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Research Article

Stability of the Stochastic Reaction-Diffusion Neural Network with Time-Varying Delays and *p*-Laplacian

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The main aim of this paper is to discuss moment exponential stability for a stochastic reaction-diffusion neural network with time-varying delays and *p*-Laplacian. Using the Itô formula, a delay differential inequality and the characteristics of the neural network, the algebraic conditions for the moment exponential stability of the nonconstant equilibrium solution are derived. An example is also given for illustration.

1. Introduction

In many neural networks, time delays cannot be avoided. For example, in electronic neural networks, time delays will be present due to the finite switching speed of amplifies. In fact, time delays are often encountered in various engineering, biological, and economical systems. On the other hand, when designing a neural network to solve a problem such as optimization or pattern recognition, we need foremost to guarantee that the neural networks model is globally asymptotically stable. However, the existence of time delay frequently causes oscillation, divergence, or instability in neural networks. In recent years, the stability of neural networks with delays or without delays has become a topic of great theoretical and practical importance (see [1–16]).

The stability of neural networks which depicted by partial differential equations was studied in [6, 7]. Stochastic differential equations were employed to research the stability of neural networks in [8–11], while [12, 13] used stochastic partial differential

equations to analysis this question. In [15], the authors studied almost exponential stability for a stochastic recurrent neural network with time-varying delays. In addition, moment exponential stability for a stochastic reaction-diffusion neural network with time-varying delays is discussed in [16].

In this paper, we consider the stochastic reaction-diffusion neural network with time-varying delays and p-Laplacian as follows:

$$\begin{aligned} du_i(t,x) = {} & \left[\sum_{k=1}^{m}\frac{\partial}{\partial x_k}\left(|\nabla u_i|^{p-2}\frac{\partial u_i}{\partial x_k}\right) - a_i(u)u_i + I_i + \sum_{j=1}^{n}T_{ij}g_j\big(u_j(t-\tau_j(t),x)\big)\right]dt \\ & + \sum_{l=1}^{m}\sigma_{il}(u_i(t,x))dW_l(t), \quad i=1,2,\ldots,n,\ t>t_0,\ x\in\Omega, \end{aligned} \tag{1.1}$$

$$\frac{\partial u_i}{\partial n} := \left(\frac{\partial u_i}{\partial x_1},\ldots,\frac{\partial u_i}{\partial x_m}\right)^T = 0, \quad i=1,2,\ldots,n,\ t\geq t_0,\ x\in\partial\Omega, \tag{1.2}$$

$$u_i(t_0+s,x) = \phi_i(s,x), \quad -\tau_i(t_0)\leq s\leq 0,\ 0\leq\tau_i(t)\leq\tau_i,\ i=1,2,\ldots,n,\ x\in\Omega. \tag{1.3}$$

In (1.1), $\nabla u_i = (\partial u_i/\partial x_1,\ldots,\partial u_i/\partial x_m)^T$, $p\geq 2$ is a common number. $\Omega\subseteq R^m$ is a bounded convex domain with smooth boundary $\partial\Omega$ and measure $\operatorname{mes}\Omega>0$. n denotes the numbers of neurons in the neural network, $u_i(t,x)$ corresponds to the state of the ith neurons at time t and in space x, the $a_i(u)$ is an amplification function. I_i is output. $g_j(u_j(t-\tau_j,x))$ denotes the output of the jth neuron at time $t-\tau_j$ and in space x, namely, activation function which shows how neurons respond to each other. $W(t) = (W_1(t),\ldots,W_m(t))^T$ is an m-dimensional Brownian motion which is defined on a complete probability space $(\mathcal{S},\mathcal{F},\mathcal{P})$ with a natural filtration $\{\mathcal{F}_t\}_{t\geq t_0}$ (i.e., $\mathcal{F}_t = \sigma\{W(s) : t_0\leq s\leq t\}$). $\sigma(u) = (\sigma_1(u_1),\ldots,\sigma_n(u_n))^T$, $\sigma_i(u_i) = (\sigma_{i1}(u_i),\ldots,\sigma_{im}(u_i))$. $\sigma_{ij}(u_i)$ denotes the intensity of the stochastic perturbation. Functions $\sigma_{ij}(u_i)$ and g_i are subject to certain conditions to be specified later. $T := (T_{ij})_{n\times n}$ is a real constant matrix and represents weight of the neuron interconnections, namely, T_{ij} denotes the strength of jth neuron on the ith neuron at time $t-\tau_j$ and in space x, and $\tau_j\in[0,\tau]$ corresponds to axonal signal transmission delay.

