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Research Article

Decentralized Control of Uncertain Fuzzy Large-Scale System with Time Delay and Optimization

Hang Zhang, Xia Wang, Yi-Jun Wang, and Zhao-Mei Sun

School of Information Science and Engineering, Central South University, Changsha 410075, China

Correspondence should be addressed to Yi-Jun Wang, xxywyj@sina.com

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This paper studies the decentralized stabilization problem for an uncertain fuzzy large-scale system with time delays. The considered large-scale system is composed of several T-S fuzzy subsystems. The decentralized parallel distributed compensation (PDC) fuzzy control for each subsystem is designed to stabilize the whole system. Based on Lyapunov criterion, some sufficient conditions are proposed. Moreover, the positive definite matrices P_i and PDC gain K_{ij} can be solved by linear matrix inequality (LMI) toolbox of Matlab. Then, the optimization design method for decentralized control is also considered with respect to a quadratic performance index. Finally, numerical examples are given and compared with those of Zhang et al., 2004 to illustrate the effectiveness and less conservativeness of our method.

1. Introduction

Many real-life problems, such as power system, economic systems, societal system, and nuclear system are frequently of high dimension. Such systems are regarded as large-scale system. They consist of a number of subsystems which serve particular functions, share resources, and are governed by a set of interrelated goals and constrains [1]. Over the past decades, many methods have been to investigate the stability and stabilization of large-scale system [2–10].

Fuzzy systems of Takagi-Sugeno (T-S) models [11] have become an effective method to represent complex nonlinear dynamic system by fuzzy sets and fuzzy reasoning. The method of T-S model is feasible since, in many situations, human experts can provide linguistic descriptions of local systems in terms of IF-THEN rules [12–14]. Reference [15] proposed a control concept "parallel distributed compensation" (PDC) for fuzzy controller design of T-S fuzzy model. Under some conditions, PDC can stabilize the closed-loop fuzzy system

asymptotically. Linear matrix inequalities (LMIs) methods to find the common positive matrix P_i always play a key role work in PDC design [16].

Now let us consider the control problem of large-scale system. Accordingly, suppose a large-scale system is composed of a number of subsystems with interconnection. Each subsystem is described by a T-S fuzzy model. That is, each subsystem dynamic is captured by a set of fuzzy implications that characterize local relations in the state space. Then, the global model of large-scale system can be achieved by smoothly connecting the local linear model in each fuzzy subspace together via the membership functions. Recently, the fuzzy decentralized control design methods and stability condition are addressed [17–24]. Wang and Luoh [17] have studied a fuzzy decentralized control design method for a fuzzy large-scale system on the assumption that all variables are available. Tseng and Chen [18] dealt with the model reference tracking control problem by using H_∞ decentralized fuzzy control, which relaxed the condition that the state variables are measurable. Wang and Zhang [19] considered the robust decentralized controller design method for nonlinear large-scale descriptor system. Hsiao et al. [20] proposed fuzzy decentralized control design methods for fuzzy large-scale system with time delays and gave the analysis of the closed-loop fuzzy large-scale system. Robust decentralized H_∞ output feedback controller was designed in [21]. Tong and Zhao [22] proposed a stabilization criterion of continuous-time interconnected fuzzy systems without uncertainties. Su and Liu [23] dealt with decentralized stabilization problem for a large-scale system in which the system is composed of several T-S fuzzy subsystems with nonlinear interconnections. Wang et al. [24] addressed robust H_∞ fuzzy controller design to overcome the parametric uncertainties of fuzzy large-scale systems without time delay and get H_∞ performance. Zhang et al. [25] proposed the stabilization problem of fuzzy large-scale system without parametric uncertainties.

However, the proposed decentralized control designs and the sufficient conditions of closed-loop system did not consider the parametric uncertainties or time delays, which is important in both theory and real-world application. In this paper, the stabilization problem of the uncertain fuzzy large-scale systems with time delays is considered. Based on decentralized control concept, we like to synthesize a PDC controller for each subsystem so that the whole system can be stabilized asymptotically. When applied to degenerated cases, (without uncertainties or time delay), the stabilization criteria are better than existing ones. Stability is one of the most important performance indecies, but it is not enough for control systems. Linear quadratic performance can reflect a lot of performance requirements, so a quadratic performance index is considered in this paper. Based on Lyapunov function stability theory, the optimization design method for decentralized control with respect to the quadratic performance index is transformed into solving linear inequality matrix by PDC controller.

This paper is organized as follows. In Section 2, the considered systems are stated and some preliminaries are presented.The problem of stabilization for fuzzy large-scale system with uncertainty is proposed in Section 3. Section 4 provides examples to illustrate the correctness of our theoretic results. Finally, a conclusion is given in Section 5.

2. System Description and Preliminaries

Consider a fuzzy large-scale system S, with uncertainties both in isolated subsystem and in their interconnection, which consists of J interconnected subsystems S_i. The ith fuzzy subsystem is described by the following equations:

$$\begin{aligned}\dot{x}_i(t) = &\sum_{j=1}^{r_i} h_{ij}(t)\left[\left(A_{ij} + \Delta A_{ij}(t)\right)x_i(t) + \left(B_{ij} + \Delta B_{ij}(t)\right)u_i(t)\right] \\ &+ \sum_{k=1}^{J}\left[(C_{ki} + \Delta C_{ki}(t))x_k(t) + (D_{ki} + \Delta D_{ki}(t))x_k(t-\tau_{ki})\right],\end{aligned} \tag{2.1}$$

where

$$\left[\Delta A_{ij}(t), \Delta B_{ij}(t)\right] = E_{ij}F_{ij}(t)\left[G_{ij}, H_{ij}\right], \qquad \left[\Delta C_{ki}(t), \Delta D_{ki}(t)\right] = \overline{E}_{ki}\overline{F}_{ki}(t)\left[L_{ki}, M_{ki}\right] \tag{2.2}$$

for for all $t \geq 0$, $j = 1, 2, \ldots, r_i$, $k = 1, 2, \ldots, J$, $x_i(t)$ is the state vector, $u_i(t)$ is the control input, $A_{ij}, B_{ij}, G_{ij}, H_{ij}, E_{ij}, C_{ki}, D_{ki}, \overline{E}_{ki}, M_{ki}, L_{ki}$ are known constant matrices with appropriate dimensions. τ_{ki} is the time delay. $h_{ij}(t)$ is the normalized weigh in (2.6). $F_{ij}(t)$ and $\overline{F}_{ki}(t)$ are time-varying matrices with appropriate dimensions satisfying

$$F_{ij}(t)F_{ij}^T(t) \leq I, \qquad \overline{F}_{ki}(t)\overline{F}_{ki}^T(t) \leq I. \tag{2.3}$$

