Hindawi Publishing Corporation
Journal of Applied Mathematics
Volume 2012, Article ID 825416, 14 pages
doi:10.1155/2012/825416

Research Article

Robust Local Regularity and Controllability of Uncertain TS Fuzzy Descriptor Systems

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Received 2 October 2012; Accepted 28 October 2012

Academic Editor: Jen Chih Yao

The robust local regularity and controllability problem for the Takagi-Sugeno (TS) fuzzy descriptor systems is studied in this paper. Under the assumptions that the nominal TS fuzzy descriptor systems are locally regular and controllable, a sufficient criterion is proposed to preserve the assumed properties when the structured parameter uncertainties are added into the nominal TS fuzzy descriptor systems. The proposed sufficient criterion can provide the explicit relationship of the bounds on parameter uncertainties for preserving the assumed properties. An example is given to illustrate the application of the proposed sufficient condition.

1. Introduction

Recently, it has been shown that the fuzzy-model-based representation proposed by Takagi and Sugeno [1], known as the TS fuzzy model, is a successful approach for dealing with the nonlinear control systems, and there are many successful applications of the TS-fuzzy-model-based approach to the nonlinear control systems (e.g., [2–19] and references therein). Descriptor systems represent a much wider class of systems than the standard systems [20]. In recent years, some researchers (e.g., [4–6, 8, 21–28] and references therein) have studied the design issue of the fuzzy parallel-distributed-compensation (PDC) controllers for each fuzzy rule of the TS fuzzy descriptor systems. Both regularity and controllability are actually two very important properties of descriptor systems with control inputs [29]. So, before the design of the fuzzy PDC controllers in the corresponding rule of the TS fuzzy descriptor

systems, it is necessary to consider both properties of local regularity and controllability for each fuzzy rule [23]. However, both regularity and controllability of the TS fuzzy systems are not considered by those mentioned-above researchers before the fuzzy PDC controllers are designed. Therefore, it is meaningful to further study the criterion that the local regularity and controllability for each fuzzy rule of the TS fuzzy descriptor systems hold [30].

On the other hand, in fact, in many cases it is very difficult, if not impossible, to obtain the accurate values of some system parameters. This is due to the inaccurate measurement, inaccessibility to the system parameters, or variation of the parameters. These parametric uncertainties may destroy the local regularity and controllability properties of the TS fuzzy descriptor systems. But, to the authors' best knowledge, there is no literature to study the issue of robust local regularity and controllability for the uncertain TS fuzzy descriptor systems.

The purpose of this paper is to present an approach for investigating the robust local regularity and controllability problem of the TS fuzzy descriptor systems with structured parameter uncertainties. Under the assumptions that the nominal TS fuzzy descriptor systems are locally regular and controllable, a sufficient criterion is proposed to preserve the assumed properties when the structured parameter uncertainties are added into the nominal TS fuzzy descriptor systems. The proposed sufficient criterion can provide the explicit relationship of the bounds on structured parameter uncertainties for preserving the assumed properties. A numerical example is given in this paper to illustrate the application of the proposed sufficient criterion.

2. Robust Local Regularity and Controllability Analysis

Based on the approach of using the sector nonlinearity in the fuzzy model construction, both the fuzzy set of premise part and the linear dynamic model with parametric uncertainties of consequent part in the exact TS fuzzy control model with parametric uncertainties can be derived from the given nonlinear control model with parametric uncertainties [5]. The TS continuous-time fuzzy descriptor system with parametric uncertainties for the nonlinear control system with structured parametric uncertainties can be obtained as the following form:

$$\begin{aligned} \widetilde{R}^i\colon\ & \text{IF } z_1 \text{ is } M_{i1} \text{ and } \ldots \text{ and } z_g \text{ is } M_{ig}, \\ & \text{then } E_i\dot{x}(t) = (A_i + \Delta A_i)x(t) + (B_i + \Delta B_i)u(t), \end{aligned} \tag{2.1}$$

or the uncertain discrete-time TS fuzzy descriptor system can be described by

$$\begin{aligned} \widetilde{R}^i\colon\ & \text{IF } z_1 \text{ is } M_{i1} \text{ and } \ldots \text{ and } z_g \text{ is } M_{ig}, \\ & \text{then } E_i x(k+1) = (A_i + \Delta A_i)x(k) + (B_i + \Delta B_i)u(k), \end{aligned} \tag{2.2}$$

with the initial state vector $x(0)$, where $\widetilde{R}^i$ $(i = 1, 2, \ldots, N)$ denotes the ith implication, N is the number of fuzzy rules, $x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^T$ and $x(k) = [x_1(k), x_2(k), \ldots, x_n(k)]^T$ denote the n-dimensional state vectors, $u(t) = [u_1(t), u_2(t), \ldots, u_p(t)]^T$ and $u(k) = [u_1(k), u_2(k), \ldots, u_p(k)]^T$ denote the p-dimensional input vectors, z_i $(i = 1, 2, \ldots, g)$ are the premise variables, E_i, A_i, and B_i $(i = 1, 2, \ldots, N)$ are, respectively, the $n \times n$, $n \times n$ and

