Hindawi Publishing Corporation
Journal of Applied Mathematics
Volume 2012, Article ID 486458, 18 pages
doi:10.1155/2012/486458

Research Article

New Traveling Wave Solutions by the Extended Generalized Riccati Equation Mapping Method of the $(2+1)$-Dimensional Evolution Equation

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Received 19 September 2012; Accepted 14 October 2012

Academic Editor: Mohamed A. Abdou

The generalized Riccati equation mapping is extended with the basic (G'/G)-expansion method which is powerful and straightforward mathematical tool for solving nonlinear partial differential equations. In this paper, we construct twenty-seven traveling wave solutions for the (2+1)-dimensional modified Zakharov-Kuznetsov equation by applying this method. Further, the auxiliary equation $G'(\eta) = w + uG(\eta) + vG^2(\eta)$ is executed with arbitrary constant coefficients and called the generalized Riccati equation. The obtained solutions including solitons and periodic solutions are illustrated through the hyperbolic functions, the trigonometric functions, and the rational functions. In addition, it is worth declaring that one of our solutions is identical for special case with already established result which verifies our other solutions. Moreover, some of obtained solutions are depicted in the figures with the aid of Maple.

1. Introduction

The study of analytical solutions for nonlinear partial differential equations (PDEs) has become more imperative and stimulating research fields in mathematical physics, engineering sciences, and other technical arena [1–47]. In the recent past, a wide range of methods have been developed to construct traveling wave solutions of nonlinear PDEs such as, the inverse scattering method [1], the Backlund transformation method [2], the Hirota bilinear transformation method [3], the bifurcation method [4, 5], the Jacobi elliptic function expansion method [6–8], the Weierstrass elliptic function method [9], the direct algebraic method [10], the homotopy perturbation method [11, 12], the Exp-function method [13–17], and others [18–28].

Recently, Wang et al. [29] presented a widely used method, called the (G'/G)-expansion method to obtain traveling wave solutions for some nonlinear evolution equations (NLEEs). Further, in this method, the second-order linear ordinary differential equation

$G''(\eta) + \lambda G'(\eta) + \mu G(\eta) = 0$ is implemented, as an auxiliary equation, where λ and μ are constant coefficients. Afterwards, many researchers investigated many nonlinear PDEs to construct traveling wave solutions via this powerful (G'/G)-expansion method. For example, Feng et al. [30] applied the same method for obtaining exact solutions of the Kolmogorov-Petrovskii-Piskunov equation. In [31], Naher et al. concerned about this method to construct traveling wave solutions for the higher-order Caudrey-Dodd-Gibbon equation. Zayed and Al-Joudi [32] studied some nonlinear partial differential equations to obtain analytical solutions by using the same method whereas Gepreel [33] executed this method and found exact solutions of nonlinear PDEs with variable coefficients in mathematical physics. Abazari and Abazari [34] constructed exact solutions for the Hirota-Ramani equation by using this method. Ozis and Aslan [35] established some traveling wave solutions for the Kawahara type equations via the same method. Naher et al. [36] investigated higher dimensional nonlinear evolution equation for obtaining traveling wave solutions by applying the improved (G'/G)-expansion method. Naher and Abdullah [37] implemented this method to construct some new traveling wave solutions of the nonlinear reaction diffusion equation whilst they [38] studied the combined KdV-MKdV equation for obtaining abundant solutions via the same method. And [39] they executed this method to establish some new traveling wave solutions of the $(2+1)$-dimensional modified Zakharov-Kuznetsov equation and so on.

Zhu [40] investigated the (2 + 1)-dimensional Boiti-Leon-Pempinelle equation by applying the generalized Riccati equation mapping with the extended tanh-function method. In addition, $G'(\eta) = w + uG(\eta) + vG^2(\eta)$ is used, as an auxiliary equation and called generalized Riccati equation, where u, v, and w are arbitrary constants. Bekir and Cevikel [41] concerned about the tanh-coth method combined with the Riccati equation to study nonlinear coupled equation in mathematical physics. Guo et al. [42] implemented the extended Riccati equation mapping method for solving the diffusion-reaction and the mKdV equation with variable coefficient whilst Li et al. [43] studied higher-dimensional Jimbo-Miwa equation via the generalized Riccati equation expansion method. Salas [44] obtained some exact solutions for the Caudrey-Dodd-Gibbon equation by applying the projective Riccati equation method whereas Naher and Abdullah [45] studied the modified Benjamin-Bona-Mahony equation via the generalized Riccati equation mapping with the basic (G'/G)-expansion method for constructing traveling wave solutions and so on.

Many researchers implemented various methods to investigate the $(2+1)$-dimensional modified Zakharov-Kuznetsov equation. For instance, Khalfallah [46] used homogeneous balance method to establish traveling wave solutions of this equation. In [47], Bekir applied basic (G'/G)-expansion method for obtaining exact traveling wave solutions for the same equation. In this basic (G'/G)-expansion method, they employed second-order linear ordinary differential equation (LODE) with constant coefficients, as an auxiliary equation instead of generalized Riccati equation.

The importance of our present work is, in order to construct many new traveling wave solutions including solitons, periodic, and rational solutions, a (2 + 1)-dimensional Modified Zakharov-Kuznetsov equation considered by applying the extended generalized Riccati equation mapping method.

2. The Extended Generalized Riccati Equation Mapping Method

Suppose the general nonlinear partial differential equation

$$H(v, v_t, v_x, v_y, v_{xt}, v_{yt}, v_{xy}, v_{tt}, v_{xx}, v_{yy}, \ldots) = 0, \tag{2.1}$$

where $v = v(x, y, t)$ is an unknown function, H is a polynomial in $v(x, y, t)$, and the subscripts indicate the partial derivatives.

The most important steps of the generalized Riccati equation mapping together with the (G'/G)-expansion method [29, 40] are as follows.