Each isolated subsystem S_i is represented by a T-S fuzzy model. The jth rule of this T-S fuzzy model is represented as follows.
Rule j. If $z_{1i}(t)$ is M_{j1i} and $\ldots$ and $z_{pi}(t)$ is M_{jpi}, then

$$\dot{x}_i(t) = \left(A_{ij} + \Delta A_{ij}(t)\right)x_i(t) + \left(B_{ij} + \Delta B_{ij}(t)\right)u_i(t), \tag{2.4}$$

where $z_i(t) = [z_{1i}(t), z_{2i}(t), \ldots, z_{pi}(t)]^T, z_{1i}, \ldots, z_{pi(t)}$ are premise variables, and M_{jli} ($l = 1, 2, \ldots, p$) are fuzzy sets. By "fuzzy blending," the final output of the ith fuzzy subsystem is described as follows:

$$\begin{aligned}\dot{x}_i(t) &= \frac{\sum_{j=1}^{r_i}\omega_{ij}(t)\left[\left(A_{ij} + \Delta A_{ij}(t)\right)x_i(t) + \left(B_{ij} + \Delta B_{ij}(t)\right)u_i(t)\right]}{\sum_{j=1}^{r_i}\omega_{ij}(t)} \\ &= \sum_{j=1}^{r_i} h_{ij}(t)\left[\left(A_{ij} + \Delta A_{ij}(t)\right)x_i(t) + \left(B_{ij} + \Delta B_{ij}(t)\right)u_i(t)\right]\end{aligned} \tag{2.5}$$

with

$$\omega_{ij}(t) = \prod_{l=1}^{p} M_{jli}(z_{li}(t)), \qquad h_{ij}(t) = \frac{\omega_{ij}(t)}{\sum_{j=1}^{r_i}\omega_{ij}(t)}, \tag{2.6}$$

where $M_{jil}(z_{il}(t))$ is the grade of membership of $z_{il}(t)$ in M_{jil}, and r_i is the number of fuzzy rules of subsystem S_i. We assumed that $\omega_{ij}(t) \geq 0$ for all t, $j = 1, 2, \ldots, r_i$. Therefore

$$\sum_{j=1}^{r_i} h_{ij}(t) = 1. \tag{2.7}$$

Define a quadratic performance index

$$J = \sum_{i=1}^{J} \int_0^{\infty} \left(x_i^T(t) Q_i x_i(t) + u_i^T(t) R_i u_i(t) \right) dt, \tag{2.8}$$

where Q_i and R_i are positive matrices.

The main propose of this paper is to synthesize a decentralized PDC fuzzy controller $u_i(t)$ for each subsystem such that the closed-loop large-scale T-S fuzzy systems (2.1) is asymptotically stable and the optimization design method for decentralized control respect to the quadratic performance index.

Before starting the main results, we need the following lemmas.

Lemma 2.1 (see [26]). *Let Q be any of $n \times n$ matrix; one will have for any constant $k \geq 0$; and any positive matrix $S > 0$ that*

$$2x^T Q y \leq k x^T Q S^{-1} Q^T x + \frac{1}{k} y^T S y \tag{2.9}$$

for all $x, y \in R^n$.

Lemma 2.2 (see [26]). *Let D, E be any constant matrices and $F^T F \leq I$, where a is a positive constant, all matrices with appropriate dimensions; one will have for any constant $k > 0$ such that*

$$2x^T DEFy \leq k x^T D D^T x + \frac{a}{k} y^T E^T E y, \tag{2.10}$$

for all $x, y \in R^n$.

Lemma 2.3 (see [27]). *Let R_i be any positive matrix; one will have for any $0 \leq h_{ij}(t) \leq 1$ such that*

$$\left(\sum_{j=1}^{r_i} h_{ij}(t) K_{ij} \right)^T R_i \left(\sum_{j=1}^{r_i} h_{ij}(t) K_{ij} \right) \leq \sum_{j=1}^{r_i} h_{ij}(t) K_{ij}^T R_i K_{ij} \tag{2.11}$$

for all $K_{ij} \in R^n$.

3. Stabilization and PDC Synthesis of Fuzzy Large-Scale System

In this section, the decentralized concept and PDC approach are applied to synthesize a local feedback controller for each local subsystem. Let the fuzzy controller be as the PDC form: *Rule j*. If $z_{1i}(t)$ is M_{j1i} and... and $z_{pi}(t)$ is M_{jpi}, then

$$u_i(t) = -K_{ij} x_i(t), \tag{3.1}$$

where $i = 1, 2, \ldots, J$, $j = 1, 2, \ldots, r_i$. The overall state feedback fuzzy control law is represented by:

$$u_i(t) = -\frac{\sum_{j=1}^{r_i} \omega_{ij}(t) K_{ij} x_i(t)}{\sum_{j=1}^{r_i} \omega_{ij}(t)} = -\sum_{j=1}^{r_i} h_{ij}(t) K_{ij} x_i(t). \tag{3.2}$$

Substituting (3.2) into (2.1), the closed-loop fuzzy subsystem becomes

$$\begin{aligned} \dot{x}_i(t) = &\sum_{j=1}^{r_i}\sum_{n=1}^{r_i} h_{ij}(t) h_{in}(t) \left[\left(A_{ij} + \Delta A_{ij}\right) - \left(B_{ij} + \Delta B_{ij}\right) K_{in} \right] x_i(t) \\ &+ \sum_{k=1}^{J} \left[(C_{ki} + \Delta C_{ki}) x_k(t) + (D_{ki} + \Delta D_{ki}) x_k(t - \tau_{ki}) \right]. \end{aligned} \tag{3.3}$$

Now, our work is to determine the local feedback gains K_{ij} such that the whole fuzzy large-scale system (3.3) is asymptotically stable.

Theorem 3.1. *The fuzzy large-scale system* (2.1) *can be asymptotically stabilized by the decentralized PDC fuzzy control* (3.2), *if there exist matrices F_{ij}, positive definite matrices X_i to satisfy the following LMIs:*

$$\begin{pmatrix} X_i A_{ij}^T + A_{ij} X_i - F_{in}^T B_{ij}^T - B_{ij} F_{in} & X_i G_{ij}^T - F_{in}^T H_{ij}^T & X_i M_i^T & \overline{C}_i & \overline{E}_i & X_i \\ G_{ij} X_i - H_{ij} F_{in} & -I & 0 & 0 & 0 & 0 \\ M_i X_i & 0 & -I & 0 & 0 & 0 \\ \overline{C}_i^T & 0 & 0 & -I & 0 & 0 \\ \overline{E}_i^T & 0 & 0 & 0 & -\frac{1}{2} I & 0 \\ X_i & 0 & 0 & 0 & 0 & -\frac{1}{2J} I \end{pmatrix} < 0, \tag{3.4}$$

for $i = 1, 2, \ldots, J$, $j = 1, 2, \ldots, r_i$, $n = 1, 2, \ldots, r_i$, where $M_i^T = [M_{i1}, M_{i2}, \ldots, M_{iJ}, L_{i1}, L_{i2}, \ldots, L_{iJ}]^T$, $\overline{C}_i = [E_{ij}, C_{1i}, C_{2i}, \ldots, C_{Ji}, D_{1i}, D_{2i}, \ldots, D_{Ji}]$, $\overline{E}_i = [\overline{E}_{1i}, \overline{E}_{2i}, \ldots, \overline{E}_{Ji}]$, $X_i = P_i^{-1}$, $F_{in} = K_{in} P_i^{-1}$.

