

Research Article

Delay-Dependent Finite-Time H_{∞} Filtering for Markovian Jump Systems with Different System Modes

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This paper is concerned with the problem of delay-dependent finite-time H_{∞} filtering for Markovian jump systems with different system modes. By using the new augmented multiple mode-dependent Lyapunov-Krasovskii functional and employing the proposed integrals inequalities in the derivation of our results, a novel sufficient condition for finite-time boundness with an H_{∞} performance index is derived. Particularly, two different Markov processes have been considered for modeling the randomness of system matrix and the state delay. Based on the derived condition, the H_{∞} filtering problem is solved, and an explicit expression of the desired filter is also given; the system trajectory stays within a prescribed bound during a specified time interval. Finally, a numerical example is given to illustrate the effectiveness and the potential of the proposed techniques.

1. Introduction

Markovian jump systems were introduced by Krasovskiĭ and Lidskii [1], which can be described by a set of systems with the transitions in a finite mode set. In the past few decades, there has been increasing interest in Markovian jump systems because this class of systems is appropriate to many physical systems which always go with random failures, repairs, and sudden environment disturbance [2–5]. Such class of systems is a special class of stochastic hybrid systems with finite operation modes, which may switch from one to another at different time, such as component failures, sudden environmental disturbance, and abrupt variations of the operating points of a nonlinear system. As a crucial factor, it is shown that such jumping can be determined by a Markovian chain [6]. For linear Markovian jumping systems, many important issues have been studied extensively such as stability, stabilization, control synthesis, and filter design [6-12]. In finite operation modes, Markovian jump systems are a special class of stochastic systems that can switch from one to another at different time.

It is worth pointing out that time delay is of interest to many researchers because of the fact that time delay is often encountered in various systems such as networked control systems, chemical processes, and communication systems. It is worth pointing out that time delay is one of the instability sources for dynamical systems and is a common phenomenon in many industrial and engineering systems. Hence, it is not surprising that much effort has been made to investigate Markovian jump systems with time delay during the last two decades [13–15]. The exponential stabilization of Markovian jump systems with time delay was firstly studied in [16] where the decay rate was estimated by solving linear matrix inequalities [17]. However, in the aforementioned works, the network-induced delays have been commonly assumed to be deterministic, which is fairly unrealistic since delays resulting from network transmissions are typically time varying [18– 24].

Generally speaking, the delay-dependent criterions are less conservative than delay-independent ones, especially when the time delay is small enough in Markovian jump systems. Thus, recent efforts were devoted to the delaydependent Markovian jump systems stability analysis by employing Lyapunov-Krasovskii functionals [25–33]. However, in most thesis, the time delay to be arbitrarily large are allowed in criterion, it always tends to be conservative.

Furthermore, though the decay rate can be computed, it is a fixed value that one cannot adjust to deduce if a larger decay rate is possible. Therefore, how to obtain the improved results without increasing the computational burden has greatly improved the current study. On the other hand, the practical problems which system described does not exceed a certain threshold over some finite time interval are considered. In finite-time interval, finite-time stability is investigated to address these transient performances of control systems. Recently, the concept of finite-time stability has been revisited in the light of linear matrix inequalities (LMIs) and Lyapunov function theory, and some results are obtained to ensure that systems are finite-time stability or finite-time boundness [34-50]. To the best of our knowledge, in most of the works about Markovian jump systems with modedependent delay, the delay mode is always assumed to be the same as the system matrices mode. However, in real systems, the delay mode may not be the same as that for jump in other system parameters. In other words, variations of delay usually depend on phenomena which may not cause abrupt changes in other systems parameters. Therefore, the work of Markovian jump systems with different system modes is not only theoretically interesting and challenging, but also very important in practical applications.

Motivated by the previous above discussions, in this paper, we present a new augmented Lyapunov functional for a class of Markovian jump systems with different system modes; in order to reduce the possible conservativeness and computational burden, some slack matrices are introduced [32]. Several sufficient conditions are derived to guarantee the finite-time stability and boundedness of the resulting closedloop system. We find that finite-time stability is an independent concept from Lyapunov stability and always can be affected by switching behavior significantly, and the finitetime boundness criteria can be tackled in the form of LMIs. Finally, a numerical example is presented to illustrate the effectiveness of the developed techniques.

Notations. Throughout this paper, we let P > 0 ($P \ge 0$, P < 0, and $P \le 0$) denote a symmetric positive definite matrix P (positive semidefinite, negative definite, and negative semidefinite). For any symmetric matrix P, $\lambda_{max}(P)$ and $\lambda_{min}(P)$ denote the maximum and minimum eigenvalues of matrix P, respectively. \mathcal{R}^n denotes the *n*-dimensional Euclidean space, and $\mathcal{R}^{n\times m}$ refers to the set of all $n \times m$ real matrices and $\mathcal{N} = \{1, 2, \dots, N\}$. The identity matrix of order *n* is denoted as I_n . * represents the elements below the main diagonal of a symmetric matrix. The superscripts T and -1 stand for matrix transposition and matrix inverse, respectively.

2. Preliminaries

In this paper, we consider the following Markov jump system described by

$$\begin{split} \dot{x}\left(t\right) &= A_{r_{t}}x\left(t\right) + A_{\tau r_{t}}x\left(t-\tau_{s_{t}}\left(t\right)\right) + D_{r_{t}}\omega\left(t\right),\\ y\left(t\right) &= C_{yr_{t}}x\left(t\right) + C_{y\tau r_{t}}x\left(t-\tau_{s_{t}}\left(t\right)\right) + D_{yr_{t}}\omega\left(t\right), \end{split}$$

$$z(t) = C_{zr_{t}}x(t) + C_{z\tau r_{t}}x(t - \tau_{s_{t}}(t)) + D_{zr_{t}}\omega(t),$$

$$x(t) = \varphi(t), \quad t = [-h, 0],$$
(1)

where $x(t) \in \mathbb{R}^n$ is the state vector of the system, $y(t) \in \mathbb{R}^l$ is the measured output, $z(t) \in \mathbb{R}^q$ is the controlled output, $\varphi(t)$, $t \in [-h, 0]$ are initial conditions of continuous state, and $r_0 \in \mathcal{N}$, $s_0 \in \mathcal{M} = \{1, 2, ..., M\}$ are initial conditions of mode. $\omega(t) \in \mathbb{R}^q$ is the disturbance input, satisfying the following condition:

$$\int_{0}^{\infty} \omega^{\mathsf{T}}(t) \,\omega(t) \,dt \le d. \tag{2}$$

Let the random form processes r_t , s_t be the Markov stochastic processes taking values on finite sets $\mathcal{N} = \{1, 2, ..., N\}$ and $\mathcal{M} = \{1, 2, ..., M\}$ with probability transition rate matrices $\Lambda = \{\lambda_{ij}\}, i, j \in \mathcal{N}$, and $\Pi = \{\pi_{m,n}\}, m, n \in \mathcal{M}$. The transition probabilities from mode j to mode j for Markov process r_t and from mode m to mode n for the Markov process s_t in time h are described as

$$\Pr(r_{t+\Delta} = j \mid r_t = i) = \varrho_{ij} + \lambda_{ij}\Delta + o(\Delta),$$

$$\Pr(s_{t+\Delta} = n \mid s_t = m) = \zeta_{mn} + \pi_{mn}\Delta + o(\Delta),$$
(3)

where

$$\varrho_{ij} = \begin{cases} 0, & \text{if } i \neq j, \\ 1, & \text{if } i = j, \end{cases} \qquad \zeta_{mn} = \begin{cases} 0, & \text{if } m \neq n, \\ 1, & \text{if } m = n, \end{cases} \tag{4}$$

and $\Delta > 0$, $\lambda_{ij} \ge 0$, for $i \ne j$, is the transition rate from mode *i* at time *t* to mode *j* at time $t + \Delta$ and

$$-\lambda_{ii} = \sum_{j=1, j \neq i}^{N} \lambda_{ij},\tag{5}$$

for each mode $i \in \mathcal{N}$, $\lim_{\Delta \to 0_+} (o(\Delta)/\Delta) = 0$. $\pi_{mn} \ge 0$ for $m \ne n$ is the transition rate from mode *m* to mode *n* at time $t + \Delta$ and

$$-\pi_{mm} = \sum_{n=1,m \neq n}^{M} \pi_{mn},$$
 (6)

for each mode $i \in \mathcal{M}$, $\lim_{\Delta \to 0_+} (o(\Delta)/\Delta) = 0$. For convenience, we denote the Markov process r_t and s_t by i and m indices, respectively. $\tau_m(t)$ denotes the mode-dependent time-varying state delay in the system and satisfies the following condition:

$$0 < \tau_m(t) \le h_m < \infty,$$

$$\dot{\tau}_m(t) \le \mu_m, \quad \forall m \in \mathcal{M},$$

(7)

where $h = \max\{h_i, i \in \mathcal{M}\}$ is prescribed integer representing the upper bounds of time-varying delay $\tau_m(t)$. Similarly, $\mu = \max\{\mu_m, m \in \mathcal{M}\}$ is prescribed integer representing the upper bounds of time-varying delay $\dot{\tau}_m(t)$. A_{r_t} , $A_{\tau r_t}$, D_{r_t} , C_{yr_t} , $C_{y\tau r_t}$, D_{yr_t} , C_{zr_t} , $C_{z\tau \tau_t}$, and D_{zr_t} are known mode-dependent matrices with appropriate dimensions functions of the random jumping process $\{r_t\}$ and represent the nominal systems for each $r_t \in \mathcal{N}$. For notation simplicity, when the system operates in the *i*-th mode $(r_t = i)$, A_{r_t} , $A_{\tau r_t}$, D_{r_t} , C_{yr_t} , $C_{y\tau r_t}$, D_{yr_t} , C_{zr_t} , $C_{z\tau r_t}$, and D_{zr_t} are denoted as A_i , $A_{\tau i}$, D_i , C_{yi} , $C_{y\tau i}$, D_{yi} , C_{zi} , $C_{z\tau i}$, and D_{zi} , respectively.

Here we are interested in designing a full-order filter described by

$$\begin{split} \dot{x}_{f}(t) &= A_{fr_{t},s_{t}} x_{f}(t) + B_{fr_{t},s_{t}} y(t) \,, \\ z_{f}(t) &= C_{fr_{t},s_{t}} x_{f}(t) \,, \end{split} \tag{8}$$

where $x_f(t) \in \mathcal{R}^n$ is the filter state, $z_f(k) \in \mathcal{R}^q$, and the matrices A_{fr_t,s_t} , B_{fr_t,s_t} , and C_{fr_t,s_t} are unknown filter parameters to be designed.

Augmenting the model of (1) to include the filter (8), we obtain the following filtering error system:

$$\dot{\eta}(t) = \overline{A}_{r_t} \eta(t) + \overline{B}_{r_t} \eta(t - \tau_m(t)) + \overline{D}_{r_t} \omega(t),$$

$$e(t) = \overline{C}_{r_t} \eta(t) + \overline{E}_{r_t} \eta(t - \tau_m(t)),$$
(9)

where

$$\eta(t) = \begin{bmatrix} x(t) \\ x_f(t) \end{bmatrix}, \quad e(t) = z(t) - z_f(t),$$

$$\overline{A}_{r_t} = \begin{bmatrix} A_{r_t} & 0 \\ B_{fr_t,s_t}C_{yr_t} & A_{fr_t,s_t} \end{bmatrix}, \quad \overline{B}_{r_t} = \begin{bmatrix} A_{\tau r_t} \\ B_{fr_t,s_t}C_{y\tau r_t} \end{bmatrix}, \quad (10)$$

$$\overline{D}_{r_t} = \begin{bmatrix} D_{r_t} \\ B_{fr_t,s_t}D_{yr_t} \end{bmatrix},$$

$$\overline{C}_{r_t} = \begin{bmatrix} C_{zr_t}, -C_{fr_t,s_t} \end{bmatrix}, \quad \overline{E}_{r_t} = \begin{bmatrix} C_{z\tau r_t}, 0 \end{bmatrix}.$$

In order to more precisely describe the main objective, we introduce the following definitions and Lemmas for the underlying system.

Definition 1. System (1) is said to be finite-time bounded with respect to (c_1, c_2, T, R, d) , if condition (2) and the following inequality hold:

$$\sup_{-h \le v \le 0} \mathbb{E} \left\{ \eta^{\mathsf{T}}(v) \, R\eta(v) \, , \dot{\eta}^{\mathsf{T}}(v) \, R\dot{\eta}(v) \right\}$$
$$\le c_1 \Longrightarrow \mathbb{E} \left\{ \eta^{\mathsf{T}}(t) \, R\eta(t) \right\} < c_2, \qquad (11)$$
$$\forall t \in [0, T] \, ,$$

where $c_2 > c_1 \ge 0$ and R > 0.

Definition 2. Consider $V(\eta_t, r_t, s_t, t > 0)$ as the stochastic positive Lyapunov function; its weak infinitesimal operator is defined as

$$\pounds V\left(\eta_{t}, r_{t}, s_{t}\right)$$

$$= \lim_{\Delta \to 0} \frac{1}{\Delta} \left[\mathbb{E} \left\{ V\left(\eta_{t+\Delta}, r_{t+\Delta}, s_{t+\Delta}\right) \mid \eta_{t}, r_{t}, s_{t} \right\} - V\left(\eta_{t}, r_{t}, s_{t}\right) \right].$$

$$(12)$$

Definition 3. Given a constant T > 0, for all admissible $\omega(t)$ subject to condition (2), under zero initial conditions, if the closed-loop Markovian jump system (1) is finite-time bounded and the control outputs satisfy condition (8) with attenuation $\gamma > 0$,

$$\mathbb{E}\left\{\int_{0}^{T} e^{\mathsf{T}}(t) e(t) dt\right\} \le \gamma^{2} e^{\eta T} \mathbb{E}\left\{\int_{0}^{T} \omega^{\mathsf{T}}(t) \omega(t) dt\right\}.$$
 (13)

Then, the controller system (1) finite-time bounded with disturbance attenuation γ .

Remark 4. It should be pointed out that the assumption of zero initial condition in system (1) is only for the purpose of technical simplification in the derivation, and it does not cause loss of generality. In fact, if this assumption is lost, the same control result can be obtained along the same line, except for adding extra manipulations in the derivation and extra terms in the control presentation. However, in real-world applications, the initial condition of the underlying system is generally not zero.

Lemma 5 (see [32]). Let $f_i : \mathscr{R}^m \to \mathscr{R}$ (i = 1, 2, ..., N) have positive values in an open subset \mathcal{D} of \mathscr{R}^m . Then, the reciprocally convex combination of f_i over \mathcal{D} satisfies

$$\min_{\{\beta_{i}\mid\beta_{i}>0,\sum_{i}\beta_{i}=1\}} \sum_{i} \frac{1}{\beta_{i}} f_{i}(t) = \sum_{i} f_{i}(t) + \max_{g_{i,j}(t)} \sum_{i\neq j} g_{i,j}(t)$$
subject to
$$\begin{cases} g_{i,j}: \mathcal{R}^{m} \longrightarrow \mathcal{R}, g_{j,i}(t) = g_{i,j}(t), \quad (14) \\ \\ g_{i,j}(t) & f_{j}(t) \end{cases}$$

Lemma 6 (Schur Complement [17]). *Given constant matrices* X, Y, Z, where $X = X^{T}$ and $0 < Y = Y^{T}$, then $X + Z^{T}Y^{-1}Z < 0$ if and only if

$$\begin{bmatrix} X & Z^{\mathsf{T}} \\ * & -Y \end{bmatrix} < 0 \quad or \quad \begin{bmatrix} -Y & Z \\ * & X \end{bmatrix} < 0. \tag{15}$$