2. Definitions and Lemmas

Throughout this paper, unless otherwise specified, let $|\cdot|$ denote Euclidean norm. Define that $|x|^p = (\sum_{i=1}^n |x_i|^2)^{p/2}$ and $|x|_p = \sum_{i=1}^n |x_i|^p$ where $x = (x_1,\ldots,x_n)^T\in R^n$. Denote by $C([-\tau,0]\times\Omega;R^n)$ the family of continuous functions φ from $[-\tau,0]\times\Omega$ to R^n. For every $t\geq t_0$ and $p\geq 2$, denote by $L^p_{\mathcal{F}_t}([-\tau,0]\times\Omega;R^n)$ the family of all $\mathcal{F}_t$-measurable $C([-\tau,0]\times\Omega;R^n)$ valued random variables such that $\|\phi\|^p_{L^p} = \sup_{-\tau\leq\theta\leq 0}E(\|\phi(\theta)\|^p_p) < \infty$, where $\|\phi(\theta)\|_p = (\int_\Omega|\phi(\theta,x)|^p dx)^{1/p}$, $E(\phi)$ denotes the expectation of random variable ϕ.

Definition 2.1. The $u(t,x) = (u_1(t,x),\ldots,u_n(t,x))^T$ is called a solution of problem (1.1)–(1.3) if it satisfies following conditions (1), (2), and (3):

(1) $u(t,x)$ adapts $\{\mathcal{F}_t\}_{t\geq t_0}$;

(2) for $T\in R^+_{t_0} := \{t\in R : t\geq t_0\}$, $u(t,x)\in C([t_0,T]\times\Omega,R^n)$, and $E(\max_{x\in\Omega}\int_{t_0}^T[|u(t,x)|^p + |\nabla u(t,x)|^p]dt) < +\infty$, where $\nabla u(x,t) = (\partial u/\partial x_1,\ldots,\partial u/\partial x_n)$;

(3) for $T \in R_{t_0}^{+}$, $t \in (t_0, T]$, it holds that

$$\begin{aligned}\int_{\Omega} u_i(t,x)dx = \int_{\Omega} \phi_i(t_0,x)dx + \int_{\Omega}\int_{t_0}^{t} \Bigg\{ \sum_{k=1}^{m} \frac{\partial}{\partial x_k}\left(|\nabla u_i|^{p-2}\frac{\partial u_i}{\partial x_k}\right) - a_i(u)u_i + I_i \\ + \sum_{j=1}^{n} T_{ij} g_j\big(u_j(\xi - \tau_j(\xi), x)\big)\Bigg\} d\xi\, dx \\ + \sum_{l=1}^{m} \int_{\Omega}\int_{t_0}^{t} \sigma_{il}(u_i(\xi,x))dW_l(\xi)dx, \\ i = 1,\ldots,n, \quad (t,x) \in (t_0,T]\times\Omega, \quad P \text{ a.s.}, \\ u_i(t_0+s,x) = \phi_i(s,x), \quad -\tau_i \le s \le 0,\ i = 1,\ldots,n,\ P \text{ a.s.}\end{aligned} \tag{2.1}$$

Definition 2.2. The $u = u^*(x)$ is called a nonconstant equilibrium solution of problem (1.1)–(1.3) if and only if $u = u^*(x)$ satisfies (1.1) and (1.2).

