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Research Article

Global Stability of Multigroup Dengue Disease Transmission Model

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We investigate a class of multigroup dengue epidemic model. We show that the global dynamics are determined by the basic reproductive number R_0. We present that when $R_0 \leq 1$, there is a unique disease-free equilibrium which is globally asymptotically stable; when $R_0 > 1$, there exists a unique endemic equilibrium and it is globally asymptotically stable proved by a graph-theoretic approach to the method of global Lyapunov function.

1. Introduction

To understand and control the spread of infectious disease in population, mathematical epidemic models have been paid more attention. One essential assumption in most classical epidemic models is that the individuals are homogeneously mixed. However, many infectious diseases, such as measles, mumps, and gonorrhea, occur in heterogeneous host population, so multigroup epidemic models seem more reasonable. One of the earliest multigroup models is analysed by Lajmanovich and Yorke [1] for gonorrhea in a nonhomogeneous population. However, because of the large scale and complexity of multigroup models, progresses in the mathematical analysis of their global dynamics have been slow, particularly, the question of uniqueness and global stability of the endemic equilibrium. Recently, a graph-theoretic approach to the method of global Lyapunov functions in [2, 3] was proposed to resolve the open problem on the uniqueness and global stability of the endemic equilibrium. Subsequently, a series of good results were produced about multigroup epidemic models in [4–8].

In this paper, we study a multigroup dengue disease transmission model by the method in [2, 3]. In the model, the population is divided into n groups. Each group is divided into five disjoint classes: susceptible individuals, infective individuals, removed individuals, susceptible mosquitoes, and infective mosquitoes whose numbers of individuals at time t are denoted by $S_{H_i}(t)$, $I_{H_i}(t)$, $R_{H_i}(t)$, $S_{V_i}(t)$, $I_{V_i}(t)$, respectively. The model to be studied takes the following form:

$$\begin{aligned}
S'_{H_i} &= A_{H_i} - \sum_{j=1}^{n} \beta_{H_{ij}} S_{H_i} I_{V_j} - \mu_{H_i} S_{H_i},\\
I'_{H_i} &= \sum_{j=1}^{n} \beta_{H_{ij}} S_{H_i} I_{V_j} - \left(\mu_{H_i} + \gamma_{H_i}\right) I_{H_i},\\
R'_{H_i} &= \gamma_{H_i} I_{H_i} - \mu_{H_i} R_{H_i},\\
S'_{V_i} &= A_{V_i} - \sum_{j=1}^{n} \beta_{V_{ij}} S_{V_i} I_{H_j} - \mu_{V_i} S_{V_i},\\
I'_{V_i} &= \sum_{j=1}^{n} \beta_{V_{ij}} S_{V_i} I_{H_j} - \mu_{V_i} I_{V_i},
\end{aligned} \tag{1.1}$$

where $i = 1, 2, \ldots, n$. Here A_{H_i} and A_{V_i} represent the recruitment rate of the humans and the mosquitoes in the ith group, $\beta_{H_{ij}}$ represents the contact rate between susceptible humans S_{H_i} and infectious mosquitoes I_{V_j}, $\beta_{V_{ij}}$ is the contact rate between infected people I_{H_j} and susceptible mosquitoes S_{V_i}, μ_{H_i} and μ_{V_i} represent the death rate of the humans and the mosquitoes in the ith group, and γ_{H_i} represents the recovery rate of the humans in the ith group. All parameter values are assumed to be nonnegative and $A_{H_i}, A_{V_i}, \mu_{H_i}, \mu_{V_i} > 0$.

Dengue fever (DF) is an acute mosquito-transmitted disease, with a recorded prevalence in 101 countries [9–11]. An estimated 50–100 million people per year are infected, with approximately 25,000 deaths annually [12]. Thus, the study of DF is perceived as signification and receives much attention. When $n = 1$, the model (1.1) had been studied extensively. For example, the global stability of the equilibria was proved with the results of the theory of competitive systems and stability of periodic orbits in [13]; in [14], the global stability of the equilibria was proved with Lyapunov functions under some conditions.

The organization of this paper is as follows. In Section 2, we quote some results from graph theory which will be used in the proof of our main results. In Section 3, we present a global analysis of the system (1.1). At Section 4, we give a further discussion.

2. Preliminaries

In this section, we will give some previous results which will be useful for our main results.

Definition 2.1 (see [15]). *Let $U = (u_{ij})_{n\times n}$. We say that $U \geq 0$ (U is nonnegative), if all its entries u_{ij} are real and nonnegative.*

If $U = (u_{ij})_{n\times n}$ and $W = (w_{ij})_{n\times n}$ are both nonnegative, we write $U \geq W$ if $u_{ij} \geq w_{ij}$ for all i and j, and $U > W$ if $u_{ij} \geq w_{ij}$ and $U \neq W$.

