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Research Article

Existence and Stability of Iterative Algorithm for a System of Random Set-Valued Variational Inclusion Problems Involving (A, m, η)-Generalized Monotone Operators

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We introduce and study a class of a system of random set-valued variational inclusion problems. Some conditions for the existence of solutions of such problems are provided, when the operators are contained in the classes of generalized monotone operators, so-called (A, m, η)-monotone operator. Further, the stability of the iterative algorithm for finding a solution of the considered problem is also discussed.

1. Introduction

It is well known that the ideas and techniques of the variational inequalities are being applied in a variety of diverse fields of pure and applied sciences and proven to be productive and innovative. It has been shown that this theory provides the most natural, direct, simple, unified, and efficient framework for a general treatment of a wide class of linear and nonlinear problems. The development of variational inequality theory can be viewed as the simultaneous pursuit of two different lines of research. On the one hand, it reveals the fundamental facts on the qualitative aspects of the solutions to important classes of problems. On the other hand, it also enables us to develop highly efficient and powerful new numerical methods for solving, for example, obstacle, unilateral, free, moving, and complex equilibrium problems. Of course, the concept of variational inequality has been extended and generalized in several directions, and it is worth to noticed that, an important and useful generalization of variational inequality problem is the concept of variational inclusion. Many efficient ways have been studied to find solutions for variational inclusions and a related technique, as resolvent operator technique, was of great concern.

In 2006, Jin [1] investigated the approximation solvability of a type of set-valued variational inclusions based on the convergence of (H, η)-resolvent operator technique, while the convergence analysis for approximate solutions much depends on the existence of Cauchy sequences generated by a proposed iterative algorithm. In the same year, Lan [2] first introduced a concept of (A, η)-monotone operators, which contains the class of (H, η)-monotonicity, A-monotonicity (see [3–5]), and other existing monotone operators as special cases. In such paper, he studied some properties of (A, η)-monotone operators and defined resolvent operators associated with (A, η)-monotone operators. Then, by using this new resolvent operator, he constructed some iterative algorithms to approximate the solutions of a new class of nonlinear (A, η)-monotone operator inclusion problems with relaxed cocoercive mappings in Hilbert spaces. After that, Verma [5] explored sensitivity analysis for strongly monotone variational inclusions using (A, η)-resolvent operator technique in a Hilbert space setting. For more examples, ones may consult [6–11].

Meanwhile, in 2001, Verma [12] introduced and studied some systems of variational inequalities and developed some iterative algorithms for approximating the solutions of such those problems. Furthermore, in 2004, Fang and Huang [13] introduced and studied some new systems of variational inclusions involving H-monotone operators. By Using the resolvent operator associated with H-monotone operators, they proved the existence and uniqueness of solutions for the such considered problem, and also some new algorithms for approximating the solutions are provided. Consequently, in 2007, Lan et al. [14] introduced and studied another system of nonlinear A-monotone multivalued variational inclusions in Hilbert spaces. Recently, based on the generalized (A, η)-resolvent operator method, Argarwal and Verma [15] considered the existence and approximation of solutions for a general system of nonlinear set-valued variational inclusions involving relaxed cocoercive mappings in Hilbert spaces. Notice that, the concept of a system of variational inequality is very interesting since it is well-known that a variety of equilibrium models, for example, the traffic equilibrium problem, the spatial equilibrium problem, the Nash equilibrium problem, and the general equilibrium programming problem, can be uniformly modelled as a system of variational inequalities. Additional researches on the approximate solvability of a system of nonlinear variational inequalities are problems; ones may see Cho et al. [16], Cho and Petrot [17], Noor [18], Petrot [19], Suantai and Petrot [20], and others.

On the other hand, the systematic study of random equations employing the techniques of functional analysis was first introduced by Špaček [21] and Hanš [22], and it has received considerable attention from numerous authors. It is well known that the theory of randomness leads to several new questions like measurability of solutions, probabilistic and statistical aspects of random solutions estimate for the difference between the mean value of the solutions of the random equations and deterministic solutions of the averaged equations. The main question concerning random operator equations is essentially the same as those of deterministic operator equations, that is, a question of existence, uniqueness, characterization, contraction, and approximation of solutions. Of course, random variational inequality theory is an important part of random function analysis. This topic has attracted many scholars and experts due to the extensive applications of the random problems. For the examples of research works in these fascinating areas, ones may see Ahmad and Bazán [23], Huang [24], Huang et al. [25], Khan et al. [26], Lan [27], and Noor and Elsanousi [28].

In this paper, inspired by the works going on these fields, we introduce a system of set-valued random variational inclusion problems and provide the sufficient conditions for the existence of solutions and the algorithm for finding a solution of proposed problems, involving a class of generalized monotone operators by using the resolvent operator technique. Furthermore, the stability of the constructed iterative algorithm is also discussed.

2. Preliminaries

Let $\mathcal{H}$ be a real Hilbert space equipped with norm $\|\cdot\|$ and inner product $\langle\cdot,\cdot\rangle$, and let $2^{\mathcal{H}}$ and $\mathrm{CB}(\mathcal{H})$ denote for the family of all the nonempty subsets of $\mathcal{H}$ and the family of all the nonempty closed bounded subsets of $\mathcal{H}$, respectively. As usual, we will define $D : \mathrm{CB}(\mathcal{H}) \times \mathrm{CB}(\mathcal{H}) \to [0,\infty)$, the Hausdorff metric on $\mathrm{CB}(\mathcal{H})$, by

$$D(A,B) = \max\left\{\sup_{x\in A}\inf_{y\in B}\|x-y\|, \sup_{y\in B}\inf_{x\in A}\|x-y\|\right\}, \quad \forall A,B\in \mathrm{CB}(\mathcal{H}). \tag{2.1}$$

Let (Ω,Σ,μ) be a complete σ-finite measure space and $\mathcal{B}(\mathcal{H})$ the class of Borel σ-fields in $\mathcal{H}$. A mapping $x : \Omega \to \mathcal{H}$ is said to be measurable if $\{t\in\Omega : x(t)\in B\}\in\Sigma$, for all $B\in\mathcal{B}(\mathcal{H})$. We will denote by $\mathcal{M}_{\mathcal{H}}$ a set of all measurable mappings on $\mathcal{H}$, that is, $\mathcal{M}_{\mathcal{H}} = \{x : \Omega\to\mathcal{H} \mid x \text{ is a measurable mapping}\}$.

Let $\mathcal{H}_1$ and $\mathcal{H}_2$ be two real Hilbert spaces. Let $F : \Omega\times\mathcal{H}_1\times\mathcal{H}_2 \to \mathcal{H}_1$ and $G : \Omega\times\mathcal{H}_1\times\mathcal{H}_2 \to \mathcal{H}_2$ be single-valued mappings. Let $U : \Omega\times\mathcal{H}_1 \to \mathrm{CB}(\mathcal{H}_1)$, $V : \Omega\times\mathcal{H}_2 \to \mathrm{CB}(\mathcal{H}_2)$, and $M_i : \Omega\times\mathcal{H}_i \to 2^{\mathcal{H}_i}$ be set-valued mappings, for $i = 1,2$. In this paper, we will consider the following problem: find measurable mappings $a,u : \Omega\to\mathcal{H}_1$ and $b,v : \Omega\to\mathcal{H}_2$ such that $u(t)\in U(t,a(t))$, $v(t)\in V(t,b(t))$ and

$$\begin{gathered} 0\in F(t,a(t),v(t)) + M_1(t,a(t)), \\ 0\in G(t,u(t),b(t)) + M_2(t,b(t)), \quad \forall t\in\Omega. \end{gathered} \tag{2.2}$$

The problem of type (2.2) is called the system of random set-valued variational inclusion problem. If $a,u : \Omega\to\mathcal{H}_1$ and $b,v : \Omega\to\mathcal{H}_2$ are solutions of problem (2.2), we will denote by $(a,u,b,v)\in \mathrm{SRSVI}_{(M_1,M_2)}(F,G,U,V)$.

Notice that, if $U : \Omega\times\mathcal{H}_1 \to \mathcal{H}_1$ and $V : \Omega\times\mathcal{H}_2 \to \mathcal{H}_2$ are two single-valued mappings, then the problem (2.2) reduces to the following problem: find $a : \Omega\to\mathcal{H}_1$ and $b : \Omega\to\mathcal{H}_2$ such that

$$\begin{gathered} 0\in F(t,a(t),V(t,b(t))) + M_1(t,a(t)), \\ 0\in G(t,U(t,a(t)),b(t)) + M_2(t,b(t)), \quad \forall t\in\Omega. \end{gathered} \tag{2.3}$$

In this case, we will denote by $(a,b)\in \mathrm{SRSI}_{(M_1,M_2)}(F,G,U,V)$. Other special cases of the problem (2.2) are presented the following.

(I) If $M_1(t,a(t)) = \partial\varphi(t,a(t))$ and $M_2(t,b(t)) = \partial\phi(t,b(t))$, where $\varphi : \Omega\times\mathcal{H}_1 \to \mathbb{R}\cup\{+\infty\}$ and $\phi : \Omega\times\mathcal{H}_2 \to \mathbb{R}\cup\{+\infty\}$ are two proper convex and lower semicontinuous functions and $\partial\varphi$ and $\partial\phi$ denoted for the subdifferential operators of φ and ϕ, respectively, then (2.2) reduces to the following problem: find $a, u : \Omega \to \mathscr{H}_1$ and $b, v : \Omega \to \mathscr{H}_2$ such that $u(t) \in U(t, a(t)), v(t) \in V(t, b(t))$ and

$$\begin{aligned} &\langle F(t, a(t), v(t)), x(t) - a(t)\rangle + \varphi(x(t)) - \varphi(a(t)) \geq 0, \quad \forall x \in \mathcal{M}_{\mathscr{H}1}, \\ &\langle G(t, u(t), b(t)), y(t) - b(t)\rangle + \phi(y(t)) - \phi(b(t)) \geq 0, \quad \forall y \in \mathcal{M}_{\mathscr{H}2}, \end{aligned} \tag{2.4}$$

for all $t \in \Omega$. The problem (2.4) is called a system of random set-valued mixed variational inequalities. A special of problem (2.4) was studied in by Agarwal and Verma [15].

