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Research Article

Solitary Wave Solutions and Periodic Wave Solutions of the $K(m,n)$ Equation with t-Dependent Coefficients

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The Exp-function method combined with F-expansion method is employed to investigate the $K(m,n)$ equation with t-dependent coefficients. The solitary wave solutions and periodic wave solutions to the equation are constructed analytically under certain circumstances. The results presented in this paper improve the previous results.

1. Introduction

The research work of nonlinear evolution equations in applied mathematics and theoretical physics has been going on for the past forty years. It is known that many physical phenomena are often described by nonlinear evolution equations. Searching for exact traveling wave solutions of nonlinear evolution equations plays an important role in the study of these nonlinear physical phenomena, for example, the wave phenomena observed in fluid dynamics, elastic media, optical fibers, and so forth. By dint of some new methods for obtaining exact solutions of nonlinear evolution equations, many new results have been published in this area for a long time. Here, it is worth to mention that the two methods, the Exp-function method [1–4] and F-expansion method [5–8], can be combined to form one method [9–15].

In this paper, by using Exp-function method combined with F-expansion method, we will study the generalized $K(m,n)$ equation having t-dependent coefficients [16]. Consider

$$\begin{aligned} &\left(u^{l}\right)_{t}+2\beta(t)u^{l}+\left[\alpha(t)+\beta(t)x\right]\left(u^{l}\right)_{x} \\ &\quad +\delta(t)u^{m}u_{x}+\gamma(t)\left(u^{n}\right)_{xxx}=0, \end{aligned} \tag{1}$$

where $\alpha(t)$ and $\beta(t)$ are functions of the time variable t related to the linear decay or growth of the wave. The functions $\gamma(t)$ and $\delta(t)$ are the time-dependent nonlinear and dispersion coefficients, respectively, with l, m, and n being integers. Generally, (1) is not integrable.

Specially, when $l=n=m=1$ and $\delta(t)=-3A\gamma(t)$ with A being a constant, (1) degenerates to the following generalized KdV equation with variable coefficient:

$$\begin{aligned} &u_{t}+2\beta(t)u+\left[\alpha(t)+\beta(t)x\right]u_{x} \\ &\quad -3A\gamma(t)uu_{x}+\gamma(t)u_{xxx}=0, \end{aligned} \tag{2}$$

which has been solved by using the Jacobi elliptic function expansion method and derived some new soliton-like solutions in [17]. Yu and Tian [18] studied the variable coefficient KdV equation (2) and obtained some new soliton-like solutions including nonsymmetrical kink solutions, compacton solutions, solitary pattern solutions, triangular function solution, and Jacobi and Weierstrass elliptic function solutions using the auxiliary equation method.

In 2009, by using solitary wave ansatz in the form of sech^{p} and $\tanh^{p}$ functions, respectively, Triki and Wazwaz [16] obtained exact bright and dark soliton solutions for (1) in the cases $l=n$ and $m>n-1$. Besides, we have recently derived some exact solutions of (1) by using Exp-function method combined with F-expansion method in the cases $l=n$ and $m\neq n-1$ [19].

In this paper, we further extend the works made in [16, 19] by investigating (1). Using Exp-function method combined with F-expansion method, we establish new solitary wave solutions and periodic solutions of (1) in the cases $m=n-1$ and $l\neq n$, which is different from those presented in the previous works [16, 19]. It is shown that the variable coefficients

$\alpha(t)$, $\beta(t)$, $\delta(t)$ and $\gamma(t)$ and the exponents l, m, and n are the main factors to cause the qualitative change in the physical structures of the solutions.

2. Description of the Method

In this section, we review the combining of the Exp-function method with F-expansion method [14, 15] at first.

Given a nonlinear partial differential equation, for instance, in two variables, as follows:

$$P(u, u_x, u_t, u_{xx}, u_{xt}, \ldots) = 0, \tag{3}$$

where P is in general a nonlinear function of its variables, we firstly use the Exp-function method to obtain new exact solutions of the following Riccati equation:

$$\phi'(\xi) = \frac{d}{d\xi}\phi(\xi) = A + h\phi^2(\xi), \tag{4}$$

where A and h are arbitrary constants, then using the Riccati equation (4) as auxiliary equation and its exact solutions, we obtain exact solutions of the nonlinear partial differential equation (3).

Seeking for the exact solutions of (4), we introduce a complex variable η, defined by

$$\eta = \rho\xi + \xi_0, \tag{5}$$

where ρ is a constant to be determined later, ξ_0 is an arbitrary constant, and Riccati equation (4) converts to

$$\rho\phi' - A - h\phi^2 = 0, \tag{6}$$

where prime denotes the derivative with respect to η.