Definition 2.2 (see [15]). *A matrix $U = (u_{ij})_{n\times n}$ is said to be reducible if either*

(i) $n = 1$ and $U = 0$; or

(ii) $n \geq 2$, there is a permutation matrix P,

$$PUP^T = \begin{pmatrix} U_1 & 0 \\ U_2 & U_3 \end{pmatrix}, \tag{2.1}$$

where U_1 and U_3 are square matrices. Otherwise, U is irreducible.

Let $\Gamma(U)$ denote the directed graph of $(u_{ij})_{n\times n}$. We have the following proposition.

Proposition 2.3 (see [16]). *For matrix U, one has*

(i) *If U is nonnegative, then the spectral radius $\rho(U)$ of U is an eigenvalue, and U has a nonnegative eigenvector corresponding to $\rho(U)$.*

(ii) *If U is nonnegative and irreducible, then $\rho(U)$ is a simple eigenvalue, and U has a positive eigenvector x corresponding to $\rho(U)$.*

(iii) *If $0 < W < U$, then $\rho(W) \leq \rho(U)$. Moreover, if $0 < W < U$ and $W + U$ is irreducible, then $\rho(W) < \rho(U)$.*

(iv) *If U is nonnegative and irreducible, and W is diagonal and positive (namely, all of its entries are positive), then UW is irreducible.*

(v) *Matrix U is irreducible if and only if $\Gamma(U)$ is strongly connected.*

3. Mathematical Analysis

From the first and the fourth equation in (1.1), we know

$$\limsup_{t\longrightarrow\infty} S_{H_i} \leq \frac{A_{H_i}}{\mu_{H_i}}, \qquad \limsup_{t\longrightarrow\infty} S_{V_i} \leq \frac{A_{V_i}}{\mu_{V_i}}. \tag{3.1}$$

For each i, adding the five equations in (1.1), we obtain

$$\begin{aligned}(S_{H_i} + I_{H_i} + R_{H_i} + S_{V_i} + I_{V_i})' &= A_{H_i} + A_{V_i} - \mu_{H_i}(S_{H_i} + I_{H_i} + R_{H_i}) - \mu_{V_i}(S_{V_i} + I_{V_i}) \\ &\leq A_{H_i} + A_{V_i} - \mu_i^*(S_{H_i} + I_{H_i} + R_{H_i} + S_{V_i} + I_{V_i}),\end{aligned} \tag{3.2}$$

where $\mu_i^* = \min\{\mu_{H_i}, \mu_{V_i}\}$. Thus,

$$\limsup_{t\longrightarrow\infty}(S_{H_i} + I_{H_i} + R_{H_i} + S_{V_i} + I_{V_i}) \leq \frac{A_{H_i} + A_{V_i}}{\mu_i^*}. \tag{3.3}$$

Before going into any detail, we simplify the system. For each i-group, since the variable R_{H_i} dose not appear in the first two and the last two equations of (1.1), it suffices to consider the following reduced system:

$$\begin{aligned} S'_{H_i} &= A_{H_i} - \sum_{j=1}^{n} \beta_{H_{ij}} S_{H_i} I_{V_j} - \mu_{H_i} S_{H_i}, \\ I'_{H_i} &= \sum_{j=1}^{n} \beta_{H_{ij}} S_{H_i} I_{V_j} - (\mu_{H_i} + \gamma_{H_i}) I_{H_i}, \\ S'_{V_i} &= A_{V_i} - \sum_{j=1}^{n} \beta_{V_{ij}} S_{V_i} I_{H_j} - \mu_{V_i} S_{V_i}, \\ I'_{V_i} &= \sum_{j=1}^{n} \beta_{V_{ij}} S_{V_i} I_{H_j} - \mu_{V_i} I_{V_i}, \end{aligned} \tag{3.4}$$

where $i = 1, 2, \ldots, n$, in the feasible region

$$\begin{aligned} D = \Bigg\{ (S, I) \ \in R_+^{4n} \mid S_{H_i} \leq \frac{A_{H_i}}{\mu_{H_i}},\ S_{V_i} \leq \frac{A_{V_i}}{\mu_{V_i}},\ S_{H_i} + I_{H_i} + S_{V_i} + I_{V_i} \\ \leq \frac{A_{H_i} + A_{V_i}}{\mu_i^*},\ i = 1, 2, \ldots, n \Bigg\}, \end{aligned} \tag{3.5}$$

where $S = (S_H, S_V)$, $I = (I_H, I_V)$, $S_H = (S_{H_1}, \ldots, S_{H_n})$, $S_V = (S_{V_1}, \ldots, S_{V_n})$, $I_H = (I_{H_1}, \ldots, I_{H_n})$, and $I_V = (I_{V_1}, \ldots, I_{V_n})$. It can be verified that D is positively invariant with respect to system (3.4). Behaviors of R_{H_i} can then be determined from the third equation in (1.1). Our results in this paper will be stated for system (3.4) in D and can be translated straightforwardly to system (1.1). Let $\mathring{D}$ denote the interior of D.