Definition 2.3. The nonconstant equilibrium solution $u^*(x)$ of (1.1) about the given norm $\|\cdot\|_{\Omega}$ is called exponential stability in pth moment, if there are constants $M > 0$, $\delta > 0$ for every stochastic field solution $u(t,x)$ of problem (1.1)–(1.3) such that

$$E[\|u(t,x) - u_*\|_{\Omega}] \le Me^{-\delta(t-t_0)}, \tag{2.2}$$

namely,

$$\limsup_{t\to\infty} \frac{1}{t} \log E[\|u(t,x) - u_0\|_{\Omega}] \le -\delta. \tag{2.3}$$

The constant $-\delta$ on the right hand side in (2.3) is called Lyapunov exponent of every solution of problem (1.1)–(1.3) converging on equilibrium about norm $\|\cdot\|_{\Omega}$.

In order to obtain pth moment exponential stability for a nonconstant equilibrium solution of problem (1.1)–(1.3), we need the following lemmas.

Lemma 2.4 (see [17]). *Let $P = (p_{ij})_{n\times n}$ and $Q = (q_{ij})_{n\times n}$ be two real matrices. The continuous function $u_i(t) \ge 0$ satisfies the delay differential inequalities*

$$D^{+}u_i(t) \le \sum_{j=1}^{n}\big[p_{ij}u_j(t) + q_{ij}u_j(t-\tau_j(t))\big], \quad 0 \le \tau_i(t) \le \tau,\ i = 1,\ldots,n. \tag{2.4}$$

If $p_{ij} > 0$ for $i \ne j$ and $q_{ij} \ge 0$ $(i,j = 1,2,\ldots,n)$ and $-(P+Q)$ is an M-matrix, then there are constants $k_i > 0$, $\alpha > 0$ such that

$$u_i(t) \le k_i\left(\sum_{j=1}^{n}\|\phi_j\|\right)\exp(-\alpha(t-t_0)), \quad i = 1,\ldots,n,\ t \ge t_0, \tag{2.5}$$

where $\phi = (\phi_1,\ldots,\phi_1)$ are initial functions. D^{+} is right-hand upper derivate. $\|\cdot\|$ represents a norm.

Lemma 2.5 (see [10]). *Let $p > 2$, then there are positive constants $e_p(n)$ and $d_p(n)$ for any $x = (x_1, \ldots, x_n)^T \in R^n$ such that*

$$e_p(n)\left(\sum_{i=1}^{n}|x_i|^2\right)^{p/2} \le \sum_{i=1}^{n}|x_i|^p \le d_p(n)\left(\sum_{i=1}^{n}|x_i|^2\right)^{p/2}. \tag{2.6}$$

Remark 2.6. If $p = 2$, Lemma 2.5 also holds with $e_p(n) = d_p(n) = 1$.

Suppose that $\sigma_{ij}(u_i)$, $a_i(u)$, and g_i are Lipschitz continuous such that the following conditions hold:

(H1) $|(\sigma_i(v_1) - \sigma_i(v_2))(\sigma_i(v_1) - \sigma_i(v_2))^T| \le 2\lambda_i|v_1 - v_2|^2$, $i = 1, 2, \ldots, n$,

(H2) $|g_i(v_1) - g_i(v_2)| \le c_i|v_1 - v_2|$, $i = 1, 2, \ldots, n$,

(H3) $(u - v)(a_i(u)u - a_i(v)v) \ge a_i|u - v|^2$, for all $u, v \in R$, $i = 1, 2, \ldots, n$,

where c_i, a_i, and λ_i $(1 \le i \le n)$ are positive constants.

3. Main Result

Set $u(t,x) = (u_1(t,x), \ldots, u_n(t,x))^T$ as a solution of the problem (1.1)–(1.3) and $u^*(x) = (u_1^*(x), \ldots, u_n^*(x))^T$ as a nonconstant equilibrium solution of the problem (1.1)–(1.3).

