Hindawi Publishing Corporation
Journal of Applied Mathematics
Volume 2012, Article ID 643418, 17 pages
doi:10.1155/2012/643418

Research Article

Periodic Solutions of a Cohen-Grossberg-Type BAM Neural Networks with Distributed Delays and Impulses

Qiming Liu and Rui Xu

Institute of Applied Mathematics, Shijiazhuang Mechanical Engineering College, Shijiazhuang 050003, China

Correspondence should be addressed to Qiming Liu, lqmmath@yahoo.com.cn

Received 14 November 2011; Revised 27 December 2011; Accepted 27 December 2011

Academic Editor: Naseer Shahzad

A class of Cohen-Grossberg-type BAM neural networks with distributed delays and impulses are investigated in this paper. Sufficient conditions to guarantee the uniqueness and global exponential stability of the periodic solutions of such networks are established by using suitable Lyapunov function, the properties of *M*-matrix, and some suitable mathematical transformation. The results in this paper improve the earlier publications.

1. Introduction

The research of neural networks with delays involves not only the dynamic analysis of equilibrium point but also that of periodic oscillatory solution. The dynamic behavior of periodic oscillatory solution is very important in learning theory due to the fact that learning usually requires repetition [1, 2].

Cohen and Grossberg proposed the Cohen-Grossberg neural networks (CGNNs) in 1983 [3]. Kosko proposed bi directional associative memory neural networks (BAMNNs) in 1988 [4]. Some important results for periodic solutions of delayed CGNNs have been obtained in [5–10]. Xiang and Cao proposed a class of Cohen-Grossberg BAM neural networks (CGBAMNNs) with distributed delays in 2007 [11]; in addition, many evolutionary processes are characterized by abrupt changes at certain time; these changes are called to be impulsive phenomena, which are included in many neural networks such as Hopfield neural networks, BAM neural networks, CGNNs, and CGBAMNNs and can affect dynamical behaviors of the systems just as time delays. The results for periodic solutions of CGBAMNNs with or without impulses are obtained in [11–15].

The objective of this paper is to study the existence and global exponential stability of periodic solutions of CGBAMNNs with distributed delays by using suitable Lyapunov function, the properties of M-matrix, and some suitable mathematical transformation. Comparing with the results in [13, 14], improved results are successively obtained, the conditions for the existence and globally exponential stability of the periodic solution of such system without impulses have nothing to do with inputs of the neurons and amplification functions; and we also point that CGBAMNNs model is a special case of CGNNs model, many results of CGBAMNNs can be directly obtained from the results of CGNNs.

The rest of this paper is organized as follows. Preliminaries are given in Section 2. Sufficient conditions which guarantee the uniqueness and global exponential stability of periodic solutions for CGBAMNNs with distributed delays and impulses are given in Section 3. Two examples are given in Section 4 to demonstrate the main results.

2. Preliminaries

Consider the following periodic CGNNs model with distributed delays and impulses:

$$\begin{aligned}\dot{x}_i(t) &= -a_i(x_i(t))\Bigg[b_i(t,x_i(t)) - \sum_{j=1}^{n} p_{ij}(t) f_j(\rho_j x_j(t)) \\ &\qquad - \sum_{j=1}^{n} u_{ij}(t)\int_0^{+\infty} k_{ij}(s) f_j(\rho_j x_j(t-s))\,ds - I_i(t)\Bigg], \quad t>0,\ t \neq t_k, \\ \Delta x_i(t_k) &= -\gamma_{ik} x_i(t_k), \quad t = t_k,\ k \in Z^+,\end{aligned} \tag{2.1}$$

where $1 \le i \le n$, $t > 0$, and $Z^+ = \{1,2,\ldots\}$. $x_i(t)$ denotes the state variable of the ith neuron, $f_j(\cdot)$ denotes the signal function of the jth neuron at time t; I_i denotes input of the ith neuron at time t; $a_i(\cdot)$ represents amplification function; $b_i(t,\cdot)$ is appropriately behaved function; $p_{ij}(t)$ and $u_{ij}(t)$ are connection weights of the neural networks at time t; respectively, ρ_j is positive constant, which corresponds to the neuronal gain associated with the neuronal activations and k_{ij} corresponds to the delay kernel function; $p_{ij}(t)$ and $u_{ij}(t)$ are continuously periodic functions on $[0,+\infty)$ with common period $T > 0$.

$\Delta x_i(t_k) = x_i(t_k^+) - x_i(t_k^-)$; t_k is called impulsive moment and satisfies $0 < t_1 < t_2 < \cdots$, $\lim_{k\to+\infty} t_k = +\infty$; $x_i(t^-)$ and $x_i(t^+)$ denote the left-hand and right-hand limits at t_k; respectively, we always assume $x_i(t_k^-) = x_i(t_k)$ and $x_i'(t_k^-) = x_i'(t_k)$, $k \in Z^+$.

For system (2.1), we assume the following.

(H_1) The amplification function $a_i(\cdot)$ is continuous, and there exist constants $\underline{a}_i$, $\overline{a}_i$ such that $0 < \underline{a}_i \le a_i(x_i(t)) \le \overline{a}_i$ for $1 \le i \le n$.

(H_2) The behaved function $b_i(t,\cdot)$ is T-periodic about the first argument; there exists continuous T-periodic function $\alpha_i(t)$ such that

$$\frac{b_i(t,x) - b_i(t,y)}{x-y} \ge \alpha_i(t) > 0, \tag{2.2}$$

for all $x \neq y$, $1 \le j \le n$.

(H_3) For activation function $f_j(\cdot)$, there exists positive constant L_j such that

$$L_j = \sup_{x \neq y} \left| \frac{f_j(x) - f_j(x)}{x - y} \right|, \tag{2.3}$$

for all $x \neq y, 1 \leq j \leq n$.

(H_4) The kernel function $k_{ij}(s)$ is nonnegative continuous function on $[0, +\infty)$ and satisfies

$$\begin{aligned} &\int_0^{+\infty} s e^{\lambda s} k_{ij}(s) ds < +\infty, \\ &K_{ij}(\lambda) = \int_0^{+\infty} e^{\lambda s} k_{ij}(s) ds \end{aligned} \tag{2.4}$$

is differentiable function for $\lambda \in [0, r_{ij}), 0 < r_{ij} < +\infty$, $K_{ij}(0) = 1$ and $\lim_{\lambda \to r_{ij}^-} K_{ij}(\lambda) = +\infty$.

(H_5) There exists positive integer k_0 such that $t_{k+k_0} = t_k + T$ and $\gamma_{i(k+k_0)} = \gamma_{ik}$ hold.

Remark 2.1. A typical example of kernel function is given by $k_{ij}(s) = (s^r/r!)\, r_{ij}^{r+1} e^{-r_{ij}s}$ for $s \in [0, +\infty)$, where $r_{ij} \in (0, +\infty)$, $r \in \{0, 1, \ldots, n\}$. These kernel functions are called as the gamma memory filter [16] and satisfy condition (H_4).