An equilibrium (S, I) of (3.4) satisfies

$$\begin{aligned} A_{H_i} - \sum_{j=1}^{n} \beta_{H_{ij}} S_{H_i} I_{V_j} - \mu_{H_i} S_{H_i} &= 0, \\ \sum_{j=1}^{n} \beta_{H_{ij}} S_{H_i} I_{V_j} - (\mu_{H_i} + \gamma_{H_i}) I_{H_i} &= 0, \\ A_{V_i} - \sum_{j=1}^{n} \beta_{V_{ij}} S_{V_i} I_{H_j} - \mu_{V_i} S_{V_i} &= 0, \\ \sum_{j=1}^{n} \beta_{V_{ij}} S_{V_i} I_{H_j} - \mu_{V_i} I_{V_i} &= 0, \end{aligned} \tag{3.6}$$

where $i = 1, 2, \ldots, n$. It is easy to see that the disease-free equilibrium denoted by $E^0 = (S^0, I^0)$ exists for all positive parameter values, where $S^0_{H_i} = A_{H_i}/\mu_{H_i}$, $S^0_{V_i} = A_{V_i}/\mu_{V_i}$, and $I^0_{H_i} = I^0_{V_i} = 0$, $i = 1, 2, \ldots, n$.

Denote

$$M(S)=\begin{pmatrix}0 & M_H(S)\\ M_V(S) & 0\end{pmatrix}, \tag{3.7}$$

where $M_H(S) = (\beta_{H_{ij}} S_{H_i}/\mu_{H_i} + \gamma_{H_i})_{n\times n}$, $M_V(S) = (\beta_{V_{ij}} S_{V_i}/\mu_{V_i})_{n\times n}$. We also denote $M_H(S_H^0) = M_{H_0}$ and $M_V(S_V^0) = M_{V_0}$

$$M_0=\begin{pmatrix}0 & M_{H_0}\\ M_{V_0} & 0\end{pmatrix}. \tag{3.8}$$

We know that for all $S \in D$, $S \le S^0$, so for all $S \in D$, $M(S) \le M_0$. We define the basic reproduction number R_0 as the spectral radius of M_0; that is $R_0 = \rho(M_0)$. We set

$$B_H=\begin{pmatrix}\beta_{H_{11}} & \beta_{H_{21}} & \cdots & \beta_{H_{n1}}\\ \beta_{H_{12}} & \beta_{H_{22}} & \cdots & \beta_{H_{n2}}\\ \vdots & \vdots & \ddots & \vdots\\ \beta_{H_{1n}} & \beta_{H_{2n}} & \cdots & \beta_{H_{nn}}\end{pmatrix}, \tag{3.9}$$

$$B_V=\begin{pmatrix}\beta_{V_{11}} & \beta_{V_{21}} & \cdots & \beta_{V_{n1}}\\ \beta_{V_{12}} & \beta_{V_{22}} & \cdots & \beta_{V_{n2}}\\ \vdots & \vdots & \ddots & \vdots\\ \beta_{V_{1n}} & \beta_{V_{2n}} & \cdots & \beta_{V_{nn}}\end{pmatrix}, \qquad B_M=\begin{pmatrix}0 & B_H\\ B_V & 0\end{pmatrix}. \tag{3.10}$$

Theorem 3.1. *Assume that B_H, B_V, and B_M are irreducible.*

(1) *If $R_0 \le 1$, then the disease-free equilibrium E^0 of system* (3.4) *is globally asymptotically stable in D.*

(2) *If $R_0 > 1$, then E^0 is unstable and system* (3.4) *is uniformly persistent in $\mathring{D}$.*

Proof. Since B_M is irreducible and nonnegative, we know that $M(S)$ and M_0 are irreducible and nonnegative. Therefore, by Proposition 2.3(ii), there exists a left eigenvector $\omega = (\omega_H, \omega_V) > 0$ of M_0 corresponding to $\rho(M_0)$, where $\omega_H = (\omega_{H_1}, \omega_{H_2}, \ldots, \omega_{H_n})$, $\omega_V = (\omega_{V_1}, \omega_{V_2}, \ldots, \omega_{V_n})$; that is, $\omega\rho(M_0) = \omega M_0$. Define

$$L=\sum_{i=1}^{n}\left(\frac{\omega_{H_i}}{\mu_{H_i}+\gamma_{H_i}} I_{H_i} + \frac{\omega_{V_i}}{\mu_{V_i}} I_{V_i}\right). \tag{3.11}$$