Proof. Let the Lyapunov functional be

$$V(t) = \sum_{i=1}^{J} V_i(t) = \sum_{i=1}^{J} \left[x_i^T P_i x_i + \sum_{k=1}^{J} \int_{t-\tau_{ki}}^{t} x_k^T(s) \left(I + M_{ki}^T M_{ki} \right) x_k(s) ds \right], \tag{3.5}$$

where $P_i > 0$ is to be selected. It is obviously that there exist σ_1 and σ_2 such that

$$\sigma_1 \|x_i(t)\|^2 \le V_i(t) \le \sigma_2 \|x_i(t)\|^2. \tag{3.6}$$

Taking the derivative of the $V_i(t)$ along the trajectories of (3.3),

$$
\begin{aligned}
&\dot{V}_i(t)\\
&= \dot{x}_i^T(t)P_ix_i(t) + x_i^T(t)P_i\dot{x}_i^T(t) + \sum_{k=1}^{J} x_k^T(t)\left(I + M_{ki}^TM_{ki}\right)x_k(t)\\
&\quad - \sum_{k=1}^{J} x_k^T(t-\tau_{ki})\left(I + M_{ki}^TM_{ki}\right)x_k(t-\tau_{ki})\\
&= \sum_{j=1}^{r_i}\sum_{n=1}^{r_i} h_{ij}(t)h_{in}(t)\Big\{ x_i^T(t)\Big[\left(A_{ij} + \Delta A_{ij}\right)^TP_i + P_i\left(A_{ij} + \Delta A_{ij}\right)\\
&\qquad\qquad -K_{in}^T\left(B_{ij} + \Delta B_{ij}\right)^TP_i - P_i\left(B_{ij} + \Delta B_{ij}\right)K_{in}\Big]x_i(t)\\
&\qquad\qquad + 2x_i^T(t)P_i\sum_{k=1}^{J}(C_{ki}+\Delta C_{ki})x_k(t) + 2x_i^T(t)P_i\sum_{k=1}^{J}(D_{ki}+\Delta D_{ki})x_k(t-\tau_{ki})\\
&\qquad\qquad +\sum_{k=1}^{J} x_k^T(t)\left(I+M_{ki}^TM_{ki}\right)x_k(t) - \sum_{k=1}^{J} x_k^T(t-\tau_{ki})\left(I+M_{ki}^TM_{ki}\right)x_k(t-\tau_{ki})\Big\}\\
&= \sum_{j=1}^{r_i}\sum_{n=1}^{r_i} h_{ij}(t)h_{in}(t)\Big\{ x_i^T(t)\Big[N_{ij} + \Delta A_{ij}^TP_i + P_i\Delta A_{ij} - K_{in}^T\Delta B_{ij}^TP_i - P_i\Delta B_{ij}K_{in}\Big]x_i(t)\\
&\qquad\qquad + 2x_i^T(t)P_i\sum_{k=1}^{J} C_{ki}x_k(t) + 2x_i^T(t)P_i\sum_{k=1}^{J}\Delta C_{ki}x_k(t) + 2x_i^T(t)P_i\sum_{k=1}^{J} D_{ki}x_k(t)\\
&\qquad\qquad + 2x_i^T(t-\tau_{ki})P_i\sum_{k=1}^{J}\Delta D_{ki}x_k(t-\tau_{ki}) + \sum_{k=1}^{J} x_k^T(t)\left(I + M_{ki}^TM_{ki}\right)x_k(t)\\
&\qquad\qquad -\sum_{k=1}^{J} x_k^T(t-\tau_{ki})\left(I + M_{ki}^TM_{ki}\right)x_k(t-\tau_{ki})\Big\},
\end{aligned}
\tag{3.7}
$$

where $N_{ij} = A_{ij}^TP_i + P_iA_{ij} - K_{in}^TB_{ij}^TP_i - P_iB_{ij}K_{in}$.

Using Lemma (2.2), we have

$$
\begin{aligned}
&x_i^T(t)\Big[\Delta A_{ij}^TP_i + P_i\Delta A_{ij} - K_{in}^T\Delta B_{ij}^TP_i - P_i\Delta B_{ij}K_{in}\Big]x_i(t)\\
&\quad = x_i^T(t)\Big[P_iE_{ij}F_{ij}(t)\left(G_{ij} - H_{ij}K_{in}\right) + \left(G_{ij} - H_{ij}K_{in}\right)^TF_{ij}^T(t)E_{ij}^TP_i\Big]x_i(t)\\
&\quad \le x_i^T(t)\left(G_{ij} - H_{ij}K_{in}\right)^T\left(G_{ij} - H_{ij}K_{in}\right)x_i(t) + x_i^T(t)P_iE_{ij}E_{ij}^TP_ix_i(t),\\
&2x_i^T(t)P_i\sum_{k=1}^{J}\Delta C_{ki}x_k(t)\\
&\quad = 2\sum_{k=1}^{J} x_i^T(t)P_i\overline{E}_{ki}\overline{F}_{ki}(t)L_{ki}x_k(t)\\
&\quad \le x_i^T(t)P_i\sum_{k=1}^{J}\overline{E}_{ki}\overline{E}_{ki}^TP_ix_i(t) + \sum_{k=1}^{J} x_k^T(t)L_{ki}L_{ki}^Tx_k(t),
\end{aligned}
$$

$$
\begin{aligned}
&2x_i^T(t)P_i\sum_{k=1}^{J}\Delta D_{ki}x_k(t-\tau_{ki})\\
&\quad=2\sum_{k=1}^{J}x_i^T(t)P_i\overline{E}_{ki}\overline{F}_{ki}(t)M_{ki}x_k(t-\tau_{ki})\\
&\quad\le x_i^T(t)P_i\sum_{k=1}^{J}\overline{E}_{ki}\overline{E}_{ki}^TP_ix_i(t)+\sum_{k=1}^{J}x_k^T(t-\tau_{ki})M_{ki}M_{ki}^Tx_k(t-\tau_{ki}).
\end{aligned}
\tag{3.8}
$$

Using Lemma 2.1, we get

$$
\begin{aligned}
2x_i^T(t)P_i\sum_{k=1}^{J}C_{ki}x_k(t)&\le x_i^T(t)P_i\sum_{k=1}^{J}C_{ki}C_{ki}^TP_ix_i(t)+\sum_{k=1}^{J}x_k^T(t)x_k(t),\\
2x_i^T(t)P_i\sum_{k=1}^{J}D_{ki}x_k(t-\tau_{ki})&\le x_i^T(t)P_i\sum_{k=1}^{J}D_{ki}D_{ki}^TP_ix_i(t)+\sum_{k=1}^{J}x_k^T(t-\tau_{ki})x_k(t-\tau_{ki}),
\end{aligned}
\tag{3.9}
$$

Noticing that the facts as follows:

$$
\begin{aligned}
\sum_{i=1}^{J}\sum_{k=1}^{J}x_k^T(t)x_k(t)&=\sum_{i=1}^{J}\sum_{k=1}^{J}x_i^T(t)x_i(t)=\sum_{i=1}^{J}x_i^T(t)(JI)x_i(t),\\
\sum_{i=1}^{J}\sum_{k=1}^{J}x_k^T(t)L_{ki}L_{ki}^Tx_k(t)&=\sum_{i=1}^{J}\sum_{k=1}^{J}x_i^T(t)L_{ik}L_{ik}^Tx_i(t),\\
\sum_{i=1}^{J}\sum_{k=1}^{J}x_k^T(t)M_{ki}M_{ki}^Tx_k(t)&=\sum_{i=1}^{J}\sum_{k=1}^{J}x_i^T(t)M_{ik}M_{ik}^Tx_i(t)
\end{aligned}
\tag{3.10}
$$