According to the Exp-function method, we assume that the solution of (6) can be expressed in the following form:

$$\phi(\eta) = \frac{a_e \exp(e\eta) + \cdots + a_{-d}\exp(-d\eta)}{b_g \exp(g\eta) + \cdots + b_{-f}\exp(-f\eta)}, \tag{7}$$

where e, d, g, and f are positive integers which are given by the homogeneous balance principle, $a_e, \ldots, a_{-d}, b_g, \ldots, b_{-f}$ are unknown constants to be determined. To determine the values of e and g, we usually balance the linear term of the highest-order in (6) with the highest-order nonlinear term. Similarly, we can determine d and f by balancing the linear term of the lowest-order in (6) with the lowest-order nonlinear term we obtain $e = g$, $d = f$. For simplicity, we set $e = g = 1$ and $d = f = 1$; then (7) becomes

$$\phi(\eta) = \frac{a_1 \exp(\eta) + a_0 + a_{-1}\exp(-\eta)}{b_1 \exp(\eta) + b_0 + b_{-1}\exp(-\eta)}. \tag{8}$$

Substituting (8) into (6), equating to zero the coefficients of all powers of $\exp(n\eta)$ $(n = -2, -1, 0, 1, 2)$ yields a set of algebraic equations for $a_1, a_0, a_{-1}, b_1, b_0, b_{-1}$, and ρ. Solving the system of algebraic equations by using Maple, we obtain the new exact solution of (4), which is read as follows:

$$\begin{aligned}\phi_1 = {} & \left(-\sqrt{-\frac{A}{h}}b_1 \exp\left(h\sqrt{-\frac{A}{h}}\xi + \xi_0\right)\right. \\ & \left. + a_{-1}\exp\left(-h\sqrt{-\frac{A}{h}}\xi - \xi_0\right)\right) \\ & \times\left(b_1 \exp\left(h\sqrt{-\frac{A}{h}}\xi + \xi_0\right)\right. \\ & \left. + \frac{a_{-1}}{\sqrt{-A/h}}\exp\left(-h\sqrt{-\frac{A}{h}}\xi - \xi_0\right)\right)^{-1},\end{aligned} \tag{9}$$

where a_{-1}, and b_1 are free parameters.

Consider

$$\begin{aligned}\phi_2 = {} & \left(\frac{\left(ha_0^2 + Ab_0^2\right)}{4h\sqrt{-A/h}b_{-1}}\exp\left(2h\sqrt{-\frac{A}{h}}\xi + \xi_0\right) + a_0\right. \\ & \left. + \sqrt{-\frac{A}{h}}b_{-1}\exp\left(-2h\sqrt{-\frac{A}{h}}\xi - \xi_0\right)\right) \\ & \times\left(\frac{\left(ha_0^2 + Ab_0^2\right)}{4Ab_{-1}}\exp\left(2h\sqrt{-\frac{A}{h}}\xi + \xi_0\right) + b_0\right. \\ & \left. + b_{-1}\exp\left(-2h\sqrt{-\frac{A}{h}}\xi - \xi_0\right)\right)^{-1},\end{aligned} \tag{10}$$

where a_0, b_0 and b_{-1} are free parameters.

By choosing properly values of a_0, a_{-1}, b_0, b_{-1}, we find many kinds of hyperbolic function solutions and triangular periodic solutions of (4), which are listed as follows.

(i) When $\xi_0 = 0$, $b_1 = 1$, $a_{-1} = \pm\sqrt{-A/h}$, $A/h < 0$, and solution (9) becomes

$$\begin{aligned}\phi &= -\sqrt{-\frac{A}{h}}\tanh\left(h\sqrt{-\frac{A}{h}}\xi\right), \\ \phi &= -\sqrt{-\frac{A}{h}}\coth\left(h\sqrt{-\frac{A}{h}}\xi\right).\end{aligned} \tag{11}$$

(ii) When $\xi_0 = 0$, $b_1 = i$, $a_{-1} = \mp\sqrt{A/h}$, $A/h > 0$, and solution (9) becomes

$$\begin{aligned}\phi &= \sqrt{\frac{A}{h}}\tan\left(h\sqrt{\frac{A}{h}}\xi\right), \\ \phi &= -\sqrt{\frac{A}{h}}\cot\left(h\sqrt{\frac{A}{h}}\xi\right).\end{aligned} \tag{12}$$

(iii) When $\xi_0 = 0$, $b_0 = 0$, $b_{-1} = 1$, $a_0 = \pm 2\sqrt{-A/h}$, $A/h < 0$, and solution (10) becomes

$$\phi = -\sqrt{-\frac{A}{h}}\left[\coth\left(2h\sqrt{-\frac{A}{h}}\xi\right) \pm \operatorname{csch}\left(2h\sqrt{-\frac{A}{h}}\xi\right)\right]. \tag{13}$$