3. Finite-Time H_{∞} Performance Analysis

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Theorem 7. System (9) is finite-time bounded with respect to (c_1, c_2, d, R, T) , if there exist matrices $P_{im} > 0$, $Q_{li} > 0$ (l = 1, 2), Q > 0, $X_{im} > 0$, X > 0, $Y_{im} > 0$, Y > 0, H > 0, S_{im} , scalars $c_1 < c_2$, $T > 0\lambda_s > 0$, (s = 1, 2, ..., 10), $\eta > 0$, and $\Lambda > 0$, such that for all $i, j \in \mathcal{N}$ and $m, n \in \mathcal{M}$, the following inequalities hold:

$$\sum_{\epsilon \in \mathcal{N}, j \neq i} \lambda_{ij} \left(Q_{1j} + Q_{2j} \right) + \sum_{n \in \mathcal{M}, m \neq n} \pi_{mn} Q_{2i} - Q < 0, \quad (16)$$

$$\sum_{n \in \mathcal{M}, m \neq n} \pi_{mn} X_{in} + \sum_{j \in \mathcal{N}, i \neq j} \lambda_{ij} X_{jm} - X < 0,$$
(17)

$$\sum_{n \in \mathcal{M}, m \neq n} \pi_{mn} Y_{in} + \sum_{j \in \mathcal{N}, i \neq j} \lambda_{ij} Y_{jm} - Y < 0, \quad (18)$$
$$\begin{bmatrix} \frac{Y_{im}}{h} & S_{im} \\ * & \frac{Y_{im}}{h} \end{bmatrix} > 0, \quad (19)$$
$$\begin{bmatrix} \Xi_{11im}, \Xi_{12im}, -\overline{S_{im}}, \overline{A}^{T} V_{i}^{T}, h \overline{A}^{T} M_{i}^{T}, P_{im} \overline{D}_{i} \end{bmatrix}$$

$$\Xi_{1im} = \begin{bmatrix} 11im & 12im & -1im & 1im & 1$$

$$\Xi_{2im} = \begin{bmatrix} \Xi_{11im} & \Xi_{12im} & -S_{im} & \overline{A}_{i}^{\mathsf{T}} V_{im}^{\mathsf{T}} & h\overline{A}_{i}^{\mathsf{T}} M_{im}^{\mathsf{T}} & P_{im} \overline{D}_{i} \\ * & \Xi_{22im} & S_{im} - \frac{Y_{im}}{h} & \overline{B}_{i}^{\mathsf{T}} V_{im}^{\mathsf{T}} & h\overline{B}_{i}^{\mathsf{T}} N_{im}^{\mathsf{T}} & 0 \\ * & * & -Q_{1i} + \frac{Y_{im}}{h} & 0 & 0 & 0 \\ * & * & * & \Xi_{44im} & -hN_{im}^{\mathsf{T}} & V_{im} \overline{D}_{i} \\ * & * & * & * & -hX_{im} & hN_{im} \overline{D}_{i} \\ * & * & * & * & * & -\delta H \end{bmatrix} < 0,$$

$$(21)$$

$$c_1\Lambda + d\delta\lambda_{10}\frac{1}{\eta}\left(1 - e^{-\eta T}\right) < \lambda_1 e^{-\eta T} c_2, \qquad (22)$$

where

$$\begin{split} \Xi_{11im} &= \sum_{n \in \mathscr{M}} \pi_{mn} P_{in} + \sum_{j \in \mathscr{N}} \lambda_{ij} P_{jm} \\ &+ \delta P_{im} + P_{im} \overline{A}_i + \overline{A}_i^{\mathsf{T}} P_{im} + e^{\delta h} Q_{1i} \\ &+ e^{\delta h} Q_{2i} + h Q \frac{e^{\delta h} - 1}{\delta} X_{im} \\ &+ \frac{e^{\delta h} - \delta h e^{\delta h} - 1}{\delta^2} X + \frac{Y_{im}}{h}, \\ \Xi_{12im} &= P_{im} \overline{B}_i - \frac{Y_{im}}{h} + S_{im}, \\ \Xi_{22im} &= r \left(\mu_m\right) Q_{2i} + \frac{2Y_{im}}{h} - S_{im} - S_{im}^{\mathsf{T}}, \\ \Xi_{44im} &= -V_{im} - V_{im}^{\mathsf{T}} + \frac{e^{\delta h} - 1}{\delta} Y_{im} + \frac{e^{\delta h} - \delta h e^{\delta h} - 1}{\delta^2} Y, \\ r \left(\tau_m\right) &= \begin{cases} -\left(1 - \mu_m\right) e^{\delta h_i}, & \text{if } \mu_m > 1, \\ -\left(1 - \mu_m\right), & \text{if } \mu_m \le 1, \end{cases} \\ \Lambda &= \lambda_2 + h e^{\delta h} \left(\lambda_3 + \lambda_4\right) + h^2 e^{\delta h} \left(\lambda_5 + \lambda_6 + \lambda_8\right) \\ &+ \frac{1}{2} h^3 e^{\delta h} \left(\lambda_7 + \lambda_9\right), \end{cases} \\ \lambda_1 &= \min_{i \in \mathscr{N}, m \in \mathscr{M}} \lambda_{\min} \left(\widetilde{P}_{im}\right), \qquad \lambda_2 &= \max_{i \in \mathscr{N}, m \in \mathscr{M}} \lambda_{\max} \left(\widetilde{P}_{im}\right), \end{split}$$

$$\lambda_{3} = \max_{i \in \mathcal{N}} \lambda_{\max} \left(\widetilde{Q}_{1i} \right),$$

$$\lambda_{4} = \max_{i \in \mathcal{N}} \lambda_{\max} \left(\widetilde{Q}_{2i} \right), \qquad \lambda_{5} = \lambda_{\max} \left(\widetilde{Q} \right),$$

$$\lambda_{6} = \max_{i \in \mathcal{N}, m \in \mathcal{M}} \lambda_{\max} \left(\widetilde{X}_{im} \right), \qquad \lambda_{7} = \lambda_{\max} \left(\widetilde{X} \right),$$

$$\lambda_{8} = \max_{i \in \mathcal{N}, m \in \mathcal{M}} \lambda_{\max} \left(\widetilde{Y}_{im} \right), \qquad \lambda_{9} = \lambda_{\max} \left(\widetilde{Y} \right),$$

$$\lambda_{10} = \lambda_{\max} \left(H \right),$$

$$\widetilde{P}_{im} = R^{-(1/2)} P_{im} R^{-(1/2)},$$

$$\widetilde{Q}_{li} = R^{-(1/2)} Q_{li} R^{-(1/2)} \left(l = 1, 2 \right),$$

$$\widetilde{Q} = R^{-(1/2)} Q R^{-(1/2)}, \qquad \widetilde{X}_{im} = R^{-(1/2)} X_{im} R^{-(1/2)},$$

$$\widetilde{X} = R^{-(1/2)} X R^{-(1/2)}, \qquad \widetilde{Y}_{im} = R^{-(1/2)} Y_{im} R^{-(1/2)},$$

$$\widetilde{Y} = R^{-(1/2)} Y R^{-(1/2)}.$$
(23)

Proof. First, in order to cast our model into the framework of the Markov processes, we define a new process $\{(\eta_t, r_t, s_t), t \ge 0\}$ by

$$\eta_t(s) = \eta(t+s), \quad s \in [-h,0].$$
 (24)

Now, we consider the following Lyapunov-Krasovskii functional:

$$V(\eta_{t}, r_{t}, s_{t}) = \sum_{l=1}^{4} V_{l}(\eta_{t}, r_{t}, t), \qquad (25)$$

where

$$V_{1}(\eta_{t}, r_{t}, s_{t}) = \eta(t)^{\mathsf{T}} e^{\delta t} P_{r_{t}, s_{t}} \eta(t),$$

$$V_{2}(\eta_{t}, r_{t}, s_{t}) = \int_{t-h}^{t} e^{\delta(s+h)} \eta^{\mathsf{T}}(s) Q_{1r_{t}} \eta(s) ds$$

$$+ \int_{t-\tau_{s_{t}}(t)}^{t} e^{\delta(s+h)} \eta^{\mathsf{T}}(s) Q_{2r_{t}} \eta(s) ds$$

$$+ \int_{-h}^{0} \int_{t+\theta}^{t} e^{\delta(s+h)} \eta^{\mathsf{T}}(s) Q\eta(s) ds d\theta,$$

$$V_{3}(\eta_{t}, r_{t}, s_{t}) = \int_{-h}^{0} \int_{t+\theta}^{t} e^{\delta(s-\theta)} \eta^{\mathsf{T}}(s) X_{r_{t}, s_{t}} \eta(s) ds d\theta$$

$$+ \int_{-h}^{0} \int_{\theta}^{0} \int_{t+v}^{t} e^{\delta(s-\theta)} \eta^{\mathsf{T}}(s) X\eta(s) ds dv d\theta,$$

$$V_{4}(\eta_{t}, r_{t}, s_{t}) = \int_{-h}^{0} \int_{t+\theta}^{t} e^{\delta(s-\theta)} \dot{\eta}^{\mathsf{T}}(s) Y_{r_{t}, s_{t}} \dot{\eta}(s) ds dv d\theta.$$

$$(26)$$

Then, for each
$$r_{t} = i$$
, $s_{t} = m$, we have

$$\pounds V_{1}(\eta_{t}, i, m)$$

$$= \lim_{\Delta \to 0^{+}} \frac{1}{\Delta} \mathbb{E} \left\{ \eta^{\mathsf{T}}(t + \Delta) e^{\delta(t + \Delta)} P_{t_{t+\Delta}, t_{t+\Delta}} \eta(t + \Delta) -\eta^{\mathsf{T}}(t) e^{\delta t} P_{im\eta}(t) \right\}$$