Based on (3.7)–(3.10), we have

$$
\begin{aligned}
\dot{V}(t)&=\sum_{i=1}^{J}\dot{V}_i(t)\\
&\le\sum_{i=1}^{J}\Bigg\{\sum_{j=1}^{r_i}\sum_{n=1}^{r_i}h_{ij}(t)h_{in}(t)x_i^T(t)\\
&\qquad\times\Bigg[A_{ij}^TP_i+P_iA_{ij}-K_{in}^TB_{ij}^TP_i-P_iB_{ij}K_{in}+\left(G_{ij}-H_{ij}K_{in}\right)^T\times\left(G_{ij}-H_{ij}K_{in}\right)\\
&\qquad+P_iE_{ij}E_{ij}^TP_i+2P_i\sum_{k=1}^{J}\overline{E}_{ki}\overline{E}_{ki}^TP_i+2JI+P_i\sum_{k=1}^{J}C_{ki}C_{ki}^TP_i\\
&\qquad+P_i\sum_{k=1}^{J}D_{ki}D_{ki}^TP_i+\sum_{k=1}^{J}M_{ik}M_{ik}^T+\sum_{k=1}^{J}L_{ik}L_{ik}^T\Bigg]x_i(t)\Bigg\}.
\end{aligned}
\tag{3.11}
$$

From Schur complement, we know

$$A_{ij}^T P_i + P_i A_{ij} - K_{in}^T B_{ij}^T P_i - P_i B_{ij} K_{in} + \left(G_{ij} - H_{ij} K_{in}\right)^T \left(G_{ij} - H_{ij} K_{in}\right) + P_i E_{ij} E_{ij}^T P_i$$
$$+ 2P_i \sum_{k=1}^{J} \overline{E}_{ki} \overline{E}_{ki}^T Pi + 2JI + P_i \sum_{k=1}^{J} C_{ki} C_{ki}^T Pi + P_i \sum_{k=1}^{J} D_{ki} D_{ki}^T Pi + \sum_{k=1}^{J} M_{ik} M_{ik}^T + \sum_{k=1}^{J} L_{ik} L_{ik}^T < 0 \tag{3.12}$$

is equivalent to

$$\begin{pmatrix} A_{ij}^T P_i + P_i A_{ij} - K_{in}^T B_{ij}^T P_i - P_i B_{ij} K_{in} + 2JI & G_{ij}^T - K_{in}^T H_{ij} & M_i^T & C_i & E_i \\ G_{ij} - H_{ij} K_{in} & -I & 0 & 0 & 0 \\ M_i & 0 & -I & 0 & 0 \\ C_i^T & 0 & 0 & -I & 0 \\ E_i^T & 0 & 0 & 0 & -\frac{1}{2} I \end{pmatrix} < 0, \tag{3.13}$$

where $M_i^T = [M_{i1}, M_{i2}, \ldots, M_{iJ}, L_{i1}, L_{i2}, \ldots, L_{iJ}]^T$, $C_i = P_i[E_{ij}, C_{1i}, C_{2i}, \ldots, C_{Ji}, D_{1i}, D_{2i}, \ldots, D_{Ji}]$, $E_i = P_i[\overline{E}_{1i}, \overline{E}_{2i}, \ldots, \overline{E}_{Ji}]$.

So we have $\dot{V}(t) < 0$ while $x_i(t)$ $(i = 1, 2, \ldots, J)$ are not all zero vectors. Note that the matrix inequalities in (3.13) can be transformed into certain forms of linear matrix inequalities (LMIs). Therefore, multiplying both sides of matrix inequalities (3.13) by $\mathrm{diag}\{P_i^{-1}, I, I, I, I\}$ and applying the change of variables such that $P_i = X_i^{-1}$, $K_{in} = F_{in} X_i^{-1}$, $(i = 1, 2, \ldots, J,\ n = 1, 2, \ldots, r_i)$, then (3.4) is obtained. □

With the similar proof of Theorem 3.1, the stabilization criterion of large-scale system (2.1) without uncertainties is also discussed. The result is presented as follows.

Corollary 3.2. *The fuzzy large-scale system* (2.1) *without uncertainties can be asymptotically stabilized by the decentralized PDC fuzzy control* (3.2)*, if there exist matrices F_{ij}, positive definite matrices X_i to satisfy the following LMIs:*

$$\begin{pmatrix} X_i A_{ij}^T + A_{ij} X_i - F_{in}^T B_{ij}^T - B_{ij} F_{in} & \overline{C}_i & X_i \\ \overline{C}_i^T & -I & 0 \\ X_i & 0 & -\frac{1}{2J} I \end{pmatrix} < 0 \tag{3.14}$$

for $i = 1, 2, \ldots, J$, $j = 1, 2, \ldots, r_i$, $n = 1, 2, \ldots, r_i$, where $\overline{C}_i = [C_{1i}, C_{2i}, \ldots, C_{Ji}, D_{1i}, D_{2i}, \ldots, D_{Ji}]$.

Proof. The proof is similar with that of Theorem 3.1; therefore details are omitted. □

Remark 3.3. This corollary is similar with Theorem 1 in [10], so the theorem in this paper is more general.

Theorem 3.4. *If there exist matrices F_{ij}, positive definite matrices X_i to satisfy the following LMIs:*

$$\begin{pmatrix} X_iA_{ij}^T + A_{ij}X_i - F_{in}^TB_{ij}^T - B_{ij}F_{in} & X_iG_{ij}^T - F_{in}^TH_{ij}^T & X_iM_i^T & \overline{C}_i & \overline{E}_i & X_i & X_i & F_{in}^T \\ G_{ij}X_i - H_{ij}F_{in} & -I & 0 & 0 & 0 & 0 & 0 & 0 \\ M_iX_i & 0 & -I & 0 & 0 & 0 & 0 & 0 \\ \overline{C}_i^T & 0 & 0 & -I & 0 & 0 & 0 & 0 \\ \overline{E}_i^T & 0 & 0 & 0 & -I & 0 & 0 & 0 \\ X_i & 0 & 0 & 0 & 0 & -\dfrac{1}{2J}I & 0 & 0 \\ X_i & 0 & 0 & 0 & 0 & 0 & -Q_i^{-1} & 0 \\ F_{in} & 0 & 0 & 0 & 0 & 0 & 0 & -R_i^{-1} \end{pmatrix} < 0, \tag{3.15}$$

the fuzzy large-scale system (2.1) *can be asymptotically stabilized by the decentralized PDC fuzzy control* (3.2)*, and the performance index* (2.8) *is satisfied the following inequality:*

$$J \le J^* = \sum_{i=1}^{J}\left[x_i^T(0)P_ix_i(0) + \sum_{k=1}^{J}\int_{-\tau_{ki}}^{0} x_k^T(s)\left(I + M_{ki}^TM_{ki}\right)x_k(s)ds\right], \tag{3.16}$$

for $i = 1,2,\ldots,J$, $j = 1,2,\ldots,r_i$, $n = 1,2,\ldots,r_i$, *where* $M_i^T = [M_{i1}, M_{i2},\ldots,M_{iJ}, L_{i1}, L_{i2},\ldots, L_{iJ}]^T$, $\overline{C}_i = [E_{ij}, C_{1i}, C_{2i},\ldots,C_{Ji}, D_{1i}, D_{2i},\ldots,D_{Ji}]$, $\overline{E}_i = [\overline{E}_{1i}, \overline{E}_{2i},\ldots,\overline{E}_{Ji}]$, $X_i = P_i^{-1}$, $F_{in} = K_{in}P_i^{-1}$.