(iv) When $\xi_0 = 0$, $b_0 = 0$, $b_{-1} = i$, $a_0 = \pm 2\sqrt{-A/h}$, $A/h < 0$, and solution (10) becomes

$$\phi = -\sqrt{-\frac{A}{h}}\left[\tanh\left(2h\sqrt{-\frac{A}{h}}\xi\right) \pm i\operatorname{sech}\left(2h\sqrt{-\frac{A}{h}}\xi\right)\right]. \tag{14}$$

(v) When $\xi_0 = 0$, $b_0 = 0$, $b_{-1} = 1$, $a_0 = \pm 2\sqrt{A/h}$, $A/h > 0$, and solution (10) becomes

$$\phi = \sqrt{\frac{A}{h}}\left[\tan\left(2h\sqrt{\frac{A}{h}}\xi\right) \pm \sec\left(2h\sqrt{\frac{A}{h}}\xi\right)\right]. \tag{15}$$

(vi) When $\xi_0 = 0$, $b_0 = 0$, $b_{-1} = i$, $a_0 = \pm 2\sqrt{A/h}$, $A/h > 0$, and solution (10) becomes

$$\phi = -\sqrt{\frac{A}{h}}\left[\cot\left(2h\sqrt{\frac{A}{h}}\xi\right) \mp \csc\left(2h\sqrt{\frac{A}{h}}\xi\right)\right]. \tag{16}$$

For simplicity, in the rest of the paper, we consider $\xi_0 = 0$.

3. Application to the $K(m,n)$ Equation with t-Dependent Coefficients

Balancing the order of the nonlinear term $(u^l)_t$ with the term $(u^n)_{xxx}$ in (1), we obtain

$$lP + 1 = nP + 3, \tag{17}$$

so that

$$P = \frac{2}{l-n}. \tag{18}$$

To get a closed-form solution, it is natural to use the transformation

$$u = v^{2/(l-n)}, \tag{19}$$

and when $m = n - 1$, (1) becomes

$$\begin{aligned} &2l(l-n)^2 v^4 v_t + 2(l-n)^3 \beta(t) v^5 \\ &\quad + 2l(l-n)^2 [\alpha(t) + \beta(t) x] v^4 v_x + 2(l-n)^2 \delta(t) v^2 v_x \\ &\quad + \gamma(t) \big[2n(3n-l)(4n-2l)(v_x)^3 \\ &\qquad + 6n(3n-l)(l-n) v v_x v_{xx} \\ &\qquad + 2n(l-n)^2 v^2 v_{xxx}\big] = 0. \end{aligned} \tag{20}$$

This means that all the evolution terms that satisfy the condition $m = n - 1$ contribute to the soliton formation.

In order to obtain new exact travelling wave solutions for (20), we use

$$v(x,t) = v(\xi), \qquad \xi = k(t)x + \omega(t), \tag{21}$$

where $k(t)$ and $\omega(t)$ are functions of t to be determined later, and substituting the (21) into (20), we obtain

$$\begin{aligned} &2l(l-n)^2 v^4 v_t + 2(l-n)^3 \beta(t) v^5 \\ &\quad + 2l(l-n)^2 k(t) [\alpha(t) + \beta(t) x] v^4 v' \\ &\quad + 2(l-n)^2 \delta(t) k(t) v^2 v' \\ &\quad + \gamma(t) \big[2n(3n-l)(4n-2l) k(t)^3 (v')^3 \\ &\qquad + 6n(3n-l)(l-n) k(t)^3 v v' v'' \\ &\qquad + 2n(l-n)^2 k(t)^3 v^2 v'''\big] = 0. \end{aligned} \tag{22}$$

Now, we assume that the solution of (22) can be expressed in the following form:

$$v = v(\xi) = \sum_{j=0}^{N} a_j(t)\phi^j(\xi) + \sum_{j=1}^{N} b_j(t)\phi^{-j}(\xi), \tag{23}$$

where N is a positive integer that is given by the homogeneous balance principle, and $\phi(\xi)$ is a solution of (4). Balancing $v^4 v'$ term with $v^2 v'''$ term in (22) gives $N = 1$. Therefore, we obtain

$$v = a_0(t) + a_1(t)\phi(\xi) + \frac{b_1(t)}{\phi(\xi)}. \tag{24}$$

Substituting (24) into (22) and using the Riccati equation (4), collecting the coefficients of $\phi(\xi)$, we have

$$\begin{aligned} &\frac{1}{D}\big[C_0(t) + C_1(t)\phi(\xi) + C_2(t)\phi^2(\xi) + \cdots \\ &\quad + C_{11}(t)\phi^{11}(\xi) + C_{12}(t)\phi^{12}(\xi)\big] = 0. \end{aligned} \tag{25}$$

Because the expresses to these coefficients $D, C_0(t), C_1(t), C_2(t), C_3(t), C_4(t), \ldots, C_{11}(t), C_{12}(t)$ of $\phi(\xi)$ in (25) are too lengthiness, so we omit them; setting the coefficients to zero yields a set of algebraic equations as follows:

$$\begin{aligned} &C_0(t) = 0,\ C_1(t) = 0,\ C_2(t) = 0,\ C_3(t) = 0, \\ &C_4(t) = 0, \ldots, C_{11}(t) = 0,\ C_{12}(t) = 0. \end{aligned} \tag{26}$$

Solving the algebraic equations obtained above, we can have the following three sets of solutions.