For any continuous function $S(t)$ on $[0, T]$, $\underline{S}$ and $\overline{S}$ denote $\min_{t \in [0,T]}\{|S(t)|\}$ and $\max_{t \in [0,T]}\{|S(t)|\}$, respectively.

For any $x(t) = (x_i(t), x_2(t), \ldots, x_k(t))^T \in R^k$, $t > 0$, define $\|x(t)\| = \sum_{i=1}^k |x_i(t)|$, and for any $\varphi(s) = (\varphi_1(s), \varphi_2(s), \ldots, \varphi_k(s))^T \in R^k$, $s \in (-\infty, 0]$, define $\|\varphi(s)\| = \sup_{s \in (-\infty, 0]} \sum_{i=1}^k |\varphi_i(s)|$.

Denote

$$\begin{aligned} \mathrm{PC}\Big((-\infty, 0], R^k\Big) = \{\varphi : [-\infty,\ 0] \longrightarrow R^k \mid \varphi(s) &\text{ is bounded and continuous for all but at} \\ &\text{most a finite number of points } s \in (-\infty, 0], \text{ and at these points } s, \\ &\varphi(s^+), \varphi(s^-) \text{ exist and } \varphi(s^-) = \varphi(s)\}. \end{aligned} \tag{2.5}$$

Then $\mathrm{PC}((-\infty, 0],\ R^k)$ is a Banach space with respect to $\|\cdot\|$.

The initial conditions of system (2.1) are given by

$$x_i(s) = \varphi_i(s), \qquad -\infty \leq s \leq 0,\ 1 \leq i \leq n, \tag{2.6}$$

where $\varphi(s) = (\varphi_1(s), \varphi_2(s), \ldots, \varphi_n(s)) \in \mathrm{PC}([-\infty, 0], R^n)$.

Let $x(t, \varphi) = (x_1(t, \varphi), x_2(t, \varphi), \ldots, x_n(t, \varphi)^T)$ denote any solution of the system (2.1) with initial value $\varphi \in \mathrm{PC}((-\infty, 0], R^n)$.

Definition 2.2. A solution $x(t,\varphi)$ of system (2.1) is said to be globally exponentially stable, if there exist two constants $\lambda > 0$, $M > 0$ such that

$$\|x(t,\psi) - x(t,\varphi)\| \leq M\|\psi - \varphi\|e^{-\lambda t}, \quad t > 0, \tag{2.7}$$

for any solutions $x(t,\psi)$ of system (2.1).

Definition 2.3. A real matrix $A = (a_{ij})_{n\times n}$ is said to be a nonsingular M-matrix if $a_{ij} \leq 0$ $(i,j = 1,2,\ldots,n, i \neq j)$, and all successive principle minors of A are positive.

Lemma 2.4 (see [17]). *A matrix with nonpositive off-diagonal elements $A = (a_{ij})_{n\times n}$ is a nonsingular M-matrix if and if only there exists a vector $p = (p_i)_{1\times n} > 0$ such that $p^T A > 0$ or Ap holds.*

Lemma 2.5. *Under assumptions* (H_1)–(H_5), *system* (2.1) *has a T-periodic solution which is globally exponentially stable, if the following conditions hold.*

(H_6) $\mathcal{M} = A - C$ *is a nonsingular M-matrix, where*

$$A = \operatorname{diag}\left(\underline{\alpha_1},\ \underline{\alpha_2},\ldots,\underline{\alpha_n}\right), \qquad C = (c_{ij})_{n\times n}, \qquad c_{ij} = \left(\overline{p}_{ij} + \overline{u}_{ij}\right)_{\rho_j L_j}. \tag{2.8}$$

(H_7) $a_i((1-\gamma_{ik})s) \geq |1-\gamma_{ik}|a_i(s)$, *for all* $s \in R$, $i = 1,2,\ldots,n$.

Proof. Let $x(t,\psi_1)$ and $x(t,\psi_2)$ be two solutions of system (2.1) with initial value $\psi_1 = (\varphi_1,\varphi_2,\ldots,\varphi_n)$ and $\psi_2 = (\zeta_1,\zeta_2,\ldots,\zeta_n) \in \mathrm{PC}((-\infty,0],\ R^n)$, respectively.

Let

$$F_i(\theta) = \mu_i\left(\underline{\alpha}_i - \frac{\theta}{\underline{a}_i}\right) - \sum_{j=1}^{n}\mu_j\left(\overline{p}_{ji} + \overline{u}_{ji}K_{ji}(\theta)\right)\rho_i L_i, \quad i = 1,2,\ldots,n. \tag{2.9}$$

Since $\mathcal{M}$ is a nonsingular M-matrix according to condition (H_6), $\mathcal{M}^T$ is also a nonsingular M-matrix; we know from Lemma 2.4 that there exists a vector $p = (\mu_1,\mu_2,\ldots,\mu_n)^T$ such that $\mathcal{M}^T p > 0$; that is,

$$\mu_i\underline{\alpha}_i - \sum_{j=1}^{n}\mu_j\left(\overline{p}_{ji} + \overline{u}_{ji}\right)\rho_i L_i > 0, \tag{2.10}$$

for $1 \leq i \leq n$, which indicates that $F_i(0) > 0$. Since $F_i(\theta)$ are continuous and differential on $[0, r_{ji})$ and $\lim_{\theta\to r_{ji}^-} F_i(\theta) = -\infty$ according to condition (H_4), $F_i'(\theta) < 0$ for $\theta \in [0, u_{ji})$. There exist constants θ_i such that $F_i(\theta_i) = 0$ for $i = 1,2,\ldots,n$. So we can choose

$$0 < \lambda \leq \min\{\theta_1,\theta_2,\ldots,\theta_n\}, \tag{2.11}$$

such that

$$F_i(\lambda) \geq 0. \tag{2.12}$$

Define

$$X_i(t) = \left| x_i(t, \varphi_2) - x_i(t, \varphi_1) \right|. \tag{2.13}$$

Now we define a Lyapunov function $V(t)$ by

$$V(t) = \sum_{i=1}^{n} \mu_i \left\{ V_i(t) + \sum_{j=1}^{n} \overline{u}_{ij} L_j \rho_j \int_0^{+\infty} k_{ij}(s) \int_{t-s}^{t} X_j(\mu) e^{\lambda(s+\mu)} d\mu\, ds \right\}, \tag{2.14}$$

in which

$$V_i(t) = e^{\lambda t} \operatorname{sign}\left(x_i(t, \varphi_2) - x_i(t, \varphi_1)\right) \int_{x_i(t,\varphi_1)}^{x_i(t,\varphi_2)} \frac{1}{a_i(s)} ds. \tag{2.15}$$

for $i = 1, 2, \ldots, n$.