Theorem 3.1. *Let $p \ge 2$ and (H1)–(H3) hold. Assume that there are positive constants $\varepsilon_1, \ldots, \varepsilon_n$ such that the matrix $M_p = -(p_{ji} + q_{ij})_{n\times n} := P^T + Q$ is an M-matrix, where*

$$\begin{gathered} p_{ij} = -p\delta_{ij}a_i + p(p-1)\delta_{ij}\lambda_i + (p-1)\left|T_{ij}\right|c_j\varepsilon_i, \quad \delta_{ii} = 1,\ \delta_{ij} = 0\ (i \ne j),\ i, j = 1, \ldots, n, \\ q_{ij} = \left|T_{ij}\right|c_j\varepsilon_i^{1-p}, \quad i, j = 1, \ldots, n, \end{gathered} \tag{3.1}$$

then the nonconstant equilibrium solution of problem (1.1)–(1.3) about L^p norm is exponential stability in pth moment, that is, there are constants $C_1 > 0$ and $\alpha > 0$, for any $t_0 \in R^+$ and any $\phi \in L^p_{F_{t_0}}$ such that

$$E\left(\|u(t,x) - u^*\|_p^p\right) \le C_1 e^{-\alpha(t-t_0)}, \quad t \ge t_0. \tag{3.2}$$

Proof. Set $V_i(u(t,x)) = |u_i(t,x) - u_i^*(x)|^p$. For every $t \ge t_0$ and $dt > 0$, by means of Itô formula and (H3), one has that

$$\begin{aligned} dV_i(u) = p\left|u_i - u_i^*\right|^{p-2}(u_i - u_i^*)\Bigg\{&\sum_{k=1}^{m}\left[\frac{\partial}{\partial x_k}\left(|\nabla u_i|^{p-2}\frac{\partial u_i}{\partial x_k}\right) - \frac{\partial}{\partial x_k}\left(\left|\nabla u_i^*\right|^{p-2}\frac{\partial u_i^*}{\partial x_k}\right)\right] \\ &- \left(a_i(u)u_i - a_i(u^*)u_i^*\right) \\ &+ \sum_{j=1}^{n}T_{ij}\left(g_j\left(u_j(t-\tau_j, x)\right) - g_j\left(u_j^*\right)\right)\Bigg\}dt \end{aligned}$$

$$
\begin{aligned}
&+\frac{1}{2}p(p-1)\left|u_i-u_i^*\right|^{p-2}\operatorname{sgn}(u_i-u_i^*)\left(\sigma_i(u_i)-\sigma_i(u_i^*)\right)\left(\sigma_i(u_i)-\sigma_i(u_i^*)\right)^T dt\\
&+p\left|u_i-u_i^*\right|^{p-2}(u_i-u_i^*)\left(\sigma_i(u_i)-\sigma_i(u_i^*)\right)dW(t)\\
\le\ & p\sum_{k=1}^{m}\left|u_i-u_i^*\right|^{p-2}(u_i-u_i^*)\left[\frac{\partial}{\partial x_k}\left(|\nabla u_i|^{p-2}\frac{\partial u_i}{\partial x_k}\right)-\frac{\partial}{\partial x_k}\left(\left|\nabla u_i^*\right|^{p-2}\frac{\partial u_i^*}{\partial x_k}\right)\right]dt\\
&+\left(-pa_i+p(p-1)\lambda_i\right)\left|u_i-u_i^*\right|^p dt\\
&+p\sum_{j=1}^{n}\left|T_{ij}\right|c_j\left|u_i-u_i^*\right|^{p-2}\left|u_i-u_i^*\right|\left|u_j(t-\tau_j,x)-u_j^*\right|dt\\
&+p\left|u_i-u_i^*\right|^{p-2}(u_i-u_i^*)\left(\sigma_i(u_i)-\sigma_i(u_i^*)\right)dW(t).
\end{aligned}
\tag{3.3}
$$