(II) Let $K_1 \subseteq \mathscr{H}_1, K_2 \subseteq \mathscr{H}_2$ be two nonempty closed and convex subsets and δ_{K_i} the indicator functions of K_i for $i = 1, 2$. If $M_1(t, x(t)) = \partial\delta_{K_1}(x(t))$ and $M_2(t, y(t)) = \partial\delta_{K_2}(y(t))$ for all $x \in \mathcal{M}_{K_1}$ and $y \in \mathcal{M}_{K_2}$. Then the problem (2.2) reduces to the following problem: find $a, u : \Omega \to \mathscr{H}_1$ and $b, v : \Omega \to \mathscr{H}_2$ such that $u(t) \in U(t, a(t)), v(t) \in V(t, b(t))$ and

$$\begin{aligned} &\langle F(t, a(t), v(t)), x(t) - a(t)\rangle \geq 0, \quad \forall x \in \mathcal{M}_{\mathscr{K}1}, \\ &\langle G(t, u(t), b(t)), y(t) - b(t)\rangle \geq 0, \quad \forall y \in \mathcal{M}_{\mathscr{K}2}, \end{aligned} \tag{2.5}$$

for all $t \in \Omega$.

(III) If $\mathscr{H}_1 = \mathscr{H}_2 = \mathscr{H}$ and $M_1(t, a(t)) = M_2(t, b(t)) = \partial\varphi(t, a(t))$, where $\varphi : \Omega \times \mathscr{H} \to \mathbb{R} \cup \{+\infty\}$ is proper convex and lower semicontinuous function and $\partial\varphi$ is denoted for the subdifferential operators of φ. Let $g : \mathscr{H} \to \mathscr{H}$ be a nonlinear mapping and $\rho, \eta > 0$. If we set $F(t, a(t), v(t)) = \rho v(t) + a(t) - g(b(t))$, and $G(t, u(t), b(t)) = \eta u(t) + b(t) - g(a(t))$ where $u(t) \in U(t, a(t)), v(t) \in V(t, b(t))$, then problem (2.2) reduces to the following system of variational inequalities: find $a, b : \Omega \to \mathscr{H}, u(t) \in U(t, a(t))$ and $v(t) \in V(t, b(t))$ such that

$$\begin{aligned} &\langle \rho v(t) + a(t) - g(b(t)), g(x(t)) - a(t)\rangle + \varphi(g(x(t))) - \varphi(a(t)) \geq 0, \\ &\langle \eta u(t) + b(t) - g(a(t)), g(x(t)) - b(t)\rangle + \varphi(g(x(t))) - \varphi(b(t)) \geq 0, \end{aligned} \tag{2.6}$$

for all $t \in \Omega$ and $g(x(t)) \in \mathcal{M}_{\mathscr{H}}$. A special of problem (2.6) was studied by Argarwal et al. [29].

(IV) Let $T : K \to \mathscr{H}$ be a nonlinear mapping and $\rho, \eta > 0$ two fixed constants. If $\mathscr{H}_1 = \mathscr{H}_2 = \mathscr{H}, K_1 = K_2 = K, F(t, a(t), v(t)) = \rho T(v(t)) + a(t) - v(t)$, and $G(t, u(t), b(t)) = \eta T(u(t)) + b(t) - u(t)$. Then (2.5) reduces to the following system of variational inequalities: find $a, u, b, v : \Omega \to \mathscr{H}$ such that $u(t) \in U(t, a(t)), v(t) \in V(t, b(t))$ and

$$\begin{aligned} &\langle \rho T(v(t)) + a(t) - v(t), x(t) - a(t)\rangle \geq 0, \\ &\langle \eta T(u(t)) + b(t) - u(t), y(t) - b(t)\rangle \geq 0, \end{aligned} \tag{2.7}$$

for all $x, y \in \mathcal{M}_{\mathscr{H}}$ and $t \in \Omega$. Notice that, if $U = V = I$, then (2.5), (2.7) are studied by Kim and Kim [30].

We now recall important basic concepts and definitions, which will be used in this work.

Definition 2.1. A mapping $f : \Omega \times \mathcal{H} \to \mathcal{H}$ is called a random single-valued mapping if for any $x \in \mathcal{H}$, the mapping $f(\cdot, x) : \Omega \to \mathcal{H}$ is measurable.

Definition 2.2. A set-valued mapping $G : \Omega \to 2^{\mathcal{H}}$ is said to be measurable if $G^{-1}(B) = \{t \in \Omega : G(t) \cap B \neq \emptyset\} \in \Sigma$, for all $B \in \mathcal{B}(\mathcal{H})$.

Definition 2.3. A set-valued mapping $F : \Omega \times \mathcal{H} \to 2^{\mathcal{H}}$ is called a random set-valued mapping if for any $x \in \mathcal{H}$, the set-valued mapping $F(\cdot, x) : \Omega \to 2^{\mathcal{H}}$ is measurable.

Definition 2.4. A single-valued mapping $\eta : \Omega \times \mathcal{H} \times \mathcal{H} \to \mathcal{H}$ is said to be random τ-Lipschitz continuous if there exists a measurable function $\tau : \Omega \to (0, \infty)$ such that

$$\|\eta(t, x(t), y(t))\| \leq \tau(t)\|x(t) - y(t)\|, \tag{2.8}$$

for all $x, y \in \mathcal{M}_{\mathcal{H}}, t \in \Omega$.

Definition 2.5. A set-valued mapping $U : \Omega \times \mathcal{H} \to \mathrm{CB}(\mathcal{H})$ is said to be random ϕ-D-Lipschitz continuous if there exists a measurable function $\phi : \Omega \to (0, \infty)$ such that

$$D(U(t, x(t)), U(t, y(t))) \leq \phi(t)\|x(t) - y(t)\|, \tag{2.9}$$

for all $x, y \in \mathcal{M}_{\mathcal{H}}$ and $t \in \Omega$, where $D(\cdot, \cdot)$ is the Hausdorff metric on $\mathrm{CB}(\mathcal{H})$.

Definition 2.6. A set-valued mapping $F : \Omega \times \mathcal{H} \to \mathrm{CB}(\mathcal{H})$ is said to be D-continuous if, for any $t \in \Omega$, the mapping $F(t, \cdot) : \mathcal{H} \to \mathrm{CB}(\mathcal{H})$ is continuous in $D(\cdot, \cdot)$, where $D(\cdot, \cdot)$ is the Hausdorff metric on $\mathrm{CB}(\mathcal{H})$.

Definition 2.7. Let $A : \Omega \times \mathcal{H} \to \mathcal{H}$ and $\eta : \Omega \times \mathcal{H} \times \mathcal{H} \to \mathcal{H}$ be two random single-valued mappings. Then A is said to be

(i) random β-Lipschitz continuous if there exists a measurable function $\beta : \Omega \to (0, \infty)$ such that

$$\|A(t, x(t)) - A(t, y(t))\| \leq \beta(t)\|x(t) - y(t)\|, \tag{2.10}$$

for all $x, y \in \mathcal{M}_{\mathcal{H}}, t \in \Omega$;

(ii) random η-monotone if

$$\langle A(t, x(t)) - A(t, y(t)), \eta(t, x(t), y(t)) \rangle \geq 0, \tag{2.11}$$

for all $x, y \in \mathcal{M}_{\mathcal{H}}, t \in \Omega$;

(iii) random strictly η-monotone if, A is a random η-monotone and

$$\langle A(t,x(t)) - A(t,y(t)), \eta(t,x(t),y(t)) \rangle = 0 \quad \text{iff } x(t) = y(t), \tag{2.12}$$

for all $x, y \in \mathcal{M}_{\mathcal{H}}, t \in \Omega$;

(iv) random (r,η)-strongly monotone if there exists a measurable function $r : \Omega \to (0,\infty)$ such that

$$\langle A(t,x(t)) - A(t,y(t)), \eta(t,x(t),y(t)) \rangle \geq r(t) \| x(t) - y(t) \|^2, \tag{2.13}$$

for all $x, y \in \mathcal{M}_{\mathcal{H}}, t \in \Omega$.

Definition 2.8. Let $A : \Omega \times \mathcal{H} \to \mathcal{H}$ be a random single-valued mapping. A single-valued mapping $F : \Omega \times \mathcal{H} \times \mathcal{H} \to \mathcal{H}$ is said to be

(i) random (c,μ)-relaxed cocoercive with respect to A in the second argument if there exist measurable functions $c, \mu : \Omega \to (0,\infty)$ such that

$$\begin{aligned} &\langle F(t,\cdot,x(t)) - F(t,\cdot,y(t)), A(t,x(t)) - A(t,y(t)) \rangle \\ &\quad \geq -c(t) \| F(t,\cdot,x(t)) - F(t,\cdot,y(t)) \|^2 + \mu(t) \| x(t) - y(t) \|^2, \end{aligned} \tag{2.14}$$

for all $x, y \in \mathcal{M}_{\mathcal{H}}$ and $t \in \Omega$;

(ii) random α-Lipschitz continuous in the second argument if there exists a measurable function $\alpha : \Omega \to (0,\infty)$ such that

$$\| F(t,\cdot,x(t)) - F(t,\cdot,y(t)) \| \leq \alpha(t) \| x(t) - y(t) \|, \tag{2.15}$$

for all $x, y \in \mathcal{M}_{\mathcal{H}}, t \in \Omega$.