$n \times p$ consequent constant matrices, ΔA_i and ΔB_i $(i = 1, 2, \ldots, N)$ are, respectively, the parametric uncertain matrices existing in the system matrices A_i and the input matrices B_i of the consequent part of the ith rule due to the inaccurate measurement, inaccessibility to the system parameters, or variation of the parameters, and M_{ij} $(i = 1, 2, \ldots, N$ and $j = 1, 2, \ldots, g)$ are the fuzzy sets. Here the matrices E_i $(i = 1, 2, \ldots, N)$ may be singular matrices with $\operatorname{rank}(E_i) \leq n$ $(i = 1, 2, \ldots, N)$. In many applications, the matrices E_i $(i = 1, 2, \ldots, N)$ are the structure information matrices; rather than parameter matrices, that is, the elements of E_i $(i = 1, 2, \ldots, N)$ contain only structure information regarding the problem considered.

In many interesting problems (e.g., plant uncertainties, constant output feedback with uncertainty in the gain matrix), we have only a small number of uncertain parameters, but these uncertain parameters may enter into many entries of the system and input matrices [31, 32]. Therefore, in this paper, we suppose that the parametric uncertain matrices ΔA_i and ΔB_i take the forms

$$\Delta A_i = \sum_{k=1}^{m} \varepsilon_{ik} A_{ik}, \qquad \Delta B_i = \sum_{k=1}^{m} \varepsilon_{ik} B_{ik}, \tag{2.3}$$

where ε_{ik} $(i = 1, 2, \ldots, N$ and $k = 1, 2, \ldots, m)$ are the elemental parametric uncertainties, and A_{ik} and B_{ik} $(i = 1, 2, \ldots, N$ and $k = 1, 2, \ldots, m)$ are, respectively, the given $n \times n$ and $n \times p$ constant matrices which are prescribed a priori to denote the linearly dependent information on the elemental parametric uncertainties ε_{ik}.

In this paper, for the uncertain TS fuzzy descriptor system in (2.1) (or (2.2)), each fuzzy-rule-nominal model $E_i\dot{x}(t) = A_i x(t) + B_i u(t)$ or $E_i x(k+1) = A_i x(k) + B_i u(k)$, which is denoted by $\{E_i, A_i, B_i\}$, is assumed to be regular and controllable. Due to inevitable uncertainties, each fuzzy-rule-nominal model $\{E_i, A_i, B_i\}$ is perturbed into the fuzzy-rule-uncertain model $\{E_i, A_i + \Delta A_i, B_i + \Delta B_i\}$. Our problem is to determine the conditions such that each fuzzy-uncertain model $\{E_i, A_i + \Delta A_i, B_i + \Delta B_i\}$ for the uncertain TS fuzzy descriptor system (2.1) (or (2.2)) is robustly locally regular and controllable. Before we investigate the robust properties of regularity and controllability for the uncertain TS fuzzy descriptor system (2.1) (or (2.2)), the following definitions and lemmas need to be introduced first.

Definition 2.1 (see [33]). The measure of a matrix $\overline{W} \in C^{n\times n}$ is defined as

$$\mu\left(\overline{W}\right) \equiv \lim_{\theta \to 0} \frac{\left(\left\| I + \theta \overline{W} \right\| - 1\right)}{\theta}, \tag{2.4}$$

where $\|\cdot\|$ is the induced matrix norm on $C^{n\times n}$.

Definition 2.2 (see [34]). The system $\{E_i, A_i, B_i\}$ is called controllable, if for any $t_1 > 0$ (or $k_1 > 0$), $x(0) \in R^n$, and $w \in R^n$, there exists a control input $u(t)$ (or $u(k)$) such that $x(t_1) = w$ (or $x(k_1) = w$).

Definition 2.3. The uncertain TS fuzzy descriptor system in (2.1) (or (2.2)) is locally regular, if each fuzzy-rule-uncertain model $\{E_i, A_i + \Delta A_i, B_i + \Delta B_i\}$ $(i = 1, 2, \ldots, N)$ is regular.

Definition 2.4. The uncertain TS fuzzy descriptor system in (2.1) (or (2.2)) is locally controllable, if each fuzzy-rule-uncertain model $\{E_i, A_i + \Delta A_i, B_i + \Delta B_i\}$ $(i = 1, 2, \ldots, N)$ is controllable.