Denote the transpose of I as I^T. Differentiating L along the solution of system (3.4), we obtain

$$\begin{aligned}
L' &= \sum_{i=1}^{n}\left\{\frac{\omega_{H_i}}{\mu_{H_i}+\gamma_{H_i}}\left[\sum_{j=1}^{n}\beta_{H_{ij}}S_{H_i}I_{V_j}-\left(\mu_{H_i}+\gamma_{H_i}\right)I_{H_i}\right]\right.\\
&\qquad\left.+\frac{\omega_{V_i}}{\mu_{V_i}}\left(\sum_{j=1}^{n}\beta_{V_{ij}}S_{V_i}I_{H_j}-\mu_{V_i}I_{V_i}\right)\right\}\\
&= \sum_{i=1}^{n}\left[\omega_{H_i}\left(\sum_{j=1}^{n}\frac{\beta_{H_{ij}}S_{H_i}I_{V_j}}{\mu_{H_i}+\gamma_{H_i}}-I_{H_i}\right)+\omega_{V_i}\left(\sum_{j=1}^{n}\frac{\beta_{V_{ij}}S_{V_i}I_{H_j}}{\mu_{V_i}}-I_{V_i}\right)\right]\\
&= \omega\left(M(S)I^T-I^T\right)\\
&\le \omega\left(M_0I^T-I^T\right)\\
&= \left(\rho(M_0)-1\right)\omega I^T\\
&\le 0.
\end{aligned}\tag{3.12}$$

Therefore, we obtain

(i) if $R_0 < 1$, $L' = 0 \Leftrightarrow I = 0$;

(ii) if $R_0 = 1$, $L' = 0 \Leftrightarrow S = S^0$ or $I = 0$.

Thus, we know that the singleton $\{E^0\}$ is the only compact invariant subset of $\{L' = 0\}$. By LaSalle's Invariance Principle [17], E^0 is globally asymptotically stable in D, if $R_0 \le 1$.

If $R_0 > 1$ and $I > 0$, it is easy to see that

$$\omega\left(M_0I^T-I^T\right) = \left(\rho(M_0)-1\right)\omega I^T > 0. \tag{3.13}$$

Then, according to continuity, there exists a neighborhood $\mathfrak{B}(E^0)$ of E^0, $\mathfrak{B}(E^0) \subseteq D$, such that for all $(S, I) \in \mathfrak{B}(E^0)$

$$L' = \omega\left(M(S)I^T-I^T\right) > 0. \tag{3.14}$$

This implies that E^0 is unstable. Using a uniform persistence result from [18] and a similar argument as in the proof of Proposition 3.3 of [19], we know that, when $R_0 > 1$, the instability of E^0 implies the uniform persistence of (3.4). The proof is complete. □

Uniform persistence of (3.4), together with uniform boundedness of solutions in $\mathring{D}$, implies the existence of an equilibrium of system (3.4) in $\mathring{D}$ [20, 21].

Corollary 3.2. *Assume B_H, B_V, and B_M are irreducible. If $R_0 > 1$, then* (3.4) *has at least one endemic equilibrium.*

Denote the endemic equilibrium by $E^* = (S^*, I^*)$, where $S^*_{H_i}, S^*_{V_i}, I^*_{H_i}, I^*_{V_i} > 0$, $i = 1, 2, \ldots, n$. One has the following result on the endemic equilibrium E^*.

Theorem 3.3. *Assume that B_H, B_V, and B_M are irreducible. If $R_0 > 1$, then the endemic equilibrium E^* of system* (3.4) *is globally asymptotically stable in $\mathring{D}$.*

Proof. The uniqueness of endemic equilibrium is obvious in $\mathring{D}$, if we prove that the endemic equilibrium E^* is globally stable when $R_0 > 1$. We denote $\overline{\beta}_{H_{ij}} = \beta_{H_{ij}} S^*_{H_i} I^*_{V_j}, \overline{\beta}_{V_{ij}} = \beta_{V_{ij}} S^*_{V_i} I^*_{H_j}$

$$\overline{B} = \begin{pmatrix} \sum_{k,j\neq 1}^{n} \overline{\beta}_{V_{1j}} \overline{\beta}_{H_{1k}} & -\sum_{k\neq 1}^{n} \overline{\beta}_{V_{21}} \overline{\beta}_{H_{k1}} & \cdots & -\sum_{k\neq 1}^{n} \overline{\beta}_{V_{n1}} \overline{\beta}_{H_{k1}} \\ -\sum_{k\neq 2}^{n} \overline{\beta}_{V_{12}} \overline{\beta}_{H_{k2}} & \sum_{k,j\neq 2}^{n} \overline{\beta}_{V_{2j}} \overline{\beta}_{H_{2k}} & \cdots & -\sum_{k\neq 2}^{n} \overline{\beta}_{V_{n2}} \overline{\beta}_{H_{k2}} \\ \vdots & \vdots & \ddots & \vdots \\ -\sum_{k\neq n}^{n} \overline{\beta}_{V_{1n}} \overline{\beta}_{H_{kn}} & -\sum_{k\neq n}^{n} \overline{\beta}_{V_{2n}} \overline{\beta}_{H_{k2}} & \cdots & \sum_{k,j\neq n}^{n} \overline{\beta}_{V_{nj}} \overline{\beta}_{H_{nk}} \end{pmatrix}. \tag{3.15}$$