Notice that, in a similar way, we can define the concepts of relaxed cocoercive and Lipschitz continuous in the third argument.

Definition 2.9. Let $\eta : \Omega \times \mathcal{H} \times \mathcal{H} \to \mathcal{H}$ and $A : \Omega \times \mathcal{H} \to \mathcal{H}$ be two random single-valued mappings. Then a set-valued mapping $M : \Omega \times \mathcal{H} \to 2^{\mathcal{H}}$ is said to be

(i) random (m,η)-relaxed monotone if there exists a measurable function $m : \Omega \to (0,\infty)$ such that

$$\langle u(t) - v(t), \eta(t,x(t),y(t)) \rangle \geq -m(t) \| x(t) - y(t) \|^2, \tag{2.16}$$

for all $x, y \in \mathcal{M}_{\mathcal{H}}, u(t) \in M(t,x(t)), v(t) \in M(t,y(t)), t \in \Omega$;

(ii) random (A, m, η)-monotone if M is a random (m, η)-relaxed monotone and $(A_t + \rho(t)M_t)(\mathcal{H}) = \mathcal{H}$ for all measurable function $\rho : \Omega \to (0, \infty)$ and $t \in \Omega$, where $A_t(x) = A(t, x(t)), M_t(x) = M(t, x(t))$.

Definition 2.10. Let $A : \Omega \times \mathcal{H} \to \mathcal{H}$ be a random single-valued mapping and $M : \Omega \times \mathcal{H} \to 2^{\mathcal{H}}$ a random (A, m, η)-monotone mapping. For each measurable function $\rho : \Omega \to (0, \infty)$, the corresponding random (A, m, η)-resolvent operator $J^{\eta, M}_{\rho, A} : \Omega \times \mathcal{H} \to \mathcal{H}$ is defined by

$$J^{\eta_t, M_t}_{\rho(t), A_t}(x) = \left(A_t + \rho(t)M_t\right)^{-1}(x), \quad \forall x \in \mathcal{M}_{\mathcal{H}},\ t \in \Omega, \tag{2.17}$$

where $A_t(x) = A(t, x(t)), M_t(x) = M(t, x(t))$, and $J^{\eta_t, M_t}_{\rho(t), A_t}(x) = J^{\eta, M}_{\rho, A}(t, x(t))$.

The following lemma, which related to $J^{\eta, M}_{\rho, A}$ operator, is very useful in order to prove our results.

Lemma 2.11. *Let $\eta : \Omega \times \mathcal{H} \times \mathcal{H} \to \mathcal{H}$ be a random single-valued mapping, $A : \Omega \times \mathcal{H} \to \mathcal{H}$ a random (r, η)-strongly monotone mapping, and $M : \Omega \times \mathcal{H} \to 2^{\mathcal{H}}$ a random (A, m, η)-monotone mapping. If $\rho : \Omega \to (0, \infty)$ is a measurable function with $\rho(t) \in (0, r(t)/m(t))$ for all $t \in \Omega$, then the following are true.*

(i) *The corresponding random (A, m, η)-resolvent operator $J^{\eta, M}_{\rho, A}$ is a random single-valued mapping.*

(ii) *If $\eta : \Omega \times \mathcal{H} \times \mathcal{H} \to \mathcal{H}$ is a random τ-Lipschitz continuous mapping, then the corresponding random (A, m, η)-resolvent operator $J^{\eta, M}_{\rho, A}$ is a random $\tau/(r - \rho m)$-Lipschitz continuous.*

Proof. The proof is similar to Proposition 3.9 in [2]. □

In order to prove our main results, we also need the following well known facts.

Lemma 2.12 (see [31]). *Let $\mathcal{H}$ be a separable real Hilbert space and $U : \Omega \times \mathcal{H} \to CB(\mathcal{H})$ be a D-continuous random set-valued mapping. Then for any measurable mapping $w : \Omega \to \mathcal{H}$, the set-valued mapping $U(\cdot, w(\cdot)) : \Omega \to CB(\mathcal{H})$ is measurable.*

Lemma 2.13 (see [31]). *Let $\mathcal{H}$ be a separable real Hilbert space and $U, V : \Omega \to CB(\mathcal{H})$ two measurable set-valued mappings; $\varepsilon > 0$ be a constant and $u : \Omega \to \mathcal{H}$ a measurable selection of U. Then there exists a measurable selection $v : \Omega \to \mathcal{H}$ of V such that*

$$\|u(t) - v(t)\| \leq (1 + \varepsilon) D(U(t), V(t)), \quad \forall t \in \Omega. \tag{2.18}$$

Lemma 2.14 (see [32]). *Let $\{\gamma_n\}$ be a nonnegative real sequence, and let $\{\lambda_n\}$ be a real sequence in $[0, 1]$ such that $\Sigma_{n=0}^{\infty} \lambda_n = \infty$. If there exists a positive integer n_1 such that*

$$\gamma_{n+1} \leq (1 - \lambda_n)\gamma_n + \lambda_n \sigma_n, \quad \forall n \geq n_1, \tag{2.19}$$

where $\sigma_n \geq 0$ for all $n \geq 0$ and $\sigma_n \to 0$ as $n \to \infty$, then $\lim_{n \to \infty} \gamma_n = 0$.

3. Existence Theorems

In this section, we will provide sufficient conditions for the existence solutions of the problem (2.2). To do this, we will begin with a useful lemma.

Lemma 3.1. *Let $\mathcal{H}_1$ and $\mathcal{H}_2$ be two real Hilbert spaces. Let $F : \Omega\times\mathcal{H}_1\times\mathcal{H}_2 \to \mathcal{H}_1$ and $G : \Omega\times\mathcal{H}_1\times\mathcal{H}_2 \to \mathcal{H}_2$ be single-valued mappings. Let $U : \Omega \times \mathcal{H}_1 \to CB(\mathcal{H}_1), V : \Omega \times \mathcal{H}_2 \to CB(\mathcal{H}_2)$, and $M_i : \Omega \times \mathcal{H}_i \to 2^{\mathcal{H}_i}$ be a set-valued mappings for $i = 1, 2$. Assume that M_i are random (A_i, m_i, η_i)-monotone mappings and $A_i : \Omega\times\mathcal{H}_i \to \mathcal{H}_i$ random (r_i, η_i)-strongly monotone mappings, for $i = 1, 2$. Then we have the following statements:*

(i) *if $(a, u, b, v) \in \mathrm{SRSVI}_{(M_1,M_2)}(F, G, U, V)$, then for any measurable functions $\rho_1, \rho_2 : \Omega \to (0, \infty)$ we have*

$$\begin{aligned} a(t) &= J^{\eta_{1_t}, M_{1_t}}_{\rho_1(t), A_{1_t}}\left[A_1(t, a(t)) - \rho_1(t)F(t, a(t), v(t))\right], \\ b(t) &= J^{\eta_{2_t}, M_{2_t}}_{\rho_2(t), A_{2_t}}\left[A_2(t, b(t)) - \rho_2(t)G(t, u(t), b(t))\right], \quad \forall t \in \Omega; \end{aligned} \tag{3.1}$$

(ii) *if there exist two measurable functions $\rho_1, \rho_2 : \Omega \to (0, \infty)$ such that*

$$\begin{aligned} a(t) &= J^{\eta_{1_t}, M_{1_t}}_{\rho_1(t), A_{1_t}}\left[A_1(t, a(t)) - \rho_1(t)F(t, a(t), v(t))\right], \\ b(t) &= J^{\eta_{2_t}, M_{2_t}}_{\rho_2(t), A_{2_t}}\left[A_2(t, b(t)) - \rho_2(t)G(t, u(t), b(t))\right], \end{aligned} \tag{3.2}$$

for all $t \in \Omega$, then $(a, u, b, v) \in \mathrm{SRSVI}_{(M_1,M_2)}(F, G, U, V)$.

Proof. (i) Let $\rho_1, \rho_2 : \Omega \to (0, \infty)$ be any measurable functions. Since $(a, u, b, v) \in \mathrm{SRSVI}_{(M_1,M_2)}(F, G, U, V)$, we have

$$\begin{aligned} &0 \in F(t, a(t), v(t)) + M_1(t, a(t)), \\ &0 \in G(t, u(t), b(t)) + M_2(t, b(t)), \quad \forall t \in \Omega. \end{aligned} \tag{3.3}$$

Let $t \in \Omega$ be fixed. By $0 \in F(t, a(t), v(t)) + M_1(t, a(t))$, we obtain

$$A_1(t, a(t)) - \rho_1(t)F(t, a(t), v(t)) \in A_1(t, a(t)) + \rho_1(t)M_1(t, a(t)). \tag{3.4}$$

This means

$$A_1(t, a(t)) - \rho_1(t)F(t, a(t), v(t)) \in \left(A_{1_t} + \rho_1(t)M_{1_t}\right)(a(t)). \tag{3.5}$$

Thus

$$a(t) = J^{\eta_{1_t}, M_{1_t}}_{\rho_1(t), A_{1_t}}\left[A_1(t, a(t)) - \rho_1(t)F(t, a(t), v(t))\right], \tag{3.6}$$

where $(A_{1_t} + \rho_1(t)M_{1_t})^{-1} = J^{\eta_{1_t}, M_{1_t}}_{\rho_1(t), A_{1_t}}$.

Similarly, if $0 \in G(t,u(t),b(t)) + M_2(t,b(t))$, we can show that $b(t) = J^{\eta_{2_t},M_{2_t}}_{\rho_2(t),A_{2_t}}[A_2(t,b(t)) - \rho_2(t)G(t,u(t),b(t))]$, where $(A_{2_t} + \rho_2(t)M_{2_t})^{-1} = J^{\eta_{2_t},M_{2_t}}_{\rho_2(t),A_{2_t}}$. Hence (i) is proved.