When $t \neq t_k,\ k \in Z^+$, calculating the upper right derivative of $V(t)$ along solution of (2.1), similar to proof of Theorem 3.1 in [10], corresponding to case in which $r \to 1$, $v_{ijl}(t) = 0$ in [10], we obtain from (2.12)–(2.15) that

$$\begin{aligned}
D^+V(t)|_{(2.1)} &\leq -e^{\lambda t} \left\{ \sum_{i=1}^{n} \mu_i \left(\underline{\alpha}_i - \frac{\lambda}{\underline{a}_i} \right) X_i(t) - \sum_{i=1}^{n} \sum_{j=1}^{n} \mu_i \left(\overline{p}_{ij} + \overline{u}_{ij} K_{ij}(\lambda) \right) \rho_j L_j X_j(t) \right\} \\
&= -e^{\lambda t} \left\{ \sum_{i=1}^{n} \mu_i \left(\underline{\alpha}_i - \frac{\lambda}{\underline{a}_i} \right) X_i(t) - \sum_{i=1}^{n} \sum_{j=1}^{n} \mu_i \left(\overline{p}_{ji} + \overline{u}_{ji} K_{ji}(\lambda) \right) \rho_i L_i X_i(t) \right\} \\
&= -e^{\lambda t} \sum_{i=1}^{n} F_i(\lambda) X_i(t) \leq 0.
\end{aligned} \tag{2.16}$$

When $t = t_k,\ k \in Z^+$, we have

$$\begin{aligned}
V_i(t_k^+) &= e^{\lambda t^+} \operatorname{sign}\left(x_i(t_k^+, \varphi_2) - x_i(t_k^+, \varphi_1)\right) \int_{x_i(t_k^+,\varphi_1)}^{x_i(t_k^+,\varphi_2)} \frac{1}{a_i(s)} ds \\
&= e^{\lambda t} \operatorname{sign}(1 - \gamma_{ik}) \left(x_i(t_k, \varphi_2) - x_i(t_k, \varphi_1)\right) \int_{(1-\gamma_{ik}) x_i(t_k,\varphi_1)}^{(1-\gamma_{ik}) x_i(t_k,\varphi_2)} \frac{1}{a_i(s)} ds, \\
&= e^{\lambda t} \operatorname{sign}\left(x_i(t_k, \varphi_2) - x_i(t_k, \varphi_1)\right) \int_{x_i(t_k,\varphi_1)}^{x_i(t_k,\varphi_2)} \frac{\left|1 - \gamma_{ik}\right|}{a_i((1 - \gamma_{ik}) s)} ds,
\end{aligned} \tag{2.17}$$

which, together with (H_7), leads to

$$\begin{aligned} V_i(t_k) - V_i(t_k^+) &= e^{\lambda t}\,\mathrm{sign}\big(x_i(t_k,\varphi_2) - x_i(t_k,\varphi_1)\big)\int_{x_i(t_k,\varphi_1)}^{x_i(t_k,\varphi_2)}\left(\frac{1}{a_i(s)} - \frac{|1-\gamma_{ik}|}{a_i((1-\gamma_{ik})s)}\right)ds, \\ &\geq e^{\lambda t}\,\mathrm{sign}\big(x_i(t_k,\varphi_2) - x_i(t_k,\varphi_1)\big)\int_{x_i(t_k,\varphi_1)}^{x_i(t_k,\varphi_2)}\frac{a_i((1-\gamma_{ik})s) - |1-\gamma_{ik}|a_i(s)}{a_i(s)a_i((1-\gamma_{ik})s)}ds \geq 0, \end{aligned} \tag{2.18}$$

that is,

$$V_i(t_k^+) \leq V_i(t_k). \tag{2.19}$$

It follows that

$$\begin{aligned} V(t_k^+) &= \sum_{i=1}^{n}\mu_i\left\{V_i(t^+) + \sum_{j=1}^{n}\overline{u}_{ij}L_j\rho_j\int_0^{+\infty}k_{ij}(s)\int_{t^+-s}^{t^+}X_j(\mu)e^{\lambda(s+\mu)}d\mu\,ds\right\} \\ &\leq \sum_{i=1}^{n}\mu_i\left\{V_i(t) + \sum_{j=1}^{n}\overline{u}_{ij}L_j\rho_j\int_0^{+\infty}k_{ij}(s)\int_{t-s}^{t}X_j(\mu)e^{\lambda(s+\mu)}d\mu\,ds\right\} = V(t_k). \end{aligned} \tag{2.20}$$

Then we have

$$V(t) \leq V(0). \tag{2.21}$$

On the other hand, from (2.14), we have

$$V(t) \geq m_0 e^{\lambda t}\sum_{i=1}^{n}|x_i(t,\varphi_2) - x_i(t,\varphi_1)|, \qquad V(0) \leq M_0 \sup_{l\in(-\infty,0]}\sum_{i=1}^{n}|\varphi_i(l) - \zeta_i(l)|, \tag{2.22}$$

in which

$$\begin{gathered} m_0 = \min_{1\leq i\leq n}\left(\frac{\mu_i}{\overline{a}_i}\right), \qquad M_0 = \max\{M_1, M_2\}, \qquad M_1 = \max_{1\leq i\leq n}\left(\frac{\mu_i}{\underline{a}_i}\right), \\ M_2 = \sum_{j=1}^{n}\mu_j\max_{1\leq i\leq n}(\overline{u}_{ji}\rho_i L_i)\int_0^{+\infty}se^{\lambda s}\max_{1\leq i\leq n}k_{ji}(s)ds. \end{gathered} \tag{2.23}$$

Hence, from (2.21) and (2.22), we know that the following inequality holds for $t > 0$:

$$\|x(t,\varphi_2) - x(t,\varphi_1)\| \leq M\|\varphi_2 - \varphi_1\|e^{-\lambda t}, \tag{2.24}$$

in which $M = M_0/m_0$.

We can always choose a positive integer N such that $e^{-\lambda_0 NT}M \leq 1/2$ and define a Poincaré mapping $P : C \to C$ by $P(\xi) = x_T(\xi)$; we have

$$\left\| P^N \varphi_2 - P^N \varphi_1 \right\| \leq \frac{1}{2} \left\| \varphi_2 - \varphi_1 \right\|, \tag{2.25}$$

which implies that P^N is a contraction mapping. Similar to [10], using contraction mapping principle, we know that system (2.1) has a T-periodic solution which is globally exponentially stable. This completes the proof. □

Remark 2.6. The result above also holds for (2.1) without impulses, and the existence and globally exponential stability of the periodic solution for (2.1) have nothing to do with amplification functions and inputs of the neuron. The results in [5] have more restrictions than Lemma 2.5 in this paper because conditions for the ones in [5] are relevant to amplification functions.