Proof. From the proof of Theorem 3.1, we know if

$$\begin{aligned} G_{ij} = {} & A_{ij}^TP_i + P_iA_{ij} - K_{in}^TB_{ij}^TP_i - P_iB_{ij}K_{in} + \left(G_{ij} - H_{ij}K_{in}\right)^T\left(G_{ij} - H_{ij}K_{in}\right) + P_iE_{ij}E_{ij}^TP_i \\ & + 2P_i\sum_{k=1}^{J}\overline{E}_{ki}\overline{E}_{ki}^TPi + 2JI + P_i\sum_{k=1}^{J}C_{ki}C_{ki}^TPi + P_i\sum_{k=1}^{J}D_{ki}D_{ki}^TPi + \sum_{k=1}^{J}M_{ik}M_{ik}^T + \sum_{k=1}^{J}L_{ik}L_{ik}^T < 0 \end{aligned} \tag{3.17}$$

then $\dot{V}(t) < 0$. Noticing that Q_i and R_i are positive matrices, if inequality

$$\begin{aligned} & A_{ij}^TP_i + P_iA_{ij} - K_{in}^TB_{ij}^TP_i - P_iB_{ij}K_{in} + \left(G_{ij} - H_{ij}K_{in}\right)^T\left(G_{ij} - H_{ij}K_{in}\right) + P_iE_{ij}E_{ij}^TP_i \\ & \quad + 2P_i\sum_{k=1}^{J}\overline{E}_{ki}\overline{E}_{ki}^TPi + 2JI + P_i\sum_{k=1}^{J}C_{ki}C_{ki}^TPi + P_i\sum_{k=1}^{J}D_{ki}D_{ki}^TPi + \sum_{k=1}^{J}M_{ik}M_{ik}^T + \sum_{k=1}^{J}L_{ik}L_{ik}^T + Q_i \\ & \quad + K_{in}^TR_iK_{in} < 0, \end{aligned} \tag{3.18}$$

holds, we obtain $\dot{V}(t) < 0$:

$$\begin{aligned} J &= \sum_{i=1}^{J} \int_0^{\infty} \left(x_i^T(t) Q_i x_i(t) + u_i^T(t) R_i u_i(t) \right) dt \\ &= \sum_{i=1}^{J} \int_0^{\infty} x_i^T(t) \left(Q_i + \sum_{n=1}^{r_i} h_{in}(t) K_{in}^T R_i \sum_{n=1}^{r_i} h_{in}(t) K_{in} \right) x_i(t) dt. \end{aligned} \tag{3.19}$$

Using Lemma 2.3, we get

$$J \leq \sum_{i=1}^{J} \int_0^{\infty} x_i^T(t) \left(Q_i + \sum_{n=1}^{r_i} h_{in}(t) K_{in}^T R_i K_{in} \right) x_i(t) dt. \tag{3.20}$$

If (3.18) holds, we have

$$\begin{aligned} J &\leq -\sum_{i=1}^{J} \sum_{j=1}^{r_i} \sum_{n=1}^{r_i} \int_0^{\infty} h_{ij}(t) h_{in}(t) x_i^T(t) G_{ij} x_i(t) dt \\ &\leq -\sum_{i=1}^{J} \int_0^{\infty} \dot{V}_i(t) dt = V(0) - V(\infty) \\ &= \sum_{i=1}^{J} \left[x_i^T(0) P_i x_i(0) + \sum_{k=1}^{J} \int_{-\tau_{ki}}^{0} \left(I + M_{ki}^T M_{ki} \right) ds \right]. \end{aligned} \tag{3.21}$$

Therefore, multiply both sides of (3.18) by P_i^{-1}, and let $X_i = P_i^{-1}$, $F_{in} = K_{in} P_i^{-1}$. From Schur complement, the proof is completed. □

4. Numerical Examples

In this section, some numerical simulations for uncertain fuzzy large-scale system will be given to illustrate the effectiveness of the proposed stabilization criteria and also compared with the existing results.

Example 4.1. Consider an uncertain FLSS time delays S composed of three fuzzy subsystems S_i, $i = 1, 2, 3$ by the following equations:

$$\begin{aligned} \dot{x}_1(t) = &\sum_{j=1}^{2} h_{1j}(t) \left[\left(A_{1j} + \Delta A_{1j} \right) x_1(t) + \left(B_{1j} + \Delta B_{1j} \right) u_1(t) \right] \\ &+ \sum_{k=1}^{3} \left[(C_{k1} + \Delta C_{k1}) x_k(t) + (D_{k1} + \Delta D_{k1}) x_k(t - \tau_{k1}) \right], \end{aligned}$$

$$\begin{aligned}
\dot{x}_2(t) &= \sum_{j=1}^{2} h_{2j}(t)\left[\left(A_{2j}+\Delta A_{2j}\right)x_2(t)+\left(B_{2j}+\Delta B_{2j}\right)u_2(t)\right] \\
&\quad + \sum_{k=1}^{3}\left[(C_{k2}+\Delta C_{k2})x_k(t)+(D_{k2}+\Delta D_{k2})x_k(t-\tau_{k2})\right], \\
\dot{x}_3(t) &= \sum_{j=1}^{2} h_{3j}(t)\left[\left(A_{3j}+\Delta A_{3j}\right)x_3(t)+\left(B_{3j}+\Delta B_{3j}\right)u_3(t)\right] \\
&\quad + \sum_{k=1}^{3}\left[(C_{k3}+\Delta C_{k3})x_k(t)+(D_{k3}+\Delta D_{k3})x_k(t-\tau_{k3})\right],
\end{aligned} \tag{4.1}$$

in which $x_1^T(t) = [x_{11}, x_{21}]^T$, $x_2^T(t) = [x_{12}, x_{22}]^T$, $x_1^T(t) = [x_{13}, x_{23}]^T$

$$\begin{aligned}
&\left[\Delta A_{1j}, \Delta B_{1j}\right] = E_{1j}F_{1j}\left[G_{1j}, H_{1j}\right], \qquad \left[\Delta A_{2j}, \Delta B_{2j}\right] = E_{2j}F_{2j}\left[G_{2j}, H_{2j}\right], \\
&\left[\Delta A_{3j}, \Delta B_{3j}\right] = E_{3j}F_{3j}\left[G_{3j}, H_{3j}\right], \qquad \left[\Delta C_{k1}, \Delta D_{k1}\right] = \overline{E}_{k1}\overline{F}_{k1}\left[L_{k1}, M_{k1}\right], \\
&\left[\Delta C_{k2}, \Delta D_{k2}\right] = \overline{E}_{k2}\overline{F}_{k2}\left[L_{k2}, M_{k2}\right], \qquad \left[\Delta C_{k3}, \Delta D_{k3}\right] = \overline{E}_{k3}\overline{F}_{k3}\left[L_{k3}, M_{k3}\right],
\end{aligned} \tag{4.2}$$

for $i, k = 1, 2, 3$, $j = 1, 2$.