Step 1. Consider the traveling wave variable:

$$v(x, y, t) = g(\eta), \qquad \eta = x + y - Ct, \tag{2.2}$$

where C is the speed of the traveling wave. Now using (2.2), (2.1) is converted into an ordinary differential equation for $g(\eta)$:

$$F(g, g', g'', g''', \ldots) = 0, \tag{2.3}$$

where the superscripts stand for the ordinary derivatives with respect to η.

Step 2. Equation (2.3) integrates term by term one or more times according to possibility and yields constant(s) of integration. The integral constant(s) may be zero for simplicity.

Step 3. Suppose that the traveling wave solution of (2.3) can be expressed in the form [29, 40]

$$g(\eta) = \sum_{j=0}^{n} e_j \left(\frac{G'}{G}\right)^j, \tag{2.4}$$

where e_j $(j = 0, 1, 2, \ldots, n)$ and $e_n \neq 0$, with $G = G(\eta)$ is the solution of the generalized Riccati equation:

$$G' = w + uG + vG^2, \tag{2.5}$$

where u, v, w are arbitrary constants and $v \neq 0$.

Step 4. To decide the positive integer n, consider the homogeneous balance between the nonlinear terms and the highest order derivatives appearing in (2.3).

Step 5. Substitute (2.4) along with (2.5) into the (2.3), then collect all the coefficients with the same order, the left hand side of (2.3) converts into polynomials in $G^k(\eta)$ and $G^{-k}(\eta)$, $(k = 0, 1, 2, \ldots)$. Then equating each coefficient of the polynomials to zero and yield a set of algebraic equations for e_j $(j = 0, 1, 2, \ldots, n)$, u, v, w, and C.

Step 6. Solve the system of algebraic equations which are found in Step 5 with the aid of algebraic software Maple to obtain values for e_j $(j = 0, 1, 2, \ldots, n)$ and C then, substitute obtained values in (2.4) along with (2.5) with the value of n, we obtain exact solutions of (2.1).

In the following, we have twenty seven solutions including four different families of (2.5).

Family 1. When $u^2 - 4vw > 0$ and $uv \neq 0$ or $vw \neq 0$, the solutions of (2.5) are:

$$
\begin{aligned}
G_1 &= \frac{-1}{2v}\left(u + \sqrt{u^2 - 4vw}\tanh\left(\frac{\sqrt{u^2 - 4vw}}{2}\eta\right)\right),\\
G_2 &= \frac{-1}{2v}\left(u + \sqrt{u^2 - 4vw}\coth\left(\frac{\sqrt{u^2 - 4vw}}{2}\eta\right)\right),\\
G_3 &= \frac{-1}{2v}\left(u + \sqrt{u^2 - 4vw}\left(\tanh\left(\sqrt{u^2 - 4vw}\ \eta\right) \pm \operatorname{isech}\left(\sqrt{u^2 - 4vw}\eta\right)\right)\right),\\
G_4 &= \frac{-1}{2v}\left(u + \sqrt{u^2 - 4vw}\left(\coth\left(\sqrt{u^2 - 4vw}\ \eta\right) \pm \operatorname{csch}\left(\sqrt{u^2 - 4vw}\eta\right)\right)\right),\\
G_5 &= \frac{-1}{4v}\left(2u + \sqrt{u^2 - 4vw}\left(\tanh\left(\frac{\sqrt{u^2 - 4vw}}{4}\eta\right) + \coth\left(\frac{\sqrt{u^2 - 4vw}}{4}\eta\right)\right)\right),\\
G_6 &= \frac{1}{2v}\left(-u + \frac{\pm\sqrt{(D^2 + E^2)(u^2 - 4vw)} - D\sqrt{u^2 - 4vw}\ \cosh\left(\sqrt{u^2 - 4vw}\eta\right)}{D\sinh\left(\sqrt{u^2 - 4vw}\ \eta\right) + E}\right),\\
G_7 &= \frac{1}{2v}\left(-u - \frac{\pm\sqrt{(D^2 + E^2)(u^2 - 4vw)} + D\sqrt{u^2 - 4vw}\cosh\left(\sqrt{u^2 - 4vw}\eta\right)}{D\sinh\left(\sqrt{u^2 - 4vw}\ \eta\right) + E}\right),
\end{aligned}
\tag{2.6}
$$

where D and E are two nonzero real constants.

$$
\begin{aligned}
G_8 &= \frac{2w\cosh\left(\left(\sqrt{u^2 - 4vw}/2\right)\eta\right)}{\sqrt{u^2 - 4vw}\sinh\left(\left(\sqrt{u^2 - 4vw}/2\right)\eta\right) - u\cosh\left(\left(\sqrt{u^2 - 4vw}/2\right)\eta\right)},\\
G_9 &= \frac{-2w\sinh\left(\left(\sqrt{u^2 - 4vw}/2\right)\eta\right)}{u\sinh\left(\left(\sqrt{u^2 - 4vw}/2\right)\eta\right) - \sqrt{u^2 - 4vw}\cosh\left(\left(\sqrt{u^2 - 4vw}/2\right)\eta\right)},\\
G_{10} &= \frac{2w\cosh\left(\sqrt{u^2 - 4vw}\eta\right)}{\sqrt{u^2 - 4vw}\sinh\left(\sqrt{u^2 - 4vw}\eta\right) - u\cosh\left(\sqrt{u^2 - 4vw}\eta\right) \pm i\sqrt{u^2 - 4vw}},\\
G_{11} &= \frac{2w\sinh\left(\sqrt{u^2 - 4vw}\eta\right)}{-u\sinh\left(\sqrt{u^2 - 4vw}\eta\right) + \sqrt{u^2 - 4vw}\cosh\left(\sqrt{u^2 - 4vw}\eta\right) \pm \sqrt{u^2 - 4vw}},\\
G_{12} &= \frac{4w\sinh\left(\left(\sqrt{u^2 - 4vw}/4\right)\eta\right)\cosh\left(\left(\sqrt{u^2 - 4vw}/4\right)\eta\right)}{-2u\sinh\left(\left(\sqrt{u^2 - 4vw}/4\right)\eta\right)\cosh\left(\left(\sqrt{u^2 - 4vw}/4\right)\eta\right) + \Delta_1},
\end{aligned}
\tag{2.7}
$$

where $\Delta_1 = 2\sqrt{u^2 - 4vw}\cosh^2((\sqrt{u^2 - 4vw}/4)\eta) - \sqrt{u^2 - 4vw}$.