Lemma 2.5 (see [34]). *The system $\{E_i, A_i, B_i\}$ is regular if and only if* $\operatorname{rank}[E_{ni} \quad B_{di}] = n^2$, *where $E_{ni} \in R^{n^2\times n}$ and $E_{di} \in R^{n^2\times n^2}$ are given by*

$$E_{ni} = \begin{bmatrix} E_i \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{bmatrix}, \qquad E_{di} = \begin{bmatrix} A_i & & & & \\ E_i & A_i & & & \\ & \cdot & \cdot & & \\ & & \cdot & \cdot & \\ & & & \cdot & \cdot \\ & & & E_i & A_i \end{bmatrix}. \tag{2.5}$$

Lemma 2.6 (see [29, 35]). *Suppose that the system $\{E_i, A_i, B_i\}$ is regular. The system $\{E_i, A_i, B_i\}$ is controllable if and only if* $\operatorname{rank}[E_{di} \quad E_{bi}] = n^2$ *and* $\operatorname{rank}[E_i \quad B_i] = n$, *where $E_{di} \in R^{n^2\times n^2}$ is given in* (2.5) *and* $E_{bi} = \operatorname{diag}\{B_i, B_i, \ldots, B_i\} \in R^{n^2\times np}$.

Lemma 2.7 (see [33]). *The matrix measures of the matrices $\overline{W}$ and $\overline{V}$, namely, $\mu(\overline{W})$ and $\mu(\overline{V})$, are well defined for any norm and have the following properties:*

(i) $\mu(\pm I) = \pm 1$, *for the identity matrix I;*

(ii) $-\|\overline{W}\| \le -\mu(-\overline{W}) \le \operatorname{Re}(\lambda(\overline{W})) \le \mu(\overline{W}) \le \|\overline{W}\|$, *for any* norm $\|\cdot\|$ *and any matrix* $\overline{W} \in C^{n\times n}$;

(iii) $\mu(\overline{W} + \overline{V}) \le \mu(\overline{W}) + \mu(\overline{V})$, *for any two matrices $\overline{W}, \overline{V} \in C^{n\times n}$;*

(iv) $\mu(\gamma\overline{W}) = \gamma\mu(\overline{W})$, *for any matrix $\overline{W} \in C^{n\times n}$ and any non-negative real number γ,*

where $\lambda(\overline{W})$ denotes any eigenvalue of $\overline{W}$, and $\operatorname{Re}(\lambda(\overline{W}))$ *denotes the real part of $\lambda(\overline{W})$.*

Lemma 2.8. *For any $\gamma < 0$ and any matrix $\overline{W} \in C^{n\times n}$, $\mu(\gamma\overline{W}) = -\gamma\mu(-\overline{W})$.*

Proof. This lemma can be immediately obtained from the property (iv) in Lemma 2.7. □

Lemma 2.9. *Let $\overline{N} \in C^{n\times n}$. If $\mu(-\overline{N}) < 1$, then* $\det(I + \overline{N}) \neq 0$.

Proof. From the property (ii) in Lemma 2.7 and since $\mu(-\overline{N}) < 1$, we can get that $\operatorname{Re}(\lambda(\overline{N})) \ge -\mu(-\overline{N}) > -1$. This implies that $\lambda(\overline{N}) \neq -1$. So, we have the stated result. □

Now, let the singular value decompositions of $R_i = [E_{ni} \quad E_{di}]$, $Q_i = [E_{di} \quad E_{bi}]$, and $P_i = [E_i \quad B_i]$ be, respectively,

$$R_i = U_i [S_i \quad 0_{n^2\times n}] V_i^H, \tag{2.6}$$

$$Q_i = U_{ri} [S_{ri} \quad 0_{n^2\times np}] V_{ri}^H, \tag{2.7}$$

$$P_i = U_{ci} [S_{ci} \quad 0_{n^2\times q}] V_{ci}^H, \tag{2.8}$$

where $U_i \in R^{n^2\times n^2}$ and $V_i \in R^{(n^2+n)\times(n^2+n)}$ are the unitary matrices, $S_i = \operatorname{diag}\{\sigma_{i1}, \sigma_{i2}, \ldots, \sigma_{in^2}\}$, and $\sigma_{i1} \geq \sigma_{i2} \geq \cdots \geq \sigma_{in^2} > 0$ are the singular values of R_i; $U_{ri} \in R^{n^2\times n^2}$ and $V_{ri} \in R^{(n^2+np)\times(n^2+np)}$ are the unitary matrices, $S_{ri} = \operatorname{diag}\{\sigma_{ri1}, \sigma_{ri2}, \ldots, \sigma_{rin^2}\}$ and $\sigma_{ri1} \geq \sigma_{ri2} \geq \cdots \geq \sigma_{rin^2} > 0$ are the singular values of Q_i; $U_{ci} \in R^{n\times n}$ and $V_{ci} \in R^{(n+p)\times(n+p)}$ are the unitary matrices, $S_{ci} = \operatorname{diag}\{\sigma_{ci1}, \sigma_{ci2}, \ldots, \sigma_{cin}\}$ and $\sigma_{ci1} \geq \sigma_{ci2} \geq \cdots \geq \sigma_{cin} > 0$ are the singular values of P_i; V_i^H, V_{ri}^H, and V_{ci}^H denote, respectively, the complex-conjugate transposes of the matrices V_i, V_{ri}, and V_{ci}.

In what follows, with the preceding definitions and lemmas, we present a sufficient criterion for ensuring that the uncertain TS fuzzy descriptor system in (2.1) or (2.2) remains locally regular and controllable.