Both sides of Inequality (3.3) are integrated about x over Ω. Set $\overline{V_i(u)}=\int_\Omega V_i(u)dx=\|u_i-u_i^*\|_p^p$. One has that

$$
\begin{aligned}
d\overline{V_i(u)}\le\ & p\sum_{k=1}^{m}\int_\Omega\left|u_i-u_i^*\right|^{p-2}(u_i-u_i^*)\left[\frac{\partial}{\partial x_k}\left(|\nabla u_i|^{p-2}\frac{\partial u_i}{\partial x_k}\right)-\frac{\partial}{\partial x_k}\left(\left|\nabla u_i^*\right|^{p-2}\frac{\partial u_i^*}{\partial x_k}\right)\right]dx\,dt\\
&+\left(-pa_i+p(p-1)\lambda_i\right)\int_\Omega\left|u_i-u_i^*\right|^p dx\,dt\\
&+p\sum_{j=1}^{n}\int_\Omega\left|u_i-u_i^*\right|^{p-2}\left|u_i-u_i^*\right|\left|T_{ij}\right|c_j\left|u_j(t-\tau_j,x)-u_j^*\right|dx\,dt\\
&+p\sum_{l=1}^{m}\int_\Omega\left|u_i-u_i^*\right|^{p-2}(u_i-u_i^*)\left(\sigma_{il}(u_i)-\sigma_{il}(u_i^*)\right)dx\,dW_l(t).
\end{aligned}
\tag{3.4}
$$

Set $(|\nabla u_i|^{p-2}(\partial u_i/\partial x_k)-|\nabla u_i^*|^{p-2}(\partial u_i^*/\partial x_k))_{k=1}^m:=(|\nabla u_i|^{p-2}(\partial u_i/\partial x_1)-|\nabla u_i^*|^{p-2}(\partial u_i^*/\partial x_1),\ldots,$ $|\nabla u_i|^{p-2}(\partial u_i/\partial x_m)-|\nabla u_i^*|^{p-2}(\partial u_i^*/\partial x_m))$. By (1.2), one has that

$$
\begin{aligned}
&\sum_{k=1}^{m}\int_\Omega\left|u_i-u_i^*\right|^{p-2}(u_i-u_i^*)\left[\frac{\partial}{\partial x_k}\left(|\nabla u_i|^{p-2}\frac{\partial u_i}{\partial x_k}\right)-\frac{\partial}{\partial x_k}\left(\left|\nabla u_i^*\right|^{p-2}\frac{\partial u_i^*}{\partial x_k}\right)\right]dx\\
&\quad=\int_\Omega\left|u_i-u_i^*\right|^{p-2}(u_i-u_i^*)\nabla\cdot\left(|\nabla u_i|^{p-2}\frac{\partial u_i}{\partial x_k}-\left|\nabla u_i^*\right|^{p-2}\frac{\partial u_i^*}{\partial x_k}\right)_{k=1}^{m}dx\\
&\quad=\int_\Omega\nabla\cdot\left(\left|u_i-u_i^*\right|^{p-2}(u_i-u_i^*)\left(|\nabla u_i|^{p-2}\frac{\partial u_i}{\partial x_k}-\left|\nabla u_i^*\right|^{p-2}\frac{\partial u_i^*}{\partial x_k}\right)_{k=1}^{m}\right)dx\\
&\qquad-\int_\Omega\left(|\nabla u_i|^{p-2}\frac{\partial u_i}{\partial x_k}-\left|\nabla u_i^*\right|^{p-2}\frac{\partial u_i^*}{\partial x_k}\right)_{k=1}^{m}\nabla\left(\left|u_i-u_i^*\right|^{p-2}(u_i-u_i^*)\right)dx
\end{aligned}
$$