Family 2. When $u^2 - 4vw < 0$ and $uv \neq 0$ or $vw \neq 0$, the solutions of (2.5) are:

$$
\begin{aligned}
G_{13} &= \frac{1}{2v}\left(-u + \sqrt{4vw - u^2}\,\tan\left(\frac{\sqrt{4vw - u^2}}{2}\eta\right)\right),\\
G_{14} &= \frac{-1}{2v}\left(u + \sqrt{4vw - u^2}\cot\left(\frac{\sqrt{4vw - u^2}}{2}\eta\right)\right),\\
G_{15} &= \frac{1}{2v}\left(-u + \sqrt{4vw - u^2}\left(\tan\left(\sqrt{4vw - u^2}\ \eta\right) \pm \sec\left(\sqrt{4vw - u^2}\eta\right)\right)\right),\\
G_{16} &= \frac{-1}{2v}\left(u + \sqrt{4vw - u^2}\left(\cot\left(\sqrt{4vw - u^2}\eta\right) \pm \csc\left(\sqrt{4vw - u^2}\eta\right)\right)\right),\\
G_{17} &= \frac{1}{4v}\left(-2u + \sqrt{4vw - u^2}\left(\tan\left(\frac{\sqrt{4vw - u^2}}{4}\eta\right) - \cot\left(\frac{\sqrt{4vw - u^2}}{4}\eta\right)\right)\right),\\
G_{18} &= \frac{1}{2v}\left(-u + \frac{\pm\sqrt{(D^2 - E^2)(4vw - u^2)} - D\sqrt{4vw - u^2}\cos\left(\sqrt{4vw - u^2}\eta\right)}{D\ \sin\left(\sqrt{4vw - u^2}\eta\right) + E}\right),\\
G_{19} &= \frac{1}{2v}\left(-u - \frac{\pm\sqrt{(D^2 - E^2)(4vw - u^2)} + D\sqrt{4vw - u^2}\cos\left(\sqrt{4vw - u^2}\eta\right)}{D\sin\left(\sqrt{4vw - u^2}\eta\right) + E}\right),
\end{aligned}
\tag{2.8}
$$

where D and E are two nonzero real constants and satisfy $D^2 - E^2 > 0$.

$$
\begin{aligned}
G_{20} &= \frac{-2w\cos\left(\left(\sqrt{4vw - u^2}/2\right)\eta\right)}{\sqrt{4vw - u^2}\sin\left(\left(\sqrt{4vw - u^2}/2\right)\eta\right) + u\cos\left(\left(\sqrt{4vw - u^2}/2\right)\eta\right)},\\
G_{21} &= \frac{2w\sin\left(\left(\sqrt{4vw - u^2}/2\right)\eta\right)}{-u\sin\left(\left(\sqrt{4vw - u^2}/2\right)\eta\right) + \sqrt{4vw - u^2}\cos\left(\left(\sqrt{4vw - u^2}/2\right)\eta\right)},\\
G_{22} &= \frac{-\ 2w\cos\left(\sqrt{4vw - u^2}\ \eta\right)}{\sqrt{4vw - u^2}\sin\left(\sqrt{4vw - u^2}\eta\right) + u\cos\left(\sqrt{4vw - u^2}\eta\right) \pm \sqrt{4vw - u^2}},\\
G_{23} &= \frac{2w\sin\left(\sqrt{4vw - u^2}\ \eta\right)}{-u\sin\left(\sqrt{4vw - u^2}\eta\right) + \sqrt{4vw - u^2}\cos\left(\sqrt{4vw - u^2}\eta\right) \pm \sqrt{4vw - u^2}},\\
G_{24} &= \frac{4w\sin\left(\left(\sqrt{4vw - u^2}/4\right)\eta\right)\cos\left(\left(\sqrt{4vw - u^2}/4\right)\eta\right)}{-2u\sin\left(\left(\sqrt{4vw - u^2}/4\right)\eta\right)\cos\left(\left(\sqrt{4vw - u^2}/4\right)\eta\right) + \Delta_2},
\end{aligned}
\tag{2.9}
$$

where $\Delta_2 = 2\sqrt{4vw - u^2}\cos^2((\sqrt{4vw - u^2}\ /4)\eta) - \sqrt{4vw - u^2}$.

Family 3. When $w = 0$ and $uv \neq 0$, the solution (2.5) becomes:

$$G_{25} = \frac{-uf_1}{v(f_1 + \cosh(u\eta) - \sinh(u\eta))},$$
$$G_{26} = \frac{-u(\cosh(u\eta) + \sinh(u\eta))}{v(f_1 + \cosh(u\eta) + \sinh(u\eta))}, \tag{2.10}$$

where f_1 is an arbitrary constant.

Family 4. when $v \neq 0$ and $w = u = 0$, the solution of (2.5) becomes:

$$G_{27} = \frac{-1}{v\eta + l_1}, \tag{2.11}$$

where l_1 is an arbitrary constant.

3. Applications of the Method

In this section, we have constructed new traveling wave solutions for the $(2+1)$-dimensional modified Zakharov-Kuznetsov equation by using the method.