$$A_{11} = \begin{pmatrix} 0.6 & 2.4 \\ 0.209 & 1.9 \end{pmatrix}, \quad A_{12} = \begin{pmatrix} 0.6 & 2.4 \\ 0 & 1.9 \end{pmatrix}, \quad A_{21} = \begin{pmatrix} -1 & -0.5 \\ 1 & 3 \end{pmatrix},$$

$$A_{22} = \begin{pmatrix} 0.6 & 2.4 \\ -0.209 & 1.9 \end{pmatrix}, \quad A_{31} = \begin{pmatrix} 0.6 & 1.2 \\ 0.209 & 1.9 \end{pmatrix}, \quad A_{32} = \begin{pmatrix} 0.6 & 1.2 \\ 0 & 1.9 \end{pmatrix},$$

$$B_{11} = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}, \quad B_{12} = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}, \quad B_{21} = \begin{pmatrix} 3 & 1 \\ 0 & 3 \end{pmatrix},$$

$$B_{22} = \begin{pmatrix} 2 & 0 \\ 3 & 2 \end{pmatrix}, \quad B_{31} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad B_{32} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix},$$

$$C_{11} = \begin{pmatrix} -0.1 & 0.1 \\ 0 & 0 \end{pmatrix}, \quad C_{21} = \begin{pmatrix} 0 & 0 \\ 0.1 & 0.1 \end{pmatrix}, \quad C_{31} = \begin{pmatrix} -0.2 & 0.15 \\ 0 & -1 \end{pmatrix},$$

$$C_{12} = \begin{pmatrix} 0 & 0 \\ 0.1 & 0.1 \end{pmatrix}, \quad C_{22} = \begin{pmatrix} 0 & 0 \\ 0 & 0.1 \end{pmatrix}, \quad C_{32} = \begin{pmatrix} -0.3 & -0.2 \\ 0.2 & 0.1 \end{pmatrix},$$

$$C_{13} = \begin{pmatrix} -0.1 & 0 \\ 0 & 0 \end{pmatrix}, \quad C_{23} = \begin{pmatrix} 0.1 & 0 \\ 0 & 0.1 \end{pmatrix}, \quad C_{33} = \begin{pmatrix} 0 & 0 \\ 0.1 & 0 \end{pmatrix},$$

$$D_{11} = \begin{pmatrix} 0.1 & 0.1 \\ 0 & 0 \end{pmatrix}, \quad D_{21} = \begin{pmatrix} 0.1 & 0 \\ 0 & 0.1 \end{pmatrix}, \quad D_{31} = \begin{pmatrix} -1.08 & 0.36 \\ 0 & -0.72 \end{pmatrix},$$

$$D_{12} = \begin{pmatrix} 0.1 & 0 \\ -0.1 & 0.1 \end{pmatrix}, \quad D_{22} = \begin{pmatrix} 0 & 0.1 \\ 0 & 0 \end{pmatrix}, \quad D_{32} = \begin{pmatrix} 0.6 & -0.3 \\ 0.1 & 0 \end{pmatrix},$$

$$D_{13} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \quad D_{23} = \begin{pmatrix} -0.1 & 0 \\ 0 & 0 \end{pmatrix}, \quad D_{33} = \begin{pmatrix} -0.1 & 0 \\ 0 & -0.1 \end{pmatrix},$$

$$E_{11} = \begin{pmatrix} -0.28 & -0.028 \\ -0.14 & 0.182 \end{pmatrix}, \quad E_{12} = \begin{pmatrix} -0.21 & -0.084 \\ -1.26 & -0.42 \end{pmatrix}, \quad E_{21} = \begin{pmatrix} -0.5 & 0.25 \\ 1.35 & 0.75 \end{pmatrix},$$

$$
\begin{aligned}
&E_{22} = \begin{pmatrix} -0.27 & 0 \\ 75.0 & 0.81 \end{pmatrix}, \quad E_{31} = \begin{pmatrix} -0.28 & 0.28 \\ 0.14 & 0.182 \end{pmatrix}, \quad E_{32} = \begin{pmatrix} 0.21 & 0.84 \\ 1.26 & -0.42 \end{pmatrix}, \\
&\overline{E}_{11} = \begin{pmatrix} -0.28 & 0.028 \\ -0.14 & 0.182 \end{pmatrix}, \quad \overline{E}_{21} = \begin{pmatrix} -0.21 & -840.0 \\ -1.26 & -0.42 \end{pmatrix}, \quad \overline{E}_{31} = \begin{pmatrix} 0.21 & -0.084 \\ -1.26 & 0.42 \end{pmatrix}, \\
&\overline{E}_{12} = \begin{pmatrix} -0.5 & 0.25 \\ 1.35 & 0.75 \end{pmatrix}, \quad \overline{E}_{22} = \begin{pmatrix} -0.27 & 0 \\ 0.81 & 0.81 \end{pmatrix}, \quad \overline{E}_{32} = \begin{pmatrix} -0.27 & 0 \\ 0.75 & 0.81 \end{pmatrix}, \\
&\overline{E}_{13} = \begin{pmatrix} -0.28 & 0.28 \\ 0.14 & 0.182 \end{pmatrix}, \quad \overline{E}_{23} = \begin{pmatrix} 0.21 & 0.84 \\ 1.26 & -0.42 \end{pmatrix}, \quad \overline{E}_{33} = \begin{pmatrix} -0.21 & 0.84 \\ 1.26 & 0.42 \end{pmatrix}, \\
&F_{11}(t) = \begin{pmatrix} 1-\sin^4 t & 0 \\ 0 & 1-\cos^4 t \end{pmatrix}, \quad \overline{F}_{11}(t) = \begin{pmatrix} \sin t^2 & 0 \\ 0 & \cos t^2 \end{pmatrix}, \\
&G_{11} = H_{11} = E_{11}, \quad G_{12} = H_{12} = E_{12}, \quad G_{21} = H_{21} = E_{21}, \quad G_{22} = H_{22} = E_{22}, \\
&G_{31} = H_{31} = E_{31}, \quad G_{32} = H_{32} = E_{32}, \quad L_{11} = M_{11} = \overline{E}_{11}, \quad L_{21} = M_{21} = \overline{E}_{21}, \\
&L_{31} = M_{31} = \overline{E}_{31}, \quad L_{12} = M_{12} = \overline{E}_{12}, \quad L_{22} = M_{22} = \overline{E}_{22}, \quad L_{32} = M_{32} = \overline{E}_{32}, \\
&L_{13} = M_{13} = \overline{E}_{13}, \quad L_{23} = M_{23} = \overline{E}_{23}, \quad L_{33} = M_{33} = \overline{E}_{33}, \\
&F_{11}(t) = F_{12}(t) = F_{21}(t) = F_{22}(t) = F_{31}(t) = F_{32}(t), \\
&\overline{F}_{11}(t) = \overline{F}_{21}(t) = \overline{F}_{31}(t) = \overline{F}_{12}(t) = \overline{F}_{22}(t) = \overline{F}_{32}(t) = \overline{F}_{13}(t) = \overline{F}_{23}(t) = \overline{F}_{33}(t).
\end{aligned}
\tag{4.3}
$$

Here, the T-S fuzzy models of the isolated subsystem are of the following:

Subsystem 1

Rule 1. If $x_{11}(t)$ is M_{111} then

$$\dot{x}_1(t) = (A_{11} + \Delta A_{11}(t))x_1(t) + (B_{11} + \Delta B_{11}(t))u_1(t). \tag{4.4}$$

Rule 2. If $x_{11}(t)$ is M_{211} then

$$\dot{x}_1(t) = (A_{12} + \Delta A_{12}(t))x_1(t) + (B_{12} + \Delta B_{12}(t))u_1(t), \tag{4.5}$$

and the membership functions for Rules 1 and 2 are, respectively, $M_{111}(x_{11}(t)) = 1/(1 + \exp(-3x_{11}(t)))$, $M_{211}(x_{11}(t)) = 1 - M_{111}(x_{11}(t))$.