Case 1. Consider

$$\begin{aligned}
a_0(t) &= 0, \qquad a_1(t) = \lambda e^{((n-l)/l)\int \beta(t)dt}, \\
b_1(t) &= \mu e^{((n-l)/l)\int \beta(t)dt}, \qquad k(t) = Ke^{-\int \beta(t)dt}, \\
\omega(t) &= -\int \left[\frac{\delta(t)l + \delta(t)n}{8\lambda n^2 \mu} e^{((2l-2n)/l)\int \beta(t)dt} + \alpha(t)\right] Ke^{-\int \beta(t)dt} dt, \\
\gamma(t) &= \frac{\lambda(l-n)^2 \delta(t)}{16n^3 \mu K^2 h^2} e^{2\int \beta(t)dt}, \\
A &= \frac{\mu}{\lambda} h, \qquad m = n-1,
\end{aligned} \tag{27}$$

where λ, μ and K are arbitrary nonzero constants.

Case 2. Consider

$$\begin{aligned}
a_0(t) &= C\lambda e^{-(4/3)\int \beta(t)dt}, \qquad a_1(t) = \lambda e^{-(4/3)\int \beta(t)dt}, \\
b_1(t) &= 0, \qquad k(t) = Ke^{-\int \beta(t)dt}, \\
\omega(t) &= -\int \left[K\alpha(t) e^{-\int \beta(t)dt} + \frac{K\delta(t)}{4nC^2\lambda^2} e^{(5/3)\int \beta(t)dt}\right] dt, \\
\gamma(t) &= -\frac{\delta(t)}{nC^2h^2K^2} e^{2\int \beta(t)dt}, \\
A &= -C^2 h, \qquad m = n-1, \qquad l = -3n,
\end{aligned} \tag{28}$$

where C, λ and K are arbitrary nonzero constants.

Case 3. Consider

$$\begin{aligned}
a_0(t) &= \lambda e^{-2\int \beta(t)dt}, \qquad a_1(t) = 0, \\
b_1(t) &= C\lambda e^{-2\int \beta(t)dt}, \qquad k(t) = Ke^{-\int \beta(t)dt}, \\
\omega(t) &= -\int \left[K\alpha(t) e^{-\int \beta(t)dt}\right] dt, \\
\gamma(t) &= -\frac{\delta(t)}{4nC^2K^2h^2} e^{2\int \beta(t)dt}, \\
A &= -C^2 h, \qquad m = n-1, \qquad l = -n,
\end{aligned} \tag{29}$$

where λ, C and K are arbitrary nonzero constants.

Thus from (24), (27), (28), and (29) we obtain families of exact solutions to (22) as follows:

$$\begin{aligned}
v &= \lambda e^{((n-l)/l)\int \beta(t)dt} \phi(\xi) + \mu e^{((n-l)/l)\int \beta(t)dt} \frac{1}{\phi(\xi)}, \\
v &= C\lambda e^{-(4/3)\int \beta(t)dt} + \lambda e^{-(4/3)\int \beta(t)dt} \phi(\xi), \\
v &= \lambda e^{-2\int \beta(t)dt} + C\lambda e^{-2\int \beta(t)dt} \frac{1}{\phi(\xi)},
\end{aligned} \tag{30}$$

where $\phi(\xi)$ is a solution of (4).

Substituting new solutions (9) and (10) of Riccati equation into solutions (30), using the transformation (19), we have the following several families of solutions to (1).

Family 1. Consider

$$\begin{aligned}
u_1(x,t) = \Bigg\{ &\lambda e^{((n-l)/l)\int \beta(t)dt} \Bigg[\left(-\sqrt{-\frac{A}{h}} b_1 \exp\left(h\sqrt{-\frac{A}{h}}\xi\right) + a_{-1} \exp\left(-h\sqrt{-\frac{A}{h}}\xi\right)\right) \\
&\times \left(b_1 \exp\left(h\sqrt{-\frac{A}{h}}\xi\right) + \frac{a_{-1}}{\sqrt{-A/h}} \exp\left(-h\sqrt{-\frac{A}{h}}\xi\right)\right)^{-1} \Bigg] \\
+ &\mu e^{((n-l)/l)\int \beta(t)dt} \Bigg[\left(b_1 \exp\left(h\sqrt{-\frac{A}{h}}\xi\right) + \frac{a_{-1}}{\sqrt{-A/h}} \exp\left(-h\sqrt{-\frac{A}{h}}\xi\right)\right) \\
&\times \left(-\sqrt{-\frac{A}{h}} b_1 \exp\left(h\sqrt{-\frac{A}{h}}\xi\right) + a_{-1} \exp\left(-h\sqrt{-\frac{A}{h}}\xi\right)\right)^{-1} \Bigg] \Bigg\}^{2/(l-n)},
\end{aligned} \tag{31}$$