(ii) Assume that there exist two measurable functions $\rho_1, \rho_2 : \Omega \to (0,\infty)$ such that

$$\begin{aligned} a(t) &= J^{\eta_{1_t},M_{1_t}}_{\rho_1(t),A_{1_t}}\left[A_1(t,a(t)) - \rho_1(t)F(t,a(t),v(t))\right], \\ b(t) &= J^{\eta_{2_t},M_{2_t}}_{\rho_2(t),A_{2_t}}\left[A_2(t,b(t)) - \rho_2(t)G(t,u(t),b(t))\right], \end{aligned} \tag{3.7}$$

for all $t \in \Omega$. Let $t \in \Omega$ be fixed. Since $a(t) = J^{\eta_{1_t},M_{1_t}}_{\rho_1(t),A_{1_t}}[A_1(t,a(t)) - \rho_1(t)F(t,a(t),v(t))]$, then by the definition of $J^{\eta_{1_t},M_{1_t}}_{\rho_1(t),A_{1_t}}$, we see that

$$a(t) = \left(A_{1_t} + \rho_1(t)M_{1_t}\right)^{-1}\left[A_1(t,a(t)) - \rho_1(t)F(t,a(t),v(t))\right]. \tag{3.8}$$

This implies that

$$-F(t,a(t),v(t)) \in M_1(t,a(t)). \tag{3.9}$$

That is,

$$0 \in F(t,a(t),v(t)) + M_1(t,a(t)). \tag{3.10}$$

Similarly, if $b(t) = J^{\eta_{2_t},M_{2_t}}_{\rho_2(t),A_{2_t}}[A_2(t,b(t)) - \rho_2(t)G(t,u(t),b(t))]$ we can show that $0 \in G(t,u(t),b(t)) + M_2(t,b(t))$. This completes the proof. □

Due to Lemma 3.1, in order to prove our main theorems, the following assumptions should be needed.

Assumption $\mathscr{A}$

- $\mathscr{A}(a)$ $\mathscr{H}_1$ and $\mathscr{H}_2$ are separable real Hilbert spaces.
- $\mathscr{A}(b)$ $\eta_i : \Omega \times \mathscr{H}_i \times \mathscr{H}_i \to \mathscr{H}_i$ are random τ_i-Lipschitz continuous single-valued mappings, for $i = 1,2$.
- $\mathscr{A}(c)$ $A_i : \Omega \times \mathscr{H}_i \to \mathscr{H}_i$ are random (r_i,η_i)-strongly monotone and random β_i-Lipschitz continuous single-valued mappings, for $i = 1,2$.
- $\mathscr{A}(d)$ $M_i : \Omega \times \mathscr{H}_i \to 2^{\mathscr{H}_i}$ are random (A_i,m_i,η_i)-monotone set-valued mappings, for $i = 1,2$.
- $\mathscr{A}(e)$ $U : \Omega \times \mathscr{H}_1 \to \mathrm{CB}(\mathscr{H}_1)$ is a random ϕ_1-D-Lipschitz continuous set-valued mapping and $V : \Omega \times \mathscr{H}_2 \to \mathrm{CB}(\mathscr{H}_2)$ is a random ϕ_2-D-Lipschitz continuous set-valued mapping.
- $\mathscr{A}(f)$ $F : \Omega \times \mathscr{H}_1 \times \mathscr{H}_2 \to \mathscr{H}_1$ is a random single-valued mapping, which has the following conditions:

(i) F is a random (c_1, μ_1)-relaxed cocoercive with respect to A_1 in the third argument and a random α_1-Lipschitz continuous in the third argument,

(ii) F is a random ζ_1-Lipschitz continuous in the second argument.

$\mathcal{A}(g)$ $G : \Omega \times \mathcal{H}_1 \times \mathcal{H}_2 \to \mathcal{H}_2$ is a random single-valued mapping, which has the following conditions:

(i) G is a random (c_2, μ_2)-relaxed cocoercive with respect to A_2 in the second argument and a random α_2-Lipschitz continuous in the second argument;

(ii) G is a random ζ_2-Lipschitz continuous in the third argument.

Now, we are in position to present our main results.

Theorem 3.2. *Assume that Assumption* ($\mathcal{A}$) *holds and there exist two measurable functions* $\rho_1, \rho_2 : \Omega \to (0, \infty)$ *such that* $\rho_i(t) \in (0, r_i(t)/m_i(t))$, *for each* $i = 1, 2$ *and*

$$\begin{aligned}
&\frac{\tau_1(t)}{r_1(t) - \rho_1(t) m_1(t)} \sqrt{\beta_1^2(t) - 2\rho_1(t)\mu_1(t) + 2\rho_1(t)\alpha_1^2(t)c_1(t) + \rho_1^2(t)\alpha_1^2(t)} < 1 - \frac{\tau_2(t)\rho_2(t)\zeta_2(t)\phi_1(t)}{r_2(t) - \rho_2(t)m_2(t)}, \\
&\frac{\tau_2(t)}{r_2(t) - \rho_2(t) m_2(t)} \sqrt{\beta_2^2(t) - 2\rho_2(t)\mu_2(t) + 2\rho_2(t)\alpha_2^2(t)c_2(t) + \rho_2^2(t)\alpha_2^2(t)} < 1 - \frac{\tau_1(t)\rho_1(t)\zeta_1(t)\phi_2(t)}{r_1(t) - \rho_1(t)m_1(t)},
\end{aligned} \tag{3.11}$$

for all $t \in \Omega$. *Then the problem* (2.2) *has a solution.*

Proof. Let $\{\varepsilon_n\}$ be a null sequence of positive real numbers. Starting with measurable mappings $a_0 : \Omega \to \mathcal{H}_1$ and $b_0 : \Omega \to \mathcal{H}_2$. By Lemma 2.12, we know that the set-valued mappings $U(\cdot, a_0(\cdot)) : \Omega \to \mathrm{CB}(\mathcal{H}_1)$ and $V(\cdot, b_0(\cdot)) : \Omega \to \mathrm{CB}(\mathcal{H}_2)$ are measurable mappings. Consequently, by Himmelberg [33], there exist measurable selections $u_0 : \Omega \to \mathcal{H}_1$ of $U(\cdot, a_0(\cdot))$ and $v_0 : \Omega \to \mathcal{H}_2$ of $V(\cdot, b_0(\cdot))$. We define now the measurable mappings $a_1 : \Omega \to \mathcal{H}_1$ and $b_1 : \Omega \to \mathcal{H}_2$ by

$$\begin{aligned}
a_1(t) &= J_{\rho_1(t), A_{1_t}}^{\eta_{1_t}, M_{1_t}} \left[A_1(t, a_0(t)) - \rho_1(t) F(t, a_0(t), v_0(t)) \right], \\
b_1(t) &= J_{\rho_2(t), A_{2_t}}^{\eta_{2_t}, M_{2_t}} \left[A_2(t, b_0(t)) - \rho_2(t) G(t, u_0(t), b_0(t)) \right],
\end{aligned} \tag{3.12}$$

where $J_{\rho_i(t), A_{i_t}}^{\eta_{i_t}, M_{i_t}}(x) = (A_{i_t} + \rho_i(t) M_{i_t})^{-1}(x)$, for all $x \in \mathcal{M}_{\mathcal{H}}$, $t \in \Omega$, and $i = 1, 2$. Further, by Lemma 2.12, the set-valued mappings $U(\cdot, a_1(\cdot)) : \Omega \to \mathrm{CB}(\mathcal{H}_1), V(\cdot, b_1(\cdot)) : \Omega \to \mathrm{CB}(\mathcal{H}_2)$ are measurable. Again, by Himmelberg [33] and Lemma 2.13, there exist measurable selections $u_1 : \Omega \to \mathcal{H}_1$ of $U(\cdot, a_1(\cdot))$ and $v_1 : \Omega \to \mathcal{H}_2$ of $V(\cdot, b_1(\cdot))$ such that

$$\begin{aligned}
\|u_0(t) - u_1(t)\| &\le (1 + \varepsilon_1) D(U(t, a_0(t)), U(t, a_1(t))), \\
\|v_0(t) - v_1(t)\| &\le (1 + \varepsilon_1) D(V(t, b_0(t)), V(t, b_1(t))),
\end{aligned} \tag{3.13}$$

for all $t \in \Omega$. Define measurable mappings $a_2 : \Omega \to \mathscr{H}_1$ and $b_2 : \Omega \to \mathscr{H}_2$ as follows:

$$\begin{aligned} a_2(t) &= J^{\eta_{1_t}, M_{1_t}}_{\rho_1(t), A_{1_t}} \left[A_1(t, a_1(t)) - \rho_1(t) F(t, a_1(t), v_1(t)) \right], \\ b_2(t) &= J^{\eta_{2_t}, M_{2_t}}_{\rho_2(t), A_{2_t}} \left[A_2(t, b_1(t)) - \rho_2(t) G(t, u_1(t), b_1(t)) \right], \end{aligned} \tag{3.14}$$

for all $t \in \Omega$. Continuing this process, inductively, we obtain the sequences $\{a_n\}, \{b_n\}, \{u_n\}$, and $\{v_n\}$ of measurable mappings satisfy the following:

$$\begin{aligned} a_{n+1}(t) &= J^{\eta_{1_t}, M_{1_t}}_{\rho_1(t), A_{1_t}} \left[A_1(t, a_n(t)) - \rho_1(t) F(t, a_n(t), v_n(t)) \right], \\ b_{n+1}(t) &= J^{\eta_{2_t}, M_{2_t}}_{\rho_2(t), A_{2_t}} \left[A_2(t, b_n(t)) - \rho_2(t) G(t, u_n(t), b_n(t)) \right], \\ \|u_n(t) - u_{n+1}(t)\| &\le (1 + \varepsilon_{n+1}) D(U(t, a_n(t)), U(t, a_{n+1}(t))), \\ \|v_n(t) - v_{n+1}(t)\| &\le (1 + \varepsilon_{n+1}) D(V(t, b_n(t)), V(t, b_{n+1}(t))), \end{aligned} \tag{3.15}$$

where $u_n(t) \in U(t, a_n(t)), v_n(t) \in V(t, b_n(t))$ and for all $t \in \Omega, n = 0, 1, 2, \ldots$.

