beta}_1} \oplus \cdots \oplus w_n \odot s_{\widetilde{\beta}_n} = s_{\widetilde{\beta}}, \tag{3.3}$$

where $\widetilde{\beta} = \sum_{i=1}^n w_i \widetilde{\beta}_i$, $(s_{\widetilde{\beta}_1}, s_{\widetilde{\beta}_2}, \ldots, s_{\widetilde{\beta}_n}) \rightarrow (s_{\widetilde{\alpha}_1}, s_{\widetilde{\alpha}_2}, \ldots, s_{\widetilde{\alpha}_n})$ being a permutation such that $s_{\widetilde{\beta}_i} \geq s_{\widetilde{\beta}_{i+1}}$, for all $i = 1, 2, \ldots, n-1$, that is, $s_{\widetilde{\beta}_i}$ is the ith highest value in the set $(s_{\widetilde{\alpha}_1}, s_{\widetilde{\alpha}_2}, \ldots, s_{\widetilde{\alpha}_n})$.

The FLIOWA operator has the following properties.

Theorem 3.4 (commutativity). *Let* $(s^*_{\widetilde{\alpha}_1}, s^*_{\widetilde{\alpha}_2}, \ldots, s^*_{\widetilde{\alpha}_n})$ *be any permutation of the fuzzy linguistic scale data* $(s_{\widetilde{\alpha}_1}, s_{\widetilde{\alpha}_2}, \ldots, s_{\widetilde{\alpha}_n})$*, then*

$$FLIOWA_w\left(s^*_{\widetilde{\alpha}_1}, s^*_{\widetilde{\alpha}_2}, \ldots, s^*_{\widetilde{\alpha}_n}\right) = FLIOWA_w(s_{\widetilde{\alpha}_1}, s_{\widetilde{\alpha}_2}, \ldots, s_{\widetilde{\alpha}_n}). \tag{3.4}$$

Proof. Let

$$\begin{aligned} \text{FLIOWA}_w(s_{\widetilde{\alpha}_1}, s_{\widetilde{\alpha}_2}, \ldots, s_{\widetilde{\alpha}_n}) &= w_1 \odot s_{\widetilde{\beta}_1} \oplus w_2 \odot s_{\widetilde{\beta}_2} \oplus \cdots \oplus w_n \odot s_{\widetilde{\beta}_n}, \\ \text{FLIOWA}_w\left(s^*_{\widetilde{\alpha}_1}, s^*_{\widetilde{\alpha}_2}, \ldots, s^*_{\widetilde{\alpha}_n}\right) &= w_1 \odot s^*_{\widetilde{\beta}_1} \oplus w_2 \odot s^*_{\widetilde{\beta}_2} \oplus \cdots \oplus w_n \odot s^*_{\widetilde{\beta}_n}. \end{aligned} \tag{3.5}$$

Since $(s^*_{\widetilde{\alpha}_1}, s^*_{\widetilde{\alpha}_2}, \ldots, s^*_{\widetilde{\alpha}_n})$ is a permutation of $(s_{\widetilde{\alpha}_1}, s_{\widetilde{\alpha}_2}, \ldots, s_{\widetilde{\alpha}_n})$, using the Definitions 2.3–2.6, we have $s_{\widetilde{\beta}_i} = s^*_{\widetilde{\beta}_i}$, $i \in N$, then

$$\text{FLIOWA}_w\left(s^*_{\widetilde{\alpha}_1}, s^*_{\widetilde{\alpha}_2}, \ldots, s^*_{\widetilde{\alpha}_n}\right) = \text{FLIOWA}_w(s_{\widetilde{\alpha}_1}, s_{\widetilde{\alpha}_2}, \ldots, s_{\widetilde{\alpha}_n}). \tag{3.6}$$

□

Theorem 3.5 (idempotency). *If* $\widetilde{\alpha}_i = \widetilde{\alpha}$ *for all* i*, and for all* $s_{\widetilde{\alpha}_i}, s_{\widetilde{\alpha}} \in \widetilde{S}$*, then*

$$FLIOWA_w(s_{\widetilde{\alpha}_1}, s_{\widetilde{\alpha}_2}, \ldots, s_{\widetilde{\alpha}_n}) = s_{\widetilde{\alpha}}. \tag{3.7}$$