Theorem 2.10. *Suppose that the each fuzzy-rule-nominal descriptor system $\{E_i, A_i, B_i\}$ is regular and controllable. The uncertain TS fuzzy descriptor system in* (2.1) *(or* (2.2)*) is still locally regular and controllable (i.e., each fuzzy-rule-uncertain descriptor system $\{E_i, A_i + \Delta A_i, B_i + \Delta B_i\}$ remains regular and controllable), if the following conditions simultaneously hold*

$$\sum_{k=1}^{m} \varepsilon_{ik}\varphi_{ik} < 1, \tag{2.9a}$$

$$\sum_{k=1}^{m} \varepsilon_{ik}\theta_{ik} < 1, \tag{2.9b}$$

$$\sum_{k=1}^{m} \varepsilon_{ik}\phi_{ik} < 1, \tag{2.9c}$$

where $i = 1, 2, \ldots, N$, and $k = 1, 2, \ldots, m$:

$$\varphi_{ik} = \begin{cases} \mu\left(-S_i^{-1}U_i^H R_{ik} V_i [I_{n^2}, 0_{n^2\times n}]^T\right), & \text{for } \varepsilon_{ik} \geq 0, \\ -\mu\left(S_i^{-1}U_i^H R_{ik} V_i [I_{n^2}, 0_{n^2\times n}]^T\right), & \text{for } \varepsilon_{ik} < 0, \end{cases}$$

$$R_{ik} = \left[0_{n^2\times n} \quad \widetilde{R}_{ik}\right] \in R^{n^2\times(n^2+n)},$$

$$\widetilde{R}_{ik} = \operatorname{diag}\{A_{ik}, \ldots, A_{ik}\} \in R^{n^2\times n^2},$$

$$\theta_{ik} = \begin{cases} \mu\left(-S_{ri}^{-1}U_{ri}^H Q_{ik} V_{ri} [I_{n^2}, 0_{n^2\times np}]^T\right), & \text{for } \varepsilon_{ik} \geq 0, \\ -\mu\left(S_{ri}^{-1}U_{ri}^H Q_{ik} V_{ri} [I_{n^2}, 0_{n^2\times np}]^T\right), & \text{for } \varepsilon_{ik} < 0, \end{cases}$$

$$Q_{ik} = \begin{bmatrix} A_{ik} & & & & & B_{ik} & & & & \\ & A_{ik} & & & & & B_{ik} & & & \\ & & \cdot & & & & & \cdot & & \\ & & & \cdot & & & & & \cdot & \\ & & & & \cdot & & & & & \cdot \\ & & & & A_{ik} & & & & & B_{ik} \end{bmatrix} \in R^{n^2\times(n^2+np)},$$

$$\phi_{ik} = \begin{cases} \mu\left(-S_{ci}^{-1}U_{ci}^{H}P_{ik}V_{ci}\left[I_n, 0_{n\times p}\right]^T\right), & \text{for } \varepsilon_{ik} \geq 0, \\ -\mu\left(S_{ci}^{-1}U_{ci}^{H}P_{ik}V_{ci}\left[I_n, 0_{n\times p}\right]^T\right), & \text{for } \varepsilon_{ik} < 0, \end{cases}$$

$$P_{ik} = \left[0_{n\times n} \quad B_{ik}\right] \in R^{n\times(n+p)}, \tag{2.10}$$

the matrices S_i, U_i,V_i, S_{ri}, U_{ri},V_{ri}, S_{ci}, U_{ci}, and V_{ci} $(i = 1,2,\ldots,N)$ are, respectively, defined in (2.6)–(2.8)*, and I_{n^2} denotes the $n^2 \times n^2$ identity matrix.*

Proof. Firstly, we show the regularity. Since each fuzzy-rule-nominal descriptor system $\{E_i, A_i, B_i\}$ $(i = 1,2,\ldots,N)$ is regular, then, from Lemma 2.5, we can get that the matrix $R_i = [E_{ni} \quad E_{di}] \in R^{n^2\times(n^2+n)}$ has full row rank (i.e., $\operatorname{rank}(R_i) = n^2$). With the uncertain matrices $A_i + \Delta A_i$ and $B_i + \Delta B_i$, each fuzzy-rule-uncertain descriptor system $\{E_i, A_i + \Delta A_i, B_i + \Delta B_i\}$ is regular if and only if

$$\widetilde{R}_i = R_i + \sum_{k=1}^{m} \varepsilon_{ik} R_{ik} \tag{2.11}$$

has full row rank, where $R_{ik} = [0_{n^2\times n} \quad \widetilde{R}_{ik}] \in R^{n^2\times(n^2+n)}$ and $\widetilde{R}_{ik} = \operatorname{diag}\{A_{ik},\ldots,A_{ik}\} \in R^{n^2\times n^2}$.

$$\text{It is known that } \operatorname{rank}\left(\widetilde{R}_i\right) = \operatorname{rank}\left(S_i^{-1}U_i^H\widetilde{R}_iV_i\right). \tag{2.12}$$