Now, since $J^{\eta_1, M_1}_{\rho_1, A_1}$ is a random $\tau_1/(r_1 - \rho_1 m_1)$-Lipschitz continuous mapping, we have

$$\begin{aligned} \|a_{n+1}(t) - a_n(t)\| &= \Big\| J^{\eta_{1_t}, M_{1_t}}_{\rho_1(t), A_{1_t}} \left[A_1(t, a_n(t)) - \rho_1(t) F(t, a_n(t), v_n(t)) \right] \\ &\qquad - J^{\eta_{1_t}, M_{1_t}}_{\rho_1(t), A_{1_t}} \left[A_1(t, a_{n-1}(t)) - \rho_1(t) F(\mathrm{t}, a_{n-1}(t), v_{n-1}(t)) \right] \Big\| \\ &\le \frac{\tau_1(t)}{r_1(t) - \rho_1(t) m_1(t)} \Big\| A_1(t, a_n(t)) - A_1(t, a_{n-1}(t)) \\ &\qquad\qquad - \rho_1(t) \left[F(t, a_n(t), v_n(t)) - F(t, a_{n-1}(t), v_{n-1}(t)) \right] \Big\| \\ &\le \frac{\tau_1(t)}{r_1(t) - \rho_1(t) m_1(t)} \Big\| A_1(t, a_n(t)) - A_1(t, a_{n-1}(t)) \\ &\qquad\qquad - \rho_1(t) \left[F(t, a_n(t), v_n(t)) - F(t, a_{n-1}(t), v_n(t)) \right] \Big\| \\ &\quad + \frac{\rho_1(t) \tau_1(t)}{r_1(t) - \rho_1(t) m_1(t)} \| F(t, a_{n-1}(t), v_n(t)) - F(t, a_{n-1}(t), v_{n-1}(t)) \|, \end{aligned} \tag{3.16}$$

for all $t \in \Omega$. On the other hand, by Assumptions $\mathscr{A}(c)$ and $\mathscr{A}(f)$, we see that

$$\begin{aligned} &\Big\| A_1(t, a_n(t)) - A_1(t, a_{n-1}(t)) - \rho_1(t) \left[F(t, a_n(t), v_n(t)) - F(t, a_{n-1}(t), v_n(t)) \right] \Big\|^2 \\ &\quad = \| A_1(t, a_n(t)) - A_1(t, a_{n-1}(t)) \|^2 \\ &\qquad - 2\rho_1(t) \langle F(t, a_n(t), v_n(t)) - F(t, a_{n-1}(t), v_n(t)), A_1(t, a_n(t)) - A_1(t, a_{n-1}(t)) \rangle \\ &\qquad + \rho_1^2(t) \| F(t, a_n(t), v_n(t)) - F(t, a_{n-1}(t), v_n(t)) \|^2 \\ &\quad \le \beta_1^2(t) \| a_n(t) - a_{n-1}(t) \|^2 + 2\rho_1(t) c_1(t) \| F(t, a_n(t), v_n(t)) - F(t, a_{n-1}(t), v_n(t)) \|^2 \\ &\qquad - 2\rho_1(t) \mu_1(t) \| a_n(t) - a_{n-1}(t) \|^2 + \rho_1^2(t) \| F(t, a_n(t), v_n(t)) - F(t, a_{n-1}(t), v_n(t)) \|^2 \end{aligned}$$

$$
\begin{aligned}
&= \left[\beta_1^2(t) - 2\rho_1(t)\mu_1(t)\right]\|a_n(t) - a_{n-1}(t)\|^2 + \left[2\rho_1(t)c_1(t) + \rho_1^2(t)\right] \\
&\quad \times \|F(t, a_n(t), v_n(t)) - F(t, a_{n-1}(t), v_n(t))\|^2 \\
&\leq \left[\beta_1^2(t) - 2\rho_1(t)\mu_1(t)\right]\|a_n(t) - a_{n-1}(t)\|^2 + \left[2\rho_1(t)c_1(t) + \rho_1^2(t)\right]\alpha_1^2(t)\|a_n(t) - a_{n-1}(t)\|^2 \\
&\leq \left[\beta_1^2(t) - 2\rho_1(t)\mu_1(t) + 2\rho_1(t)c_1(t)\alpha_1^2(t) + \rho_1^2(t)\alpha_1^2(t)\right]\|a_n(t) - a_{n-1}(t)\|^2,
\end{aligned}
\tag{3.17}
$$

for all $t \in \Omega$. This gives

$$
\begin{aligned}
&\left\| A_1(t, a_n(t)) - A_1(t, a_{n-1}(t)) - \rho_1(t)[F(t, a_n(t), v_n(t)) - F(t, a_{n-1}(t), v_n(t))] \right\| \\
&\quad \leq \sqrt{\beta_1^2(t) - 2\rho_1(t)\mu_1(t) + 2\rho_1(t)c_1(t)\alpha_1^2(t) + \rho_1^2(t)\alpha_1^2(t)}\|a_n(t) - a_{n-1}(t)\|,
\end{aligned}
\tag{3.18}
$$

for all $t \in \Omega$.

Meanwhile, since F is a random ζ_1-Lipschitz continuous mapping in the second argument, we get

$$
\|F(t, a_{n-1}(t), v_n(t)) - F(t, a_{n-1}(t), v_{n-1}(t))\| \leq \zeta_1(t)\|v_n(t) - v_{n-1}(t)\|,
\tag{3.19}
$$

for all $t \in \Omega$. From (3.16), (3.18), and (3.19), we obtain that

$$
\begin{aligned}
\|a_{n+1}(t) - a_n(t)\| &\leq \frac{\tau_1(t)\|a_n(t) - a_{n-1}(t)\|}{r_1(t) - \rho_1(t)m_1(t)}\sqrt{\beta_1^2(t) - 2\rho_1(t)\mu_1(t) + 2\rho_1(t)c_1(t)\alpha_1^2(t) + \rho_1^2(t)\alpha_1^2(t)} \\
&\quad + \frac{\tau_1(t)\rho_1(t)\zeta_1(t)}{r_1(t) - \rho_1(t)m_1(t)}\|v_n(t) - v_{n-1}(t)\| \\
&= \Delta_1(t)\|a_n(t) - a_{n-1}(t)\| + \frac{\tau_1(t)\rho_1(t)\zeta_1(t)}{r_1(t) - \rho_1(t)m_1(t)}\|v_n(t) - v_{n-1}(t)\|,
\end{aligned}
\tag{3.20}
$$

where

$$
\Delta_1(t) = \frac{\tau_1(t)}{r_1(t) - \rho_1(t)m_1(t)}\sqrt{\beta_1^2(t) - 2\rho_1(t)\mu_1(t) + 2\rho_1(t)c_1(t)\alpha_1^2(t) + \rho_1^2(t)\alpha_1^2(t)}
\tag{3.21}
$$

for all $t \in \Omega$.

Similarly, by using Assumptions $\mathcal{A}(c)$ and $\mathcal{A}(g)$, we know that

$$
\|b_{n+1}(t) - b_n(t)\| \leq \Delta_2(t)\|b_n(t) - b_{n-1}(t)\| + \frac{\tau_2(t)\rho_2(t)\zeta_2(t)}{r_2(t) - \rho_2(t)m_2(t)}\|u_n(t) - u_{n-1}(t)\|,
\tag{3.22}
$$

where

$$\Delta_2(t) = \frac{\tau_2(t)}{r_2(t) - \rho_2(t)m_2(t)}\sqrt{\beta_2^2(t) - 2\rho_2(t)\mu_2(t) + 2\rho_2(t)c_2(t)\alpha_2^2(t) + \rho_2^2(t)\alpha_2^2(t)} \tag{3.23}$$

for all $t \in \Omega$.

Next, since U is a random ϕ_1-D-Lipschitz continuous mapping and V is a random ϕ_2-D-Lipschitz continuous mapping, by the choices of $\{u_n\}$ and $\{v_n\}$, we have

$$\begin{aligned} \|v_n(t) - v_{n-1}(t)\| &\le (1+\varepsilon_n)D(V(t,b_n(t)), V(t,b_{n-1}(t))) \\ &\le (1+\varepsilon_n)\phi_2(t)\|b_n(t) - b_{n-1}(t)\|, \\ \|u_n(t) - u_{n-1}(t)\| &\le (1+\varepsilon_n)D(U(t,a_n(t)), U(t,a_{n-1}(t))) \\ &\le (1+\varepsilon_n)\phi_1(t)\|a_n(t) - a_{n-1}(t)\|, \end{aligned} \tag{3.24}$$

for all $t \in \Omega$. Now, by (3.20), (3.22), and (3.24), we obtain that

$$\begin{aligned} \|a_{n+1}(t) - a_n(t)\| + \|b_{n+1}(t) - b_n(t)\| \le{}& \left(\Delta_1(t) + (1+\varepsilon_n)\frac{\tau_2(t)\rho_2(t)\zeta_2(t)\phi_1(t)}{r_2(t) - \rho_2(t)m_2(t)}\right)\|a_n(t) - a_{n-1}(t)\| \\ &+ \left(\Delta_2(t) + (1+\varepsilon_n)\frac{\tau_1(t)\rho_1(t)\zeta_1(t)\phi_2(t)}{r_1(t) - \rho_1(t)m_1(t)}\right) \\ &\times \|b_n(t) - b_{n-1}(t)\|, \end{aligned} \tag{3.25}$$

for all $t \in \Omega$. This implies that

$$\|a_{n+1}(t) - a_n(t)\| + \|b_{n+1}(t) - b_n(t)\| \le \theta_n(t)(\|a_n(t) - a_{n-1}(t)\| + \|b_n(t) - b_{n-1}(t)\|), \tag{3.26}$$

where

$$\theta_n(t) = \max\left\{\Delta_1(t) + (1+\varepsilon_n)\frac{\tau_2(t)\rho_2(t)\zeta_2(t)\phi_1(t)}{r_2(t) - \rho_2(t)m_2(t)}, \Delta_2(t) + (1+\varepsilon_n)\frac{\tau_1(t)\rho_1(t)\zeta_1(t)\phi_2(t)}{r_1(t) - \rho_1(t)m_1(t)}\right\}, \tag{3.27}$$

for all $t \in \Omega$.