Proof. Since $\widetilde{\alpha}_i = \widetilde{\alpha}$ for all i, according to the Definition 2.5, $s_{\widetilde{\alpha}_i} = s_{\widetilde{\alpha}}$ for all i, then

$$\begin{aligned} \text{FLIOWA}_w(s_{\widetilde{\alpha}_1}, s_{\widetilde{\alpha}_2}, \ldots, s_{\widetilde{\alpha}_n}) &= w_1 \odot s_{\widetilde{\beta}_1} \oplus w_2 \odot s_{\widetilde{\beta}_2} \oplus \cdots \oplus w_n \odot s_{\widetilde{\beta}_n} \\ &= w_1 \odot s_{\widetilde{\alpha}} \oplus w_2 \odot s_{\widetilde{\alpha}} \oplus \cdots \oplus w_n \odot s_{\widetilde{\alpha}} \\ &= (w_1 + w_2 + \cdots + w_n) \odot s_{\widetilde{\alpha}} \\ &= s_{\widetilde{\alpha}}. \end{aligned} \tag{3.8}$$

□

Theorem 3.6 (monotonicity). *If* $\tilde{\alpha}_i \leq \tilde{\alpha}_i^*, s_{\tilde{\alpha}_i}, s_{\tilde{\alpha}_i^*} \in \tilde{S}$, *for all* i, *then*

$$FLIOWA_w(s_{\tilde{\alpha}_1}, s_{\tilde{\alpha}_2}, \ldots, s_{\tilde{\alpha}_n}) \leq FLIOWA_w\left(s_{\tilde{\alpha}_1^*}, s_{\tilde{\alpha}_2^*}, \ldots, s_{\tilde{\alpha}_n^*}\right). \tag{3.9}$$

Proof. Since $\tilde{\alpha}_i \leq \tilde{\alpha}_i^*$ for all i, according to the Definition 2.5, $s_{\tilde{\alpha}_i} \leq s_{\tilde{\alpha}_i^*}$ for all i. Let

$$\begin{aligned}
\text{FLIOWA}_w(s_{\tilde{\alpha}_1}, s_{\tilde{\alpha}_2}, \ldots, s_{\tilde{\alpha}_n}) &= w_1 \odot s_{\tilde{\beta}_1} \oplus \cdots \oplus w_n \odot s_{\tilde{\beta}_n} = s_{\tilde{\beta}}, \\
\text{FLIOWA}_w\left(s_{\tilde{\alpha}_1^*}, s_{\tilde{\alpha}_2^*}, \ldots, s_{\tilde{\alpha}_n^*}\right) &= w_1 \odot s_{\tilde{\beta}_1^*} \oplus \cdots \oplus w_n \odot s_{\tilde{\beta}_n^*} = s_{\tilde{\beta}^*},
\end{aligned} \tag{3.10}$$

where $\tilde{\beta} = \sum_{i=1}^n w_i \tilde{\beta}_i$, $\tilde{\beta}^* = \sum_{i=1}^n w_i \tilde{\beta}_i^*$, since $(s_{\tilde{\beta}_1}, s_{\tilde{\beta}_2}, \ldots, s_{\tilde{\beta}_n})$ is a permutation of $(s_{\tilde{\alpha}_1}, s_{\tilde{\alpha}_2}, \ldots, s_{\tilde{\alpha}_n})$, and $(s_{\tilde{\beta}_1^*}, s_{\tilde{\beta}_2^*}, \ldots, s_{\tilde{\beta}_n^*})$ is a permutation of $(s_{\tilde{\alpha}_1^*}, s_{\tilde{\alpha}_2^*}, \ldots, s_{\tilde{\alpha}_n^*})$, respectively, so, we have $s_{\tilde{\beta}_i} \leq s_{\tilde{\beta}_i^*}$ for all i, then $s_{\tilde{\beta}} \leq s_{\tilde{\beta}^*}$, that is,

$$\text{FLIOWA}_w(s_{\tilde{\alpha}_1}, s_{\tilde{\alpha}_2}, \ldots, s_{\tilde{\alpha}_n}) \leq \text{FLIOWA}_w\left(s_{\tilde{\alpha}_1^*}, s_{\tilde{\alpha}_2^*}, \ldots, s_{\tilde{\alpha}_n^*}\right). \tag{3.11}$$