$$
\begin{aligned}
&= \int_{\partial\Omega} \left(\left|u_i - u_i^*\right|^{p-2} (u_i - u_i^*) \left(|\nabla u_i|^{p-2} \frac{\partial u_i}{\partial x_k} - \left|\nabla u_i^*\right|^{p-2} \frac{\partial u_i^*}{\partial x_k} \right)_{k=1}^{m} \right) \cdot \vec{n} ds \\
&\quad - \int_{\Omega} (p-1) \left|u_i - u_i^*\right|^{p-2} \nabla (u_i - u_i^*) \cdot \left(|\nabla u_i|^{p-2} \frac{\partial u_i}{\partial x_k} - \left|\nabla u_i^*\right|^{p-2} \frac{\partial u_i^*}{\partial x_k} \right)_{k=1}^{m} dx \\
&= -\sum_{k=1}^{m} \int_{\Omega} (p-1) \left|u_i - u_i^*\right|^{p-2} \left(|\nabla u_i|^{p-2} \frac{\partial u_i}{\partial x_k} - \left|\nabla u_i^*\right|^{p-2} \frac{\partial u_i^*}{\partial x_k} \right) \cdot \left(\frac{\partial u_i}{\partial x_k} - \frac{\partial u_i^*}{\partial x_k} \right) dx \\
&\le 0,
\end{aligned}
\tag{3.5}
$$

where $\vec{n}$ is unit outer cotangent vector on $\partial\Omega$. By (3.4), (3.5), (H1), and Young's inequality, one has that

$$
\begin{aligned}
d\overline{V_i(u)} &\le -\sum_{k=1}^{m} \int_{\Omega} (p-1) \left|u_i - u_i^*\right|^{p-2} \left(|\nabla u_i|^{p-2} \frac{\partial u_i}{\partial x_k} - \left|\nabla u_i^*\right|^{p-2} \frac{\partial u_i^*}{\partial x_k} \right) \cdot \left(\frac{\partial u_i}{\partial x_k} - \frac{\partial u_i^*}{\partial x_k} \right) dx \\
&\quad + \left(-p a_i + p(p-1)\lambda_i\right) \left\|u_i(t) - u_i^*\right\|_p^p dt \\
&\quad + p \sum_{j=1}^{n} \int_G \left|u_i - u_i^*\right|^{p-2} \left|u_i - u_i^*\right| \left|T_{ij}\right| c_j \left|u_j(t-\tau_j, x) - u_j^*\right| dx\, dt \\
&\quad + p \sum_{l=1}^{m} \int_G \left|u_i - u_i^*\right|^{p-2} (u_i - u_i^*) \left(\sigma_{il}(u_i) - \sigma_{il}(u_i^*)\right) dx\, dW_l(t) \\
&\le \sum_{j=1}^{n} p_{ij} \left\|u_i(t) - u_i^*\right\|_p^p dt + \sum_{j=1}^{n} q_{ij} \left\|u_j(t-\tau_j, x) - u_j^*\right\|_p^p dt \\
&\quad + p \sum_{l=1}^{m} \int_G \left|u_i - u_i^*\right|^{p-2} (u_i - u_i^*) \left(\sigma_{il}(u_i) - \sigma_{il}(u_i^*)\right) dx\, dW_l(t),
\end{aligned}
\tag{3.6}
$$

where p_{ij} and q_{ij} are defined by (3.1).

For $\Delta t > 0$, both sides of (3.6) are integrated about t from t to $t + \Delta t$, then both sides of (3.6) are calculated expectation. By the properties of Brownian motion, one has that

$$
\begin{aligned}
&E\left[\left\|u_i(t+\Delta t) - u_i^*\right\|_p^p\right] - E\left[\left\|u_i(t) - u_i^*\right\|_p^p\right] \\
&\quad \le \sum_{j=1}^{n} \left\{ p_{ij} E\left[\int_t^{t+\Delta t} \left\|u_i(s) - u_i^*\right\|_p^p ds\right] + q_{ij} E\left[\int_t^{t+\Delta t} \left\|u_j(s-\tau_j(s)) - u_j^*\right\|_p^p ds\right] \right\}.
\end{aligned}
\tag{3.7}
$$