3. Periodic Solutions of CGBAMNNs with Distributed Delays and Impulses

Consider the following periodic CGBAMNNs model with distributed delays:

$$\begin{aligned}
\dot{x}_i(t) &= -a_i(x_i(t)) \Bigg[b_i(t, x_i(t)) - \sum_{j=1}^{m} p_{ij}(t) f_j(\rho_j y_j(t)) \\
&\qquad - \sum_{j=1}^{m} u_{ij}(t) \int_0^{+\infty} k_{ij}(s) f_j(\rho_j y_j(t-s)) ds - I_i(t) \Bigg], \qquad t > 0,\ t \neq t_k, \\
\Delta x_i(t_k) &= -\gamma_{ik} x_i(t_k), \qquad t = t_k,\ k \in Z^+, \\
\dot{y}_j(t) &= -c_j(y_j(t)) \Bigg[d_j(t, y_j(t)) - \sum_{i=1}^{n} q_{ji}(t) g_i(\tilde{\rho}_i x_i(t)) \\
&\qquad - \sum_{i=1}^{n} v_{ji}(t) \int_0^{+\infty} \tilde{k}_{ji}(s) g_i(\tilde{\rho}_i x_i(t-s)) ds - J_j(t) \Bigg], \qquad t > 0, t \neq t_k, \\
\Delta y_j(t_k) &= -\delta_{jk} y_j(t_k), \qquad t = t_k,\ k \in Z^+
\end{aligned} \tag{3.1}$$

for $1 \leq i \leq n,\ 1 \leq j \leq m$, and $Z^+ = \{1, 2, \ldots\}$; $x_i(t)$ and $y_j(t)$ denote the state variable of the ith neuron from the neural field F_X and the jth neuron from the neural field F_Y at time t; $f_j(\cdot)$ and $g_i(\cdot)$ denote the signal functions of the jth neuron from the neural field F_Y and the ith neuron from the neural field F_X at time t; respectively, I_i and J_j denote inputs of the ith neuron from the neural field F_X and the jth neuron from the neural field F_Y at time t; respectively, $a_i(\cdot)$ and $c_j(\cdot)$ represent amplification functions; $b_i(t, \cdot)$ and $d_j(t, \cdot)$ are appropriately behaved functions; $p_{ij}(t), q_{ji}(t), u_{ij}(t)$, and $v_{ji}(t)$ are the connection weights; $\rho_j, \tilde{\rho}_i$ are positive constants, which correspond to the neuronal gains associated with the neuronal activations; k_{ij} and $\tilde{k}_{ji}$ correspond to the delay kernel functions; $u_{ij}(t), v_{ji}(t)$, $p_{ij}(t)$, $q_{ji}(t)$, $I_i(t)$, and $J_j(t)$ are all continuously periodic functions on $[0, +\infty)$ with common period $T > 0$.

$\Delta x_i(t_k) = x_i(t_k^+) - x_i(t_k^-)$, $\Delta y_j(t_k) = y_j(t_k^+) - y_j(t_k^-)$; t_k is called impulsive moment and satisfies $0 < t_1 < t_2 < \cdots$, $\lim_{k\to+\infty} t_k = +\infty$; $x_i(t^-)$, $y_j(t^-)$ and $x_i(t^+), y_j(t^+)$ denote the left-hand and right-hand limits at t_k; respectively, we always assume $x_i(t_k^-) = x_i(t_k)$, $y_j(t_k^-) = y_j(t_k)$, $x_i'(t_k^-) = x_i'(t_k)$, and $y_j'(t_k^-) = y_j'(t_k)$, $k \in Z^+$.

For system (3.1), we assume the following.

(H_8) Amplification functions $a_i(\cdot)$ and $c_j(\cdot)$ are continuous and there exist constants $\underline{a}_i$, $\overline{a}_i$ and $\underline{c}_j$, $\overline{c}_j$ such that $0 < \underline{a}_i \le a_i(x_i(t)) \le \overline{a}_i$, $0 < \underline{c}_j \le c_j(y_j(t)) \le \overline{c}_j$, $1 \le i \le n$, $1 \le j \le m$.

(H_9) $b_i(t,u)$, $d_j(t,u)$ are T-periodic about the first argument, there exist continuous, T-periodic functions $\alpha_i(t)$ and $\beta_j(t)$ such that

$$\frac{b_i(t,x) - b_i(t,y)}{x-y} \ge \alpha_i(t) > 0, \qquad \frac{d_j(t,x) - d_j(t,y)}{x-y} \ge \beta_j(t) > 0 \tag{3.2}$$

for all $x \ne y$, $1 \le i \le n$, $1 \le j \le m$.

(H_{10}) For activation functions $f_j(\cdot)$ and $g_i(\cdot)$, there exist constant L_j and $\widetilde{L}_i$ such that

$$L_j = \sup_{x\ne y}\left|\frac{f_j(x) - f_j(y)}{x-y}\right|, \quad \widetilde{L}_i = \sup_{x\ne y}\left|\frac{g_i(x) - g_i(y)}{x-y}\right|, \quad \forall x \ne y \in R,\ 1 \le i \le n,\ 1 \le j \le m. \tag{3.3}$$

(H_{11}) The kernel functions $k_{ij}(s)$ and $\widetilde{k}_{ji}(s)$ are nonnegative continuous functions on $[0,+\infty)$ and satisfy

$$\begin{aligned} &\int_0^{+\infty} s e^{\lambda s} k_{ij}(s)\,ds < +\infty, &&\int_0^{+\infty} s e^{\lambda s} \widetilde{k}_{ji}(s)\,ds < +\infty, \\ &K_{ij}(\lambda) = \int_0^{+\infty} e^{\lambda s} k_{ij}(s)\,ds, &&\widetilde{K}_{ji}(\lambda) = \int_0^{+\infty} e^{\lambda s} \widetilde{k}_{ji}(s)\,ds, \end{aligned} \tag{3.4}$$

are differentiable functions for $\lambda \in [0, r_{ij})$ and $\lambda \in [0, \widetilde{r}_{ji})$; respectively, $0 < r_{ij} < +\infty$, $0 < \widetilde{r}_{ji} < +\infty$, $K_{ij}(0) = 1$, $\widetilde{K}_{ji}(0) = 1$, $\lim_{\lambda\to r_{ij}^-} K_{ij}(\lambda) = +\infty$ and $\lim_{\lambda\to \widetilde{r}_{ji}^-} \widetilde{K}_{ji}(\lambda) = +\infty$.

(H_{12}) There exists positive integer k_0 such that $t_{k+k_0} = t_k + T$ and $\gamma_{i(k+k_0)} = \gamma_{ik}$, $\delta_{j(k+k_0)} = \delta_{jk}$ hold.

We assume that system (3.1) has the following initial conditions:

$$x_i(s) = \varphi_i(s), \quad y_j(s) = \phi_j(s), \quad i = 1,2,\ldots,n,\ j = 1,2,\ldots,m,\ -\infty \le s \le 0, \tag{3.5}$$

where $\psi = (\varphi, \phi) \in \mathrm{PC}((-\infty,0], R^{n+m})$, $\varphi(s) = (\varphi_1(s), \varphi_2(s), \ldots, \varphi_n(s))$, $\phi(s) = (\phi_1(s), \phi_2(s), \ldots, \phi_m(s))$.

Let $Z(t,\psi) = (x(t,\psi), y(t,\psi))$ denote any solution of the system (3.1) with initial value $\psi = (\varphi, \phi) \in \mathrm{PC}$, $x(t,\psi) = (x_1(t,\psi), x_2(t,\psi), \ldots, x(t_n,\psi))$, $y(t,\psi) = (y_1(t,\psi), y_2(t,\psi), \ldots, y_m(t,\psi))$.