□

Theorem 3.7 (boundedness). *Let* $s_{\tilde{\alpha}} = \min_i(s_{\tilde{\alpha}_1}, s_{\tilde{\alpha}_2}, \ldots, s_{\tilde{\alpha}_n})$, $s_{\tilde{\beta}} = \min_i(s_{\tilde{\alpha}_1}, s_{\tilde{\alpha}_2}, \ldots, s_{\tilde{\alpha}_n})$, *then*

$$s_{\tilde{\alpha}} \leq FLIOWA_w(s_{\tilde{\alpha}_1}, s_{\tilde{\alpha}_2}, \ldots, s_{\tilde{\alpha}_n}) \leq s_{\tilde{\beta}}. \tag{3.12}$$

Proof. Since $s_{\tilde{\alpha}} \leq s_{\tilde{\alpha}_i} \leq s_{\tilde{\beta}}$ for all i, using the Theorems 3.5 and 3.6, then

$$\begin{aligned}
\text{FLIOWA}_w(s_{\tilde{\alpha}_1}, s_{\tilde{\alpha}_2}, \ldots, s_{\tilde{\alpha}_n}) &= w_1 \odot s_{\tilde{\beta}_1} \oplus w_2 \odot s_{\tilde{\beta}_2} \oplus \cdots \oplus w_n \odot s_{\tilde{\beta}_n} \\
&\geq w_1 \odot s_{\tilde{\alpha}} \oplus w_2 \odot s_{\tilde{\alpha}} \oplus \cdots \oplus w_n \odot s_{\tilde{\alpha}} \\
&= (w_1 + w_2 + \cdots + w_n) \odot s_{\tilde{\alpha}} \\
&= s_{\tilde{\alpha}}, \\
\text{FLIOWA}_w(s_{\tilde{\alpha}_1}, s_{\tilde{\alpha}_2}, \ldots, s_{\tilde{\alpha}_n}) &= w_1 \odot s_{\tilde{\beta}_1} \oplus w_2 \odot s_{\tilde{\beta}_2} \oplus \cdots \oplus w_n \odot s_{\tilde{\beta}_n} \\
&\leq w_1 \odot s_{\tilde{\beta}} \oplus w_2 \odot s_{\tilde{\beta}} \oplus \cdots \oplus w_n \odot s_{\tilde{\beta}} \\
&= (w_1 + w_2 + \cdots + w_n) \odot s_{\tilde{\beta}} \\
&= s_{\tilde{\beta}}.
\end{aligned} \tag{3.13}$$

So $s_{\tilde{\alpha}} \leq \text{FLIOWA}_w(s_{\tilde{\alpha}_1}, s_{\tilde{\alpha}_2}, \ldots, s_{\tilde{\alpha}_n}) \leq s_{\tilde{\beta}}$. □

Especially, if $w = (w_1, w_2, \ldots, w_n)^T = (1/n, 1/n, \ldots, 1/n)^T$, then FLIOWA is reduced to the fuzzy linguistic extended arithmetical averaging (FEAA) operator; if $\alpha_{i1} = \alpha_{i2} = \alpha_{i3}$ for all i, then $\widetilde{\alpha}_i = \alpha_i$, the FLIOWA is reduced to the extended order weighted averaging (EOWA) operator.

4. A New Approach to Decision Making with Fuzzy Linguistic Scale Information

In the following, we are going to develop a new approach in a decision making problem about selection of strategies with fuzzy linguistic scale information. Let $X = \{x_1, x_2, \ldots, x_k\}$ be a discrete set of alternatives, we suppose that the decision makers compare these six companies with respect to a single criterion by using the fuzzy linguistic scale

$$\widetilde{S} = \left\{ s_{\widetilde{9}} = P, s_{\widetilde{8}} = EH, s_{\widetilde{7}} = VH, s_{\widetilde{6}} = H, s_{\widetilde{5}} = M, s_{\widetilde{4}} = L, s_{\widetilde{3}} = VL, s_{\widetilde{2}} = EL, s_{\widetilde{1}} = N \right\}. \tag{4.1}$$

In the following, we shall apply the FLIOWA operator to decision making with fuzzy linguistic scale information.