Next, let us define a norm $\|\cdot\|^+$ on $\mathscr{H}_1 \times \mathscr{H}_2$ by

$$\|(x,y)\|^+ = \|x\| + \|y\|, \quad \forall (x,y) \in \mathscr{H}_1 \times \mathscr{H}_2. \tag{3.28}$$

It is well known that $(\mathscr{H}_1 \times \mathscr{H}_2, \|\cdot\|^+)$ is a Hilbert space. Moreover, for each $n \in \mathbb{N}$, we have

$$\|(a_{n+1}(t), b_{n+1}(t)) - (a_n(t), b_n(t))\|^+ \le \theta_n(t)\|(a_n(t), b_n(t)) - (a_{n-1}(t), b_{n-1}(t))\|^+, \tag{3.29}$$

for all $t \in \Omega$.

Let

$$\theta(t)=\max\left\{\Delta_1(t)+\frac{\tau_2(t)\rho_2(t)\zeta_2(t)\phi_1(t)}{r_2(t)-\rho_2(t)m_2(t)},\Delta_2(t)+\frac{\tau_1(t)\rho_1(t)\zeta_1(t)\phi_2(t)}{r_1(t)-\rho_1(t)m_1(t)}\right\},\quad \text{for each } t\in\Omega. \tag{3.30}$$

We see that $\theta_n(t)\downarrow\theta(t)$ as $n\to\infty$. Moreover, condition (3.11) yields that $0<\theta(t)<1$ for all $t\in\Omega$. This allows us to choose $\vartheta\in(\theta(t),1)$ and a natural number N such that $\theta_n(t)<\vartheta$ for all $n\geq N$. Using this one together with (3.30), we get

$$\|(a_{n+1}(t),b_{n+1}(t))-(a_n(t),b_n(t))\|^+\leq\vartheta\|(a_n(t),b_n(t))-(a_{n-1}(t),b_{n-1}(t))\|^+, \tag{3.31}$$

for all $t\in\Omega$ and $n\geq N$. Thus, for each $n>N$, we obtain

$$\|(a_{n+1}(t),b_{n+1}(t))-(a_n(t),b_n(t))\|^+\leq\vartheta^{n-N}\|(a_{N+1}(t),b_{N+1}(t))-(a_N(t),b_N(t))\|^+, \tag{3.32}$$

for all $t\in\Omega$. So, for any $m\geq n>N$, we have

$$\begin{aligned}\|(a_m(t),b_m(t))-(a_n(t),b_n(t))\|^+&\leq\Sigma_{i=n}^{m-1}\|(a_{i+1}(t),b_{i+1}(t))-(a_i(t),b_i(t))\|^+\\&\leq\Sigma_{i=n}^{m-1}\vartheta^{i-N}\|(a_{N+1}(t),b_{N+1}(t))-(a_N(t),b_N(t))\|^+\\&\leq\frac{\vartheta^n}{\vartheta^N(1-\vartheta)}\|(a_{N+1}(t),b_{N+1}(t))-(a_N(t),b_N(t))\|^+,\end{aligned} \tag{3.33}$$

for all $t\in\Omega$. Since $\vartheta\in(0,1)$, it follows that $\{\vartheta^n\}_{n=N+1}^{\infty}$ converges to 0, as $n\to\infty$. This means that $\{(a_n(t),b_n(t))\}$ is a Cauchy sequence, for each $t\in\Omega$. Thus, there are $a(t)\in\mathscr{H}_1$ and $b(t)\in\mathscr{H}_2$ such that $a_n(t)\to a(t)$ and $b_n(t)\to b(t)$ as $n\to\infty$, for each $t\in\Omega$.

Next, we will show that $\{u_n(t)\}$ and $\{v_n(t)\}$ converge to an element of $U(t,a(t))$ and $V(t,b(t))$, for all $t\in\Omega$. Indeed, for $m\geq n>N$, we have from (3.24) and (3.33) that

$$\begin{aligned}\|(u_m(t),v_m(t))-(u_n(t),v_n(t))\|^+&=\|u_m(t)-u_n(t)\|+\|v_m(t)-v_n(t)\|\\&\leq\Sigma_{i=n}^{m-1}\|u_{i+1}(t)-u_i(t)\|+\Sigma_{i=n}^{m-1}\|v_{i+1}(t)-v_i(t)\|\\&\leq\Sigma_{i=n}^{m-1}(1+\varepsilon_{i+1})\phi_2(t)\|a_{i+1}(t)-a_i(t)\|\\&\quad+\Sigma_{i=n}^{m-1}(1+\varepsilon_{i+1})\phi_1(t)\|b_{i+1}(t)-b_i(t)\|\\&\leq2\phi(t)\Sigma_{i=n}^{m-1}\|(a_{i+1}(t),b_{i+1}(t))-(a_i(t),b_i(t))\|^+\\&=\frac{2\phi(t)\vartheta^n}{\vartheta^N(1-\vartheta)}\|(a_{N+1}(t),b_{N+1}(t))-(a_N(t),b_N(t))\|^+,\end{aligned} \tag{3.34}$$

where $\phi(t) = \max\{\phi_1(t), \phi_2(t)\}$, for each $t \in \Omega$. This implies that $\{(u_n(t), v_n(t))\}$ is a Cauchy sequence in $(\mathcal{H}_1 \times \mathcal{H}_2, \|\cdot\|^+)$, for all $t \in \Omega$. Therefore, there exist $u(t) \in \mathcal{H}_1$ and $v(t) \in \mathcal{H}_2$ such that $u_n(t) \to u(t)$ and $v_n(t) \to v(t)$ as $n \to \infty$, for each $t \in \Omega$. Furthermore,

$$\begin{aligned}\inf\{\|u(t) - u'(t)\| : u'(t) \in U(t, a(t))\} &\le \|u(t) - u_n(t)\| + \inf_{u(t) \in U(t,a(t))} \|u_n(t) - u(t)\| \\ &\le \|u(t) - u_n(t)\| + D(U(t, a_n(t)), U(t, a(t))) \\ &\le \|u(t) - u_n(t)\| + \phi_1(t)\|a_n(t) - a(t)\|.\end{aligned} \tag{3.35}$$

Since $u_n(t) \to u(t)$ and $a_n(t) \to a(t)$ as $n \to \infty$, we have from the closedness property of $U(t, a(t))$ and (3.35) that $u(t) \in U(t, a(t))$, for all $t \in \Omega$. Similarly, we can show that $v(t) \in V(t, b(t))$, for all $t \in \Omega$.

Finally, in view of (3.15) and applying the continuity of A_i, F, G and $J^{\eta_i, M_i}_{\rho_i, A_i}$, for $i = 1, 2$, we see that

$$\begin{aligned}a(t) &= J^{\eta_{1_t}, M_{1_t}}_{\rho_1(t), A_{1_t}}\left[A_1(t, a(t)) - \rho_1(t) F(t, a(t), v(t))\right], \\ b(t) &= J^{\eta_{2_t}, M_{2_t}}_{\rho_2(t), A_{2_t}}\left[A_2(t, b(t)) - \rho_2(t) G(t, u(t), b(t))\right],\end{aligned} \tag{3.36}$$

for all $t \in \Omega$. Thus Lemma 3.1(ii) implies that (a, b, u, v) is a solution to problem (2.2). This completes the proof. □

In particular, we have the following result.

Theorem 3.3. *Let $U : \Omega \times \mathcal{H}_1 \to \mathcal{H}_1$ and $V : \Omega \times \mathcal{H}_2 \to \mathcal{H}_2$ be two random single-valued mappings. Assume that Assumption $\mathcal{A}$ holds and there exist measurable functions ρ_1, ρ_2 satisfing* (3.11). *Then problem* (2.3) *has a unique solution.*

Proof. From Theorem 3.2, we know that the problem (2.3) has a solution. So it remains to prove that, in fact, it has the unique solution. Assume that $a, a^* : \Omega \to \mathcal{H}_1$ and $b, b^* : \Omega \to \mathcal{H}_2$ such that $(a, b), (a^*, b^*)$ are solutions of the problem (2.3). Using the same lines as obtaining (3.20) and (3.22), by replacing a_n with a and a_{n+1} with a^*, we have

$$\|a(t) - a^*(t)\| \le \Delta_1(t)\|a(t) - a^*(t)\| + \frac{\tau_1(t)\rho_1(t)\zeta_1(t)\phi_2(t)}{r_1(t) - \rho_1(t)m_1(t)}\|b(t) - b^*(t)\|, \quad \forall t \in \Omega, \tag{3.37}$$

and, by replacing b_n with b and b_{n+1} with b^*, we obtain that

$$\|b(t) - b^*(t)\| \le \Delta_2(t)\|b(t) - b^*(t)\| + \frac{\tau_2(t)\rho_2(t)\zeta_2(t)\phi_1(t)}{r_2(t) - \rho_2(t)m_2(t)}\|a(t) - a^*(t)\|, \quad \forall t \in \Omega, \tag{3.38}$$

where $\Delta_1(t)$ and $\Delta_2(t)$ are defined as in (3.21) and (3.23), respectively. From (3.37) and (3.38), we get

$$\begin{aligned}\|(a(t),b(t))-(a^*(t),b^*(t))\|^+ &\le \left[\Delta_1(t)+\frac{\tau_2(t)\rho_2(t)\zeta_2(t)\phi_1(t)}{r_2(t)-\rho_2(t)m_2(t)}\right]\|a(t)-a^*(t)\| \\ &\quad+\left[\Delta_2(t)+\frac{\tau_1(t)\rho_1(t)\zeta_1(t)\phi_2(t)}{r_1(t)-\rho_1(t)m_1(t)}\right]\|b(t)-b^*(t)\| \\ &\le \theta(t)\|(a(t),b(t))-(a^*(t),b^*(t))\|^+, \quad \forall t\in\Omega,\end{aligned} \tag{3.39}$$

where $\theta(t)$ is defined as in (3.30). Since $0<\theta(t)<1$, it follows that $(a(t),b(t))=(a^*(t),b^*(t))$, for all $t\in\Omega$. This completes the proof. □

4. Stability Analysis

In the proof of Theorem 3.3, in fact, we have constructed a sequence of measurable mappings $\{(a_n,b_n)\}$ and show that its limit point is nothing but the unique element of $\mathrm{SRSI}_{(M_1,M_2)}(F,G,U,V)$. In this section, we will consider the stability of such a constructed sequence.