Thus, instead of $\operatorname{rank}(\widetilde{R}_i)$, we can discuss the rank of

$$[I_{n^2}, 0_{n^2\times n}] + \sum_{k=1}^{m} \varepsilon_{ik}\widehat{R}_{ik}, \tag{2.13}$$

where $\widehat{R}_{ik} = S_i^{-1}U_i^H R_{ik}V_i$, for $i = 1,2,\ldots,N$ and $k = 1,2,\ldots,m$. Since a matrix has at least rank n^2 if it has at least one nonsingular $n^2\times n^2$ submatrix, a sufficient condition for the matrix in (2.13) to have rank n^2 is the nonsingularity of

$$L_i = I_{n^2} + \sum_{k=1}^{m} \varepsilon_{ik}\overline{R}_{ik}, \tag{2.14}$$

where $\overline{R}_{ik} = S_i^{-1}U_i^H R_{ik}V_i[I_{n^2}, 0_{n^2\times n}]^T$ (for $i = 1,2,\ldots,N$ and $k = 1,2,\ldots,m$).

Using the properties in Lemmas 2.7 and 2.8 and from (2.9a), we get

$$\begin{aligned}\mu\left(-\sum_{k=1}^{m}\varepsilon_{ik}\overline{R}_{ik}\right) &= \mu\left(-\sum_{k=1}^{m}\varepsilon_{ik}S_i^{-1}U_i^H R_{ik}V_i[I_{n^2},0_{n^2\times n}]^T\right)\\ &\le \sum_{k=1}^{m}\mu\left(-\varepsilon_{ik}S_i^{-1}U_i^H R_{ik}V_i[I_{n^2},0_{n^2\times n}]^T\right)\\ &= \sum_{k=1}^{m}\varepsilon_{ik}\varphi_{ik} < 1.\end{aligned} \tag{2.15}$$

From Lemma 2.9, we have that

$$\det(L_i) = \det\left(I_{n^2} + \sum_{k=1}^{m}\varepsilon_{ik}\overline{R}_{ik}\right) \neq 0. \tag{2.16}$$

Hence, the matrix L_i in (2.14) is nonsingular. That is, the matrix $\widetilde{R}_i$ in (2.11) has full row rank n^2. Thus, from the Lemma 2.5, the regularity of each fuzzy-rule-uncertain descriptor system $\{E_i, A_i + \Delta A_i, B_i + \Delta B_i\}$ is ensured.

Next, we show the controllability. Since each fuzzy-rule-nominal descriptor system $\{E_i, A_i, B_i\}$ $(i = 1, 2, \ldots, N)$ is controllable, then from Lemma 2.6, we have that the matrix $Q_i = [E_{di} \quad E_{bi}]$ has full row rank (i.e., $\operatorname{rank}(Q_i) = n^2$) and $P_i = [E_i \quad B_i]$ has full row rank (i.e., $\operatorname{rank}(P_i) = n$). With the uncertain matrices $A_i + \Delta A_i$ and $B_i + \Delta B_i$, each fuzzy-rule-uncertain descriptor system $\{E_i, A_i + \Delta A_i, B_i + \Delta B_i\}$ is controllable if and only if

$$\widetilde{Q}_i = Q_i + \sum_{k=1}^{m}\varepsilon_{ik}Q_{ik}, \tag{2.17}$$

$$\widetilde{P}_i = P_i + \sum_{k=1}^{m}\varepsilon_{ik}P_{ik} \tag{2.18}$$

have full row rank, where

$$Q_{ik} = \begin{bmatrix} A_{ik} & & & & B_{ik} & & & \\ & A_{ik} & & & & B_{ik} & & \\ & & \cdot & & & & \cdot & \\ & & & \cdot & & & & \cdot \\ & & & & \cdot & & & \\ & & & A_{ik} & & & & B_{ik} \end{bmatrix} \in R^{n^2\times(n^2+np)}, \tag{2.19}$$

and $P_{ik} = [0_{n\times n} \quad B_{ik}] \in R^{n\times(n+p)}$.

It is known that

$$\operatorname{rank}\left(\tilde{Q}_i\right) = \operatorname{rank}\left(S_{ri}^{-1}U_{ri}^H\tilde{Q}_iV_{ri}\right). \tag{2.20}$$

Thus, instead of $\operatorname{rank}(\tilde{Q}_i)$, we can discuss the rank of

$$\left[I_{n^2}, 0_{n^2\times np}\right] + \sum_{k=1}^{m}\varepsilon_{ik}\widehat{Q}_{ik}, \tag{2.21}$$

where $\widehat{Q}_{ik} = S_{ri}^{-1}U_{ri}^HQ_{ik}V_{ri}$, for $i = 1,2,\ldots,N$ and $k = 1,2,\ldots,m$. Since a matrix has at least rank n^2 if it has at least one nonsingular $n^2\times n^2$ submatrix, a sufficient condition for the matrix in (2.21) to have rank n^2 is the nonsingularity of

$$G_i = I_{n^2} + \sum_{k=1}^{m}\varepsilon_{ik}\overline{Q}_{ik}, \tag{2.22}$$

where $\overline{Q}_{ik} = S_{ri}^{-1}U_{ri}^HQ_{ik}V_{ri}\left[I_{n^2}, 0_{n^2\times np}\right]^T$ (for $i = 1,2,\ldots,N$ and $k = 1,2,\ldots,m$).

