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Research Article

Extended "Mononobe-Okabe" Method for Seismic Design of Retaining Walls

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Mononobe-Okabe (M-O) method is still employed as the first option to estimate lateral earth pressures during earthquakes by geotechnical engineers. Considering some simple assumptions and using a closed form method, M-O solves the equations of equilibrium and suggests seismic active and passive lateral earth pressures. Therefore, the results are true in its assumption range only, and in many other practical cases, M-O method is not applicable. Noncontinues backfill slopes, cohesive soils, and rising water behind the wall are some well-known examples in which the M-O theory is irrelevant. Using the fundamental framework of M-O method, this study proposes an iterative method to overcome the limits of the M-O method. Based on trial and error process, the proposed method is able to cover many of the defects which regularly occur in civil engineering when M-O has no direct answer.

1. Introduction

Retaining walls are those structures which are usually constructed to form roads, stabilize trenches and soil slopes, and support unstable structures. Figure 1 shows one of the common configurations of retaining structures, schematically.

Lateral earth pressure model is belonging to the first group of theories in classical soil mechanics. Coulomb [1] and Rankine [2] proposed their theories to estimate active and passive lateral earth pressures. These kinds of theories propose a coefficient which is a ratio between horizontal and vertical stress behind retaining walls. Using the ratio, lateral pressure is simply calculated by the horizontal stress integration.

Mononobe-Okabe method (M-O), a seismic version of coulomb theory, was proposed based on pseudostatic earthquake loading for granular soils. This method applies earthquake force components using two coefficients called seismic horizontal and vertical coefficients. Beside other complex theoretical models and numerical methods, M-O theory is one of the best initial estimates.

Although M-O is the first choice for engineers to design retaining walls, some limitations make it incapable to model most civil engineering projects. This problem rises according to the simplifier assumptions in M-O method to solve the equations in a closed form fashion. The contribution of this paper is primarily to remove these limits and to cover other problems that M-O has no answer for them. On the other hand, reports on retaining wall failures during major or minor earthquakes confirm the necessity of immune design of retaining structures. Since the stability of retaining walls plays an important role during and right after an earthquake, this study strives to provide a reliable tool for quick engineering designs. The methodology given in this paper can also be used as a model to study the effect of earthquake parameters on retaining structures with a specific geometry or can be reshaped for any other unusual retaining structures.

2. Mononobe-Okabe Method

Mononobe and Matsuo [3] and Okabe [4] proposed a method to determine lateral earth pressure of granular cohesionless soils during earthquake [5]. The method was a modified version of Coulomb theory [1] in which earthquake forces are

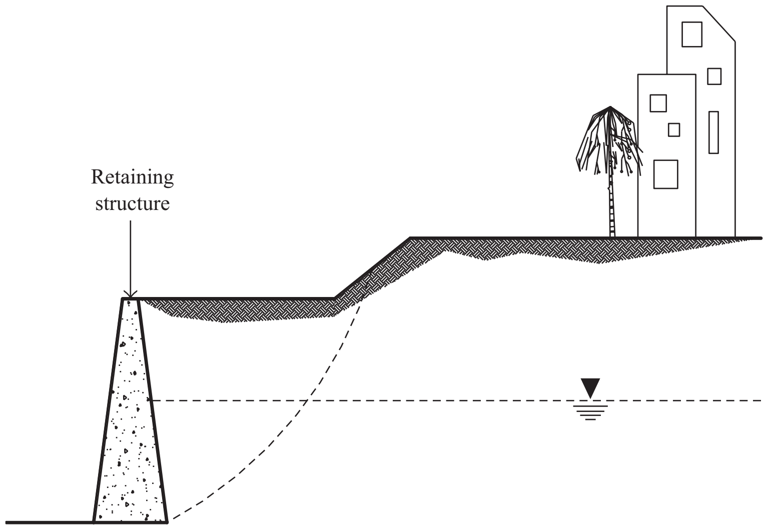

Figure 1: Application of retaining walls in civil engineering.