Since the integrals $\int_t^{t+\Delta t} E[\|u_i(s)-u_i^*\|_p^p]ds$ and $\int_t^{t+\Delta t} E[\|u_j(s-\tau_j)-u_i^*\|_p^p]ds$ are finite, by Fubini theorem [18] and (3.7), one obtain that

$$E\left[\|u_i(t+\Delta t)-u_i^*\|_p^p\right]-E\left[\|u_i(t)-u_i^*\|_p^p\right]\le\sum_{j=1}^n p_{ij}\int_t^{t+\Delta t}E\left[\|u_i(s)-u_i^*\|_p^p\right]ds + \sum_{j=1}^n q_{ij}\int_t^{t+\Delta t}E\left[\left\|u_j(s-\tau_j(s))-u_j^*\right\|_p^p\right]ds. \tag{3.8}$$

Set $v_i(t)=E[\|u_i(t)-u_i^*\|_p^p]$. Both sides of Inequality (3.8) are divided by Δt, let $\Delta t\to 0$, one has the following inequality:

$$D^+v_i(t)\le\sum_{j=1}^n\left[p_{ij}v_i(t)+q_{ij}v_j(t-\tau_j(t))\right]. \tag{3.9}$$

By Lemma 2.4, there are positive constants K_i, α such that

$$E\left[\|u_i(t)-u_i^*\|_p^p\right]\le K_i\sum_{j=1}^n\left\|E\left[\left\|\phi_j-u_j^*\right\|_p^p\right]\right\|\exp(-\alpha(t-t_0)),\quad i=1,\dots,n,\ t\ge t_0, \tag{3.10}$$

where ϕ_j is initial value. Set $K=\max\{K_i:1\le i\le n\}$, then

$$E\left[\sum_{j=1}^n\left\|u_j-u_j^*\right\|_p^p\right]\le nK\sum_{j=1}^n\left\|E\left[\left\|\phi_j-u_j^*\right\|_p^p\right]\right\|\exp(-\alpha(t-t_0)),\quad t\ge t_0. \tag{3.11}$$

By (3.11) and Lemma 2.5, one obtains that

$$E\left[\|u-u^*\|_p^p\right]\le e_p^{-1}(n)d_p(n)nK\sum_{j=1}^n\left\|E\left[\left\|\phi_j-u_j^*\right\|_p^p\right]\right\|\exp(-\alpha(t-t_0)),\quad t\ge t_0. \tag{3.12}$$

In order to prove Theorem 3.1, we need the following lemma. □

Lemma 3.2. *The nonconstant equilibrium solution of the problem* (1.1)–(1.3), $u^*(x)$ *satisfies* $E(\|u^*\|_p^p)=0$.

Proof. Set $F_i(u^*(x))=|u_i^*(x)|^p$. Similar to (3.8) in proof of Theorem 3.1, one has that

$$\sum_{j=1}^n p_{ij}\int_t^{t+\Delta t}E\left[\|u_i^*\|_p^p\right]ds+\sum_{j=1}^n q_{ij}\int_t^{t+\Delta t}E\left[\left\|u_j^*\right\|_p^p\right]ds\ge 0. \tag{3.13}$$

By (3.13) and the assumption that $-(p_{ji}+q_{ij})_{n\times n} := P^T + Q$ is an M-matrix, one obtains that

$$-\sum_{j=1}^{n}(p_{ji}+q_{ij})E\left[\left\|u_j^*\right\|_p^p\right]\Delta t \le 0, \quad i=1,2,\ldots,n. \tag{3.14}$$

Because of $E[\|u_j^*\|_p^p] \ge 0$ and $\Delta t > 0$, one has that $E(\|u^*\|_p^p) = 0$.

We continue the proof of Theorem 3.1 as the following.

By Lemma 3.2, one has that

$$\left\|E\left[\left\|\phi_j - u_j^*\right\|_p^p\right]\right\| \le M, \tag{3.15}$$

where $M > 0$ is a common number. We derive every solution of problem (1.1)–(1.3) such that

$$\limsup_{t\to\infty}\frac{1}{t}\log E\left[\|u-u^*\|_p^p\right] \le -\alpha, \tag{3.16}$$

then a nonconstant equilibrium solution of problem (1.1)–(1.3) about L^p norm is exponential stability in pth moment. The proof of Theorem 3.1 is complete. □

In order to illustrate the application of the theorem, we give an example.