It is easy to see that

$$\begin{aligned}\overline{B} = &\begin{pmatrix} \sum_{k,j\neq 1}^{n} \overline{\beta}_{V_{1j}} \overline{\beta}_{H_{1k}} & 0 & \cdots & 0 \\ 0 & \sum_{k,j\neq 2}^{n} \overline{\beta}_{V_{2j}} \overline{\beta}_{H_{2k}} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sum_{k,j\neq n}^{n} \overline{\beta}_{V_{nj}} \overline{\beta}_{H_{nk}} \end{pmatrix} \\ &- \begin{pmatrix} \sum_{k\neq 1}^{n} \overline{\beta}_{H_{k1}} & 0 & \cdots & 0 \\ 0 & \sum_{k\neq 2}^{n} \overline{\beta}_{H_{k2}} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sum_{k\neq n}^{n} \overline{\beta}_{H_{kn}} \end{pmatrix} \begin{pmatrix} 0 & \overline{\beta}_{V_{21}} & \cdots & \overline{\beta}_{V_{n1}} \\ \overline{\beta}_{V_{12}} & 0 & \cdots & \overline{\beta}_{V_{n2}} \\ \vdots & \vdots & \ddots & \vdots \\ \overline{\beta}_{V_{1n}} & \overline{\beta}_{V_{2n}} & \cdots & 0 \end{pmatrix}.\end{aligned} \tag{3.16}$$

Since B_H is irreducible and nonnegative, we get $\sum_{k\neq j}^{n} \overline{\beta}_{H_{kj}} \neq 0$, $j = 1, 2, \ldots, n$. Together with B_V being irreducible and nonnegative, by Proposition 2.3(iv), we know that $\overline{B}$ is irreducible. Let C_{ij} denote the cofactor of the (i, j) entry of $\overline{B}$. According to Lemma 2.1 in [2], we have that the equation $\overline{B}v = 0$ has a positive solution $v = (v_1, v_2, \ldots, v_n)$, where $v_i = C_{ii} > 0$ for $i = 1, 2, \ldots, n$. Define a Lyapunov function as follows:

$$\begin{aligned} V = \sum_{i=1}^{n} v_i \Bigg[&\sum_{k=1}^{n} \overline{\beta}_{V_{ik}} \left(S_{H_i} - S_{H_i}^* \ln S_{H_i} + I_{H_i} - I_{H_i}^* \ln I_{H_i} \right) \\ &+ \sum_{k=1}^{n} \overline{\beta}_{H_{ik}} \left(S_{V_i} - S_{V_i}^* \ln S_{V_i} + I_{V_i} - I_{V_i}^* \ln I_{V_i} \right) \Bigg]. \end{aligned} \tag{3.17}$$

Together with (3.6), we get the derivative of V along the solution of system (3.4)

$$\begin{aligned} V' = \sum_{i=1}^{n} v_i \Bigg\{ &\sum_{k=1}^{n} \overline{\beta}_{V_{ik}} \Bigg[A_{H_i} - \mu_{H_i} S_{H_i} - \frac{S_{H_i}^*}{S_{H_i}} \left(A_{H_i} - \sum_{j=1}^{n} \beta_{H_{ij}} S_{H_i} I_{V_j} - \mu_{H_i} S_{H_i} \right) \\ &\quad - (\mu_{H_i} + \gamma_{H_i}) I_{H_i} - \frac{I_{H_i}^*}{I_{H_i}} \left(\sum_{j=1}^{n} \beta_{H_{ij}} S_{H_i} I_{V_j} - (\mu_{H_i} + \gamma_{H_i}) \, I_{H_i} \right) \Bigg] \\ &+ \sum_{k=1}^{n} \overline{\beta}_{H_{ik}} \Bigg[A_{V_i} - \mu_{V_i} S_{V_i} - \frac{S_{V_i}^*}{S_{V_i}} \left(A_{V_i} - \sum_{j=1}^{n} \beta_{V_{ij}} S_{V_i} I_{H_j} - \mu_{V_i} S_{V_i} \right) \\ &\quad - \mu_{V_i} I_{V_i} - \frac{I_{V_i}^*}{I_{V_i}} \left(\sum_{j=1}^{n} \beta_{V_{ij}} S_{V_i} I_{H_j} - \mu_{V_i} I_{V_i} \right) \Bigg] \Bigg\}, \\ = \sum_{i,k=1}^{n} v_i &\left[\overline{\beta}_{V_{ik}} \mu_{H_i} S_{H_i}^* \left(2 - \frac{S_{H_i}}{S_{H_i}^*} - \frac{S_{H_i}^*}{S_{H_i}} \right) + \overline{\beta}_{H_{ik}} \mu_{V_i} S_{V_i}^* \left(2 - \frac{S_{V_i}}{S_{V_i}^*} - \frac{S_{V_i}^*}{S_{V_i}} \right) \right] \\ + \sum_{i,k=1}^{n} v_i \overline{\beta}_{V_{ik}} &\left(2 \sum_{j=1}^{n} \overline{\beta}_{H_{ij}} - \frac{S_{H_i}^*}{S_{H_i}} \sum_{j=1}^{n} \overline{\beta}_{H_{ij}} + \sum_{j=1}^{n} \overline{\beta}_{H_{ij}} \frac{I_{V_j}}{I_{V_j}^*} - \frac{I_{H_i}}{I_{H_i}^*} \sum_{j=1}^{n} \overline{\beta}_{H_{ij}} - \frac{I_{H_i}^*}{I_{H_i}} \sum_{j=1}^{n} \beta_{H_{ij}} S_{H_i} I_{V_j} \right) \\ + \sum_{i,k=1}^{n} v_i \overline{\beta}_{H_{ik}} &\left(2 \sum_{j=1}^{n} \overline{\beta}_{V_{ij}} - \frac{S_{V_i}^*}{S_{V_i}} \sum_{j=1}^{n} \overline{\beta}_{V_{ij}} + \sum_{j=1}^{n} \overline{\beta}_{V_{ij}} \frac{I_{H_j}}{I_{H_j}^*} - \frac{I_{V_i}}{I_{V_i}^*} \sum_{j=1}^{n} \overline{\beta}_{V_{ij}} - \frac{I_{V_i}^*}{I_{V_i}} \sum_{j=1}^{n} \beta_{V_{ij}} S_{V_i} I_{H_j} \right). \end{aligned} \tag{3.18}$$