where $\xi = Ke^{-\int \beta(t)dt} x - \int [((\delta(t)l + \delta(t)n)/8\lambda n^2 \mu) e^{((2l-2n)/l)\int \beta(t)dt} + \alpha(t)] Ke^{-\int \beta(t)dt} dt$,

$$\begin{aligned}
\gamma(t) &= \frac{\lambda(l-n)^2 \delta(t)}{16n^3 \mu K^2 h^2} e^{2\int \beta(t)dt}, \\
A &= \frac{\mu}{\lambda} h, \qquad m = n-1.
\end{aligned} \tag{32}$$

If we set $b_1 = 1$, $a_{-1} = \sqrt{-A/h}$, and $A/h < 0$ in (31), we obtain

$$u_{1(1)}(x,t) = \left\{ \lambda e^{((n-l)/l)\int \beta(t)dt} \sqrt{-\frac{A}{h}} \tanh\left(h\sqrt{-\frac{A}{h}}\xi\right) + \frac{\mu e^{((n-l)/l)\int \beta(t)dt}}{\sqrt{-A/h}} \coth\left(h\sqrt{-\frac{A}{h}}\xi\right) \right\}^{2/(l-n)}. \tag{33}$$

Setting $b_1 = i$, $a_{-1} = -\sqrt{A/h}$, and $A/h > 0$ in (31), we get

$$u_{1(2)}(x,t) = \left\{ \lambda e^{((n-l)/l)\int \beta(t)dt} \sqrt{\frac{A}{h}} \tan\left(h\sqrt{\frac{A}{h}}\xi\right) + \frac{\mu e^{((n-l)/l)\int \beta(t)dt}}{\sqrt{A/h}} \cot\left(h\sqrt{\frac{A}{h}}\xi\right) \right\}^{2/(l-n)}. \tag{34}$$

Family 2. Consider

$$\begin{aligned}
u_2(x,t) &= \Bigg\{\lambda e^{((n-l)/l)\int\beta(t)dt}\Bigg[\Bigg(\frac{\left(ha_0^2+Ab_0^2\right)}{4h\sqrt{-A/h}b_{-1}}\exp\left(2h\sqrt{-\frac{A}{h}}\xi\right)+a_0 \\
&\quad +\sqrt{-\frac{A}{h}}b_{-1}\exp\left(-2h\sqrt{-\frac{A}{h}}\xi\right)\Bigg) \\
&\quad \times\Bigg(\frac{\left(ha_0^2+Ab_0^2\right)}{4Ab_{-1}}\exp\left(2h\sqrt{-\frac{A}{h}}\xi\right)+b_0 \\
&\quad +b_{-1}\exp\left(-2h\sqrt{-\frac{A}{h}}\xi\right)\Bigg)^{-1}\Bigg] \\
&\quad +\mu e^{((n-l)/l)\int\beta(t)dt} \\
&\quad \times\Bigg[\Bigg(\frac{\left(ha_0^2+Ab_0^2\right)}{4Ab_{-1}}\exp\left(2h\sqrt{-\frac{A}{h}}\xi\right)+b_0 \\
&\quad +b_{-1}\exp\left(-2h\sqrt{-\frac{A}{h}}\xi\right)\Bigg) \\
&\quad \times\Bigg(\frac{\left(ha_0^2+Ab_0^2\right)}{4h\sqrt{-A/h}b_{-1}}\exp\left(2h\sqrt{-\frac{A}{h}}\xi\right)+a_0 \\
&\quad +\sqrt{-\frac{A}{h}}b_{-1}\exp(-2h\sqrt{-\frac{A}{h}}\xi)\Bigg)^{-1}\Bigg]\Bigg\}^{2/(l-n)},
\end{aligned} \tag{35}$$

where $\xi = Ke^{-\int\beta(t)dt}x - \int[((\delta(t)l+\delta(t)n)/8\lambda n^2\mu)e^{((2l-2n)/l)\int\beta(t)dt}+\alpha(t)]Ke^{-\int\beta(t)dt}dt$, $\gamma(t) = (\lambda(l-n)^2\delta(t)/16n^3\mu K^2h^2)e^{2\int\beta(t)dt}$, and $A=(\mu/\lambda)h$, $m=n-1$.