Example 3.3. Discuss the stochastic reaction-diffusion neural network with time-varying delays and p-Laplacian as the following:

$$\begin{aligned} du_1(t,x) = {} & \left[\sum_{k=1}^{2}\frac{\partial}{\partial x_k}\left(|\nabla u_1|^{p-2}\frac{\partial u_1}{\partial x_k}\right) - a_1u_1 + I_1 + \sum_{j=1}^{2}T_{1j}g_j\big(u_j(t-\tau_j(t),x)\big)\right]dt \\ & + \sigma_1(u_1(t,x))dW(t), \end{aligned} \tag{3.17}$$

$$\begin{aligned} du_2(t,x) = {} & \left[\sum_{k=1}^{2}\frac{\partial}{\partial x_k}\left(|\nabla u_2|^{p-2}\frac{\partial u_2}{\partial x_k}\right) - a_2u_2 + I_2 + \sum_{j=1}^{2}T_{2j}g_j\big(u_j(t-\tau_j(t),x)\big)\right]dt \\ & + \sigma_2(u_2(t,x))dW(t), \end{aligned} \tag{3.18}$$

$$\frac{\partial u_i}{\partial n} := \left(\frac{\partial u_i}{\partial x_1}, \frac{\partial u_i}{\partial x_2}\right)^T = 0, \quad i=1,2,\ t\ge t_0\ge 0,\ x\in\partial\Omega, \tag{3.19}$$

$$u_i(t_0+s,x) = \phi_i(s,x), \quad -\tau_i(t_0)\le s\le 0,\ 0\le\tau_i(t)\le\tau_i,\ i=1,2,\ x\in\Omega, \tag{3.20}$$

where

$$\begin{gathered} a_1 = 1.2, \qquad a_2 = 1.8, \qquad \sigma_1(u_1) = 0.2(u_1 - u_1^*), \qquad \sigma_2(u_2) = 0.3(u_2 - u_2^*), \\ T_{11} = 1.5, \qquad T_{12} = 0.5, \qquad T_{21} = 0.6, \qquad T_{22} = 1, \\ g_1(u_1(t-\tau_1(t),x)) = 0.3u_1(t-\tau_1(t),x), \\ g_2(u_2(t-\tau_2(t),x)) = 0.2u_2(t-\tau_2(t),x)\cos(u_2(t-\tau_2(t),x)). \end{gathered} \tag{3.21}$$

Set $u^*(x) = (u_1^*(x), u_1^*(x))^T$ as a nonconstant equilibrium solution of (3.17) and (3.18). One can derive that

$$\begin{gathered} \left|g_1(v_1) - g_1(v_2)\right| \le 0.3|v_1 - v_2|, \quad c_1 = 0.3; \\ \left|g_2(v_1) - g_2(v_2)\right| \le 0.4|v_1 - v_2|, \quad c_2 = 0.4; \\ \left|(\sigma_1(v_1) - \sigma_1(v_2))(\sigma_1(v_1) - \sigma_1(v_2))^T\right| \le 0.04|v_1 - v_2|, \quad \lambda_1 = 0.02; \\ \left|(\sigma_2(v_1) - \sigma_2(v_2))(\sigma_2(v_1) - \sigma_2(v_2))^T\right| \le 0.09|v_1 - v_2|, \quad \lambda_2 = 0.045. \end{gathered} \tag{3.22}$$

Taking $\varepsilon_i = 1$ $(i = 1, 2)$ and $p = 3$, one has that

$$M_3 = \begin{pmatrix} 2.58 & -0.56 \\ -0.58 & 1.53 \end{pmatrix}, \tag{3.23}$$

and M_3 is an M-matrix. The nonconstant equilibrium solution of (3.17) and (3.18) about L^3 norm is exponential stability in the 3rd moment.

Remark 3.4. The Theorem 3.1 extends the correlative results in [12, 13, 16] to the situation related to the p-Laplacian.

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