According to $(x_1/x_2) + (x_2/x_1) \geq 2$ for each $x_1, x_2 > 0$, with equality holding if and only if $x_1 = x_2$, we have

$$\begin{aligned} \mu_{H_i} S_{H_i}^* \left(2 - \frac{S_{H_i}^*}{S_{H_i}} - \frac{S_{H_i}}{S_{H_i}^*} \right) &\leq 0, \\ \mu_{V_i} S_{V_i}^* \left(2 - \frac{S_{V_i}^*}{S_{V_i}} - \frac{S_{V_i}}{S_{V_i}^*} \right) &\leq 0, \end{aligned} \tag{3.19}$$

where $i = 1, 2, \ldots, n$ and equalities hold, respectively, if and only if

$$S_{H_i} = S_{H_i}^*, \quad S_{V_i} = S_{V_i}^*, \quad i = 1, 2, \ldots, n. \tag{3.20}$$

Hence,

$$\begin{aligned}
V' \le{}& \sum_{i,k=1}^{n} v_i \overline{\beta}_{V_{ik}} \left(2\sum_{j=1}^{n} \overline{\beta}_{H_{ij}} - \frac{S_{H_i}^*}{S_{H_i}} \sum_{j=1}^{n} \overline{\beta}_{H_{ij}} + \sum_{j=1}^{n} \overline{\beta}_{H_{ij}} \frac{I_{V_j}}{I_{V_j}^*} - \frac{I_{H_i}}{I_{H_i}^*} \sum_{j=1}^{n} \overline{\beta}_{H_{ij}} - \frac{I_{H_i}^*}{I_{H_i}} \sum_{j=1}^{n} \beta_{H_{ij}} S_{H_i} I_{V_j} \right) \\
&+ \sum_{i,k=1}^{n} v_i \overline{\beta}_{H_{ik}} \left(2\sum_{j=1}^{n} \overline{\beta}_{V_{ij}} - \frac{S_{V_i}^*}{S_{V_i}} \sum_{j=1}^{n} \overline{\beta}_{V_{ij}} + \sum_{j=1}^{n} \overline{\beta}_{V_{ij}} \frac{I_{H_j}}{I_{H_j}^*} - \frac{I_{V_i}}{I_{V_i}^*} \sum_{j=1}^{n} \overline{\beta}_{V_{ij}} - \frac{I_{V_i}^*}{I_{V_i}} \sum_{j=1}^{n} \beta_{V_{ij}} S_{V_i} I_{H_j} \right) \\
={}& \sum_{i=1}^{n} v_i \left[\sum_{k=1}^{n} \overline{\beta}_{V_{ik}} \left(\sum_{j=1}^{n} \overline{\beta}_{H_{ij}} \frac{I_{V_j}}{I_{V_j}^*} - \frac{I_{H_i}}{I_{H_i}^*} \sum_{j=1}^{n} \overline{\beta}_{H_{ij}} \right) + \sum_{k=1}^{n} \overline{\beta}_{H_{ik}} \left(\sum_{j=1}^{n} \overline{\beta}_{V_{ij}} \frac{I_{H_j}}{I_{H_j}^*} - \frac{I_{V_i}}{I_{V_i}^*} \sum_{j=1}^{n} \overline{\beta}_{V_{ij}} \right) \right] \\
&+ \sum_{i,j,k=1}^{n} v_i \overline{\beta}_{V_{ik}} \overline{\beta}_{H_{ij}} \left(4 - \frac{S_{H_i}^*}{S_{H_i}} - \frac{S_{V_i}^*}{S_{V_i}} - \frac{I_{H_i}^* S_{H_i} I_{V_j}}{I_{H_i} S_{H_i}^* I_{V_j}^*} - \frac{I_{V_i}^* S_{V_i} I_{H_j}}{I_{V_i} S_{V_i}^* I_{H_j}^*} \right) \\
=:{}& K_1 + K_2.
\end{aligned} \tag{3.21}$$