3.1. The $(2+1)$*-Dimensional Modified Zakharov-Kuznetsov Equation*

We consider the (2 + 1)-dimensional Modified Zakharov-Kuznetsov equation followed by Bekir [47]

$$u_t + u^2\, u_x + \, u_{xxx} + u_{xyy} = 0. \tag{3.1}$$

Now, we use the wave transformation (2.2) into the (3.1), which yields:

$$-Cg' + u^2 g' + 2g''' = 0. \tag{3.2}$$

Equation (3.2) is integrable, therefore, integrating with respect η once yields:

$$Q - Cg + \frac{1}{3}g^3 + 2g'' = 0, \tag{3.3}$$

where Q is an integral constant which is to be determined later.

Taking the homogeneous balance between g^3 and g'' in (3.3), we obtain $n = 1$.

Therefore, the solution of (3.3) is of the form:

$$g(\eta) = e_1\left(\frac{G'}{G}\right) + e_0, \quad e_1 \neq 0. \tag{3.4}$$

Using (2.5), (3.4) can be rewritten as

$$g(\eta) = e_1\left(u + wG^{-1} + vG\right) + e_0, \tag{3.5}$$

where u, v, and w are free parameters.

By substituting (3.5) into (3.3), the left hand side is converted into polynomials in G^k and G^{-k} $(k = 0, 1, 2, \ldots)$. Setting each coefficient of these resulted polynomials to zero, we obtain a set of algebraic equations for e_0, e_1, u, v, w, Q, and C (algebraic equations are not shown, for simplicity). Solving the system of algebraic equations with the help of algebraic software Maple, we obtain

$$e_0 = \mp ui\sqrt{3}, \qquad e_1 = \pm 2i\sqrt{3}, \qquad C = -u^2 - 8vw, \qquad Q = 8uvwi\sqrt{3}. \tag{3.6}$$

Family 5. The soliton and soliton-like solutions of (3.1) (when $u^2 - 4vw > 0$ and $uv \neq 0$ or $vw \neq 0$) are:

$$g_1 = \pm 2i\sqrt{3}\frac{2\Psi^2 \operatorname{sech}^2(\Psi\eta)}{u + 2\Psi \tanh(\Psi\eta)} \mp ui\sqrt{3}, \tag{3.7}$$

where $\Psi = (1/2)\sqrt{u^2 - 4vw}$, $\eta = x + y + (u^2 + 8vw)t$ and u, v, w are arbitrary constants.

$$\begin{gathered}
g_2 = \mp 2i\sqrt{3}\,\frac{2\Psi^2 \operatorname{csch}^2(\Psi\eta)}{u + 2\Psi \coth(\Psi\eta)} \mp ui\sqrt{3}, \\
g_3 = \pm 2i\sqrt{3}\,\frac{4\Psi^2 \operatorname{sech}(2\Psi\eta)(1 \mp i\sinh(2\Psi\eta))}{u\cosh(2\Psi\eta) + 2\Psi\sinh(2\Psi\eta) \pm i2\Psi} \mp ui\sqrt{3}, \\
g_4 = \mp 2i\sqrt{3}\,\frac{2\Psi^2 \operatorname{csch}(\Psi\eta)}{u\sinh(\Psi\eta) + 2\Psi\cosh(\Psi\eta)} \mp ui\sqrt{3}, \\
g_5 = \mp 2i\sqrt{3}\,\frac{4\Psi^2 \operatorname{csch}(2\Psi\eta)}{u\tanh(\Psi\eta) + 2\Psi} \mp ui\sqrt{3}, \\
g_6 = \mp 2i\sqrt{3}\,\frac{4D\Psi^2\left(D - E\sinh(2\Psi\eta) - \sqrt{(D^2 + E^2)}\cosh(2\Psi\eta)\right)}{(D\sinh(2\Psi\eta) + E)\Omega_1} \mp ui\sqrt{3}, \\
g_7 = \mp 2i\sqrt{3}\,\frac{4D\Psi^2\left(D - E\sinh(2\Psi\eta) + \sqrt{(D^2 + E^2)}\cosh(2\Psi\eta)\right)}{(D\sinh(2\Psi\eta) + E)\Omega_2} \mp ui\sqrt{3},
\end{gathered} \tag{3.8}$$

where $\Omega_1 = uD\sinh(2\Psi\eta) + uE - 2\Psi\sqrt{(D^2+E^2)} + 2D\Psi\cosh(2\Psi\eta)$, $\Omega_2 = uD\sinh(2\Psi\eta) + uE + 2\Psi\sqrt{(D^2+E^2)} + 2D\Psi\cosh(2\Psi\eta)$, D and E are two nonzero real constants.

$$
\begin{aligned}
g_8 &= \mp 2i\sqrt{3}\frac{2\Psi^{\ 2}\operatorname{sech}(\Psi\eta)}{2\Psi\sinh(\Psi\eta) - u\cosh(\Psi\eta)} \mp ui\sqrt{3},\\
g_9 &= \pm 2i\sqrt{3}\frac{2\Psi^{\ 2}\operatorname{csch}(\Psi\eta)}{2\Psi\cosh(\Psi\eta) - u\sinh(\Psi\eta)} \mp ui\sqrt{3},\\
g_{10} &= \pm 2i\sqrt{3}\frac{4\Psi^2\operatorname{sech}(2\Psi\ \eta)(1 \mp i\sinh(2\Psi\eta))}{u\cosh(2\Psi\eta) - 2\Psi\sinh(2\Psi\eta) \mp i2\Psi} \mp ui\sqrt{3},\\
g_{11} &= \pm 2i\sqrt{3}\frac{4\Psi^2\operatorname{csch}(2\Psi\eta)(1 \pm \cosh(2\Psi\eta))}{2\Psi\cosh(2\Psi\eta) - u\sinh(2\Psi\eta) \pm 2\Psi} \mp ui\sqrt{3},\\
g_{12} &= \pm 2i\sqrt{3}\ \frac{2\Psi^2\operatorname{csch}(\Psi\eta)}{2\Psi\cosh(\Psi\eta) - u\sinh(\Psi\eta)} \mp ui\sqrt{3}.
\end{aligned}
\tag{3.9}
$$