We start with a definition for stability analysis.

Definition 4.1. Let $\mathscr{H}_1,\mathscr{H}_2$ be real Hilbert spaces. Let $Q:\Omega\times\mathscr{H}_1\times\mathscr{H}_2\to\mathscr{H}_1\times\mathscr{H}_2$, $(a_0(t),b_0(t))\in\mathscr{H}_1\times\mathscr{H}_2$, and let $(a_{n+1}(t),b_{n+1}(t))=h(Q,a_n(t),b_n(t))$ define an iterative procedure which yields a sequence of points $\{(a_n(t),b_n(t))\}$ in $\mathscr{H}_1\times\mathscr{H}_2$, where h is an iterative procedure involving the mapping Q. Let $F(Q)=\{(a,b)\in\mathcal{M}_{\mathscr{H}1}\times\mathcal{M}_{\mathscr{H}2}:Q(t,a(t),b(t))=(a(t),b(t)),$ for all $t\in\Omega\}\neq\emptyset$ and that $\{(a_n,b_n)\}$ converges to a random fixed point (a,b) of Q. Let $\{(x_n,y_n)\}$ be an arbitrary sequence in $\mathcal{M}_{\mathscr{H}1}\times\mathcal{M}_{\mathscr{H}2}$ and let $\delta_n(t)=\|(x_{n+1}(t),y_{n+1}(t))-h(Q,x_n(t),y_n(t))\|$, for each $n\ge 0$ and $t\in\Omega$. For each $t\in\Omega$, if $\lim_{n\to\infty}\delta_n(t)=0$ implies that $\lim_{n\to\infty}(x_n(t),y_n(t))\to(a(t),b(t))$, then the iteration procedure defined by $(a_{n+1}(t),b_{n+1}(t))=h(Q,a_n(t),b_n(t))$ is said to be Q-stable or stable with respect to Q.

Let F,G,M_i,η_i,A_i, and ρ_i, for $i=1,2$, be random mappings defined as in Theorem 3.2. Now, for each $t\in\Omega$, if $\{(x_n(t),y_n(t))\}$ is any sequence in $\mathscr{H}_1\times\mathscr{H}_2$. We will consider the sequence $\{(S_n(t),T_n(t))\}$, which is defined by

$$\begin{aligned}S_n(t)&=J^{\eta_{1_t},M_{1_t}}_{\rho_1(t),A_{1_t}}\left[A_1(t,x_n(t))-\rho_1(t)F(t,x_n(t),V(t,y_n(t)))\right],\\ T_n(t)&=J^{\eta_{2_t},M_{2_t}}_{\rho_2(t),A_{2_t}}\left[A_2(t,y_n(t))-\rho_2(t)G(t,U(t,x_n(t)),y_n(t))\right],\end{aligned} \tag{4.1}$$

where $U:\Omega\times\mathscr{H}_1\to\mathscr{H}_1$ and $V:\Omega\times\mathscr{H}_2\to\mathscr{H}_2$ and $t\in\Omega$. Consequently, we put

$$\delta_n(t)=\left\|(x_{n+1}(t),y_{n+1}(t))-(S_n(t),T_n(t))\right\|^+. \tag{4.2}$$

Meanwhile, let $Q : \Omega \times \mathscr{H}_1 \times \mathscr{H}_2 \to \mathscr{H}_1 \times \mathscr{H}_2$ be defined by

$$Q(t, a(t), b(t)) = \Big(J^{\eta_{1_t}, M_{1_t}}_{\rho_1(t), A_{1_t}} \big[A_1(t, a(t)) - \rho_1(t) F(t, a(t), b(t))\big], \\ J^{\eta_{2_t}, M_{2_t}}_{\rho_2(t), A_{2_t}} \big[A_2(t, b(t)) - \rho_2(t) G(t, a(t), b(t))\big] \Big) \tag{4.3}$$

for all $a \in \mathcal{M}_{\mathscr{H}1}, b \in \mathcal{M}_{\mathscr{H}2}$, $t \in \Omega$. In view of Lemma 3.1, we see that $(a, b) \in \mathrm{SRSI}_{(M_1, M_2)}(F, G, U, V)$ if and only if $(a, b) \in F(Q)$.

Now, we prove the stability of the sequence $\{(a_n, b_n)\}$ with respect to mapping Q, defined by (4.3).

Theorem 4.2. *Assume that Assumption $\mathscr{A}$ holds and there exist ρ_1, ρ_2 satisfing* (3.11). *Then for each $t \in \Omega$, we have $\lim_{n\to\infty} \delta_n(t) = 0$ if and only if $\lim_{n\to\infty}(x_n(t), y_n(t)) = (a(t), b(t))$, where $\delta_n(t)$ are defined by* (4.2) *and $(a(t), b(t)) \in F(Q)$.*

Proof. According to Theorem 3.3, the solution set $\mathrm{SRSI}_{(M_1, M_2)}(F, G, U, V)$ of problem (2.3) is a singleton set, that is, $\mathrm{SRSI}_{(M_1, M_2)}(F, G, U, V) = \{(a, b)\}$. For each $t \in \Omega$, let $\{(x_n(t), y_n(t))\}$ be any sequence in $\mathscr{H}_1 \times \mathscr{H}_2$. By (4.1) and (4.2), we have

$$\begin{aligned} &\big\| (x_{n+1}(t), y_{n+1}(t)) - (a(t), b(t)) \big\|^+ \\ &\quad \le \big\| (x_{n+1}(t), y_{n+1}(t)) - (S_n(t), T_n(t)) \big\|^+ + \| (S_n(t), T_n(t)) - (a(t), b(t)) \|^+ \\ &\quad = \| (S_n(t), T_n(t)) - (a(t), b(t)) \|^+ + \delta_n(t) \\ &\quad = \Big\| J^{\eta_{1_t}, M_{1_t}}_{\rho_1(t), A_{1_t}} \big[A_1(t, x_n(t)) - \rho_1(t) F\big(t, x_n(t), V\big(t, y_n(t)\big)\big)\big] - a(t) \Big\| \\ &\qquad + \Big\| J^{\eta_{2_t}, M_{2_t}}_{\rho_2(t), A_{2_t}} \big[A_2(t, y_n(t)) - \rho_2(t) G\big(t, U(t, x_n(t)), y_n(t)\big)\big] - b(t) \Big\| + \delta_n(t). \end{aligned} \tag{4.4}$$

Since $J^{\eta_1, M_1}_{\rho_1, A_1}$ is a random $\tau_1/(r_1 - \rho_1 m_1)$-Lipschitz continuous mapping, by Assumptions $\mathscr{A}(c), \mathscr{A}(f)$ and Lemma 3.1(i), we get

$$\begin{aligned} &\Big\| J^{\eta_{1_t}, M_{1_t}}_{\rho_1(t), A_{1_t}} \big[A_1(t, x_n(t)) - \rho_1(t) F\big(t, x_n(t), V\big(t, y_n(t)\big)\big)\big] - a(t) \Big\| \\ &\quad = \Big\| J^{\eta_{1_t}, M_{1_t}}_{\rho_1(t), A_{1_t}} \big[A_1(t, x_n(t)) - \rho_1(t) F\big(t, x_n(t), V\big(t, y_n(t)\big)\big)\big] \\ &\qquad - J^{\eta_{1_t}, M_{1_t}}_{\rho_2(t), A_{1_t}} \big[A_1(t, a(t)) - \rho_1(t) F(t, a(t), V(t, b(t)))\big] \Big\| \\ &\quad \le \frac{\tau_1(t)}{r_1(t) - \rho_1(t) m_1(t)} \\ &\qquad \times \big\| A_1(t, x_n(t)) - A_1(t, a(t)) - \rho_1(t) \big[F\big(t, x_n(t), V\big(t, y_n(t)\big)\big) - F(t, a(t), V(t, b(t)))\big] \big\| \\ &\quad \le \frac{\tau_1(t)}{r_1(t) - \rho_1(t) m_1(t)} \\ &\qquad \times \big\| A_1(t, x_n(t)) - A_1(t, a(t)) - \rho_1(t) \big[F\big(t, x_n(t), V\big(t, y_n(t)\big)\big) - F\big(t, a(t), V\big(t, y_n(t)\big)\big)\big] \big\| \\ &\qquad + \frac{\rho_1(t) \tau_1(t)}{r_1(t) - \rho_1(t) m_1(t)} \big\| F\big(t, a(t), V\big(t, y_n(t)\big)\big) - F(t, a(t), V(t, b(t))) \big\|. \end{aligned} \tag{4.5}$$