Theorem 3.1. *Under assumptions* (H_8)–(H_{12})*, there exists a T-periodic solution which is asymptotically stable, if the following conditions hold.*

(H_{13}) *The following* $\mathcal{M}$ *is a nonsingular M-matrix, and*

$$\mathcal{M} = \begin{pmatrix} A & -C \\ -\widetilde{C} & \widetilde{A} \end{pmatrix}, \tag{3.6}$$

in which

$$\begin{gathered} A = \operatorname{diag}\left(\underline{\alpha}_1, \underline{\alpha}_2, \ldots, \underline{\alpha}_n\right), \qquad \widetilde{A} = \operatorname{diag}\left(\underline{\beta}_1, \underline{\beta}_2, \ldots, \underline{\beta}_m\right), \\ C = \left(e_{ij}\right)_{m\times n}, \qquad \widetilde{C} = \left(\widetilde{e}_{ij}\right)_{n\times m}, \qquad \widetilde{e}_{ij} = \left(\overline{q}_{ji} + \overline{v}_{ji}\right)\widetilde{\rho}_i \widetilde{L}_i, \qquad e_{ij} = \left(\overline{p}_{ij} + \overline{u}_{ij}\right)\rho_j L_j. \end{gathered} \tag{3.7}$$

(H_{14}) $a_i((1-\gamma_{ik})s) \ge |1-\gamma_{ik}| a_i(s)$, $c_j((1-\delta_{jk})s) \ge |1-\delta_{jk}| c_j(s)$, $\forall s \in R, i = 1,2,\ldots,n,\ j = 1,2,\ldots,m$.

Proof. Let

$$\begin{gathered} x_{n+j}(t) = y_j(t), \qquad a_{n+j}(t, x_{n+j}(t)) = c_j(t, y_j(t)), \qquad b_{n+j}(t, x_{n+j}(t)) = d_j(t, y_j(t)), \\ p_{n+j,i}(t) = q_{ji}(t), \qquad p_{i,n+j}(t) = p_{ij}(t), \qquad u_{n+j,i}(t) = v_{ji}(t), \qquad u_{i,n+j}(t) = u_{ij}(t), \\ S_i(x_i(t)) = g_i(x_i(t)), \qquad S_{n+j}(x_{n+j}(t)) = f_j(x_j(t)), \qquad \varphi_{n+j}(s) = \psi_j(s), \\ I_{n+j} = J_j(t), \qquad k_{n+j,i}(s) = \widetilde{k}_{ji}(s), \qquad k_{ij}(s) = k_{i,n+j}(s), \\ \alpha_{n+j}(t) = \beta_j(t), \qquad \widetilde{L}_{n+j} = L_j, \qquad \widetilde{\rho}_{n+j} = \rho_j. \end{gathered} \tag{3.8}$$

It follows that system (3.1) can be rewritten as

$$\begin{aligned} \dot{x}_i(t) &= -a_i(x_i(t)) \Bigg[b_i(t, x_i(t)) - \sum_{j=1}^{m} p_{i,n+j}(t) S_{n+j}\left(\widetilde{\rho}_{n+j} x_{n+j}(t)\right) \\ &\qquad - \sum_{j=1}^{m} u_{i,n+j}(t) \int_0^{+\infty} k_{i,n+j}(s) S_{n+j}\left(\widetilde{\rho}_{n+j} x_{n+j}(t-s)\right) ds - I_i(t) \Bigg], \quad t \ne t_k, \\ \Delta x_i(t_k) &= -\gamma_{ik} x_i(t_k), \quad t = t_k,\ k \in Z^+, \\ \dot{x}_{n+j}(t) &= -a_{n+j}\left(x_{n+j}(t)\right) \Bigg[b_{n+j}\left(t, x_{n+j}(t)\right) - \sum_{i=1}^{n} p_{n+j,i}(t) S_i\left(\widetilde{\rho}_i x_i(t)\right) \\ &\qquad - \sum_{i=1}^{n} u_{n+j,i}(t) \int_0^{+\infty} k_{n+j,i}(s) S_i\left(\widetilde{\rho}_i x_i(t-s)\right) ds - I_{n+j}(t) \Bigg], \quad t \ne t_k, \\ \Delta x_{n+j}(t_k) &= -\gamma_{n+j,k} x_{n+j}(t_k), \quad t = t_k,\ k \in Z^+, \end{aligned} \tag{3.9}$$

for $1 \le i \le n,\ 1 \le j \le m$.

Initial conditions are given by

$$x_i(s) = \varphi_i(s), \qquad s \in (-\infty, 0],\ i = 1, 2, \ldots, (n+m). \tag{3.10}$$

Thus system (3.9) is a special case of system (2.1) in mathematical form, under conditions (H_8)–(H_{14}), we obtain from Lemma 2.5 that system (3.9) has a T-periodic solution which is globally exponentially stable if $a_i((1-\gamma_{ik})s) \geq |1-\gamma_{ik}|a_i(s)$ and the following matrix $\mathscr{M}'$ is a M-matrix, and

$$\mathscr{M}' = A' - C', \tag{3.11}$$

where

$$A' = \operatorname{diag}\left(\underline{\alpha}_1, \underline{\alpha}_2, \ldots, \underline{\alpha}_{n+m}\right),$$

$$D' = \begin{pmatrix} 0 & \cdots & 0 & w'_{1,n+1} & \cdots & w'_{1,n+m} \\ \vdots & & \vdots & \vdots & & \vdots \\ 0 & \cdots & 0 & w'_{n,n+1} & \cdots & w'_{n,n+m} \\ w'_{n+1,1} & \cdots & w'_{n+1,n} & 0 & \cdots & 0 \\ \vdots & & \vdots & \vdots & & \vdots \\ w'_{n+m,1} & \cdots & w'_{n+m,n} & 0 & \cdots & 0 \end{pmatrix}, \tag{3.12}$$

in which $w'_{ij} = (\overline{p}_{ij} + \overline{u}_{ij})\widetilde{\rho}_j\widetilde{L}_j$.

Then, we know from (3.8) and (3.11) that Theorem 3.1 holds.