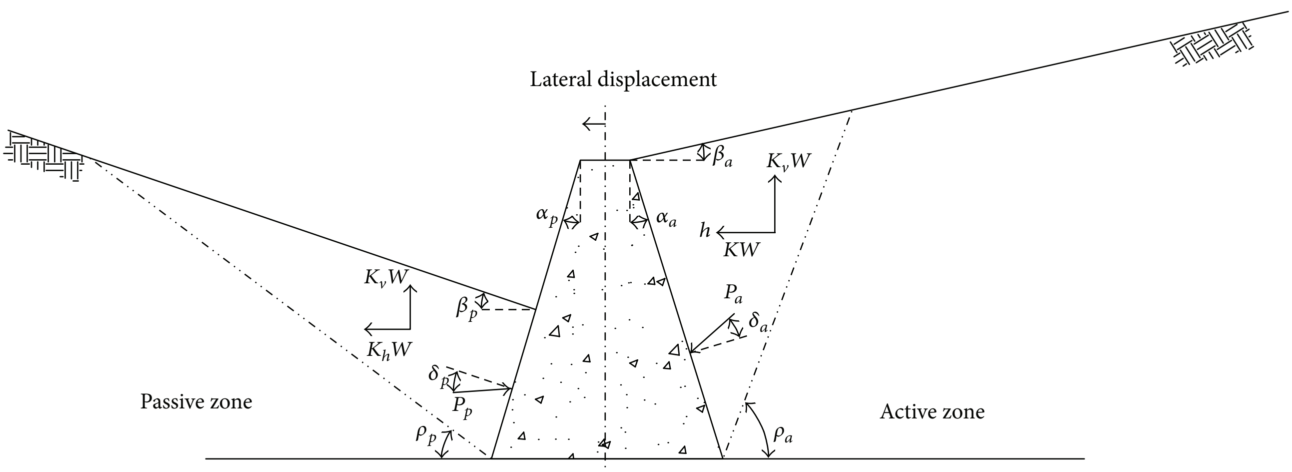

Figure 2: Geometry and parameters of M-O method.

applied to the failure mass by pseudostatic method. To get a final simple formulation like other closed form solutions in geotechnical engineering, M-O uses exact form solution with simple assumption such as simplicity in geometry, material behavior, or dynamic loading to make the equations solvable.

Because of the old age of M-O method, tens of studies have been focused on this area (e.g., [6–8]). An important study on M-O was carried out by Seed and Whitman [9]. They confirmed M-O active pressure after long laboratory runs. However, they recommended more studies on passive theory of M-O. They also proposed a method to find the location of resultant force which acts on 1/3 of height in M-O method. M-O had been studied by others such as Fang and Chen [10] on the direction of seismic force components on the failure mass.

Figure 2 shows the parameters and characteristics of M-O method. In M-O, static force equilibrium is satisfied for a rigid wedge placed on a failure plane with elastic-perfectly plastic behavior based on Mohr-Coulomb failure criteria. Active and passive forces, P_a and P_p, are then calculated using the following equations:

$$\begin{Bmatrix} P_a \\ P_p \end{Bmatrix} = \frac{1}{2}\gamma H^2 \left(1 - K_v\right) \begin{Bmatrix} K_a \\ K_p \end{Bmatrix},$$

$$\begin{aligned} \begin{Bmatrix} K_a \\ K_p \end{Bmatrix} &= \left(\cos^2\left(\varphi \mp \alpha - \theta\right)\right) \\ &\quad \times \left(\cos\theta \times \cos^2\alpha \times \cos\left(\delta \pm \alpha + \theta\right) \right. \\ &\quad \left. \times \left[1 \pm \left(\frac{\sin\left(\varphi + \delta\right) \times \sin\left(\varphi \mp \beta - \theta\right)}{\cos\left(\delta \pm \alpha + \theta\right) \times \cos\left(\beta - \alpha\right)} \right)^{1/2} \right]^2 \right)^{-1}, \\ \theta &= \tan^{-1}\left(\frac{K_h}{1 - K_v}\right). \end{aligned} \tag{1}$$

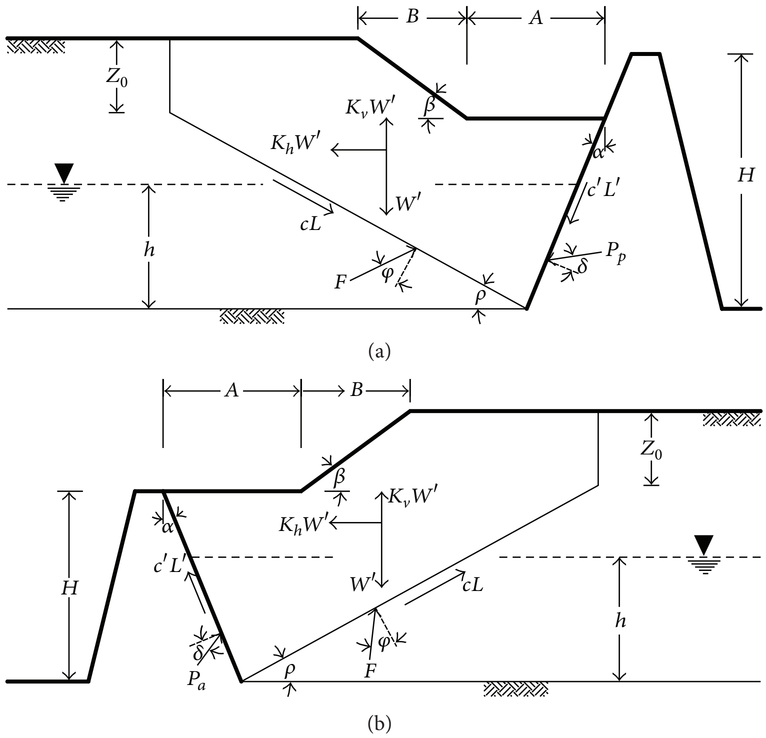

Figure 3: Proposed model for passive (a) and active (b) pressure.

It should be noticed that the above formulations were derived assuming planar failure surfaces. This assumption has significantly simplified the resultant equations that can practically capture the contrast between different design scenarios and parameters. However, in general, planar failure surfaces usually overestimate passive pressures and might underestimate active pressure. Moreover, in reality curved surfaces are more often useable. Therefore, the results of the current study can be used as upper limits while it should be considered with caution for active pressure. The works by Morrison and Ebeling [11], Kumar [12], Subba Rao and Choudhury [13] and Yazdani and Azad [14] explain how the planar models can be extended.