$$= \lim_{\Delta \to 0^{+}} \frac{1}{\Delta} \mathbb{E} \left\{ \eta^{\mathsf{T}}(t + \Delta) \left[(1 + \lambda_{ii}\Delta + o(\Delta)) \times (1 + \pi_{mm}\Delta + o(\Delta)) e^{\delta(t + \Delta)} P_{im} + (1 + \lambda_{ii}\Delta + o(\Delta)) \times (\sum_{n \in \mathscr{M}} (\pi_{nm}\Delta + o(\Delta))) e^{\delta(t + \Delta)} P_{in} + (1 + \pi_{mm}\Delta + o(\Delta)) \right\}$$

$$\times \left(\sum_{n \in \mathscr{M}} (\pi_{nm}\Delta + o(\Delta)) \right) e^{\delta(t + \Delta)} P_{jm} + (\sum_{n \in \mathscr{M}} (\pi_{nm}\Delta + o(\Delta))) e^{\delta(t + \Delta)} P_{jm} + (\sum_{j \in \mathscr{N}} (\lambda_{ij}\Delta + o(\Delta))) e^{\delta(t + \Delta)} P_{jm} \right]$$

$$\times \eta(t + \Delta)$$

$$- \eta^{\mathsf{T}}(t) e^{\delta t} P_{im}\eta(t) \right\}$$

$$= \delta e^{\delta t} \eta^{\mathsf{T}}(t) P_{im}\eta(t) + 2e^{\delta t} \eta^{\mathsf{T}}(t) P_{im}\dot{\eta}(t)$$

$$+ e^{\delta t} \eta^{\mathsf{T}}(t) \left(\sum_{n \in \mathscr{M}} \pi_{mn} P_{in} + \sum_{j \in \mathscr{N}} \lambda_{ij} P_{jm} + \delta P_{im} \right)$$

$$+ P_{im}\overline{A}_{i} + \overline{A}_{i}^{\mathsf{T}} P_{im} \right) \eta(t)$$

$$+ 2e^{\delta t} \eta^{\mathsf{T}}(t) P_{im}\overline{B}_{i}\eta(t - \tau_{m}(t)) + 2e^{\delta t} \eta^{\mathsf{T}}(t) P_{im}\overline{D}_{i}\omega(t),$$
(27)

$$= \lim_{\Delta \to 0^{+}} \frac{1}{\Delta} \mathbb{E} \left\{ \int_{t+\Delta-h}^{t+\Delta} \eta^{\mathsf{T}}(s) e^{\delta(s+h)} Q_{1r_{t+\Delta}} \eta(s) \, ds - \int_{t-h}^{t} \eta^{\mathsf{T}}(s) e^{\delta(s+h)} Q_{1i} \eta(s) \, ds \right\}$$

$$\begin{split} &+ \lim_{\Delta \to 0^+} \frac{1}{\Delta} \mathbb{E} \left\{ \int_{t+\Delta-\tau_{t+\Delta}(t)}^{t+\Delta} \eta^{\mathsf{T}}(s) e^{\delta(s+h)} Q_{2t_{t+\Delta}} \eta(s) \, ds \right. \\ &\quad - \int_{t-\tau_m(t)}^t \eta^{\mathsf{T}}(s) e^{\delta(s+h)} Q_{2i} \eta(s) \, ds \right\} \\ &\quad + \lim_{\Delta \to 0^+} \frac{1}{\Delta} \mathbb{E} \left\{ \int_{-h}^0 \int_{t+\Delta+\theta}^{t+\Delta} e^{\delta(s+h)} \eta^{\mathsf{T}}(s) \, Q\eta(s) \, ds \, d\theta \right. \\ &\quad - \int_{-h}^0 \int_{t+\theta}^t e^{\delta(s+h)} \eta^{\mathsf{T}}(s) \, Q\eta(s) \, ds \, d\theta \right\} \\ &= \lim_{\Delta \to 0^+} \frac{1}{\Delta} \mathbb{E} \left\{ \int_{t+\Delta-h}^{t+\Delta} \eta^{\mathsf{T}}(s) e^{\delta(s+h)} Q_{1i} \eta(s) \, ds \right. \\ &\quad + \int_{t+\Delta-h}^{t+\Delta} \eta^{\mathsf{T}}(s) e^{\delta(s+h)} Q_{1i} \eta(s) \, ds \right. \\ &\quad + \int_{t+\Delta-h}^{t+\Delta} \eta^{\mathsf{T}}(s) Q_{1i} \eta(s) \, ds \\ &\quad + \int_{t+\Delta-\eta^{\mathsf{T}}}^{t+\Delta} \eta^{\mathsf{T}}(s) Q_{1i} \eta(s) \, ds \\ &\quad + \int_{t+\Delta-\tau_i(t+\Delta)}^{t+\Delta} \eta^{\mathsf{T}}(s) e^{\delta(s+h)} Q_{2i} \eta(s) \, ds \\ &\quad + \sum_{n \in \mathcal{M}} (\pi_{nn} \Delta + o(\Delta)) \\ &\quad \times \int_{t+\Delta-\tau_n(t+\Delta)}^{t} \eta^{\mathsf{T}}(s) e^{\delta(s+h)} Q_{2i} \eta(s) \, ds \\ &\quad + \int_{t+\Delta}^{t} -\tau_j(t+\Delta)^{t+\Delta} \eta^{\mathsf{T}}(s) \\ &\quad \times \sum_{j \in \mathcal{N}} (\lambda_{ij} \Delta + o(\Delta)) \\ &\quad \times e^{\delta(s+h)} Q_{2j} \eta(s) \, ds \\ &\quad - \int_{t-\tau_m(t)}^{t} \eta^{\mathsf{T}}(s) e^{\delta(s+h)} Q_{2i} \eta(s) \, ds \\ &\quad + \int_{t+\Delta}^{t} 0 (s) e^{\delta(s+h)} Q_{2i} \eta(s) \, ds \\ &\quad + \int_{t+\Delta}^{t} 0 (s) e^{\delta(s+h)} Q_{2i} \eta(s) \, ds \\ &\quad + \int_{t+\Delta}^{t} 0 (s) e^{\delta(s+h)} Q_{2i} \eta(s) \, ds \\ &\quad + \int_{t-\tau_m(t)}^{t} 0 (s) e^{\delta(s+h)} Q_{2i} \eta(s) \, ds \\ &\quad + e^{\delta t} \eta^{\mathsf{T}}(t) h Q \eta(t) - \int_{t-h} e^{\delta(s+h)} \eta^{\mathsf{T}}(s) Q \eta(s) \, ds \\ &\leq e^{\delta t} \eta^{\mathsf{T}}(t) (e^{\delta h} Q_{1i} + e^{\delta h} Q_{2i} + hQ) \eta(t) \\ &\quad - e^{\delta t} \eta^{\mathsf{T}}(t-h) Q_{1i} \eta(t-h) \\ &\quad - (1-\tau_m(t)) e^{\delta t} \eta^{\mathsf{T}}(t-\tau_m(t)) Q_{2i} \eta(t-\tau_m(t)) \\ &\quad + \int_{t-h}^{t} e^{\sigma(s+h)} \eta^{\mathsf{T}}(s) \left(\sum_{j \in \mathcal{N}} \lambda_{ij} (Q_{1j} + Q_{2j}) \right) \end{split}$$

$$+\sum_{n\in\mathcal{M},m\neq n}\pi_{mn}Q_{2i}-Q\right)$$

 $\times \eta(s) ds.$

(28)

Since

$$0 \le \tau_m(t) \le h_m,\tag{29}$$

we define

$$r(\tau_m) = \begin{cases} -(1-\mu_m) e^{\delta h_m}, & \text{if } \mu_m > 1, \\ -(1-\mu_m), & \text{if } \mu_m \le 1. \end{cases}$$
(30)