If $a_i(x_i(t)) = c_j(y_j(t)) = 1$, $b_i(t, x_i(t)) = b_i(t)x_i(t)$ and $d_j(t, y_j(t)) = d_j(t)y_j(t)$, where $b_i(t)$ and $d_j(t)$ are positive continuous T-periodic functions for $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, m$. System (3.1) reduces to the following Hopfield-type BAM neural networks model:

$$\begin{aligned} \dot{x}_i(t) &= -b_i(t)x_i(t) + \sum_{j=1}^{m} p_{ij}(t) f_j\left(\rho_j y_j(t)\right) \\ &\quad + \sum_{j=1}^{m} u_{ij}(t) \int_0^{+\infty} k_{ij}(s) f_j\left(\rho_j y_j(t-s)\right) ds + I_i(t), \qquad t > 0,\ t \neq t_k, \\ \Delta x_i(t_k) &= -\gamma_{ik} x_i(t_k), \qquad t = t_k,\ k \in Z^+, \\ \dot{y}_j(t) &= -d_j(t)y_j(t) + \sum_{i=1}^{n} q_{ji}(t) g_i\left(\widetilde{\rho}_i x_i(t)\right) \\ &\quad + \sum_{i=1}^{n} v_{ji}(t) \int_0^{+\infty} \widetilde{k}_{ji}(s) g_i\left(\widetilde{\rho}_i x_i(t-s)\right) ds + J_j(t), \qquad t > 0,\ t \neq t_k, \\ \Delta y_j(t_k) &= -\delta_{jk} y_j(t_k), \qquad t = t_k,\ k \in Z^+. \end{aligned} \tag{3.13}$$

□

Corollary 3.2. *Under assumptions* (H_9)–(H_{12}), *there exists a* T*-periodic solution which is globally asymptotically stable, if the following conditions hold.*

(H_{13}') *The following* $\mathcal{M}$ *is a nonsingular M-matrix, and*

$$\mathcal{M} = \begin{pmatrix} A & -C \\ -\widetilde{C} & \widetilde{A} \end{pmatrix}, \tag{3.14}$$

in which

$$\begin{gathered} A = \operatorname{diag}\left(\underline{b}_1,\ \underline{b}_2, \ldots, \underline{b}_n\right), \qquad \widetilde{A} = \operatorname{diag}\left(\underline{d}_1,\ \underline{d}_2, \ldots, \underline{d}_m\right), \\ C = \left(e_{ij}\right)_{m\times n}, \qquad \widetilde{C} = \left(\widetilde{e}_{ij}\right)_{n\times m}, \qquad e_{ij} = \left(\overline{p}_{ij} + \overline{u}_{ij}\right)\rho_j L_j, \qquad \widetilde{e}_{ij} = \left(\overline{q}_{ji} + \overline{v}_{ji}\right)\widetilde{\rho}_i \widetilde{L}_i. \end{gathered} \tag{3.15}$$

(H_{14}') $0 \le \gamma_{ik} \le 2,\ 0 \le \delta_{jk} \le 2$ *for* $i = 1, 2, \ldots, n,\ j = 1, 2, \ldots, m,\ k \in Z^+$.

Proof. As $b_i(t, x_i(t)) = b_i(t)x_i(t)$ and $d_j(t, y_j(t)) = d_j(t)y_j(t)$, we obtain $\alpha_i(t) = b_i(t)$ and $\beta_j(t) = d_j(t)$ in (H_2), (H_{13}') implies (H_{13}) holds. Since $a_i(x_i(t)) = c_j(y_j(t)) \equiv 1$, then condition ($H_{14}$) reduces to ($H_{14}'$). Corollary 3.2 Holds from Theorem 3.1. □

Remark 3.3. The conditions for the existence and globally exponential stability of the periodic solution of (3.1) without impulses have nothing to do with inputs of the neuron and amplification functions. The results in [13, 14] have more restrictions than Theorem 3.1 in this paper because conditions for the ones in [13, 14] are relevant to amplification functions and inputs of neurons our results should be better. In addition, Corollary 3.2 is similar to Theorem 2.1 in [15]; our results generalize the results in [15].

Remark 3.4. In view of proof of Theorem 3.1, since CGBAMNNs model is a special case of CGNNs model in form as BAM neural networks model is a special case of Hopfield neural networks model, many results of CGBAMNNs can be directly obtained from the ones of CGNNs, needing no repetitive discussions. Since system (3.1) reduces to autonomous system, Theorem 3.1 still holds, which means that system (3.1) has a equilibrium which is globally asymptotically stable; we know that many results in [18] can be directly obtained from the results in [19].

4. Two Simple Examples

Example 4.1. Consider the following CGNNs model with distributed delays:

$$\begin{aligned} \dot{x}_1(t) &= -2\left[x_1(t) - 0.2\tanh(x_1(t)) - \sin t\int_0^{+\infty} e^{-s}\tanh(x_2(t-s))ds\right], \\ \dot{x}_2(t) &= -(2+\cos(x_2(t)))\left[x_2(t) - 0.3\int_0^{+\infty} e^{-s}\tanh(x_1(t-s))ds - 5\right]. \end{aligned} \tag{4.1}$$

Obviously, system (4.1) satisfies (H_1)–(H_5).

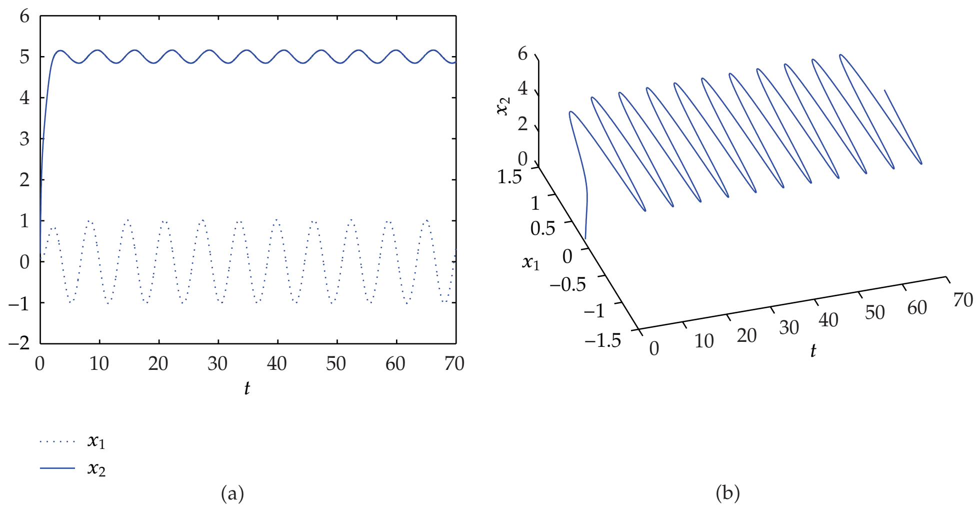

Figure 1: Time response of state variables $x_1,\ x_2$ and phase plot in space $(t,\ x_1,\ x_2)$ for system (4.1).

Note that

$$\mathcal{M} = \begin{pmatrix} 1 & -1.2 \\ -0.3 & 1 \end{pmatrix}, \tag{4.2}$$

it is a nonsingular M-matrix and system (4.1) also satisfies condition (H_6). According to Lemma 2.5, system (4.1) has a 2π-periodic solution which is globally exponentially stable. Figure 1 shows the dynamic behaviors of system (4.1) with initial condition $(0.1, 0.2)$.

However, It is easy to check that system (4.1) does not satisfy Theorem 4.3 or 4.4 in [5], so theorems in [5] cannot are used to ascertain the existence and stability of periodic solutions of system (4.1).