Family 6. The periodic form solutions of (3.1) (when $u^2 - 4vw < 0$ and $uv \neq 0$ or $vw \neq 0$) are:

$$
g_{13} = \pm 2i\sqrt{3}\frac{2\Theta^2\sec^2(\Theta\eta)}{-u + 2\Theta\ \tan(\Theta\eta)} \mp ui\sqrt{3},
\tag{3.10}
$$

where $\Theta = (1/2)\sqrt{4\ vw - \ u^2}$, $\eta = x + y + (u^2 + 8vw)t$ and u, v, w are arbitrary constants.

$$
\begin{aligned}
g_{14} &= \mp 2i\sqrt{3}\frac{2\Theta^2\csc^2(\Theta\eta)}{u + 2\Theta\ \cot(\Theta\eta)} \mp ui\sqrt{3},\\
g_{15} &= \pm 2i\sqrt{3}\frac{4\Theta^2\sec(2\Theta\eta)(1 \pm \sin(2\Theta\eta))}{-u\cos(2\Theta\eta) + 2\Theta\sin(2\Theta\eta) \pm 2\Theta} \mp ui\sqrt{3},\\
g_{16} &= \mp 2i\sqrt{3}\frac{2\Theta^2\sec(\Theta\eta)}{u\cos(\Theta\eta) + 2\Theta\sin(\Theta\eta)} \mp ui\sqrt{3},\\
g_{17} &= \mp 2i\sqrt{3}\frac{2\Theta^2\csc(\Theta\eta)}{u\sin(\Theta\eta) + 2\Theta\cos(\Theta\eta)} \mp ui\sqrt{3},\\
g_{18} &= \mp 2i\sqrt{3}\frac{4D\Theta^2\left(\sqrt{(D^2-E^2)}\cos(2\Theta\eta) - E\sin(2\Theta\eta) - D\right)}{(D\sin(2\Theta\eta) + E)\Omega_3} \mp ui\sqrt{3}.\\
g_{19} &= \mp 2i\sqrt{3}\frac{4D\Theta^2\left(\sqrt{(D^2-E^2)}\cos(2\Theta\eta) + E\sin(2\Theta\eta) + D\right)}{(D\sin(2\Theta\eta) + E)\Omega_4} \mp ui\sqrt{3},
\end{aligned}
\tag{3.11}
$$

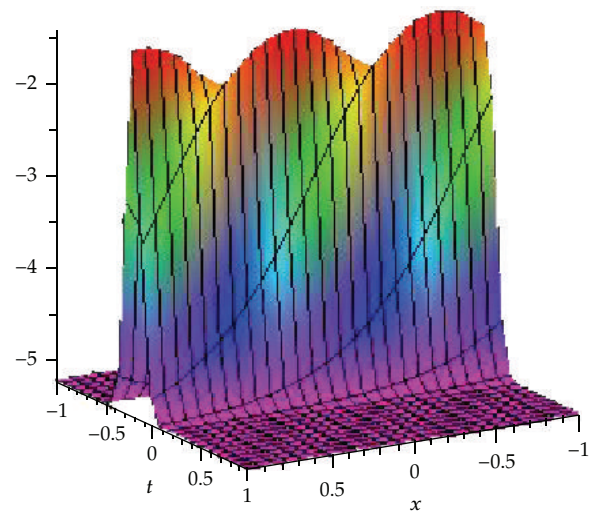

Figure 1: Solitons solution for $u = 3$, $v = 0.5$, $w = 0.25$.

where $\Omega_3 = uD\sin(2\Theta\eta) + 2D\Theta\cos(2\Theta\eta) + uE - 2\Theta\sqrt{(D^2 - E^2)}$, $\Omega_4 = uD\sin(2\Theta\eta) + 2D\Theta\cos(2\Theta\eta) + uE + 2\Theta\sqrt{(D^2 - E^2)}$, D and E are two nonzero real constants and satisfies $D^2 - E^2 > 0$.

$$
\begin{aligned}
g_{20} &= \mp 2i\sqrt{3}\,\frac{4\Theta^2\csc(2\Theta\eta)}{u\cot(\Theta\eta) + 2\Theta} \mp ui\sqrt{3},\\
g_{21} &= \mp 2i\sqrt{3}\,\frac{4\Theta^2\csc(2\Theta\eta)}{u\tan(\Theta\eta) + 2\Theta} \mp ui\sqrt{3},\\
g_{22} &= \mp 2i\sqrt{3}\frac{2\Theta^2\sec(2\Theta\eta)\left(1 \pm \sin(2\Theta\eta)\right)\left(u\cos(2\Theta\eta) + 2\Theta\sin(2\Theta\eta) \pm 2\Theta\right)}{(u^2 - 2vw)\cos^2(2\Theta\eta) + 2\Theta\left(1 \pm \sin(2\Theta\eta)\right)\left(2\Theta \pm u\cos(2\Theta\eta)\right)} \mp ui\sqrt{3},\\
g_{23} &= \pm 2i\sqrt{3}\frac{2\Theta^2\csc(2\Theta\eta)\left(-u\sin(2\Theta\eta) + 2\Theta\cos(2\Theta\eta) \pm 2\Theta\right)}{(2vw - u^2)\cos(2\Theta\eta) - 2u\Theta\sin(2\Theta\eta) \pm 2vw} \mp ui\sqrt{3},\\
g_{24} &= \mp 2i\sqrt{3}\,\frac{2\Theta^2\csc(\Theta\eta)}{u\sin(\Theta\eta) + 2\Theta\cos(\Theta\eta)} \mp ui\sqrt{3}.
\end{aligned}
\tag{3.12}
$$