Applying the properties in Lemmas 2.7 and 2.8 and from (2.9b), we get

$$\begin{aligned}
\mu\left(-\sum_{k=1}^{m}\varepsilon_{ik}\overline{Q}_{ik}\right) &= \mu\left(-\sum_{k=1}^{m}\varepsilon_{ik}S_{ri}^{-1}U_{ri}^HQ_{ik}V_{ri}\left[I_{n^2}, 0_{n^2\times np}\right]^T\right)\\
&\le \sum_{k=1}^{m}\mu\left(-\varepsilon_{ik}S_{ri}^{-1}U_{ri}^HQ_{ik}V_{ri}\left[I_{n^2}, 0_{n^2\times np}\right]^T\right)\\
&= \sum_{k=1}^{m}\varepsilon_{ik}\theta_{ik} < 1.
\end{aligned} \tag{2.23}$$

From Lemma 2.9, we have that

$$\det(G_i) = \det\left(I_{n^2} + \sum_{k=1}^{m}\varepsilon_{ik}\overline{Q}_{ik}\right) \neq 0. \tag{2.24}$$

Hence, the matrix G_i in (2.22) is nonsingular. That is, the matrix $\tilde{Q}_i$ in (2.17) has full row rank n^2.

And then, it is also known that

$$\operatorname{rank}\left(\tilde{P}_i\right) = \operatorname{rank}\left(S_{ci}^{-1}U_{ci}^H\tilde{P}_iV_{ci}\right). \tag{2.25}$$

Thus, instead of $\operatorname{rank}(\widetilde{P}_i)$, we can discuss the rank of

$$[I_n, 0_{n\times p}] + \sum_{k=1}^{m} \varepsilon_{ik}\widehat{P}_{ik}, \tag{2.26}$$

where $\widehat{P}_{ik} = S_{ci}^{-1}U_{ci}^{H}P_{ik}V_{ci}$, for $i = 1,2,\ldots,N$ and $k = 1,2,\ldots,m$. Since a matrix has at least rank n if it has at least one nonsingular $n \times n$ submatrix, a sufficient condition for the matrix in (2.26) to have rank n is the nonsingularity of

$$H_i = I_n + \sum_{k=1}^{m} \varepsilon_{ik}\overline{P}_{ik}, \tag{2.27}$$

where $\overline{P}_{ik} = S_{ci}^{-1}U_{ci}^{H}P_{ik}V_{ci}[I_n, 0_{n\times p}]^T$ (for $i = 1,2,\ldots,N$ and $k = 1,2,\ldots,m$).

Adopting the properties in Lemmas 2.7 and 2.8 and from (2.9c), we obtain

$$\begin{aligned}
\mu\left(-\sum_{k=1}^{m} \varepsilon_{ik}\overline{P}_{ik}\right) &= \mu\left(-\sum_{k=1}^{m} \varepsilon_{ik}S_{ci}^{-1}U_{ci}^{H}P_{ik}V_{ci}\left[I_n,\ 0_{n\times p}\right]^T\right) \\
&\le \sum_{k=1}^{m} \mu\left(-\varepsilon_{ik}S_{ci}^{-1}U_{ci}^{H}P_{ik}V_{ci}\left[I_n,\ 0_{n\times p}\right]^T\right) \\
&= \sum_{k=1}^{m} \varepsilon_{ik}\phi_{ik} < 1.
\end{aligned} \tag{2.28}$$

From Lemma 2.9, we get that

$$\det(H_i) = \det\left(I_n + \sum_{k=1}^{m} \varepsilon_{ik}\overline{P}_{ik}\right) \neq 0. \tag{2.29}$$

Hence, the matrix H_i in (2.27) is nonsingular. That is, the matrix $\widetilde{P}_i$ in (2.18) has full row rank n. Thus, from the Lemma 2.6 and the results mentioned above, the controllability of each fuzzy-rule-uncertain descriptor system $\{E_i, A_i + \Delta A_i, B_i + \Delta B_i\}$ is ensured. Therefore, we can conclude that the uncertain TS fuzzy descriptor system in (2.1) (or (2.2)) is locally regular and controllable, if the inequalities (2.9a), (2.9b), and (2.9c) are simultaneously satisfied. Thus, the proof is completed. □

Remark 2.11. The proposed sufficient conditions in (2.9a)–(2.9c) can give the explicit relationship of the bounds on ε_{ik} ($i = 1,2,\ldots,N$ and $k = 1,2,\ldots,m$) for preserving both regularity and controllability. In addition, the bounds, that are obtained by using the proposed sufficient conditions, on ε_{ik} are not necessarily symmetric with respect to the origin of the parameter space regarding ε_{ik} ($i = 1,2,\ldots,N$ and $k = 1,2,\ldots,m$).