3. Mononobe-Okabe Method Defects and Limitations

Some of the limits of M-O method that cause the method not to cover many of the usual engineering problems are as follows.

(a) M-O method is applicable for cohesionless soils only.

(b) Effect of water table behind the wall has not been considered directly in the formula.

(c) M-O method has no answer when $\varphi - \beta - \theta \leq 0$.

(d) The conventional problems in civil engineering are not always wall with continues backfill. Sometimes, one has to use equivalent forms of M-O method to model a real problem.

4. Overcome Mononobe-Okabe Method Limits

To overcome M-O limits, the fundamental basis of limit equilibrium analysis can be used. The difference is the solution type which is carried out using an iterative process for various values of ρ_a and ρ_b to find the minimum and maximum values of the function instead of using closed form solutions of differential equations.

4.1. Problem Framework. According to Figure 3, wall displacement and also its direction produce effective forces on failure mass in both sides of a retaining wall. Variable parameters have been listed in Table 1. It can be seen that the main differences between current method and M-O method are as follows.

(a) Geometry of backfill soil has been modeled in an engineering configuration. Two real cases with the same geometry have been illustrated in Figures 4 and 5. Both examples are in north side of Tehran, Iran, which are located in a high seismic risk zone. Another similar geometry has been plotted in Figure 6. The figure also shows the failure mechanism of the wall in Chi-Chi Earthquake, Taiwan [15].

(b) In addition to soil cohesion, virtual cohesion between soil and wall material (adhesion) is included in the model.
Seismic active earth pressure considering $c{-}\varphi$ backfill has been already evaluated by Prakash and Saran [16] as well as Saran and Prakash [17]. In their methods, adhesion was considered identical to cohesion. Das

Table 1: Parameters of the proposed method.

Parameter	Symbol
Wall height	H
Water depth	h
Wall angle	α
Soil-wall friction angle	δ
Backfill angle	β
Soil special weight	γ
Saturated soil special weight	γ_{sat}
Ref. to Figure 3	A
Ref. to Figure 3	B
Soil internal friction angle	φ
Soil cohesion	c
Soil-wall cohesion	c'
Horizontal earthquake coefficient	K_h
Vertical earthquake coefficient	K_v

and Puri [18] improved the analysis by considering different value of cohesion and adhesion. Shukla et al. [19] presented an idea to extend Mononobe-Okabe concept for $c - \varphi$ backfill in such a way to get single critical wedge surface. Ghosh and Sengupta [20] presented a formulation to evaluate seismic active earth pressure including the influence of both adhesion and cohesion for a nonvertical retaining wall.

(c) Static water table is included in the model to affect the earth pressure, directly.

(d) The effect of tension crack has been considered. This effect is quite important in active earth pressure on retaining wall for cohesive soil backfill [21]. Shukla et al. [19] showed that for soil backfill with tension cracks, the total active earth pressure in static condition will increase up to 20%–40% over the case without tension cracks. Therefore, the effect of tension cracks in cohesive soil backfill should not be neglected in the calculation of active earth pressure. Ghosh and Sharma [22] used the following equation in their analysis to compute the depth of tension cracked zone in seismic condition:

$$Z_0 = \frac{2c}{\gamma\sqrt{K_a}}; \quad K_a\,(\text{Rankine}) = \frac{1-\sin\varphi}{1+\sin\varphi}. \tag{2}$$

This equation is based on the Rankine theory of active earth pressure for cohesive backfill under static condition. The effect of seismic acceleration on the depth of tension crack is neglected in that analysis. Given that the inclination of the stress characteristics depends on acceleration level, a Rankine condition is valid for the vast majority of cantilever wall configurations under strong seismic action [23]. This is applicable even to short heel walls, with an error of about 5% [24, 25].

Shao-jun et al. [21] made an effort to determine the depth of tension cracked zone under seismic loading and used the pseudodynamic approach to compute seismic active force on retaining wall with cohesive backfills.

Figure 4: Geometry of natural slope behind a retaining wall, Tehran, Iran.

Figure 5: Geometry of backfill behind a retaining wall, Tehran, Iran.

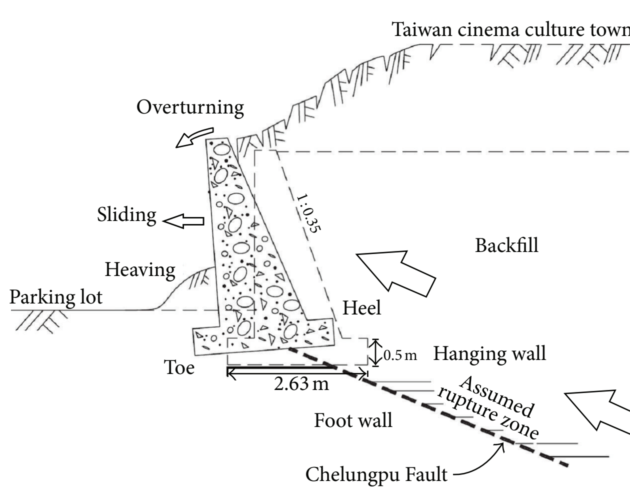

Figure 6: Geometry and failure mechanism of a retaining wall in Chi-Chi Earthquake, Taiwan (after [15]).