Then,

$$\pounds V_{2}(\eta_{t}, i, m) \leq e^{\delta t} \eta^{\mathsf{T}}(t) \left(e^{\delta h} Q_{1i} + e^{\delta h} Q_{2i} + hQ \right) \eta(t)$$

$$- e^{\delta t} \eta^{\mathsf{T}}(t - h) Q_{1i} \eta(t - h)$$

$$+ e^{\delta t} \eta^{\mathsf{T}}(t - \tau_{m}(t)) r(\tau_{m}) Q_{2i} \eta(t - \tau_{m}(t))$$

$$+ \int_{t-h}^{t} e^{\sigma(s+h)} \eta^{\mathsf{T}}(s)$$

$$\times \left(\sum_{j \in \mathcal{N}, j \neq i} \lambda_{ij} \left(Q_{1j} + Q_{2j} \right)$$

$$+ \sum_{n \in \mathcal{M}, m \neq n} \pi_{mn} Q_{2i} - Q \right)$$

$$\times \eta(s) \, ds.$$

$$(31)$$

Similar to the previous process, we can obtain

$$\begin{split} \pounds V_{3}\left(\eta_{t}, i, m\right) &\leq \int_{-h}^{0} \int_{t+\theta}^{t} e^{\delta(s-\theta)} \eta^{\mathsf{T}}\left(s\right) \\ &\times \left(\sum_{n \in \mathscr{M}, m \neq n} \pi_{mn} X_{in} \right. \\ &\left. + \sum_{j \in \mathscr{N}, i \neq j} \lambda_{ij} X_{jm} - X\right) \\ &\times \eta\left(s\right) ds d\theta \end{split}$$
(32)
$$&+ e^{\delta t} \eta^{\mathsf{T}}\left(t\right) X_{im} \eta\left(t\right) \int_{-h}^{0} e^{-\delta v} dv \\ &\left. - e^{\delta t} \int_{t-h}^{t} \eta^{\mathsf{T}}\left(s\right) X_{im} \eta\left(s\right) ds \\ &\left. + e^{\delta t} \eta^{\mathsf{T}}\left(t\right) X\eta\left(t\right) \int_{-h}^{0} \int_{v}^{0} e^{-\delta v} d\theta dv. \end{split}$$

By using Lemma 5, it yields that

$$-\int_{t-h}^{t} \eta^{\mathsf{T}}(s) X_{im} \eta(s) ds = -\int_{t-\tau_m(t)}^{t} \eta^{\mathsf{T}}(s) X_{im} \eta(s) ds$$
$$-\int_{t-h}^{t-\tau_m(t)} \eta^{\mathsf{T}}(s) X_{im} \eta(s) ds$$
$$\leq -\tau_m(t) U_1^{\mathsf{T}} X_{im} U_1$$
$$-(h-\tau_m(t)) U_2^{\mathsf{T}} X_{im} U_2,$$
(33)

where

$$U_{1} = \frac{1}{\tau_{m}(t)} \int_{t-\tau_{m}(t)}^{t} \eta(s) ds,$$

$$U_{2} = \frac{1}{h-\tau_{m}(t)} \int_{t-h}^{t-\tau_{m}(t)} \eta(s) ds,$$

$$\lim_{\tau_{m}(t)\to 0} \frac{1}{\tau_{m}(t)} \int_{t-\tau_{m}(t)}^{t} \eta(s) ds = \eta(t),$$

$$\lim_{\tau_{m}(t)\to h} \frac{1}{h-\tau_{m}(t)} \int_{t-h}^{t-\tau_{m}(t)} \eta(s) ds = \eta(t-h).$$
(34)

From the Newton-Leibniz formula, the following equation is true for any matrices M_{im} , N_{im} , and V_{im} with appropriate dimensions:

$$(2\tau_{m}(t) U_{1}^{\mathsf{T}} M_{im} + 2(h - \tau_{m}(t)) U_{2}^{\mathsf{T}} N_{im} + 2\dot{\eta}^{\mathsf{T}}(t) V_{im})$$

$$\times \left[-\dot{\eta}(t) + \overline{A}_{i}\eta(t) + \overline{B}_{\tau i}\eta(t - \tau_{m}(t)) + \overline{D}_{i}\omega(t) \right] = 0,$$

$$\pounds V_{4}(\eta_{t}, i, m)$$

$$\leq \int_{-h}^{0} \int_{t+\theta}^{t} e^{\delta(s-\theta)} \dot{\eta}^{\mathsf{T}}(s)$$

$$\times \left(\sum_{n \in \mathscr{M}, m \neq n} \pi_{mn} Y_{in} + \sum_{j \in \mathscr{N}, i \neq j} \lambda_{ij} Y_{jm} - Y \right)$$

$$\times \dot{\eta}(s) \, ds \, d\theta$$

$$+ e^{\delta t} \dot{\eta}^{\mathsf{T}}(t) Y_{im} \dot{\eta}(t) \int_{-h}^{0} e^{-\delta v} dv$$

$$- e^{\delta t} \int_{-h}^{t} \dot{\eta}^{\mathsf{T}}(s) Y_{im} \dot{\eta}(s) \, ds$$

$$(35)$$

 $-e^{\delta t}\int_{t-h}\dot{\eta}^{\mathsf{T}}(s)Y_{im}\dot{\eta}(s)\,ds$ $+e^{\delta t}\dot{\eta}^{\mathsf{T}}(t)Y\dot{\eta}(t)\int_{-h}^{0}\int_{v}^{0}e^{-\delta v}d\theta\,dv.$

From Lemma 5, it yields that

$$\begin{aligned} &-\int_{t-h}^{t} \dot{\eta}^{\mathsf{T}}\left(s\right) Y_{im} \dot{\eta}\left(s\right) ds \\ &= -\int_{t-\tau_{m}(t)}^{t} \dot{\eta}^{\mathsf{T}}\left(s\right) Y_{im} \dot{\eta}\left(s\right) ds \\ &-\int_{t-h}^{t-\tau_{m}(t)} \dot{\eta}^{\mathsf{T}}\left(s\right) Y_{im} \dot{\eta}\left(s\right) ds \\ &\leq -\frac{h}{\tau_{m}\left(t\right)} \left[\int_{t-\tau_{m}(t)}^{t} \dot{\eta}\left(s\right) ds\right]^{\mathsf{T}} \frac{Y_{im}}{h} \left[\int_{t-\tau_{m}(t)}^{t} \dot{\eta}\left(s\right) ds \end{aligned}$$

$$-\frac{h}{h-\tau_{m}(t)}\left[\int_{t-h}^{t-\tau_{m}(t)}\dot{\eta}(s)\,ds\right]^{\mathsf{T}}\frac{Y_{im}}{h}\left[\int_{t-h}^{t-\tau_{m}(t)}\dot{\eta}(s)\,ds\right]$$

$$\leq -\left[\int_{t-\tau_{m}(t)}^{t}\dot{\eta}(s)\,ds\right]^{\mathsf{T}}\left[\frac{Y_{im}}{h}\quad S_{im}\right]_{t-\tau_{m}(t)}\left[\int_{t-h}^{t}\dot{\eta}(s)\,ds\right]^{\mathsf{T}}\left[\frac{Y_{im}}{h}\quad S_{im}\right]_{*}\left[\int_{t-h}^{t-\tau_{m}(t)}\dot{\eta}(s)\,ds\right].$$
(37)

From (25)–(37), we can eventually obtain

$$\pounds V\left(\eta_{t}, r_{t}, s_{t}\right) - \delta\omega^{\mathsf{T}}\left(t\right) H\omega\left(t\right) \le e^{\delta t} \xi^{\mathsf{T}}\left(t\right) \overline{\Xi}_{im} \xi\left(t\right), \quad (38)$$

where

$$\xi^{\mathsf{T}}(t) = \left[\eta^{\mathsf{T}}(t), \eta^{\mathsf{T}}(t-\tau_{m}(t)), \eta^{\mathsf{T}}(t-h), \dot{\eta}^{\mathsf{T}}(t), U_{1}^{\mathsf{T}}, U_{2}^{\mathsf{T}}, \omega^{\mathsf{T}}(t)\right],$$

$$\Xi_{11im} \Xi_{12im} -S_{im} A_{i}^{\mathsf{T}} V_{im}^{\mathsf{T}} \tau_{m}(t) A_{i}^{\mathsf{T}} M_{im}^{\mathsf{T}} (h-\tau_{m}(t)) A_{i}^{\mathsf{T}} N_{im}^{\mathsf{T}} P_{im} D_{i}$$

$$* \Xi_{22im} S_{im} - \frac{Y_{im}}{h} A_{\tau i}^{\mathsf{T}} V_{im}^{\mathsf{T}} \tau_{m}(t) A_{\tau i}^{\mathsf{T}} M_{im}^{\mathsf{T}} (h-\tau_{m}(t)) A_{\tau i}^{\mathsf{T}} N_{im}^{\mathsf{T}} 0$$

$$* * -Q_{1i} + \frac{Y_{im}}{h} 0 0 0 0 0$$

$$* * * \Xi_{44im} -\tau_{m}(t) M_{im}^{\mathsf{T}} - (h-\tau_{m}(t)) N_{im}^{\mathsf{T}} V_{im} D_{i}$$