Remark 2.12. This paper studies the problem of robust local regularity and controllability analysis. If the proposed conditions in (2.9a)–(2.9c) are satisfied, each rule of the uncertain TS fuzzy descriptor system $\{E_i, A_i + \Delta A_i, B_i + \Delta B_i\}$ is guaranteed to be robustly locally regular and controllable. This implies that, in the fuzzy PDC controller design, if the proposed conditions in (2.9a)–(2.9c) are satisfied, the PDC controller of each fuzzy rule can control every state variable in the corresponding rule of the uncertain TS fuzzy descriptor system $\{E_i, A_i + \Delta A_i, B_i + \Delta B_i\}$. However, here, it should be noticed that although the PDC controller of each control rule can control every state variable in the corresponding rule under the presented conditions being held, the PDC controller gains should be determined using global design criteria that are needed to guarantee the global stability and control performance [5], where many useful global design criteria have been proposed by some researchers (e.g., [4-6, 8, and 21-28] and references therein).

3. Illustrative Example

Consider a two-rule fuzzy descriptor system as that considered by Wang et al. [21]. The TS fuzzy descriptor system with the elemental parametric uncertainties is described by

$$\begin{aligned} &\widetilde{R}^1\colon \text{ IF } z_1 \text{ is } M_{11}, \\ &\qquad \text{then } E_1 x(k+1) = (A_1 + \Delta A_1)x(k) + (B_1 + \Delta B_1)\ u(k); \end{aligned} \tag{3.1a}$$

$$\begin{aligned} &\widetilde{R}^2\colon \text{ IF } z_1 \text{ is } M_{21}, \\ &\qquad \text{then } E_2 x(k+1) = (A_2 + \Delta A_2)x(k) + (B_2 + \Delta B_2)\ u(k), \end{aligned} \tag{3.1b}$$

where

$$\begin{gathered} x(k) = \begin{bmatrix} x_1(k) & x_2(k) \end{bmatrix}^T, \qquad E_1 = E_2 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \qquad A_1 = \begin{bmatrix} 0.848 & 0 \\ 0 & -0.315 \end{bmatrix}, \\ A_2 = \begin{bmatrix} -0.236 & 0 \\ 0 & 0.113 \end{bmatrix}, \qquad B_1 = B_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \qquad \Delta A_i = \sum_{k=1}^{2} \varepsilon_{ik} A_{ik}, \\ \Delta B_i = \sum_{k=1}^{2} \varepsilon_{ik} B_{ik}, \qquad A_{11} = \begin{bmatrix} 0 & 0 \\ -0.1 & 0 \end{bmatrix}, \qquad A_{21} = \begin{bmatrix} 0 & 0 \\ 0.2 & 0 \end{bmatrix}, \qquad A_{12} = A_{22} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \\ B_{i1} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \qquad B_{i2} = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}, \qquad M_{11} = \frac{1}{1 + \exp(-0.5(z_1 - 0.3))}, \\ M_{21} = \frac{\exp(-0.5(z_1 - 0.3))}{1 + \exp(-0.5(z_1 - 0.3))}, \quad \varepsilon_{i1} \in [-1 \quad 1.1], \quad \varepsilon_{i2} \in [-1.2 \quad 10], \text{ in which } i = 1, 2. \end{gathered} \tag{3.2}$$

Now, applying the sufficient conditions in (2.9a)–(2.9c) with the two-norm-based matrix measure, we can get the following:

(I) for the fuzzy rule 1:

$$\text{(i)} \sum_{k-1}^{2} \varepsilon_{ik}\varphi_{ik} \leq 0.17460 < 1, \quad \text{for } \varepsilon_{i1} \in [0 \quad 1.1],\ \varepsilon_{i2} \in [-1.2 \quad 10], \tag{3.3a}$$

$$\text{(ii)} \sum_{k-1}^{2} \varepsilon_{ik}\varphi_{ik} \leq 0.15873 < 1, \quad \text{for } \varepsilon_{i1} \in [-1 \quad 0],\ \varepsilon_{i2} \in [-1.2 \quad 10], \tag{3.3b}$$

$$\text{(iii)} \sum_{k-1}^{2} \varepsilon_{ik}\theta_{ik} \leq 0.03297 < 1, \quad \text{for } \varepsilon_{i1} \in [0 \quad 1.1],\ \varepsilon_{i2} \in [0 \quad 10], \tag{3.3c}$$

$$\text{(iv)} \sum_{k-1}^{2} \varepsilon_{ik}\theta_{ik} \leq 0.13861 < 1, \quad \text{for } \varepsilon_{i1} \in [-1 \quad 0],\ \varepsilon_{i2} \in [0 \quad 10], \tag{3.3d}$$