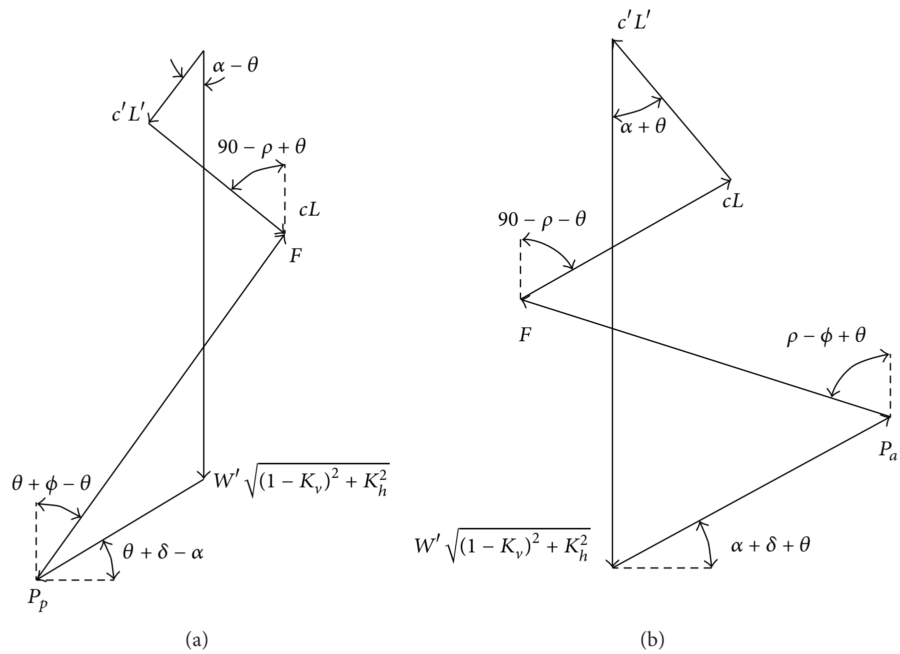

Figure 7: Effective force diagrams on failure mass (wedge).

In the current research, (2) is adopted to take into account the effect of tension crack. By this way, the shape of active wedge in original M-O method is changed from triangular to trapezoidal, and the resultant active earth pressure will be computed more realistically. However, in passive condition, the failure wedge was considered triangular similar to the conventional M-O method, though this assumption causes the passive earth pressure to be overestimated.

4.2. Free Body Diagrams. According to limit analysis on the failure surface which is bilinear in this study, shear stresses are developed based on Mohr-Coulomb theory. A 2D static set of equilibriums can simply connect stresses in a force (or stress) diagram. The diagrams for active and passive conditions have been shown in Figure 7.

4.3. Solution Methodology. As mentioned, using a closed form solution is not applicable herein according to the large number of parameters and nonlinearity of equations. Therefore, semianalytical iterative calculations for searching the favorite conditions (ρ, P_a, and P_p) have been chosen in this study.

The continuity of the equations is plotted in Figure 8, where the active pressure is the maximum while the passive pressure is the minimum points in relevant curves. Since the equations have one unique global maximum or minimum value, no sophisticated search algorithm is needed. A simple scanning search method was coded to pick the critical (extremum) conditions.

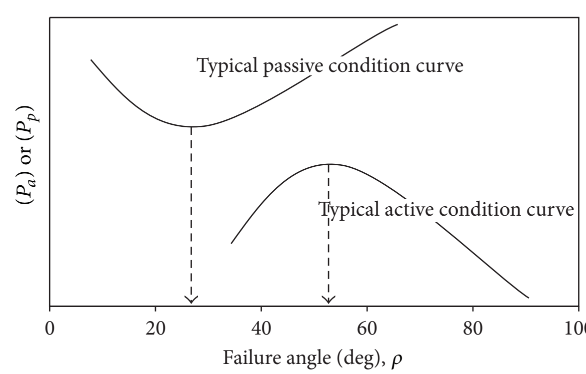

Figure 8: Typical active and passive condition curves.

5. Parametric Study

Because of the large number of parameters in each analysis, deriving complete series of calculation and results seems useless. Therefore, this section just reflects results of a parametric study on a 10-meter high wall. Backfill soil has specific weight of 2 g/cm^3. Other variables have been explained in the following subsections. Other undefined parameters like parameter "A" have been assumed to be zero.

5.1. Effects of Backfill Soil Geometry. To assess the effects of backfill soil geometry, a backfill soil with various values of β and B/H (the ratio of slope width to wall height as shown in Figure 3) and internal friction of soil equal to 30° was considered. Horizontal component of earthquake was set to $K_h = 0.2$. This problem was solved with the standard M-O and the proposed method. It should be noted that the values of slope angle in M-O were chosen equal to β.