We first show $K_1 \equiv 0$ for all $(S, I) \in \overset{\circ}{D}$. It follows from $\overline{B} v = 0$ that

$$\sum_{k,j=1}^{n} \overline{\beta}_{V_{ij}} \overline{\beta}_{H_{ik}} v_i = \sum_{k,j=1}^{n} \overline{\beta}_{V_{ji}} \overline{\beta}_{H_{ki}} v_j, \tag{3.22}$$

$i = 1, 2, \ldots, n$. This implies that

$$\begin{aligned}
\sum_{i=1}^{n} v_i \sum_{k=1}^{n} \overline{\beta}_{V_{ik}} \sum_{j=1}^{n} \overline{\beta}_{H_{ij}} \frac{I_{V_j}}{I_{V_j}^*} &= \sum_{j=1}^{n} \frac{I_{V_j}}{I_{V_j}^*} \sum_{k=1}^{n} \sum_{i=1}^{n} v_i \overline{\beta}_{V_{ik}} \overline{\beta}_{H_{ij}} \\
&= \sum_{j=1}^{n} \frac{I_{V_j}}{I_{V_j}^*} v_j \sum_{k,i=1}^{n} \overline{\beta}_{V_{jk}} \overline{\beta}_{H_{ji}}.
\end{aligned} \tag{3.23}$$

Thus,

$$\sum_{i=1}^{n} v_i \sum_{k=1}^{n} \overline{\beta}_{V_{ik}} \sum_{j=1}^{n} \overline{\beta}_{H_{ij}} \frac{I_{V_j}}{I_{V_j}^*} - \sum_{i=1}^{n} v_i \sum_{k=1}^{n} \overline{\beta}_{H_{ik}} \frac{I_{V_i}}{I_{V_i}^*} \sum_{j=1}^{n} \overline{\beta}_{V_{ij}} = 0. \tag{3.24}$$

Similarly, we produce

$$\sum_{i=1}^{n} v_i \sum_{k=1}^{n} \overline{\beta}_{H_{ik}} \sum_{j=1}^{n} \overline{\beta}_{V_{ij}} \frac{I_{H_j}}{I_{H_j}^*} - \sum_{i=1}^{n} v_i \sum_{k=1}^{n} \overline{\beta}_{V_{ik}} \frac{I_{H_i}}{I_{H_i}^*} \sum_{j=1}^{n} \overline{\beta}_{H_{ij}} = 0. \tag{3.25}$$

Therefore, $K_1 \equiv 0$ for all $(S, I) \in \overset{\circ}{D}$.

$\Gamma(\overline{B})$ has vertices $\{1,2,\ldots,n\}$ with a directed arc (i,j) from i to j if and only if $\sum_{k\neq j}^{n}\overline{\beta}_{V_{ij}}\overline{\beta}_{H_{kj}}\neq 0$. Since $\overline{B}$ is irreducible, by a similar argument in [2], we obtain $K_2\leq 0$ for all $(S,I)\in \overset{\circ}{D}$. Furthermore, we produce that

$$V'\leq 0. \tag{3.26}$$

If (3.20) holds, we have

$$K_2=0 \Longleftrightarrow I_{H_i}=\eta I_{H_i}^{*},\quad I_{V_i}=\eta I_{V_i}^{*},\quad i=1,2,\ldots,n, \tag{3.27}$$

where η is arbitrary positive numbers.

According to (3.20) and (3.27), we know that $V'=0 \Leftrightarrow S_{H_i}=S_{H_i}^{*}$, $S_{V_i}=S_{V_i}^{*}$, $I_{H_i}=\eta I_{H_i}^{*}$, $I_{V_i}=\eta I_{V_i}^{*}$, $i=1,2,\ldots,n$. Substituting (3.20) and (3.27) into system (3.4), we obtain

$$\begin{aligned} 0&=A_{H_i}-\eta\sum_{j=1}^{n}\beta_{H_{ij}}S_{H_i}^{*}I_{V_j}^{*}-\mu_{H_i}S_{H_i}^{*},\\ 0&=A_{V_i}-\eta\sum_{j=1}^{n}\beta_{V_i}S_{V_i}^{*}I_{H_j}^{*}-\mu_{V_i}S_{V_i}^{*}. \end{aligned} \tag{3.28}$$

Since the right-hand side of (3.28) is strictly decreasing in η, by (3.6), we get that (3.28) holds if and only if $\eta=1$, namely, at E^{*}. By LaSalle's Invariance Principle, E^{*} is globally asymptotically stable in $\overset{\circ}{D}$. The proof is complete. □

From the process of proof of Theorem 3.3 and the definition of matrix $\overline{B}$, it is easy to get a corollary as follows.