$$\text{(v)} \sum_{k-1}^{2} \varepsilon_{ik}\theta_{ik} \leq 0.25078 < 1, \quad \text{for } \varepsilon_{i1} \in [-1 \quad 0],\ \varepsilon_{i2} \in [-1.2 \quad 0], \tag{3.3e}$$

$$\text{(vi)} \sum_{k-1}^{2} \varepsilon_{ik}\theta_{ik} \leq 0.14514 < 1, \quad \text{for } \varepsilon_{i1} \in [0 \quad 1.1],\ \varepsilon_{i2} \in [-1.2 \quad 0], \tag{3.3f}$$

$$\text{(vii)} \sum_{k-1}^{2} \varepsilon_{ik}\phi_{ik} = 0 < 1, \quad \text{for } \varepsilon_{i1} \in [-1 \quad 1.1],\ \varepsilon_{i2} \in [0 \quad 10], \tag{3.3g}$$

$$\text{(viii)} \sum_{k-1}^{2} \varepsilon_{ik}\phi_{ik} \leq 0.1200 < 1, \quad \text{for } \varepsilon_{i1} \in [-1 \quad 1.1],\ \varepsilon_{i2} \in [-1.2 \quad 0]; \tag{3.3h}$$

$$\text{(i)} \sum_{k-1}^{2} \varepsilon_{ik}\varphi_{ik} \leq 0.97345 < 1, \quad \text{for } \varepsilon_{i1} \in [0 \quad 1.1],\ \varepsilon_{i2} \in [-1.2 \quad 10], \tag{3.4a}$$

$$\text{(ii)} \sum_{k-1}^{2} \varepsilon_{ik}\varphi_{ik} \leq 0.88496 < 1, \quad \text{for } \varepsilon_{i1} \in [-1 \quad 0],\ \varepsilon_{i2} \in [-1.2 \quad 10], \tag{3.4b}$$

$$\text{(iii)} \sum_{k-1}^{2} \varepsilon_{ik}\theta_{ik} \leq 0.56719 < 1, \quad \text{for } \varepsilon_{i1} \in [0 \quad 1.1],\ \varepsilon_{i2} \in [0 \quad 10], \tag{3.4c}$$

$$\text{(iv)} \sum_{k-1}^{2} \varepsilon_{ik}\theta_{ik} \leq 0.87768 < 1, \quad \text{for } \varepsilon_{i1} \in [-1 \quad 0],\ \varepsilon_{i2} \in [0 \quad 10], \tag{3.4d}$$

$$\text{(v)} \sum_{k-1}^{2} \varepsilon_{ik}\theta_{ik} \leq 0.99740 < 1, \quad \text{for } \varepsilon_{i1} \in [-1 \quad 0],\ \varepsilon_{i2} \in [-1.2 \quad 0], \tag{3.4e}$$

$$\text{(vi)} \sum_{k-1}^{2} \varepsilon_{ik}\theta_{ik} \leq 0.168691 < 1, \quad \text{for } \varepsilon_{i1} \in [0 \quad 1.1],\ \varepsilon_{i2} \in [-1.2 \quad 0], \tag{3.4f}$$

$$\text{(vii)} \quad \sum_{k-1}^{2} \varepsilon_{ik}\phi_{ik} = 0 < 1, \quad \text{for } \varepsilon_{i1} \in [-1 \quad 1.1],\ \varepsilon_{i2} \in [0 \quad 10], \tag{3.4g}$$

$$\text{(viii)} \quad \sum_{k-1}^{2} \varepsilon_{ik}\phi_{ik} \leq 0.1200 < 1, \quad \text{for } \varepsilon_{i1} \in [-1 \quad 1.1],\ \varepsilon_{i2} \in [-1.2 \quad 0]. \tag{3.4h}$$

From the results in (3.3a)–(3.3h) and (3.4a)–(3.4h), we can conclude that the uncertain TS fuzzy descriptor system (3.1a) and (3.1b) is locally robustly regular and controllable.

4. Conclusions

The robust local regularity and controllability problem for the uncertain TS fuzzy descriptor systems has been investigated. The rank preservation problem for robust local regularity and controllability of the uncertain TS fuzzy descriptor systems is converted to the nonsingularity analysis problem. Under the assumption that each fuzzy rule of the nominal TS fuzzy descriptor system has the full row rank for its related regularity and controllability matrices, a sufficient criterion has been proposed to preserve the assumed properties when the elemental parameter uncertainties are added into the nominal TS fuzzy descriptor systems. The proposed sufficient conditions in (2.9a)–(2.9c) can provide the explicit relationship of the bounds on elemental parameter uncertainties for preserving the assumed properties. One example has been given to illustrate the application of the proposed sufficient conditions. On the other hand, the issue of robust global regularity and controllability with evolutionary computation [36] for the uncertain TS fuzzy descriptor systems will be an interesting and important topic for further research.

Acknowledgment

This work was in part supported by the National Science Council, Taiwan, under Grants nos. NSC 100-2221-E-151-009, NSC 101-2221-E-151-076, and NSC 101-2320-B-037-022.

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