Family 7. The soliton and soliton-like solutions of (3.1) (when $w = 0$ and $uv \neq 0$) are:

$$
\begin{aligned}
g_{25} &= \pm 2i\sqrt{3}\frac{u\left(\cosh(u\eta) - \sinh(u\eta)\right)}{f_1 + \cosh(u\eta) - \sinh(u\eta)} \mp ui\sqrt{3},\\
g_{26} &= \pm 2i\sqrt{3}\,\frac{uf_1}{f_1 + \cosh(u\eta) + \sinh(u\eta)} \mp ui\sqrt{3},
\end{aligned}
\tag{3.13}
$$

where f_1 is an arbitrary constant, $\eta = x + y + (u^2 + 8vw)t$.

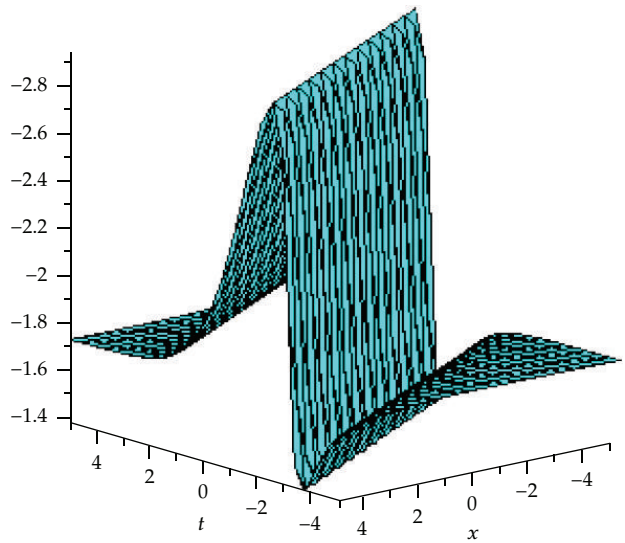

Figure 2: Periodic solution for $u = 1$, $v = 1$, $w = 0.125$.

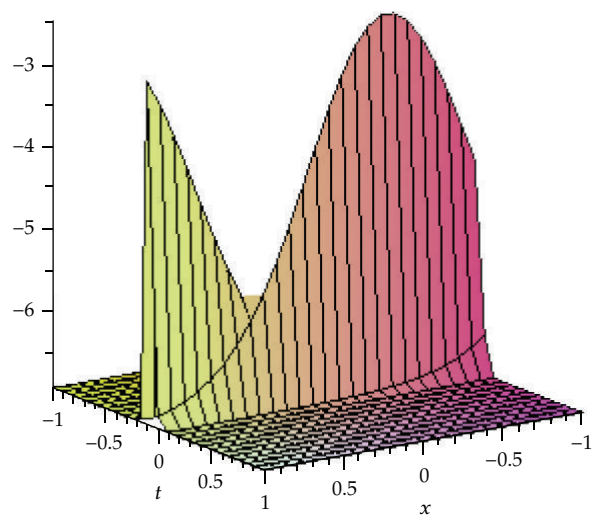

Figure 3: Solitons solution for $u = 4$, $v = 0.5$, $w = 1$.

Family 8. The rational function solution (when $v \neq 0$ and $w = u = 0$) is:

$$g_{27} = \frac{\mp 2iv\sqrt{3}}{v\eta + l_1}, \tag{3.14}$$

where l_1 is an arbitrary constant and $\eta = x + y + (u^2 + 8vw)t$.

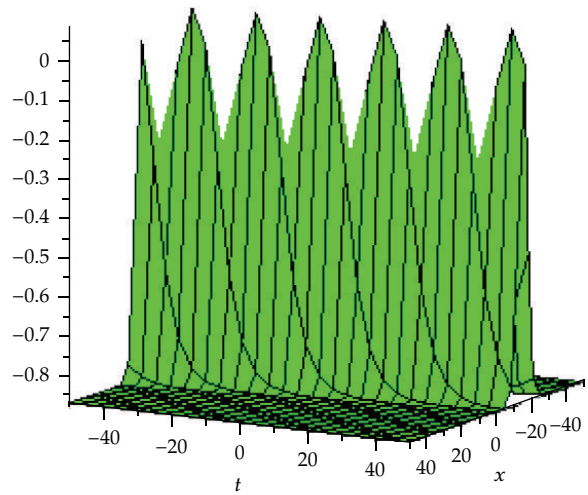

Figure 4: Solitons solution for $u = 0.5$, $v = 25.10^{-6}$, $w = 25.10^{-5}$.

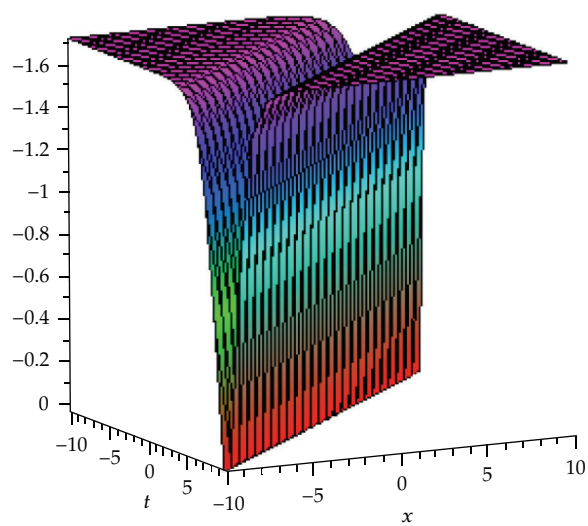

Figure 5: Periodic solution for $u = 1$, $v = 25.10^{-5}$, $w = 25.10^{-3}$.