Subsystem 2

Rule 1. If $x_{12}(t)$ is M_{112} Then

$$\dot{x}_2(t) = (A_{21} + \Delta A_{21}(t))x_2(t) + (B_{21} + \Delta B_{21}(t))u_2(t). \tag{4.6}$$

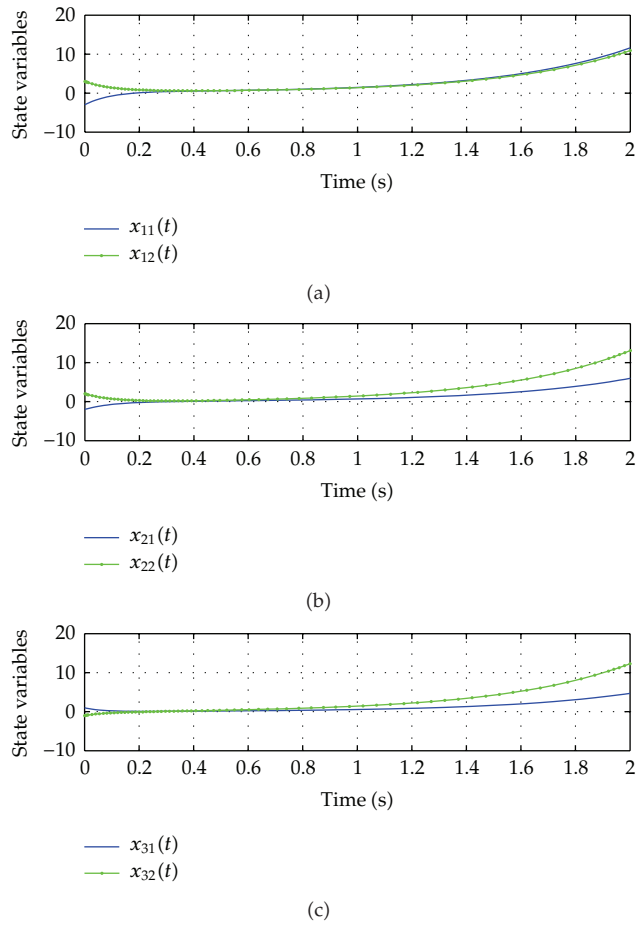

Figure 1: The state response with $u_i(t) = 0$.

Rule 2. If $x_{12}(t)$ is M_{212} then

$$\dot{x}_2(t) = (A_{22} + \Delta A_{22}(t))x_2(t) + (B_{22} + \Delta B_{22}(t))u_2(t), \tag{4.7}$$

and the membership functions for Rules 1 and 2 are, respectively, $M_{112}(x_{12}(t)) = \exp(-2x_{12}^2(t))$, $M_{212}(x_{12}(t)) = 1 - M_{112}(x_{12}(t))$.

Subsystem 3

Rule 1. If $x_{13}(t)$ is M_{113} then

$$\dot{x}_3(t) = (A_{31} + \Delta A_{31}(t))x_3(t) + (B_{31} + \Delta B_{31}(t))u_3(t). \tag{4.8}$$

Rule 2. If $x_{13}(t)$ is M_{213} then

$$\dot{x}_3(t) = (A_{32} + \Delta A_{32}(t))x_3(t) + (B_{32} + \Delta B_{32}(t))u_3(t), \tag{4.9}$$

and the membership functions for Rules 1 and 2 are, respectively, $M_{113}(x_{13}(t)) = 1/(1 + \exp(-1.5x_{13}(t)))$, $M_{213}(x_{13}(t)) = 1 - M_{113}(x_{13}(t))$.

It is noted that the above large-scale system without control ($u_i(t) = 0$) has unstable responses with initial conditions $x_1(t) = [-3, 3]^T$, $x_2(t) = [-2, 2]^T$, $x_3(t) = [1, -1]^T$ as shown in Figure 1.

In order to stabilize the large-scale fuzzy system, three decentralized PDC fuzzy controllers are designed in the following.

Fuzzy Controller C_1

Rule 1. If $x_{11}(t)$ is M_{111} then

$$u_1(t) = -K_{11}x_1(t). \tag{4.10}$$

Rule 2. If $x_{11}(t)$ is M_{211} then

$$u_1(t) = -K_{12}x_1(t). \tag{4.11}$$

Fuzzy Controller C_2

Rule 1. If $x_{12}(t)$ is M_{112} then

$$u_2(t) = -K_{21}x_2(t). \tag{4.12}$$

Rule 2. If $x_{12}(t)$ is M_{212} then

$$u_2(t) = -K_{22}x_2(t). \tag{4.13}$$

Fuzzy Controller C_3

Rule 1. If $x_{13}(t)$ is M_{113} then

$$u_3(t) = -K_{31}x_3(t). \tag{4.14}$$

Rule 2. If $x_{13}(t)$ is M_{213} Then

$$u_3(t) = -K_{32}x_3(t). \tag{4.15}$$

By using the approaches of Theorem 3.1, we obtain the matrices K_{ij} for subsystems S_1, S_2, and S_3:

$$K_{11} = \begin{pmatrix} 1.5681 & 0.4733 \\ 0.5296 & 3.0430 \end{pmatrix}, \quad K_{12} = \begin{pmatrix} 2.2261 & 0.2680 \\ 0.5801 & 1.7275 \end{pmatrix}, \quad K_{21} = \begin{pmatrix} 0.8717 & 0.7769 \\ -0.4856 & 1.2398 \end{pmatrix},$$
$$K_{22} = \begin{pmatrix} 1.4831 & 0.5179 \\ 0.2098 & 1.7072 \end{pmatrix}, \quad K_{31} = \begin{pmatrix} 1.6548 & 0.7281 \\ 0.4122 & 2.9992 \end{pmatrix}, \quad K_{32} = \begin{pmatrix} 1.5769 & 0.4958 \\ 0.5823 & 2.0029 \end{pmatrix}. \tag{4.16}$$

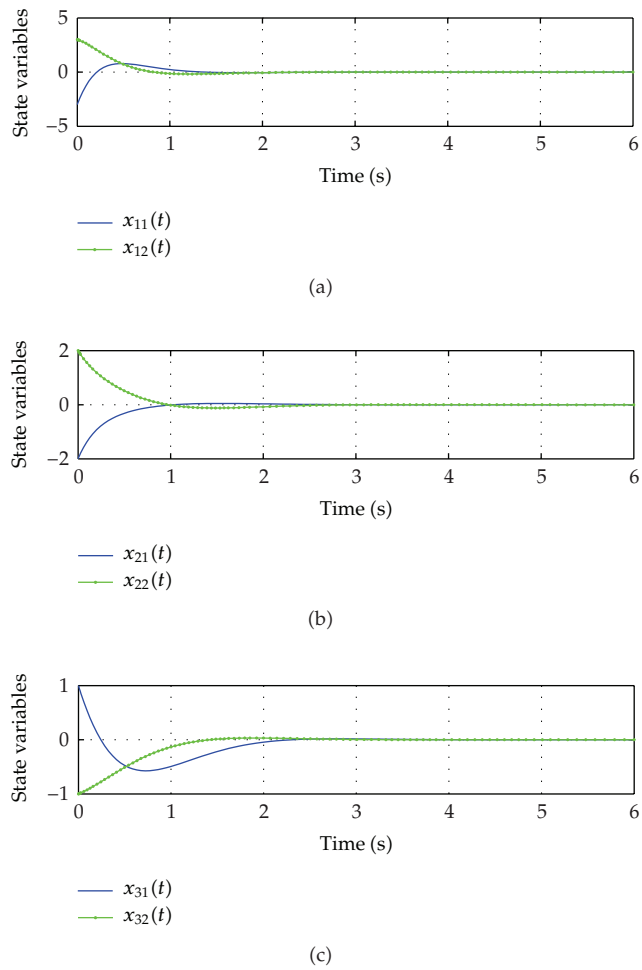

Figure 2: The state response with fuzzy control by Theorem 3.1.