$$* * * * * -\tau_{m}(t) X_{im} 0 \tau_{m}(t) M_{im} D_{i}$$

$$* * * * * * -\tau_{m}(t) X_{im} 0 - \tau_{m}(t) N_{im} D_{i}$$

$$* * * * * * -\tau_{m}(t) X_{im} (h-\tau_{m}(t)) N_{im} D_{i}$$

The LMIs (20) and (21) lead to $\tau_m(t) \to h$ and $\tau_m(t) \to 0$, respectively. It is easy to see that Ξ_{1im} results from $\overline{\Xi}_{im} | \tau_m(t) = h$ and $\overline{\Xi}_{im} | \tau_m(t) = 0$. Thus, we obtain

$$\mathbb{E}\left\{ \pounds V\left(\eta_{t}, r_{t}, s_{t}\right) \right\} \leq \eta \mathbb{E}\left[V\left(\eta_{t}, r_{t}, s_{t}\right) \right] + \delta \omega^{\mathsf{T}}\left(t\right) H \omega\left(t\right).$$

$$\tag{40}$$

Multiplying the aforementioned inequality by $e^{-\eta t},$ we can get

$$\mathbb{E}\left\{ \pounds\left[e^{-\eta t}V\left(\eta_{t}, r_{t}, s_{t}\right)\right]\right\} \leq e^{-\eta t}\delta\omega^{\mathsf{T}}\left(t\right)H\omega\left(t\right).$$

$$(41)$$

By integrating the aforementioned inequality between 0 and *t*, it follows that

$$e^{-\eta t} \mathbb{E} \left[V \left(\eta_t, r_t, s_t \right) \right] - \mathbb{E} \left[V \left(\eta_0, r_0, s_0 \right) \right]$$

$$\leq \delta \int_0^t e^{-\eta s} \omega^{\mathsf{T}} \left(s \right) H \omega \left(s \right) ds.$$
(42)

Denote that $\widetilde{P}_{im} = R^{-(1/2)}P_{im}R^{-(1/2)}$, $\widetilde{Q}_{si} = R^{-(1/2)}Q_{si}R^{-(1/2)}$ (s = 1, 2), $\widetilde{Q} = R^{-(1/2)}QR^{-(1/2)}$, $\widetilde{X}_{im} = R^{-(1/2)}X_{im}R^{-(1/2)}$, $\widetilde{X} = R^{-(1/2)}XR^{-(1/2)}$,

 $\tilde{Y}_{im} = R^{-(1/2)} Y_{im} R^{-(1/2)}$, and $\tilde{Y} = R^{-(1/2)} Y R^{-(1/2)}$; it yields that

$$\begin{split} & \mathbb{E}\left[V\left(\eta_{0}, r_{0}, s_{0}\right)\right] \\ &\leq \max_{i \in \mathcal{N}, m \in \mathscr{M}} \lambda_{\max}\left(\widetilde{P}_{im}\right)\eta^{\mathsf{T}}\left(0\right) R\eta\left(0\right) \\ &+ \left(\max_{i \in \mathcal{N}} \lambda_{\max}\left(\widetilde{Q}_{1i}\right) + \max_{i \in \mathcal{N}} \lambda_{\max}\left(\widetilde{Q}_{2i}\right)\right) e^{\delta h} \\ &\times \int_{-h}^{0} e^{\delta s} \eta^{\mathsf{T}}\left(s\right) R\eta\left(s\right) ds \\ &+ e^{\delta h} \lambda_{\max}\left(\widetilde{Q}\right) \int_{-h}^{0} \int_{\theta}^{0} e^{\delta s} \eta^{\mathsf{T}}\left(s\right) R\eta\left(s\right) ds \\ &+ e^{\delta h} \max_{i \in \mathcal{N}, m \in \mathscr{M}} \lambda_{\max}\left(\widetilde{X}_{im}\right) \\ &\times \int_{-h}^{0} \int_{\theta}^{0} e^{-\delta \theta} \eta^{\mathsf{T}}\left(s\right) R\eta\left(s\right) ds d\theta \\ &+ e^{\delta h} \lambda_{\max}\left(\widetilde{X}\right) \\ &\times \int_{-h}^{0} \int_{\theta}^{0} \int_{v}^{0} e^{-\delta \theta} \eta^{\mathsf{T}}\left(s\right) R\eta\left(s\right) ds d\theta dv \\ &+ e^{\delta h} \max_{i \in \mathcal{N}, m \in \mathscr{M}} \lambda_{\max}\left(\widetilde{Y}_{im}\right) \end{split}$$

$$\times \int_{-h}^{0} \int_{\theta}^{0} e^{-\delta\theta} \dot{\eta}^{\mathsf{T}}(s) R\dot{\eta}(s) ds d\theta$$

$$+ e^{\delta h} \lambda_{\max}\left(\tilde{Y}\right)$$

$$\times \int_{-h}^{0} \int_{\theta}^{0} \int_{v}^{0} e^{-\delta\theta} \dot{\eta}^{\mathsf{T}}(s) R\dot{\eta}(s) ds d\theta dv$$

$$\leq \left\{ \max_{i \in \mathcal{N}, m \in \mathcal{M}} \lambda_{\max}\left(\tilde{P}_{im}\right)$$

$$+ he^{\delta h} \left(\max_{i \in \mathcal{N}} \lambda_{\max}\left(Q_{1i} + \max_{i \in \mathcal{N}} \max\lambda_{\max}\left(Q_{2i}\right)\right) \right)$$

$$+ h^{2} e^{\delta h} \left(\max_{i \in \mathcal{N}, m \in \mathcal{M}} \lambda_{\max}\left(\tilde{X}_{im}\right) + \lambda_{\max}\left(\tilde{Q}\right)$$

$$+ \max_{i \in \mathcal{N}, m \in \mathcal{M}} \lambda_{\max}\left(\tilde{X}_{im}\right) \right)$$

$$+ \frac{1}{2} h^{3} e^{\delta h} \left(\lambda_{\max}\left(\tilde{X}\right) + \lambda_{\max}\left(\tilde{Y}_{im}\right) \right)$$

$$\times \sup_{-h \leq s \leq 0} \left\{ \eta^{\mathsf{T}}(s) R\eta(s), \dot{\eta}^{\mathsf{T}}(s) R\dot{\eta}(s) \right\}$$

$$= c_{1} \Lambda.$$

$$(43)$$

For given $\eta > 0$ and $0 \le t \le T$, we have

$$\mathbb{E}\left[V\left(\eta_{t}, r_{t}, s_{t}\right)\right] \leq \mathbb{E}\left[e^{\eta t}V\left(\eta_{0}, r_{0}, s_{0}\right)\right] \\ + e^{\eta t}\delta\int_{0}^{t}e^{-\eta s}\omega^{\mathsf{T}}\left(s\right)H\omega\left(s\right)ds \\ \leq e^{\eta T}c_{1}\Lambda + d\delta e^{\eta T}\lambda_{\max}\left(H\right)\int_{0}^{T}e^{-\eta s}ds \qquad (44) \\ \leq e^{\eta T}\left\{c_{1}\Lambda + d\delta\lambda_{10}\frac{1}{\eta}\left(1 - e^{-\eta T}\right)\right\}.$$

On the other hand, it follows from (25) that

$$\mathbb{E}\left[V\left(\eta_{t}, r_{t}, s_{t}\right)\right] \geq \mathbb{E}\left[\eta^{\mathsf{T}}\left(t\right) e^{\lambda t} P_{im}\eta\left(t\right)\right]$$
$$\geq \max_{i \in \mathcal{N}} \lambda_{\min}\left(P_{im}\right) \mathbb{E}\left[\eta^{\mathsf{T}}\left(t\right) R\eta\left(t\right)\right] \qquad (45)$$
$$= \lambda_{1} \mathbb{E}\left[\eta^{\mathsf{T}}\left(t\right) R\eta\left(t\right)\right].$$

It can be derived from (43)-(45) that

$$\mathbb{E}\left[\eta^{\mathsf{T}}(t)\,R\eta(t)\right] \leq \frac{e^{\eta T}}{\lambda_{1}}\left\{c_{1}\Lambda + d\delta\lambda_{10}\frac{1}{\eta}\left(1 - e^{-\eta T}\right)\right\}.$$
 (46)

From (22) and (46), we have

$$\mathbb{E}\left[\eta^{\mathsf{T}}\left(t\right)R\eta\left(t\right)\right] < c_{2}.\tag{47}$$

Then, the system is finite-time bounded with respect to (c_1, c_2, d, R, T) .