Example 4.2. Consider the following CGBAMNNs model with distributed delays and impulses:

$$\begin{gathered} \dot{x}_1(t) = -(2+\sin(x_1(t)))\left[2x_1(t) - \sin t\int_0^{+\infty} e^{-s}\left|y_1(t-s)\right|ds - 1\right], \quad t>0,\ t \neq t_k, \\ \Delta x_1(t_k) = -\gamma_{1k}x_i(t_k), \quad t = t_k,\ k \in Z^+, \\ \dot{y}_1(t) = -\left(3+\cos\left(y_1(t)\right)\right)\left[(3+\cos t)y_1(t) - \sin t\int_0^{+\infty} e^{-s}|x_1(t-s)|ds - 1\right], \quad t>0,\ t \neq t_k, \\ \Delta y_1(t_k) = -\delta_{1k}y_1(t_k), \quad t = t_k,\ k \in Z^+, \end{gathered} \tag{4.3}$$

where $t_k = \pi k,\ k \in Z^+$.

Obviously, system (4.3) satisfies (H_8)–(H_{12}).

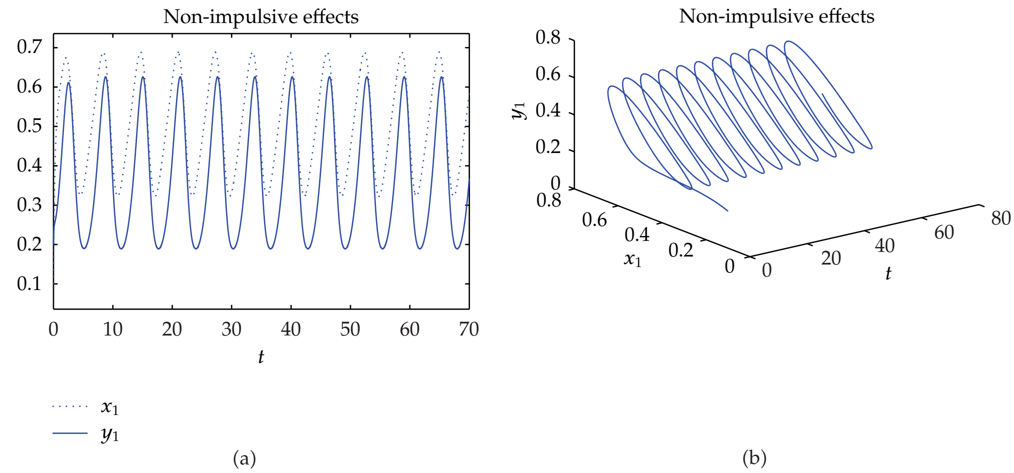

Figure 2: Time response of state variables x_1, y_1 and phase plot in space (t, x_1, y_1) for system (4.3) without impulsive effects.

Case 1. $\gamma_{1k} = 0$, $\delta_{1k} = 0$. Note that

$$\mathcal{M} = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}, \tag{4.4}$$

it is a nonsingular M-matrix and system (4.3) also satisfies condition (H_{13}). According to Theorem 3.1, system (4.3) without impulses has a 2π-periodic solution which is globally exponentially stable. Figure 2 shows the dynamic behaviors of system (4.3) with initial condition $(0.1, 0.2)$.

However, it is easy to check that system (4.3) without impulses does not satisfy Theorem 1 in [13] and theorems in [14]; so theorems in [13, 14] cannot be used to ascertain the existence and stability of periodic solutions of system (4.3).

Case 2. $\gamma_{1k} = 0.7$, $\delta_{1k} = (1 - 0.5\sin(t_k + 1))$. Note that $a_1(s) = 2 + \sin s$, $c_1(s) = 3 + \cos s$, and $\underline{a}_1/\overline{a}_1 = 0.5 > |1 - \gamma_{1k}| = 0.3$ and $|1 - \delta_{1k}| < 0.5 < \underline{c}_1/\overline{c}_1 = 2/3$, which means condition (H_{14}) also holds for system (4.3). Hence, system (4.3) with impulses still has that there exists a 2π-periodic solution which is globally asymptotically stable. Figure 3 shows the dynamic behaviors of system (4.3) with initial condition $(0.1, 0.2)$.

This example illustrates the feasibility and effectiveness of the main results obtained in this paper, and it also shows that the conditions for the existence and globally exponential stability of the periodic solutions of CGBAMNNs without impulses have nothing to do with inputs of the neurons and amplification functions. If impulsive perturbations exist, the periodic solutions still exist and they are globally exponentially stable when we give some restrictions on impulsive perturbations.

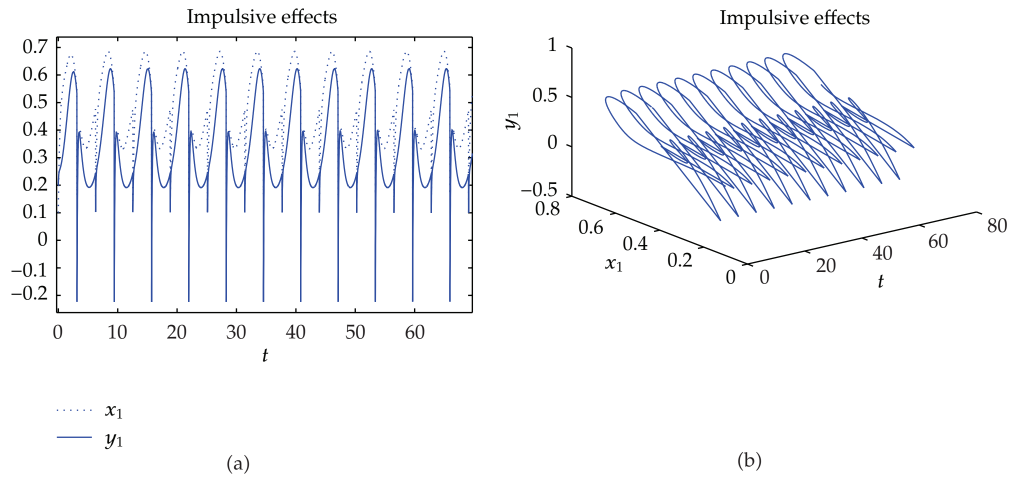

Figure 3: Time response of state variables $x_1,\ y_1$ and phase plot in space $(t,\ x_1,\ y_1)$ for system (4.3) with impulsive effects.

5. Conclusions

A class of CGBAMNNs with distributed delays and impulses are investigated by using suitable Lyapunov functional, the properties of M-matrix, and some suitable mathematical transformation in this paper. Sufficient conditions to guarantee the uniqueness and global exponential stability of the periodic solutions of such networks are established without assuming the boundedness of the activation functions. Lemma 2.5 improves the results in [5], and Theorem 3.1 improves the results in [13, 14] and generalize the results in [15]. In addition, we point that CGBAMNNs model is a special case of CGNNs model; many results of CGBAMNNs can be directly obtained from the ones of CGNNs, needing no repetitive discussions. Our results are new, and two examples have been provided to demonstrate the effectiveness of our results.