Corollary 3.4. *Assume that B_V and B_M are irreducible and $\sum_{k\neq j}^{n}\beta_{H_{kj}}\neq 0$ (or B_H, B_M are irreducible and $\sum_{k\neq j}^{n}\beta_{V_{kj}}\neq 0$), $j=1,2,\ldots,n$. If $R_0>1$, then the endemic equilibrium E^{*} of system* (3.4) *is globally asymptotically stable in $\overset{\circ}{D}$.*

4. Discussion

Taking the basic reproduction number R_0 as a sharp threshold parameter, we establish the global dynamics of system (3.4). Our result implies that, if $R_0\leq 1$, then the dengue disease always dies out in all groups; if $R_0>1$, then the dengue disease always persists at the unique endemic equilibrium level in all groups, independent of the initial condition.

Biologically, our assumptions in Theorem 3.3 and Corollary 3.4 mean that mosquitoes in I_{V_j} can infect ones in individuals S_{H_i} directly or indirectly; individuals in I_{H_j} can infect ones in mosquitoes S_{V_i} directly or indirectly, and individuals in I_{H_j} can infect ones in S_{H_i} by mosquitoes indirectly, respectively.

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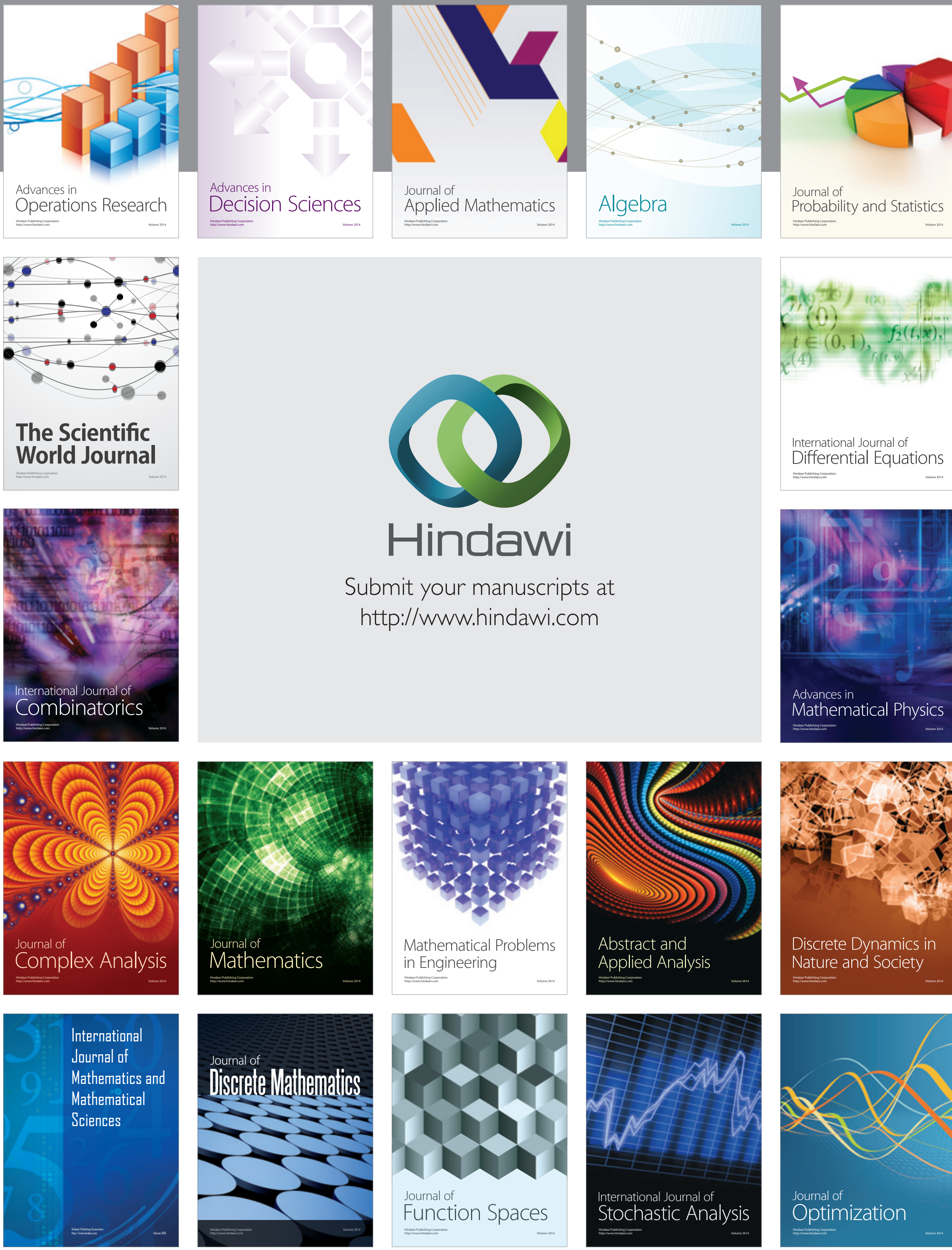
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