Remark 8. In this paper, $\tau_m(t)$ and $\dot{\tau}_m(t)$ may have different upper bounds in various delay intervals satisfying (7), respectively. However, in previous work such as [20, 21], $\tau_m(t)$ and $\dot{\tau}_m(t)$ are enlarged to $\tau_m(t) \leq h = \max\{h_i, m \in \mathcal{M}\}$ and $\dot{\tau}_m(t) \leq \mu = \max\{\mu_m, m \in \mathcal{M}\}$, respectively, which may lead to conservativeness inevitably. However, the previouse case can be taken fully into account by employing the Lyapunov-Krasovskii functional (25).

Remark 9. When dealing with term $-\int_{t-h}^{t} \dot{\eta}^{\mathsf{T}}(s)Y_{im}\dot{\eta}(s)ds$, the convex combination is not employed, Lemma 5 is used in this paper, and then the free-weighting matrices-dependent null add items are necessary to be introduced in our proof, which lead to the decrease of the number of LMIs and LMIs scalar decision variables.

Remark 10. The feature of this paper is the way to deal with the integral term. Many researchers have enlarged the derivative of the Lyapunov functional in order to deal with the integral term in mathematical operations. In this paper, we propose a novel delay-dependent sufficient criterion, which ensures that the Markovian jump system with different mode systems is finite-time stable.

Remark 11. It should be pointed out that the novelty of the Lyapunov functional (25) lies in distinct Lyapunov matrices (P_{im}, X_{im}, Y_{im}) which is chosen for different system modes i (i = 1, 2, ..., N) and m (m = 1, 2, ..., M).

Theorem 12. System (9) is finite-time bounded with respect to (c_1, c_2, d, R, T) , if there exist matrices $P_i > 0$, $Q_{li} > 0$ (l = 1, 2), Q > 0, $X_{im} > 0$, X > 0, $Y_{im} > 0$, Y > 0, S_{im} , scalars $c_1 < c_2$, T > 0, $\lambda_s > 0$ (s = 1, 2, ..., 9), $\eta > 0$, $\gamma > 0$, and $\Lambda > 0$, such that for all $i, j \in \mathcal{N}$ and $m, n \in \mathcal{M}$ and (16)–(19), the following inequalities hold:

$$\Sigma_{1im} = \begin{bmatrix} \Xi_{11im} & \Xi_{12im} & -S_{im} & \overline{A}_{i}^{\mathsf{T}} \mathbf{V}_{im}^{\mathsf{T}} & h\overline{A}_{i}^{\mathsf{T}} \mathbf{M}_{im}^{\mathsf{T}} & P_{im} \overline{D}_{i} & \overline{C}_{i}^{\mathsf{T}} \\ * & \Xi_{22im} & S_{im} - \frac{Y_{im}}{h} & \overline{B}_{i}^{\mathsf{T}} \mathbf{V}_{im}^{\mathsf{T}} & h\overline{B}_{i}^{\mathsf{T}} \mathbf{M}_{im}^{\mathsf{T}} & 0 & \overline{E}_{i}^{\mathsf{T}} \\ * & * & -Q_{1i} + \frac{Y_{im}}{h} & 0 & 0 & 0 & 0 \\ * & * & * & \Xi_{44im} & -hM_{im}^{\mathsf{T}} & V_{im} \overline{D}_{i} & 0 \\ * & * & * & * & -hX_{im} & hM_{im} \overline{D}_{i} & 0 \\ * & * & * & * & * & -hX_{im} & hM_{im} \overline{D}_{i} & 0 \\ * & * & * & * & * & * & -\gamma^{2}I & 0 \\ * & * & * & * & * & * & * & -I \end{bmatrix}$$
(48)
$$\Sigma_{2im} = \begin{bmatrix} \Xi_{11im} & \Xi_{12im} & -S_{im} & \overline{A}_{i}^{\mathsf{T}} V_{im}^{\mathsf{T}} & h\overline{A}_{i}^{\mathsf{T}} N_{im}^{\mathsf{T}} & P_{im} \overline{D}_{i} & \overline{C}_{i}^{\mathsf{T}} \\ * & \Xi_{22im} & S_{im} - \frac{Y_{im}}{h} & \overline{B}_{i}^{\mathsf{T}} V_{im}^{\mathsf{T}} & h\overline{B}_{i}^{\mathsf{T}} N_{im}^{\mathsf{T}} & 0 & \overline{E}_{i}^{\mathsf{T}} \\ * & * & -Q_{1i} + \frac{Y_{im}}{h} & 0 & 0 & 0 & 0 \\ * & * & * & * & -hX_{im} & hN_{im} \overline{D}_{i} & 0 \\ * & * & * & * & * & -hX_{im} & hN_{im} \overline{D}_{i} & 0 \\ * & * & * & * & * & -hX_{im} & hN_{im} \overline{D}_{i} & 0 \\ * & * & * & * & * & * & * & -I \end{bmatrix}$$
(49)

$$c_1\Lambda + d\delta\gamma^2 \frac{1}{\eta} \left(1 - e^{-\eta T} \right) < \lambda_1 e^{-\eta T} c_2.$$
(50)

Proof. We now consider the H_{∞} performance of system (9). Select the same Lyapunov-Krasovskii functional as Theorem 7; it yields that

$$\mathcal{L}V\left(\eta_{t}, r_{t}, s_{t}\right) + e^{\mathsf{T}}\left(t\right)e\left(t\right) - \gamma^{2}\omega^{\mathsf{T}}\left(t\right)\omega\left(t\right) \le \xi^{\mathsf{T}}\left(t\right)\Sigma_{lim}\xi\left(t\right)$$

$$(l = 1, 2).$$
(51)

It follows from (49)-(50) that

$$\mathbb{E}\left\{ \pounds V\left(\eta_{t}, r_{t}, s_{t}\right)\right\}$$

$$\leq \mathbb{E}\left[\eta V\left(\eta_{t}, r_{t}, s_{t}\right)\right] + \gamma^{2} \omega^{\mathsf{T}}\left(t\right) \omega\left(t\right) - \mathbb{E}\left[e^{\mathsf{T}}\left(t\right) e\left(t\right)\right].$$
(52)

Multiplying the aforementioned inequality by $e^{-\eta t}$, one has

$$\mathbb{E}\left\{ \pounds\left[e^{-\eta t}V\left(\eta_{t},r_{t},s_{t}\right)\right]\right\} \leq e^{-\eta t}\left[\gamma^{2}\omega^{\mathsf{T}}\left(t\right)\omega\left(t\right)-e^{\mathsf{T}}\left(t\right)e\left(t\right)\right].$$
(53)

In zero initial condition and $\mathbb{E}[V(\eta_t, r_t, s_t)] > 0$, by integrating the aforementioned inequality between 0 and T, we can get

$$\int_{0}^{T} e^{-\eta t} \left[\gamma^{2} \omega^{\mathsf{T}}(t) \omega(t) - e^{\mathsf{T}}(t) e(t) \right] dt$$

$$\leq \mathbb{E} \left\{ \int_{0}^{T} \mathfrak{t} \left[e^{-\eta t} V\left(\eta_{t}, r_{t}, s_{t}\right) \right] dt \right\} \leq V\left(\eta_{0}, r_{0}, s_{0}\right) = 0.$$
(54)

Using Dynkins formula, it results that

$$\mathbb{E}\left[\int_{0}^{T} e^{-\eta v} e^{\mathsf{T}}(v) e(v) dv\right] \leq \gamma^{2} \mathbb{E}\left[\int_{0}^{T} e^{-\eta v} \omega^{\mathsf{T}}(v) \omega(v) dv\right].$$
(55)

Then, it yields that

$$\mathbb{E}\left[\int_{0}^{T} e^{\mathsf{T}}(v) e(v) dv\right] \leq \gamma^{2} e^{\eta T} \mathbb{E}\left[\int_{0}^{T} \omega^{\mathsf{T}}(v) \omega(v) dv\right].$$
(56)

Thus, it is concluded by Definition 3 that system (9) is finite-time bounded with an H_{∞} performance γ . This completes the proof. Remark 13. From the proof process of Theorems 7 and 12, it is easy to see that neither bounding technique for crossterms nor model transformation is involved. In other words, the obtained result is expected to be less conservative.

Remark 14. The Lyapunov asymptotic stability and finitetime stability of a class of system are independent concepts. A Lyapunov asymptotic stability system may not be finitetime stability. Moreover, finite-time stability system may also not be Lyapunov asymptotic stability. There exist some results on Lyapunov stability, while finite-time stability also needs our full investigation, which was neglected by most previous work.