Step 1. Construct a fuzzy linguistic scale decision making matrix $\widetilde{R} = (\widetilde{r}_{ij})_{k\times k}$ according to the fuzzy linguistic scale evaluation for the alternative $x_i \in X$, $(i = 1, 2, \ldots, k)$ under the attribute $u_i \in U$, $(i = 1, 2, \ldots, k)$.

Step 2. Calculate the overall preference triangular fuzzy values of the alternative by utilizing the FLIOWA operator

$$s_{\widetilde{\alpha}_i} = s_{(\alpha_{i1}, \alpha_{i2}, \alpha_{i3})} = \mathrm{FLIOWA}_w(\widetilde{r}_{i1}, \widetilde{r}_{i2}, \ldots, \widetilde{r}_{ik}), \quad (i = 1, 2, \ldots, k). \tag{4.2}$$

Step 3. Calculate the graded mean values of triangular fuzzy number $\widetilde{\alpha}_i$, $(i = 1, 2, \ldots, k)$

$$P(\widetilde{a}_i) = \frac{a_{i1} + 4a_{i2} + a_{i3}}{6}. \tag{4.3}$$

Step 4. Rank all the alternatives x_i $(i = 1, 2, \ldots, k)$ and select the best one(s) in accordance with by using Definition 2.5.

Step 5. End.

5. Numerical Example

In this section, we are going to give a brief illustrative example of the new approach in a decision making problem. Let us suppose an investment company that it is operating in Europe and North America is analyzing its general policy for the next year, which wants to invest strategies in the best option [17]. In order to evaluate these strategies, the group of experts of the company considers that the key factor for the next year is the economic situation. Then, depending on the situation, the expected benefits for the company will be different. The experts have considered 6 possible situations for the next year. There is a panel with six possible alternatives to invest the strategies One main criterion used is the growth analysis. In this example, we assume that the experts use the following weighting vector

Table 1: Fuzzy linguistic scale decision matrix.

	u_1	u_2	u_3	u_4	u_5	u_6
x_1	$s_{\tilde{6}}$	$s_{\tilde{6}}$	$s_{\tilde{8}}$	$s_{\tilde{6}}$	$s_{\tilde{8}}$	$s_{\tilde{5}}$
x_2	$s_{\tilde{5}}$	$s_{\tilde{7}}$	$s_{\tilde{6}}$	$s_{\tilde{7}}$	$s_{\tilde{6}}$	$s_{\tilde{7}}$
x_3	$s_{\tilde{4}}$	$s_{\tilde{4}}$	$s_{\tilde{9}}$	$s_{\tilde{5}}$	$s_{\tilde{7}}$	$s_{\tilde{8}}$
x_4	$s_{\tilde{7}}$	$s_{\tilde{6}}$	$s_{\tilde{7}}$	$s_{\tilde{7}}$	$s_{\tilde{4}}$	$s_{\tilde{6}}$
x_5	$s_{\tilde{5}}$	$s_{\tilde{6}}$	$s_{\tilde{7}}$	$s_{\tilde{6}}$	$s_{\tilde{5}}$	$s_{\tilde{7}}$
x_6	$s_{\tilde{4}}$	$s_{\tilde{6}}$	$s_{\tilde{8}}$	$s_{\tilde{3}}$	$s_{\tilde{6}}$	$s_{\tilde{6}}$

for the FLIOWA operators: $W = (0.2, 0.2, 0.2, 0.2, 0.1, 0.1)^T$. Then, we utilize the approach developed to get the most desirable alternative(s).