Figure 9 shows the results of this analysis. It is clear that for $\beta \geq 20°$, M-O method has no result. Diagrams explain that when β increases, the difference between the

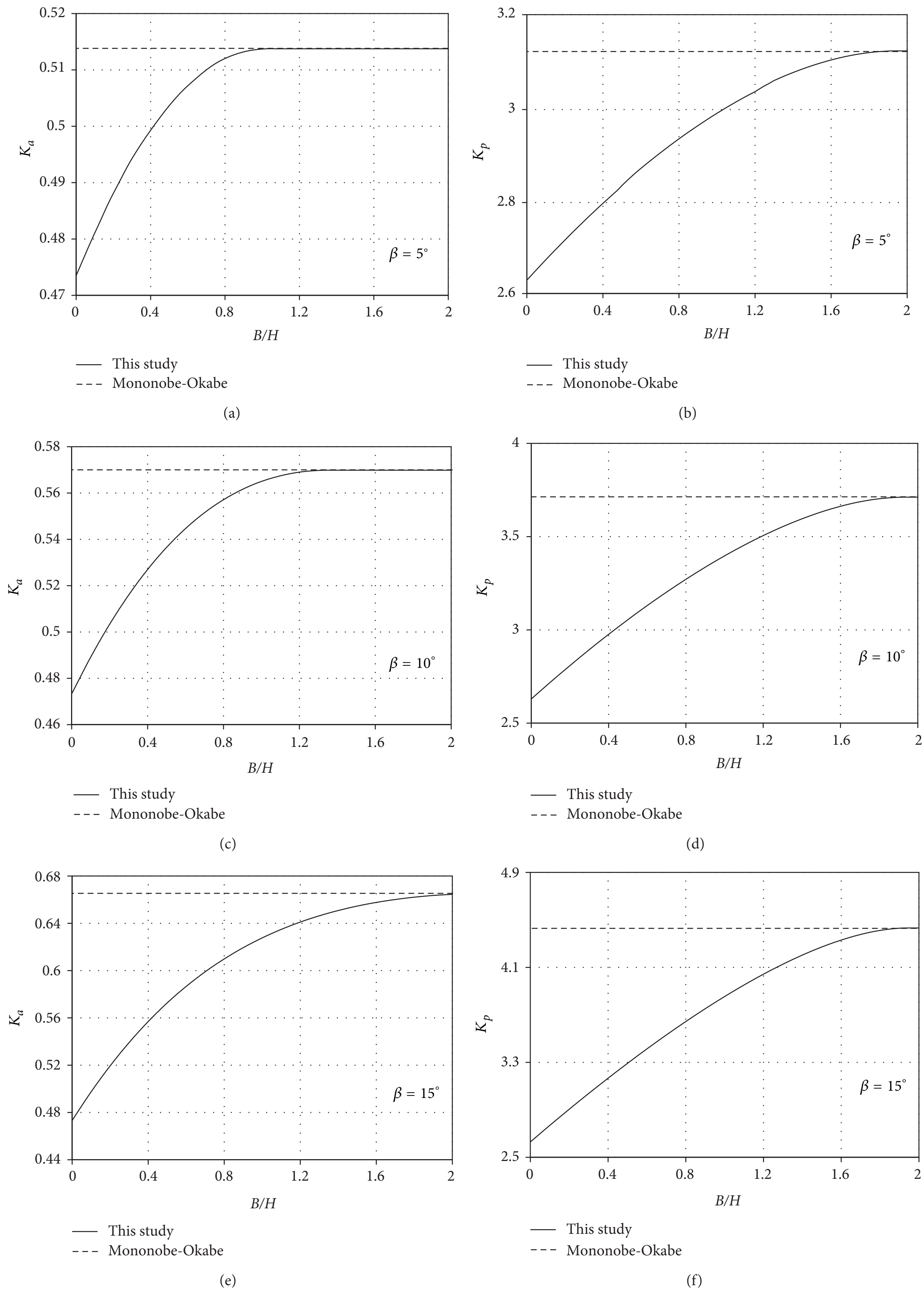

Figure 9: Continued.