4. Results and Discussion

It is significant to mention that one of our solutions is coincided for some special case with already published results which are presented in Table 1. Furthermore, some of newly constructed solutions are illustrated in Figures 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, and 12.

Table 1: Comparison between Bekir [47] solutions and newly obtained solutions.

Bekir [47] solutions	New solutions
(i) If $C_1 = 1, C_2 = \frac{1}{2}$ and $\lambda^2 - 4\mu = 0$, solution Equation (4.9) (from Section 4) becomes: $u_{5,6}(\xi) = \pm\, i\sqrt{3}\frac{2}{2+x}$.	(i) If $v = \frac{1}{2}, l_1 = 1, y = 0$ and $g_{27}(\eta) = u_{5,6}(\xi)$, solution g_{27} becomes: $u_{5,6}(\xi) = \pm\, i\sqrt{3}\frac{2}{2+x}$.
(ii) $C_1 = 1, C_2 = \frac{-1}{2}$ and $\lambda^2 - 4\mu = 0$, solution Equation (4.9) (from Section 4) becomes: $u_{5,6}(\xi) = \mp\, i\sqrt{3}\frac{2}{2-x}$.	(ii) If $v = \frac{-1}{2}, l_1 = 1, y = 0$ and $g_{27}(\eta) = u_{5,6}(\xi)$, solution g_{27} becomes: $u_{5,6}(\xi) = \mp\, i\sqrt{3}\frac{2}{2-x}$.
(iii) $C_1 = 0, C_2 = 1$ and $\lambda^2 - 4\mu = 0$, solution Equation (4.9) (from Section 4) becomes: $u_{5,6}(\xi) = \mp\, i\sqrt{3}\left(\frac{2}{x}\right)$.	(iii) If $v = 1, l_1 = 0, y = 0$ and $g_{27}(\eta) = u_{5,6}(\xi)$, solution g_{27} becomes: $u_{5,6}(\xi) = \mp\, i\sqrt{3}\left(\frac{2}{x}\right)$.

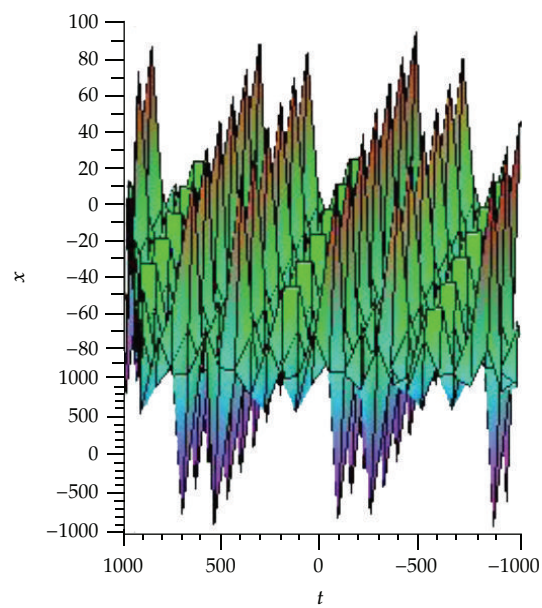

Figure 6: Solitons solution for $u = 1$, $v = 0.45$, $w = 3$.

As in Table 1, we have newly constructed traveling wave solutions g_1 to g_{26} which are not being stated in the earlier literature.

4.1. Graphical Depictions of Newly Obtained Traveling Wave Solutions

The graphical descriptions of some solutions are represented in Figures 1–12 with the aid of commercial software Maple.

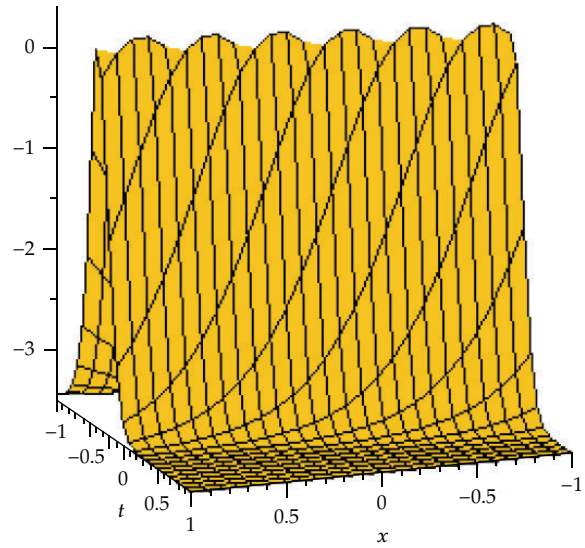

Figure 7: Solitons solution for $u = 2$, $v = 25.10^{-4}$, $w = 25.10^{-3}$.

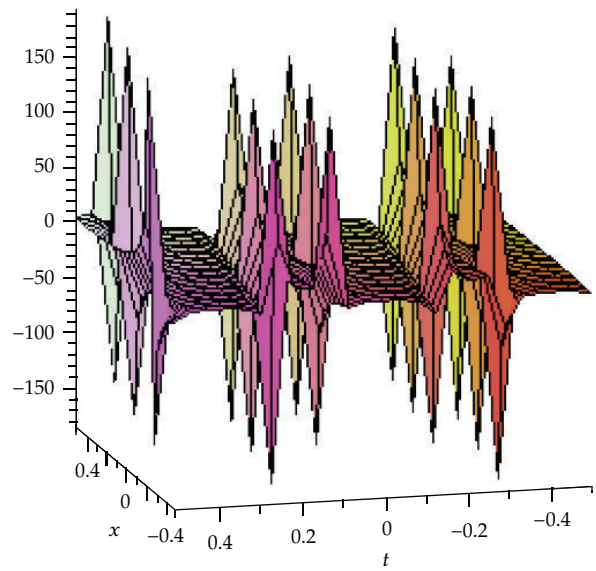

Figure 8: Solitons solution for $u = 1$, $v = 1$, $w = 1$.