If we set $b_0=0$, $b_{-1}=1$, $a_0=\pm2\sqrt{-A/h}$, and $A/h<0$ in (35), we obtain

$$\begin{aligned}
u_{2(1)}(x,t) &= \Bigg\{\lambda e^{((n-l)/l)\int\beta(t)dt}\sqrt{-\frac{A}{h}} \\
&\quad \times\left[\coth\left(2h\sqrt{-\frac{A}{h}}\xi\right)\pm\operatorname{csch}\left(2h\sqrt{-\frac{A}{h}}\xi\right)\right] \\
&\quad +\left(\mu e^{((n-l)/l)\int\beta(t)dt}\right) \\
&\quad \times\Bigg(\sqrt{-\frac{A}{h}}\Bigg[\coth\left(2h\sqrt{-\frac{A}{h}}\xi\right) \\
&\quad \pm\operatorname{csch}\left(2h\sqrt{-\frac{A}{h}}\xi\right)\Bigg]\Bigg)^{-1}\Bigg\}^{2/(l-n)}.
\end{aligned} \tag{36}$$

Setting $b_0=0$, $b_{-1}=i$, $a_0=\pm2\sqrt{-A/h}$, and $A/h<0$ in (35), we get

$$\begin{aligned}
u_{2(2)}(x,t) &= \Bigg\{\lambda e^{((n-l)/l)\int\beta(t)dt}\sqrt{-\frac{A}{h}} \\
&\quad \times\left[\tanh\left(2h\sqrt{-\frac{A}{h}}\xi\right)\pm i\operatorname{sech}\left(2h\sqrt{-\frac{A}{h}}\xi\right)\right] \\
&\quad +\left(\mu e^{((n-l)/l)\int\beta(t)dt}\right) \\
&\quad \times\Bigg(\sqrt{-\frac{A}{h}}\Bigg[\tanh\left(2h\sqrt{-\frac{A}{h}}\xi\right) \\
&\quad \pm i\operatorname{sech}\left(2h\sqrt{-\frac{A}{h}}\xi\right)\Bigg]\Bigg)^{-1}\Bigg\}^{2/(l-n)}.
\end{aligned} \tag{37}$$

Setting $b_0=0$, $b_{-1}=1$, $a_0=\pm2\sqrt{A/h}$, and $A/h>0$ in (35), we have

$$\begin{aligned}
u_{2(3)}(x,t) &= \Bigg\{\lambda e^{((n-l)/l)\int\beta(t)dt}\sqrt{\frac{A}{h}} \\
&\quad \times\left[\tan\left(2h\sqrt{\frac{A}{h}}\xi\right)\pm\sec\left(2h\sqrt{\frac{A}{h}}\xi\right)\right] \\
&\quad +\frac{\mu e^{((n-l)/l)\int\beta(t)dt}}{\sqrt{A/h}\left[\tan\left(2h\sqrt{A/h}\xi\right)\pm\sec\left(2h\sqrt{A/h}\xi\right)\right]}\Bigg\}^{2/(l-n)}.
\end{aligned} \tag{38}$$

Setting $b_0=0$, $b_{-1}=i$, $a_0=\pm2\sqrt{A/h}$, and $A/h>0$ in (35), we have

$$\begin{aligned}
u_{2(4)}(x,t) &= \Bigg\{\lambda e^{((n-l)/l)\int\beta(t)dt}\sqrt{\frac{A}{h}} \\
&\quad \times\left[\cot\left(2h\sqrt{\frac{A}{h}}\xi\right)\mp\csc\left(2h\sqrt{\frac{A}{h}}\xi\right)\right] \\
&\quad +\frac{\mu e^{((n-l)/l)\int\beta(t)dt}}{\sqrt{A/h}\left[\cot\left(2h\sqrt{A/h}\xi\right)\mp\csc\left(2h\sqrt{A/h}\xi\right)\right]}\Bigg\}^{2/(l-n)}.
\end{aligned} \tag{39}$$

Family 3. Consider

$$\begin{aligned}
u_3(x,t) &= \Bigg\{C\lambda e^{-(4/3)\int\beta(t)dt}+\lambda e^{-(4/3)\int\beta(t)dt} \\
&\quad \times\Bigg[\Bigg(-\sqrt{-\frac{A}{h}}b_1\exp\left(h\sqrt{-\frac{A}{h}}\xi\right)
\end{aligned}$$

$$+a_{-1}\exp\left(-h\sqrt{-\frac{A}{h}}\xi\right)\Bigg)$$
$$\times\left(b_1\exp\left(h\sqrt{-\frac{A}{h}}\xi\right)\right.$$
$$\left.\left.\left.+\frac{a_{-1}}{\sqrt{-A/h}}\exp\left(-h\sqrt{-\frac{A}{h}}\xi\right)\right)^{-1}\right]\right\}^{2/(l-n)},\tag{40}$$

where $\xi = Ke^{-\int\beta(t)dt}x - \int[K\alpha(t)e^{-\int\beta(t)dt} + (K\delta(t)/4nC^2\lambda^2)e^{(5/3)\int\beta(t)dt}]dt$, $\gamma(t) = -(\delta(t)/nC^2h^2K^2)e^{2\int\beta(t)dt}$, and $A = -C^2h$, $m = n-1$, $l = -3n$.