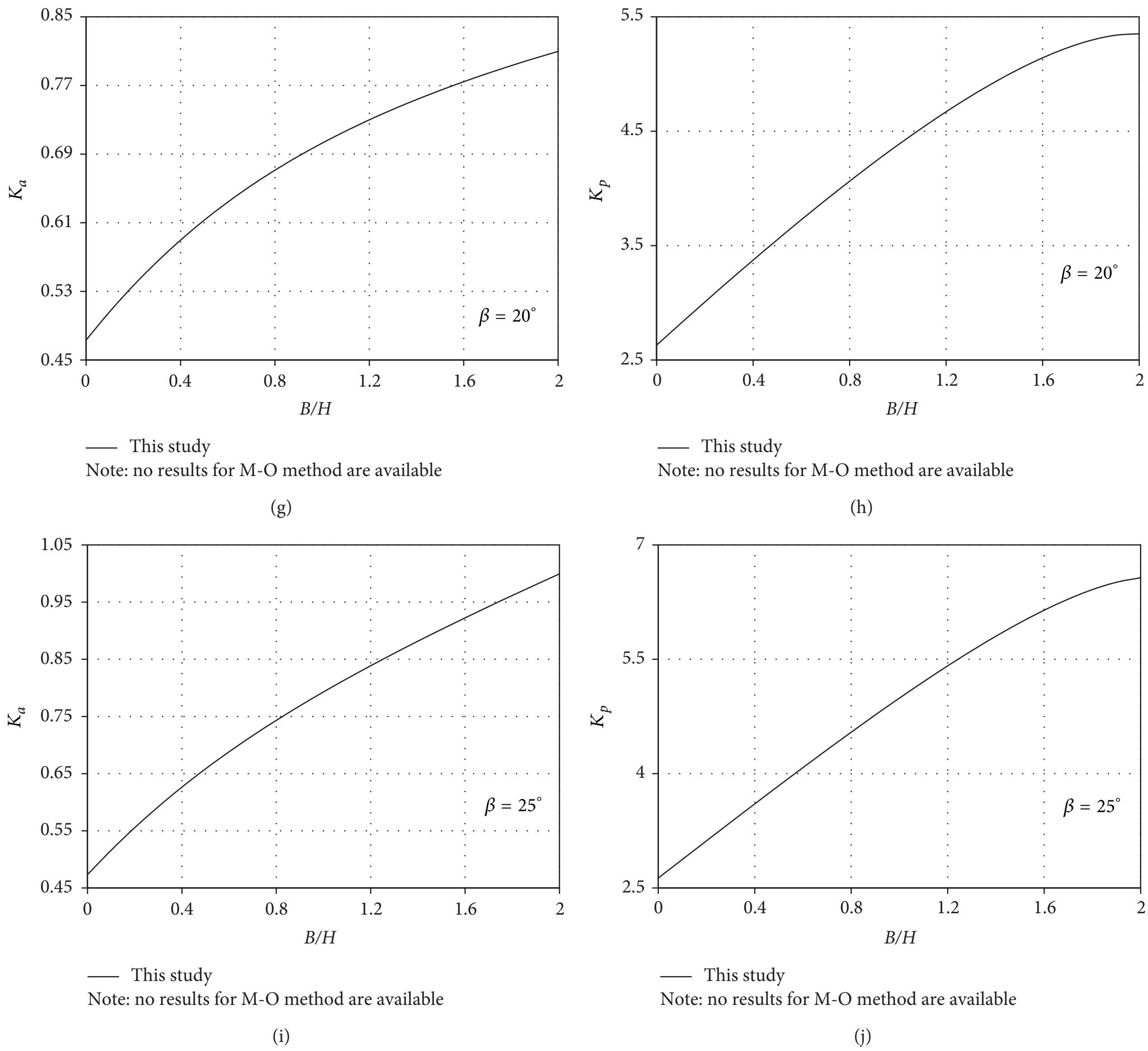

Figure 9: Lateral pressure coefficient for various backfill soil geometry.

proposed method and M-O results is significant. However, this difference decreases when B/H decreases. It means that if variation of β ($0\sim\beta$) occurs near the wall (approximately for $B/H < 1$), between the results of these methods a large difference appears. However, the proposed method is more accurate. As a result, when β is near the wall, M-O method is not economical for active conditions and is not accurate for passive cases.

5.2. Effects of Water Table behind Wall. In the original M-O method, water table is not considered directly in the model, and the earth pressure is given only for the dry condition. To overcome this deficiency, either a correctness factor recommended by some design codes should be utilized or the following relationship must be applied in which the lateral earth pressure ratio in each dry or saturated region is imposed on the relevant unit weight:

$$\text{Total earth pressure (M-O)} = \left\{\frac{1}{2}\gamma(H-h)^2 + \gamma h\,(H-h) + \frac{1}{2}\gamma' \cdot h^2\right\} \times K_a. \tag{3}$$

However, in the proposed method, the total earth pressure can be simply obtained by the following equation:

$$\text{Total earth pressure (this study)} = \frac{1}{2}\gamma \cdot H^2 \times K_a. \tag{4}$$

It should be noted that K_a in the above equations has no similar value. The reason is hidden in the philosophy

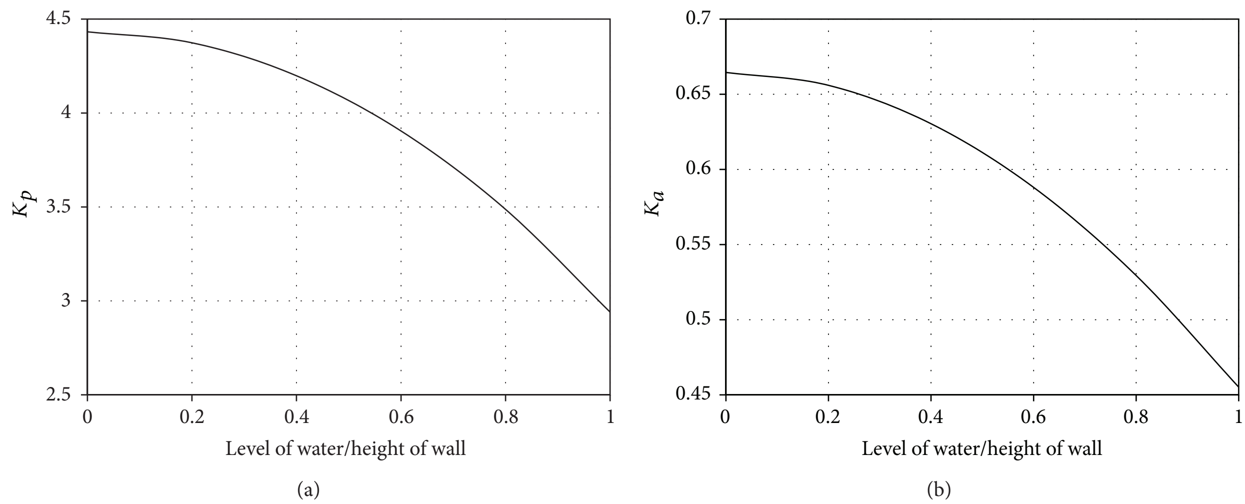

Figure 10: Effects of water table on lateral earth pressure coefficients.