Firstly, construct a fuzzy linguistic scale decision making matrix $\widetilde{R} = (\widetilde{r}_{ij})_{n\times n}$. We suppose that the decision makers compare these six alternatives with respect to the criterion growth analysis by using the fuzzy linguistic scale and the fuzy linguistic decision matrix $\widetilde{R} = (\widetilde{r}_{ij})_{6\times 6}$ is in Table 1 [20].

Secondly, calculate the overall preference triangular fuzzy values of the alternative by utilizing the FLIOWA operator

$$\begin{aligned}
s_{\tilde{\alpha}_1} &= \text{FLIOWA}_w\left(s_{\tilde{6}}, s_{\tilde{6}}, s_{\tilde{8}}, s_{\tilde{6}}, s_{\tilde{8}}, s_{\tilde{5}}\right) = s_{(0.57,0.67,0.77)},\\
s_{\tilde{\alpha}_2} &= \text{FLIOWA}_w\left(s_{\tilde{5}}, s_{\tilde{7}}, s_{\tilde{6}}, s_{\tilde{7}}, s_{\tilde{6}}, s_{\tilde{7}}\right) = s_{(0.55,0.65,0.75)},\\
s_{\tilde{\alpha}_3} &= \text{FLIOWA}_w\left(s_{\tilde{4}}, s_{\tilde{4}}, s_{\tilde{9}}, s_{\tilde{5}}, s_{\tilde{7}}, s_{\tilde{8}}\right) = s_{(0.56,0.66,0.76)},\\
s_{\tilde{\alpha}_4} &= \text{FLIOWA}_w\left(s_{\tilde{7}}, s_{\tilde{6}}, s_{\tilde{7}}, s_{\tilde{7}}, s_{\tilde{4}}, s_{\tilde{6}}\right) = s_{(0.54,0.64,0.74)},\\
s_{\tilde{\alpha}_5} &= \text{FLIOWA}_w\left(s_{\tilde{5}}, s_{\tilde{6}}, s_{\tilde{7}}, s_{\tilde{6}}, s_{\tilde{5}}, s_{\tilde{7}}\right) = s_{(0.52,0.62,0.72)},\\
s_{\tilde{\alpha}_6} &= \text{FLIOWA}_w\left(s_{\tilde{4}}, s_{\tilde{6}}, s_{\tilde{8}}, s_{\tilde{3}}, s_{\tilde{6}}, s_{\tilde{6}}\right) = s_{(0.49,0.59,0.69)}.
\end{aligned} \tag{5.1}$$

Thirdly, calculate the grade mean value of triangular fuzzy number $\widetilde{\alpha}_i$ as follows:

$$\begin{gathered}
P(\widetilde{a}_1) = 0.67, \qquad P(\widetilde{a}_2) = 0.65, \qquad P(\widetilde{a}_3) = 0.66,\\
P(\widetilde{a}_4) = 0.64, \qquad P(\widetilde{a}_5) = 0.62, \qquad P(\widetilde{a}_6) = 0.58333,
\end{gathered} \tag{5.2}$$

then we have $\widetilde{a}_1 > \widetilde{a}_3 > \widetilde{a}_2 > \widetilde{a}_4 > \widetilde{a}_5 > \widetilde{a}_6$.

Finally, rank all the alternatives x_i $(i = 1, 2, 3, 4, 5, 6)$. According to the grade mean value of the linguistic scale and the Definition 2.5, we have $s_{\tilde{a}_1} > s_{\tilde{a}_3} > s_{\tilde{a}_2} > s_{\tilde{a}_4} > s_{\tilde{a}_5} > s_{\tilde{a}_6}$, and thus the most desirable alternative is x_1.

6. Conclusion

The traditional induced aggregation operators are generally suitable for aggregating the information taking the form of numerical values, and yet they will fail in dealing with fuzzy linguistic scale information. In this paper, we develop a new FLIOWA operator and study some desirable properties of the operator, such as commutativity, idempotency, and monotonicity and applied the FIOWA operators to decision making with fuzzy linguistic scale information. The proposed operator can be used to aggregate the fuzzy linguistic scale information in MAGDM problems. It also enriches the existing approaches to aggregating fuzzy linguistic information. Finally an illustrative example has been given to show the developed method.

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