The complete simulation results with initial conditions $x_1(t) = [-3,3]^T$, $x_2(t) = [-2,2]^T$, $x_3(t) = [1,-1]^T$ are shown in Figure 2. It is obvious that they are stabilized asymptotically.

Example 4.2. Consider the fuzzy large-scale system S composed of two subsystems S_i as follows [25]:

$$\begin{aligned}\dot{x}_1(t) &= \sum_{j=1}^{2} h_{1j}(t)\left(A_{1j}x_1(t) + B_{1j}u_1(t)\right) + \sum_{h=1}^{2} C_{h1}x_h(t),\\ \dot{x}_2(t) &= \sum_{j=1}^{2} h_{2j}(t)\left(A_{2j}x_2(t) + B_{2j}u_2(t)\right) + \sum_{h=1}^{2} C_{h2}x_h(t),\end{aligned} \tag{4.17}$$

where $x_1(t) = [x_{11}(t), x_{12}(t)]^T, x_2(t) = [x_{21}(t), x_{22}(t)]^T$

$$\begin{gathered} A_{11} = \begin{pmatrix} 5 & 1 \\ 0 & -4 \end{pmatrix}, \qquad A_{12} = \begin{pmatrix} -4 & 0 \\ 2 & -8 \end{pmatrix}, \qquad B_{11} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \\ B_{12} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \qquad C_{11} = C_{22} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \qquad A_{21} = \begin{pmatrix} -3 & 0 \\ 4 & -2 \end{pmatrix}, \\ A_{22} = \begin{pmatrix} -4 & -5 \\ 1 & -3 \end{pmatrix}, \qquad B_{21} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}, \qquad B_{22} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}, \\ C_{21} = \begin{pmatrix} 5 & 1 \\ 0 & 0 \end{pmatrix}, \qquad C_{12} = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix} \end{gathered} \tag{4.18}$$

and the membership functions are, respectively, $M_{111}(x_{11}(t)) = \exp(-x_{11}^2(t))$, $M_{211}(x_{11}(t)) = 1 - M_{111}(x_{11}(t))$, $M_{112}(x_{21}(t)) = 1/(1 + \exp(-x_{21}(t)))$, $M_{212}(x_{21}(t)) = 1 - M_{112}(x_{21}(t))$.

In order to stabilize the large-sale fuzzy system, three decentralized PDC fuzzy controllers are designed in the following.

Fuzzy Controller C_1

Rule 1. If $x_{11}(t)$ is M_{111} then

$$u_1(t) = -K_{11}x_1(t). \tag{4.19}$$

Rule 2. If $x_{11}(t)$ is M_{211} then

$$u_1(t) = -K_{12}x_1(t). \tag{4.20}$$

Fuzzy Controller C_2

Rule 1. If $x_{21}(t)$ is M_{112} then

$$u_2(t) = -K_{21}x_2(t). \tag{4.21}$$

Rule 2. If $x_{21}(t)$ is M_{212} then

$$u_2(t) = -K_{22}x_2(t). \tag{4.22}$$

We let $Q_1 = Q_2 = \left(\begin{smallmatrix} 5 & 0 \\ 0 & 1 \end{smallmatrix}\right)$, $R_1 = 1$, $R_2 = 2$. By using the approaches of Theorem 3.4, we obtain

$$P_1 = \begin{pmatrix} 5.5084 & 1.6329 \\ 1.6329 & 1.5120 \end{pmatrix}, \qquad P_2 = \begin{pmatrix} 0.9692 & -0.0406 \\ -0.0406 & 1.0681 \end{pmatrix}. \tag{4.23}$$

The control gain K_{ij} for subsystem S_1, S_2 is compared with [24] in Table 1. The complete simulation results with initial conditions $x_1(t) = [1, -1]^T, x_2(t) = [2, -2]^T$ are shown in Figure 3.

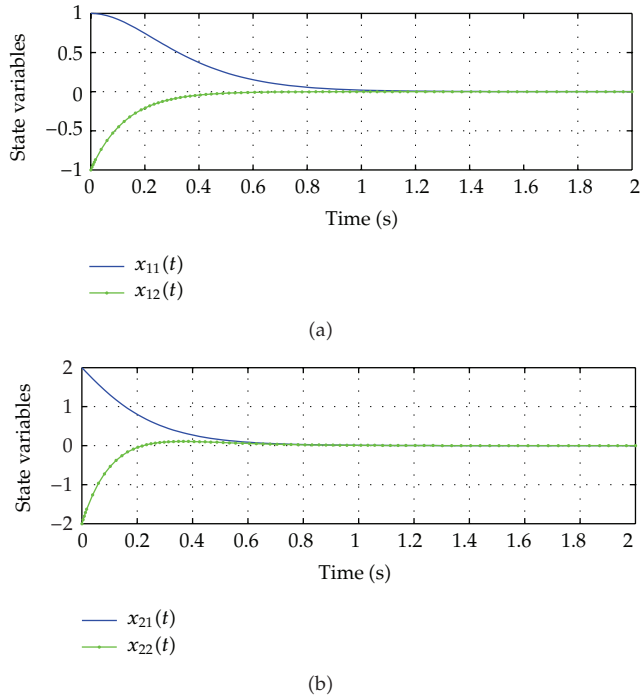

Figure 3: The state response with fuzzy control by Theorem 3.4.

Table 1: Comparison of fuzzy controller, for example, with Zhang et al. [25].

Index	K_{11}	K_{12}	K_{21}	K_{22}	J^*	$\|K\|_2$
Zhang et al. [25]	[10.7602 4.8931]	[7.1424 1.0899]	[2.2702 0.1348]	[1.9527 0.8715]	21.0606	14.2011
This paper	[9.0414 1.0000]	[2.0000 1.8284]	[1.1604 0.0740]	[0.3515 2.2722]	12.2288	9.8351

Remark 4.3. From Figure 3, we can see that the system can be stabilized through appropriate decentralized control. Obviously, in Table 1, we can see the method of this paper has smaller gain matrices and performance index, so it has better control effectiveness.

5. Conclusions

In this paper we explore the stabilization problems for uncertain fuzzy large-scale system with time delays. The decentralized PDC fuzzy controller has been designed under some conditions such that the whole closed-loop large-scale fuzzy system is asymptotically stable. Then, the optimization design method for decentralized control is also considered with respect to a quadratic performance index. Finally, numerical examples are provided to demonstrate the correctness and less conservativeness of the theoretical results. However, there are still some other problems to be addressed, such that time-varying delays and delay-dependent stability and stabilization of fuzzy large-scale system and the results developed in the paper can be extended to the case that the underlying systems are invovled with any switching dynamics.

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