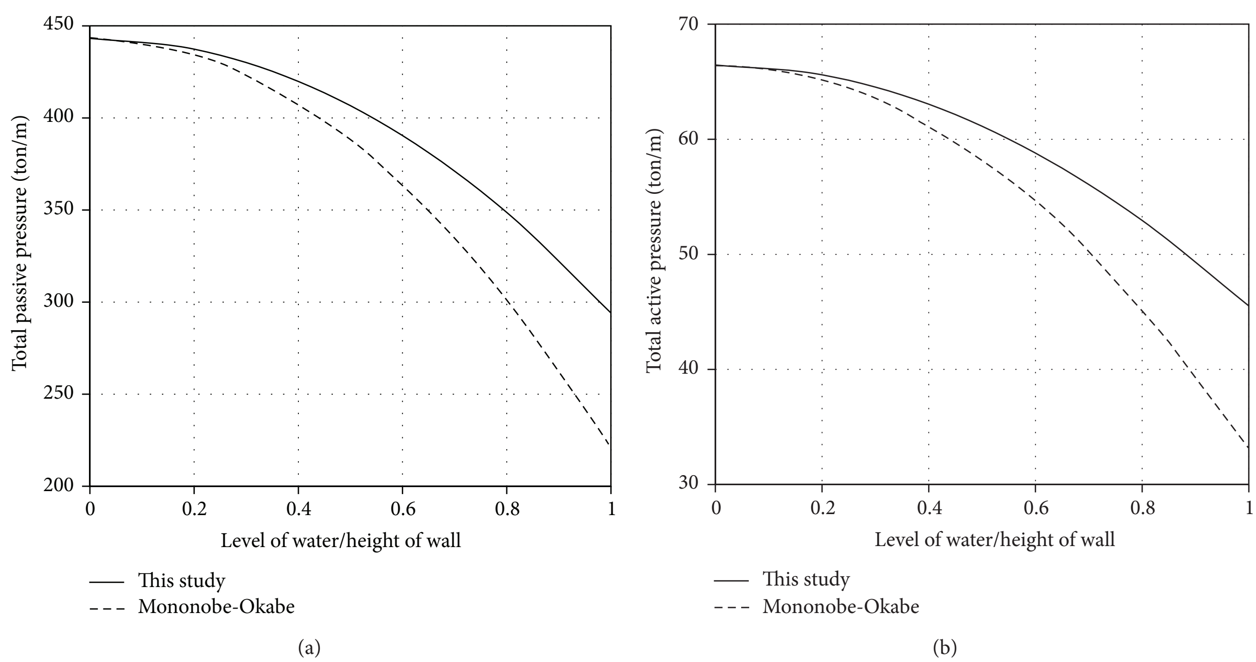

Figure 11: Effects of water table on total lateral earth pressure compared to M-O.

of calculations. In fact, in (4) the weight of the submerged part of the sliding wedge is considered through the effective unit weight of soil which is reflected in the force equilibrium diagram, and then K_a is calculated. The two equations (3) and (4) can be regenerated in the same way for passive condition as well.

To investigate the ability of the proposed method in accounting for water effect on lateral earth pressures, a simple case has first been solved with the following parameters with no water table: $\beta = 15^\circ$, $B/H = 2$, $\phi = 30^\circ$, and $K_h = 0.2$. The results are plotted in Figure 9 that shows perfect agreement between M-O and the proposed model for the given geometry. In the second step, the same model was used with water table varying and $\gamma_{\text{sat}} = \gamma = 2\,\text{g/cm}^3$. The sensitivity analysis results are illustrated in Figure 10. This figure shows that increasing water depth will decrease lateral pressure in both active and passive conditions. The comparison between M-O based on (3) and this study based on (4) for passive case is illustrated in Figure 11. These diagrams reveal that the M-O method outputs are not in the safe side of design. Since the proposed model honors the physics of the problem with more details, it offers a better tool for design purposes.