If we set $b_1 = 1$, $a_{-1} = \pm\sqrt{-A/h}$, and $A/h < 0$ in (40), we obtain

$$u_{3(1)}(x,t) = \left\{C\lambda e^{-(4/3)\int\beta(t)dt} - \lambda e^{-(4/3)\int\beta(t)dt}\sqrt{-\frac{A}{h}}\tanh\left(h\sqrt{-\frac{A}{h}}\xi\right)\right\}^{2/(l-n)},$$
$$u_{3(2)}(x,t) = \left\{C\lambda e^{-(4/3)\int\beta(t)dt} - \lambda e^{-(4/3)\int\beta(t)dt}\sqrt{-\frac{A}{h}}\coth\left(h\sqrt{-\frac{A}{h}}\xi\right)\right\}^{2/(l-n)}.\tag{41}$$

Family 4. Consider

$$u_4(x,t) = \left\{C\lambda e^{-(4/3)\int\beta(t)dt} + \lambda e^{-(4/3)\int\beta(t)dt}\right.$$
$$\times\left[\left(\frac{\left(ha_0^2+Ab_0^2\right)}{4h\sqrt{-A/h}b_{-1}}\exp\left(2h\sqrt{-\frac{A}{h}}\xi\right)+a_0\right.\right.$$
$$\left.+\sqrt{-\frac{A}{h}}b_{-1}\exp\left(-2h\sqrt{-\frac{A}{h}}\xi\right)\right)\tag{42}$$
$$\times\left(\frac{\left(ha_0^2+Ab_0^2\right)}{4Ab_{-1}}\exp\left(2h\sqrt{-\frac{A}{h}}\xi\right)+b_0\right.$$
$$\left.\left.\left.+b_{-1}\exp\left(-2h\sqrt{-\frac{A}{h}}\xi\right)\right)^{-1}\right]\right\}^{2/(l-n)},$$

where $\xi = Ke^{-\int\beta(t)dt}x - \int[K\alpha(t)e^{-\int\beta(t)dt} + (K\delta(t)/4nC^2\lambda^2)e^{(5/3)\int\beta(t)dt}]dt$, $\gamma(t) = -(\delta(t)/nC^2h^2K^2)e^{2\int\beta(t)dt}$, and $A = -C^2h$, $m = n-1$, $l = -3n$.

If we set $b_0 = 0$, $b_{-1} = 1$, $a_0 = \pm2\sqrt{-A/h}$, and $A/h < 0$ in (42), we obtain

$$u_{4(1)}(x,t) = \left\{C\lambda e^{-(4/3)\int\beta(t)dt} - \lambda e^{-(4/3)\int\beta(t)dt}\sqrt{-\frac{A}{h}}\right.$$
$$\left.\times\left[\coth\left(2h\sqrt{-\frac{A}{h}}\xi\right)\pm\operatorname{csch}\left(2h\sqrt{-\frac{A}{h}}\xi\right)\right]\right\}^{2/(l-n)}.\tag{43}$$

Setting $b_0 = 0$, $b_{-1} = i$, $a_0 = \pm2\sqrt{-A/h}$, and $A/h < 0$ in (42), we get

$$u_{4(2)}(x,t) = \left\{C\lambda e^{-(4/3)\int\beta(t)dt} - \lambda e^{-(4/3)\int\beta(t)dt}\sqrt{-\frac{A}{h}}\right.$$
$$\left.\times\left[\tanh\left(2h\sqrt{-\frac{A}{h}}\xi\right)\pm i\operatorname{sech}\left(2h\sqrt{-\frac{A}{h}}\xi\right)\right]\right\}^{2/(l-n)}.\tag{44}$$

Family 5. Consider

$$u_5(x,t) = \left\{\lambda e^{-2\int\beta(t)dt} + C\lambda e^{-2\int\beta(t)dt}\right.$$
$$\times\left[\left(b_1\exp\left(h\sqrt{-\frac{A}{h}}\xi\right)\right.\right.$$
$$\left.+\frac{a_{-1}}{\sqrt{-A/h}}\exp\left(-h\sqrt{-\frac{A}{h}}\xi\right)\right)\tag{45}$$
$$\times\left(-\sqrt{-\frac{A}{h}}b_1\exp\left(h\sqrt{-\frac{A}{h}}\xi\right)\right.$$
$$\left.\left.\left.+a_{-1}\exp\left(-h\sqrt{-\frac{A}{h}}\xi\right)\right)^{-1}\right]\right\}^{2/(l-n)},$$

where $\xi = Ke^{-\int\beta(t)dt}x - \int[K\alpha(t)e^{-\int\beta(t)dt}]dt$, $\gamma(t) = -(\delta(t)/4nC^2K^2h^2)e^{2\int\beta(t)dt}$, $A = -C^2h$, $m = n-1$, and $l = -n$.