On the other hand, by Assumptions $\mathscr{A}(c)$ and $\mathscr{A}(f)$, we see that

$$\begin{aligned}
&\left\| A_1(t,x_n(t)) - A_1(t,a(t)) - \rho_1(t)\left[F(t,x_n(t),V(t,y_n(t))) - F(t,a(t),V(t,y_n(t)))\right]\right\|^2 \\
&\quad= \|A_1(t,x_n(t)) - A_1(t,a(t))\|^2 \\
&\qquad - 2\rho_1(t)\langle F(t,x_n(t),V(t,y_n(t))) - F(t,a(t),V(t,y_n(t))), A_1(t,x_n(t)) - A_1(t,a(t))\rangle \\
&\qquad + \rho_1^2(t)\left\| F(t,x_n(t),V(t,y_n(t))) - F(t,a(t),V(t,y_n(t)))\right\|^2 \\
&\quad\le \beta_1^2(t)\|x_n(t) - a(t)\|^2 + 2\rho_1(t)c_1(t)\left\| F(t,x_n(t),V(t,y_n(t))) - F(t,a(t),V(t,y_n(t)))\right\|^2 \\
&\qquad - 2\rho_1(t)\mu_1(t)\|x_n(t) - a(t)\|^2 \\
&\qquad + \rho_1^2(t)\left\| F(t,x_n(t),V(t,y_n(t))) - F(t,a(t),V(t,y_n(t)))\right\|^2 \\
&\quad= \left[\beta_1^2(t) - 2\rho_1(t)\mu_1(t)\right]\|x_n(t) - a(t)\|^2 + \left[2\rho_1(t)c_1(t) + \rho_1^2(t)\right] \\
&\qquad \times \left\| F(t,x_n(t),V(t,y_n(t))) - F(t,a(t),V(t,y_n(t)))\right\|^2 \\
&\quad\le \left[\beta_1^2(t) - 2\rho_1(t)\mu_1(t)\right]\|x_n(t) - a(t)\|^2 + \left[2\rho_1(t)c_1(t) + \rho_1^2(t)\right]\alpha_1^2(t)\|x_n(t) - a(t)\|^2 \\
&\quad\le \left[\beta_1^2(t) - 2\rho_1(t)\mu_1(t) + 2\rho_1(t)c_1(t)\alpha_1^2(t) + \rho_1^2(t)\alpha_1^2(t)\right]\|x_n(t) - a(t)\|^2.
\end{aligned} \tag{4.6}$$

This gives

$$\begin{aligned}
&\left\| A_1(t,x_n(t)) - A_1(t,a(t)) - \rho_1(t)\left[F(t,x_n(t),V(t,y_n(t))) - F(t,a(t),V(t,y_n(t)))\right]\right\| \\
&\quad\le \sqrt{\beta_1^2(t) - 2\rho_1(t)\mu_1(t) + 2\rho_1(t)c_1(t)\alpha_1^2(t) + \rho_1^2(t)\alpha_1^2(t)}\,\|x_n(t) - a(t)\|.
\end{aligned} \tag{4.7}$$

Meanwhile, since F is a random ζ_1-Lipschitz continuous mapping in the second argument, we get

$$\left\| F(t,a(t),V(t,y_n(t))) - F(t,a(t),V(t,b(t)))\right\| \le \zeta_1(t)\phi_2(t)\left\| y_n(t) - b(t)\right\|. \tag{4.8}$$

From (4.5)–(4.8), we obtain that

$$\begin{aligned}
&\left\| J^{\eta_{1_t},M_{1_t}}_{\rho_1(t),A_{1_t}}\left[A_1(t,x_n(t)) - \rho_1(t)F(t,x_n(t),V(t,y_n(t)))\right] - a(t)\right\| \\
&\quad\le \frac{\tau_1(t)\sqrt{\beta_1^2(t) - 2\rho_1(t)\mu_1(t) + 2\rho_1(t)c_1(t)\alpha_1^2(t) + \rho_1^2(t)\alpha_1^2(t)}}{r_1(t) - \rho_1(t)m_1(t)}\|x_n(t) - a(t)\| \\
&\qquad + \frac{\rho_1(t)\tau_1(t)\zeta_1(t)\phi_2(t)}{r_1(t) - \rho_1(t)m_1(t)}\left\| y_n(t) - b(t)\right\|,
\end{aligned} \tag{4.9}$$

where

$$\Delta_1(t) = \frac{\tau_1(t)}{r_1(t) - \rho_1(t)m_1(t)}\sqrt{\beta_1^2(t) - 2\rho_1(t)\mu_1(t) + 2\rho_1(t)c_1(t)\alpha_1^2(t) + \rho_1^2(t)\alpha_1^2(t)}. \tag{4.10}$$

Similarly, since $J_{\rho_2,A_2}^{\eta_2,M_2}$ is a random $\tau_2/(r_2 - \rho_2 m_2)$-Lipschitz continuous mapping, by Assumption $\mathcal{A}(c), \mathcal{A}(g)$, and Lemma 3.1, we obtain that

$$\begin{aligned} &\left\| J_{\rho_2(t),A_{2_t}}^{\eta_{2_t},M_{2_t}}\left[A_2(t,y_n(t)) - \rho_2(t)G(t,U(t,x_n(t)),y_n(t))\right] - b(t)\right\| \\ &\quad \leq \Delta_2(t)\left\|y_n(t) - b(t)\right\| + \frac{\tau_2(t)\rho_2(t)\zeta_2(t)\phi_1(t)}{r_2(t) - \rho_2(t)m_2(t)}\|x_n(t) - a(t)\|, \end{aligned} \tag{4.11}$$

where

$$\Delta_2(t) = \frac{\tau_2(t)}{r_2(t) - \rho_2(t)m_2(t)}\sqrt{\beta_2^2(t) - 2\rho_2(t)\mu_2(t) + 2\rho_2(t)c_2(t)\alpha_2^2(t) + \rho_2^2(t)\alpha_2^2(t)}. \tag{4.12}$$

Thus

$$\begin{aligned} \left\|(x_{n+1}(t),y_{n+1}(t)) - (a(t),b(t))\right\|^+ &\leq \left[\Delta_1(t) + \frac{\tau_2(t)\rho_2(t)\zeta_2(t)\phi_1(t)}{r_2(t) - \rho_2(t)m_2(t)}\right]\|x_n(t) - a(t)\| \\ &\quad + \left[\Delta_2(t) + \frac{\tau_1(t)\rho_1(t)\zeta_1(t)\phi_2(t)}{r_1(t) - \rho_1(t)m_1(t)}\right]\left\|y_n(t) - b(t)\right\| \\ &\leq \theta(t)\left\|(x_n(t),y_n(t)) - (a(t),b(t))\right\|^+ + \delta_n(t) \\ &= (1 - (1 - \theta(t)))\left\|(x_n(t),y_n(t)) - (a(t),b(t))\right\|^+ + \delta_n(t), \end{aligned} \tag{4.13}$$

where $\theta(t) = \max\{\Delta_1(t) + \tau_2(t)\rho_2(t)\zeta_2(t)\phi_1(t)/(r_2(t) - \rho_2(t)m_2(t)), \Delta_2(t) + \tau_1(t)\rho_1(t)\zeta_1(t)\phi_2(t)/(r_1(t) - \rho_1(t)m_1(t))\}$, for all $t \in \Omega$.

So

$$\begin{aligned} &\left\|(x_{n+1}(t),y_{n+1}(t)) - (a(t),b(t))\right\|^+ \\ &\quad \leq (1 - (1 - \theta(t)))\left\|(x_n(t),y_n(t)) - (a(t),b(t))\right\|^+ + (1 - \theta(t)) \cdot \frac{\delta_n(t)}{(1 - \theta(t))}. \end{aligned} \tag{4.14}$$

In view of (4.14), if $\lim_{n\to\infty}\delta_n(t) = 0$, we see that Lemma 2.14 implies

$$\lim_{n\to\infty}\left(x_n(t),y_n(t)\right) = (a(t),b(t)). \tag{4.15}$$

On the other hand, by using (4.5) and (4.11), we see that

$$\begin{aligned} \delta_n(t) &\leq \left\|(x_{n+1}(t),y_{n+1}(t)) - (S_n(t),T_n(t))\right\|^+ \\ &\leq \left\|(x_{n+1}(t),y_{n+1}(t)) - (a(t),b(t))\right\|^+ + \|(a(t),b(t)) - (S_n(t),T_n(t))\|^+ \\ &\leq \left\|(x_{n+1}(t),y_{n+1}(t)) - (a(t),b(t))\right\|^+ + \theta(t)\left\|(x_n(t),y_n(t)) - (a(t),b(t))\right\|^+ \end{aligned} \tag{4.16}$$

for all $t \in \Omega$. Consequently, if for each $t \in \Omega$ we assume $\lim_{n\to\infty}(x_n(t), y_n(t)) = (a(t), b(t))$, we will have $\lim_{n\to\infty}\delta_n(t) = 0$. This completes the proof. □

Remark 4.3. Theorem 4.2 shows that the iterative sequence $\{(a_n, b_n)\}$, which has constructed in Theorem 3.3, is Q-stable.

5. Conclusion

We have introduced a new system of set-valued random variational inclusions involving (A, m, η)-monotone operator and random relaxed cocoercive operators in Hilbert space. By using the resolvent operator technique, we have constructed an iterative algorithm and then the approximation solvability of a aforesaid problem is examined. Moreover, we have considered the stability of such iterative algorithm. It is worth noting that for a suitable and appropriate choice of the operators, as F, G, M, η, A, one can obtain a large number of various classes of variational inequalities; this means that problem (2.2) is quite general and unifying. Consequently, the results presented in this paper are very interesting and improve some known corresponding results in the literature.

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