5.3. Effects of Cohesion and Surface Tension Crack. Almost all soils have naturally a very small amount of cohesion. Also, in

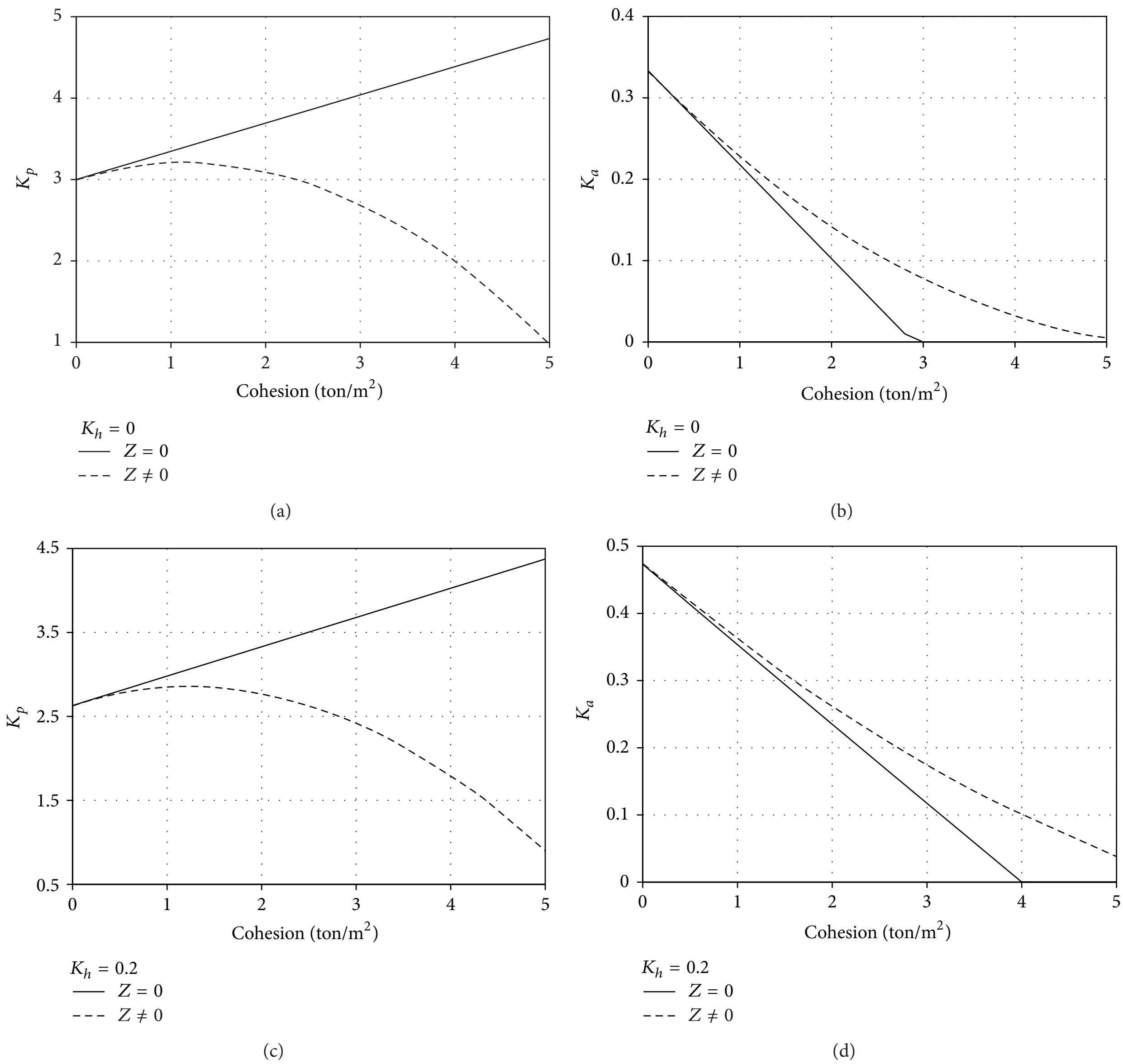

Figure 12: Effects of cohesion on lateral earth pressure.

many projects there is no access to a clean granular material or it is not economical to use such soils. Therefore, usage of cohesive soil is inevitable.

To better evaluate the effects of cohesion factor on lateral earth pressure, a wall with horizontal backfill and $\phi = 30^\circ$ was analyzed. Analyses have been repeated in the cases of with and without considering tension cracks for two different categories of $K_h = 0$ and $K_h = 0.2$. Results are reported in Figure 12 which shows the significant difference in active coefficient between cohesionless ($C = 0$) and cohesive ($C > 0$) soils. Where $K_h = 0.2$, for minimum 2 t/m^2 of cohesion, the active pressure changes 50% and the passive pressure changes 27%. These two values for $K_h = 0$ are 70% for the active pressure and 23% for the passive pressure. Also, it seems that tension cracks have small effects on the passive pressure while they are more appropriate for the active conditions.

6. Conclusion

In this paper, the Mononobe-Okabe (M-O) method was revised, and a new approach with more general picture of problems in civil engineering was proposed. Based on the limit equilibrium analysis and a semianalytical procedure, the proposed model can go over the limitations of closed form solutions of M-O method. The modified version is also capable of considering different backfill geometry, cohesion of backfill soil, soil-wall interaction, and water table behind the wall. Using the method explained in this paper, seismic active and passive earth pressure could be calculated in many usual engineering problems without any approximation.

The parametric study on a 10 m wall was also performed in the paper to explain the methodology more clearly. The results reveal this fact that the standard M-O method, in some cases, is incapable of offering an answer. Because of its simple assumptions, M-O sometimes stands in an unsafe side of design or it overestimates and directs the problem into an uneconomical design. However, the proposed methodology relives engineers from some approximations and equivalent methods. This methodology can be easily rederived to any other simple or sophisticated problems.

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