If we set $b_1 = 1$, $a_{-1} = \pm\sqrt{-A/h}$, and $A/h < 0$ in (45), we obtain

$$u_{5(1)}(x,t) = \left\{\lambda e^{-2\int\beta(t)dt} - \frac{C\lambda e^{-2\int\beta(t)dt}}{\sqrt{-A/h}}\coth\left(h\sqrt{-\frac{A}{h}}\xi\right)\right\}^{2/(l-n)},$$

$$u_{5(2)}(x,t)=\left\{\lambda e^{-2\int\beta(t)dt}-\frac{C\lambda e^{-2\int\beta(t)dt}}{\sqrt{-A/h}}\tanh\left(h\sqrt{-\frac{A}{h}}\xi\right)\right\}^{2/(l-n)}. \tag{46}$$

Family 6. Consider

$$\begin{aligned}u_6(x,t)&=\left\{\lambda e^{-2\int\beta(t)dt}+C\lambda e^{-2\int\beta(t)dt}\right.\\&\quad\times\left[\left(\frac{\left(ha_0^2+Ab_0^2\right)}{4Ab_{-1}}\exp\left(2h\sqrt{-\frac{A}{h}}\xi\right)+b_0\right.\right.\\&\quad\left.+b_{-1}\exp\left(-2h\sqrt{-\frac{A}{h}}\xi\right)\right)\\&\quad\times\left(\frac{\left(ha_0^2+Ab_0^2\right)}{4h\sqrt{-A/h}b_{-1}}\exp\left(2h\sqrt{-\frac{A}{h}}\xi\right)+a_0\right.\\&\quad\left.\left.\left.+\sqrt{-\frac{A}{h}}b_{-1}\exp\left(-2h\sqrt{-\frac{A}{h}}\xi\right)\right)^{-1}\right]\right\}^{2/(l-n)},\end{aligned} \tag{47}$$

where $\xi=Ke^{-\int\beta(t)dt}x-\int[K\alpha(t)e^{-\int\beta(t)dt}]dt$, $\gamma(t)=-(\delta(t)/4nC^2K^2h^2)e^{2\int\beta(t)dt}$, $A=-C^2h$, $m=n-1$, and $l=-n$.

If we set $b_0=0$, $b_{-1}=1$, $a_0=\pm2\sqrt{-A/h}$, and $A/h<0$ in (47), we obtain

$$\begin{aligned}u_{6(1)}(x,t)&=\left\{\lambda e^{-2\int\beta(t)dt}\right.\\&\quad-\left(C\lambda e^{-2\int\beta(t)dt}\right)\\&\quad\times\left(\sqrt{-\frac{A}{h}}\left[\coth\left(2h\sqrt{-\frac{A}{h}}\xi\right)\right.\right.\\&\quad\left.\left.\left.\pm\operatorname{csch}\left(2h\sqrt{-\frac{A}{h}}\xi\right)\right]\right)^{-1}\right\}^{2/(l-n)}.\end{aligned} \tag{48}$$

Setting $b_0=0$, $b_{-1}=i$, $a_0=\pm2\sqrt{-A/h}$, and $A/h<0$ in (47), we get

$$\begin{aligned}u_{6(2)}(x,t)&=\left\{\lambda e^{-2\int\beta(t)dt}\right.\\&\quad-\left(C\lambda e^{-2\int\beta(t)dt}\right)\\&\quad\times\left(\sqrt{-\frac{A}{h}}\left[\tanh\left(2h\sqrt{-\frac{A}{h}}\xi\right)\right.\right.\\&\quad\left.\left.\left.\pm i\operatorname{sech}\left(2h\sqrt{-\frac{A}{h}}\xi\right)\right]\right)^{-1}\right\}^{2/(l-n)}.\end{aligned} \tag{49}$$

4. Conclusions

The Exp-function method combined with F-expansion method is used to investigate the $K(m,n)$ equation with t-dependent coefficients. We acquire the exact solutions of Exp-function type of (1) in the cases $m=n-1$ and $l\neq n$. The solitary wave solutions and periodic wave solutions of the equation are obtained under different circumstances. It is shown that many solutions in this work are different from those presented in [16, 19]. These solutions may be useful to explain some physical phenomena in genuinely nonlinear dynamical systems that are described by the $K(m,n)$-type models. The approach applied may be employed in further works to find new solutions for other types of nonlinear partial differential equations.

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