5. Conclusions

In this paper, we have investigated the (2 + 1)-dimensional modified Zakharov-Kuznetsov equation via the extended generalized Riccati equation mapping method. Twenty seven exact traveling wave solutions are constructed including solitons and periodic wave solutions by applying this powerful method. In addition, newly obtained solutions are depicted in terms of the hyperbolic, the trigonometric, and the rational functional form. The obtained solutions

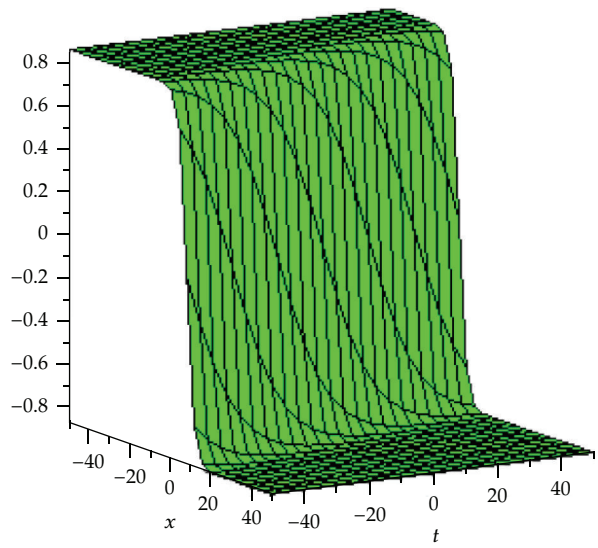

Figure 9: Periodic solution for $u = 0.5$, $v = 7$, $w = 0$, $f_1 = 0.125$.

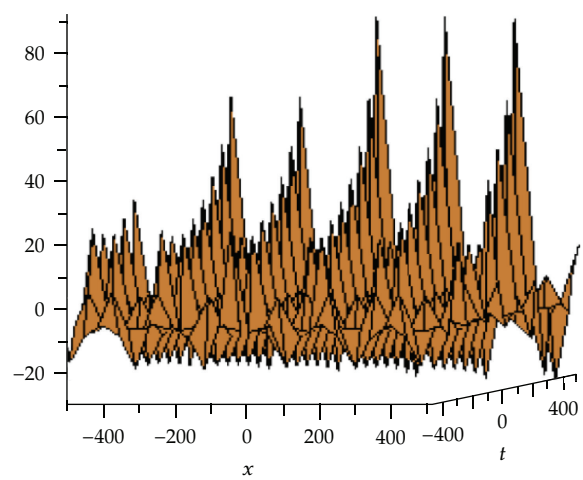

Figure 10: Solitons solution for $u = 1$, $v = 0.5$, $w = 1$.

reveal that this method is a promising mathematical tool because it can establish a variety of new solutions of dissimilar physical structures if compared with existing methods. The correctness of newly constructed solutions is verified to be compared with already published results. Consequently, nonlinear evolution equations which regularly arise in many scientific real-time application fields can be studied by applying the extended generalized Riccati equation mapping method.

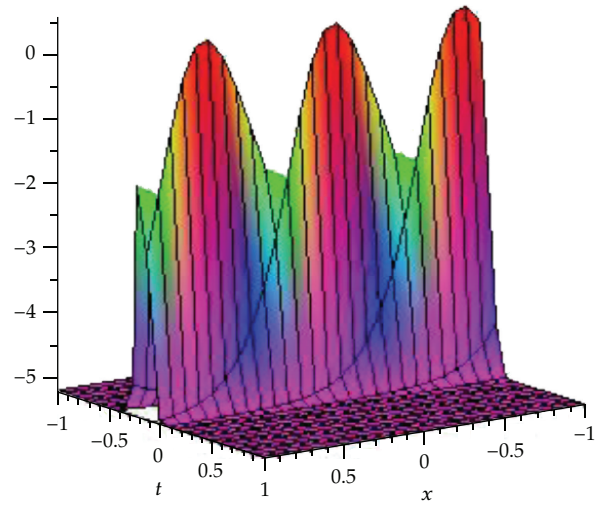

Figure 11: Solitons solution for $u = 3$, $v = 25.10^{-4}$, $w = 25.10^{-3}$.

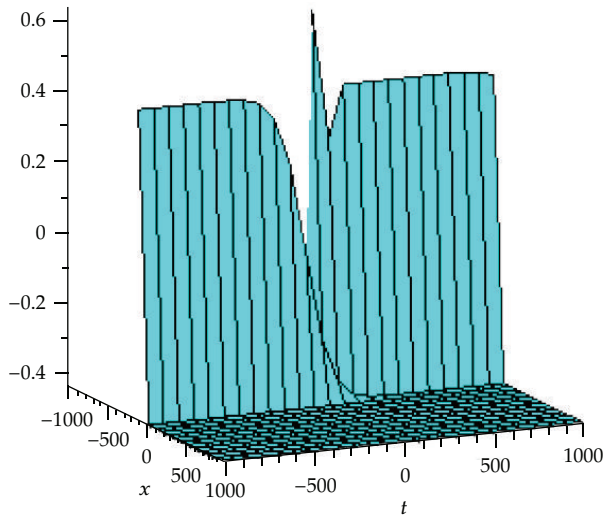

Figure 12: Periodic solution for $u = 0.25$, $v = -5$, $w = 0$, $f_1 = 0.25$.

Acknowledgments

This paper is supported by the USM short-term Grant (Ref. no. 304/PMATHS/6310072), and the authors would like to express their thanks to the School of Mathematical Sciences, USM for providing related research facilities.

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