Appendix

The source program (MATLAB 7.0) of Figure 1 is given as follows [14].

```
clear
T=70;
N=7000;
h=T/N;
m=40/h;
for i=1:m
U(:,i)=[0.1; 0.2];
end
```

```
for i=(m+1):(N+m)
r(i)=i*h-40;
x(i)=r(i);
I=2;
J=2+cos(U(2,i-1));
A=[-I,0; 0,-J];
B=[0,sin(x(i))*I; 0.3*J,0];
U(:,i)=h*A*[(U(1,i-1)-0.2*tanh(U(1,i-1))); U(2,i-1)]+U(:,i-1);
P(:,1)=[0; 0];
for k=1:m
P(:,1)=P(:,1)+h*exp(-(40-(k-1)*h))*[(tanh(U(1,i-m+k-1)));(tanh(U(2,i-m+k-1)))];
end
U(:,i)=U(:,i)+B*h*[(P(1,1));(P(2,1))]+h*[0; 5*J];
end
y=U(1,:);
z=U(2,:);
hold on
plot(r,y,':')
hold on
plot(r,z)
hold on
plot3(r,y,z)
```

The source program (MATLAB 7.0) of Figures 2 and 3 is given as follows [14].

```
clear
T=70;
N=7000;
h=T/N;
m=40/h;
for i=1:m
U(:,i)=[0.1;0.2];
end
for i=(m+1):(N+m)
r(i)=i*h-40;
x(i)=r(i);
I=2+sin(U(1,i-1));
J=3+cos(U(2,i-1));
```

```
A=[-I,0;0,-J];
B=[0,I*sin(x(i)); J*sin(x(i)),0];
U(:,i)=h*A*[2*U(1,i-1);(3+cos(x(i)))*U(2,i-1)]+U(:,i-1);
P(:,1)=[0;0];
for k=1:m
P(:,1)=P(:,1)+h*exp(-(40-(k-1)*h))*[(abs(U(1,i-m+k-1)));(abs(U(2,i-m+k-1)))];
end
U(:,i)=U(:,i)+B*h*[(P(1,1));(P(2,1))]+[I; J]*h;
if mod(i-m,314)==0
U(:,i)=[0.3,0; 0,1/2*(sin(x(i)+1))]*U(:,i);
end
end
y=U(1,:);
z=U(2,:)
hold on
plot(r,y,':')
hold on
plot(r,z)
hold on
plot3(r,y,z)
```

Acknowledgment

The authors would like to thank the editor and the reviewers for their valuable suggestions and comments which greatly improved the original paper. Projects supported by the National Natural Science Foundation of China (no. 11071254).

References

[1] Z. Huang and Y. Xia, "Exponential periodic attractor of impulsive BAM networks with finite distributed delays," *Chaos, Solitons and Fractals*, vol. 39, no. 1, pp. 373–384, 2009.
[2] S. Townley, A. Ilchmann, M. G. Weiß et al., "Existence and learning of oscillations in recurrent neural networks," *IEEE Transactions on Neural Networks*, vol. 11, no. 1, pp. 205–214, 2000.
[3] M. A. Cohen and S. Grossberg, "Absolute stability of global pattern formation and parallel memory storage by competitive neural networks," *IEEE Transactions on Systems, Man, and Cybernetics*, vol. 13, no. 5, pp. 815–826, 1983.
[4] B. Kosko, "Bidirectional associative memories," *IEEE Transactions on Systems, Man, and Cybernetics*, vol. 18, no. 1, pp. 49–60, 1988.
[5] J. Sun and L. Wan, "Global exponential stability and periodic solutions of Cohen-Grossberg neural networks with continuously distributed delays," *Physica D*, vol. 208, no. 1-2, pp. 1–20, 2005.
[6] Y. Li, "Existence and stability of periodic solutions for Cohen-Grossberg neural networks with multiple delays," *Chaos, Solitons and Fractals*, vol. 20, no. 3, pp. 459–466, 2004.

[7] C.-H. Li and S.-Y. Yang, "Existence and attractivity of periodic solutions to non-autonomous Cohen-Grossberg neural networks with time delays," *Chaos, Solitons and Fractals*, vol. 41, no. 3, pp. 1235–1244, 2009.

[8] Q. Song, J. Cao, and Z. Zhao, "Periodic solutions and its exponential stability of reaction-diffusion recurrent neural networks with continuously distributed delays," *Nonlinear Analysis. Real World Applications*, vol. 7, no. 1, pp. 65–80, 2006.

[9] X. Yang, "Existence and global exponential stability of periodic solution for Cohen-Grossberg shunting inhibitory cellular neural networks with delays and impulses," *Neurocomputing*, vol. 72, no. 10-12, pp. 2219–2226, 2009.

[10] Q. Liu and R. Xu, "Periodic solutions of high-order Cohen-Grossberg neural networks with distributed delays," *Communications in Nonlinear Science and Numerical Simulation*, vol. 16, no. 7, pp. 2887–2893, 2011.

[11] H. Xiang and J. Cao, "Exponential stability of periodic solution to Cohen-Grossberg-type BAM networks with time-varying delays," *Neurocomputing*, vol. 72, no. 7-9, pp. 1702–1711, 2009.

[12] Y. Li, X. Chen, and L. Zhao, "Stability and existence of periodic solutions to delayed Cohen-Grossberg BAM neural networks with impulses on time scales," *Neurocomputing*, vol. 72, no. 7–9, pp. 1621–1630, 2009.

[13] A. Chen and J. Cao, "Periodic bi-directional Cohen-Grossberg neural networks with distributed delays," *Nonlinear Analysis. Theory, Methods & Applications*, vol. 66, no. 12, pp. 2947–2961, 2007.

[14] X. Li, "Existence and global exponential stability of periodic solution for impulsive Cohen-Grossberg-type BAM neural networks with continuously distributed delays," *Applied Mathematics and Computation*, vol. 215, no. 1, pp. 292–307, 2009.

[15] Y.-t. Li and J. Wang, "An analysis on the global exponential stability and the existence of periodic solutions for non-autonomous hybrid BAM neural networks with distributed delays and impulses," *Computers & Mathematics with Applications*, vol. 56, no. 9, pp. 2256–2267, 2008.

[16] J. Principle, J. Kuo, and S. Celebi, "An analysis of the gamma memory in dynamics neural networks," *IEEE Transactions on Neural Networks*, vol. 5, pp. 337–361, 1994.

[17] R. S. Varga, *Matrix Iterative Analysis*, vol. 27, Springer, Berlin, Germany, 2000.

[18] K. Li, L. Zhang, X. Zhang, and Z. Li, "Stability in impulsive Cohen-Grossberg-type BAM neural networks with distributed delays," *Applied Mathematics and Computation*, vol. 215, no. 11, pp. 3970–3984, 2010.

[19] K. Li, "Stability analysis for impulsive Cohen-Grossberg neural networks with time-varying delays and distributed delays," *Nonlinear Analysis. Real World Applications*, vol. 10, no. 5, pp. 2784–2798, 2009.