4. Finite-Time H_{∞} **Filtering**

Theorem 15. *System* (9) *is finite-time bounded with respect to* (c_1, c_2, d, R, T) , if there exist matrices $P_{im} > 0$, M_{im} , N_{im} , V_{im} , $Q_{li} > 0 \ (l = 1, 2), Q > 0, X_{im} > 0, X > 0, Y_{im} > 0, Y > 0,$ $\overline{A}_{fim}, \overline{B}_{fim}, \overline{C}_{fim}, \overline{M}_{Aim}, \overline{M}_{Bim}, \overline{N}_{Aim}, \overline{N}_{Bim}, \overline{V}_{Aim}, \overline{V}_{Bim}, S_{im} \\ scalars c_1 < c_2, T > 0, \sigma_s > 0 \ (s = 1, 2, \dots, 9), \ \delta > 0, \ \eta > 0,$ and $\overline{\Lambda} > 0$, such that for all $i, j \in \mathcal{N}$ and $m, n \in \mathcal{M}$, the following inequalities hold:

$$P_{im} = \begin{bmatrix} P_{1im} & P_{2im} \\ P_{2im} & P_{2im} \end{bmatrix} > 0, \qquad M_{im} = \begin{bmatrix} M_{1im} & M_{2im} \\ M_{2im} & M_{2im} \end{bmatrix}, \quad (57)$$

$$N_{im} = \begin{bmatrix} N_{1im} & N_{2im} \\ N_{2im} & N_{2im} \end{bmatrix}, \qquad V_{im} = \begin{bmatrix} V_{1im} & V_{2im} \\ V_{2im} & V_{2im} \end{bmatrix},$$
(58)

$$\Gamma_{1im} = \begin{bmatrix} I_{11im} & I_{12im} & -S_{im} & I_{14im} & I_{15im} & I_{16im} & I_{17im} \\ * & \Xi_{22im} & S_{im} - \frac{Y_{im}}{h} & \Gamma_{24im} & \Gamma_{25im} & 0 & \begin{bmatrix} C_{z}^{\mathsf{r}} \tau_i \\ 0 \end{bmatrix} \\ * & * & -Q_{1i} + \frac{Y_{im}}{h} & 0 & 0 & 0 & 0 \\ * & * & * & E_{44im} & -hM_{im}^{\mathsf{T}} & \Gamma_{46im} & 0 \\ * & * & * & * & -hX_{im} & \Gamma_{56im} & 0 \\ * & * & * & * & * & -hX_{im} & \Gamma_{56im} & 0 \\ * & * & * & * & * & * & -I \end{bmatrix} < 0,$$

$$(59)$$

$$\Gamma_{2im} = \begin{bmatrix} \Gamma_{11im} & \Gamma_{12im} & -S_{im} & \Gamma_{14im} & \Gamma_{15im}' & \Gamma_{16im} & \Gamma_{17im} \\ * & \Xi_{22im} & S_{im} - \frac{Y_{im}}{h} & \Gamma_{24im} & \Gamma_{25im}' & 0 & \begin{bmatrix} C_{z}^{\mathsf{T}} \\ 0 \end{bmatrix} \\ * & * & -Q_{1i} + \frac{Y_{im}}{h} & 0 & 0 & 0 & 0 \\ * & * & * & \Xi_{44im} & -hN_{im}^{\mathsf{T}} & \Gamma_{46im}' & 0 \\ * & * & * & * & -hX_{im} & \Gamma_{56im}' & 0 \\ * & * & * & * & * & -\gamma^2 I & 0 \\ * & * & * & * & * & * & -I \end{bmatrix} < 0,$$

where

$$\Gamma_{11im} = \sum_{n \in \mathcal{M}} \pi_{mn} P_{in} + \sum_{j \in \mathcal{N}} \lambda_{ij} P_{jm} + e^{\delta h} Q_{1i} + e^{\delta h} Q_{2i} + hQ + \frac{e^{\delta h} - 1}{\delta} X_{im} + \frac{e^{\delta h} - \delta h e^{\delta h} - 1}{\delta^2} X$$

$$+ \frac{Y_{im}}{h} + \begin{bmatrix} \delta P_{1im} + P_{1im} A_i + A_i^{\mathsf{T}} P_{1im} + \overline{B}_{fim} C_{yi} + C_{yi}^{\mathsf{T}} \overline{B}_{fim}^{\mathsf{T}} & \delta P_{2im} + \overline{A}_{fim} + \overline{A}_{fim}^{\mathsf{T}} \\ \delta P_{2im} + P_{2im} A_i + A_i^{\mathsf{T}} P_{2im} + \overline{B}_{fim} C_{yi} + C_{yi}^{\mathsf{T}} \overline{B}_{fim}^{\mathsf{T}} & \delta P_{2im} + \overline{A}_{fim} + \overline{A}_{fim}^{\mathsf{T}} \end{bmatrix},$$

$$(61)$$

$$\begin{split} \Gamma_{12im} &= -\frac{Y_{im}}{h} + S_{im} + \begin{bmatrix} P_{1im}A_{\tau i} + B_{fim}C_{y\tau i} \\ P_{2im}A_{\tau i} + B_{fim}C_{y\tau i} \end{bmatrix}, \\ \Gamma_{14im} &= \begin{bmatrix} A_{i}^{\mathsf{T}} V_{1im}^{\mathsf{T}} + C_{yi}^{\mathsf{T}} \overline{V}_{Bim}^{\mathsf{T}} \overline{V}_{Aim}^{\mathsf{T}} \\ A_{i}^{\mathsf{T}} V_{2im}^{\mathsf{T}} + C_{yi}^{\mathsf{T}} \overline{V}_{Bim}^{\mathsf{T}} h \overline{M}_{Aim}^{\mathsf{T}} \\ A_{i}^{\mathsf{T}} N_{2im}^{\mathsf{T}} + h C_{yi}^{\mathsf{T}} \overline{M}_{Bim}^{\mathsf{T}} h \overline{M}_{Aim}^{\mathsf{T}} \end{bmatrix}, \\ \Gamma_{15im} &= \begin{bmatrix} hA_{i}^{\mathsf{T}} N_{1im}^{\mathsf{T}} + hC_{yi}^{\mathsf{T}} \overline{N}_{Bim}^{\mathsf{T}} h \overline{M}_{Aim}^{\mathsf{T}} \\ hA_{i}^{\mathsf{T}} M_{2im}^{\mathsf{T}} + hC_{yi}^{\mathsf{T}} \overline{N}_{Bim}^{\mathsf{T}} h \overline{N}_{Aim}^{\mathsf{T}} \end{bmatrix}, \\ \Gamma_{15im} &= \begin{bmatrix} hA_{i}^{\mathsf{T}} N_{1im}^{\mathsf{T}} + hC_{yi}^{\mathsf{T}} \overline{N}_{Bim}^{\mathsf{T}} h \overline{N}_{Aim}^{\mathsf{T}} \\ hA_{i}^{\mathsf{T}} N_{2im}^{\mathsf{T}} + hC_{yi}^{\mathsf{T}} \overline{N}_{Bim}^{\mathsf{T}} h \overline{N}_{Aim}^{\mathsf{T}} \end{bmatrix}, \\ \Gamma_{16im} &= \begin{bmatrix} P_{1im} D_{i} + \overline{B}_{fim} D_{yi} \\ P_{2im} D_{i} + \overline{B}_{fim} D_{yi} \end{bmatrix}, \\ \Gamma_{17im} &= \begin{bmatrix} C_{zi} \\ -\overline{C}_{fim} \end{bmatrix}, \\ \Gamma_{24im} &= \begin{bmatrix} A_{\tau i}^{\mathsf{T}} V_{1im}^{\mathsf{T}} + C_{y\tau i}^{\mathsf{T}} \overline{V}_{Bim} \\ A_{\tau i}^{\mathsf{T}} V_{2im}^{\mathsf{T}} + C_{y\tau i}^{\mathsf{T}} \overline{N}_{Bim}^{\mathsf{T}} \end{bmatrix}, \\ \Gamma_{25im} &= \begin{bmatrix} hA_{\tau i}^{\mathsf{T}} M_{1im}^{\mathsf{T}} + hC_{y\tau i}^{\mathsf{T}} \overline{M}_{Bim}^{\mathsf{T}} \\ hA_{\tau i}^{\mathsf{T}} M_{2im}^{\mathsf{T}} + hC_{y\tau i}^{\mathsf{T}} \overline{N}_{Bim}^{\mathsf{T}} \end{bmatrix}, \\ \Gamma_{46im} &= \begin{bmatrix} V_{1im} D_{i} + \overline{V}_{Bim} D_{yi} \\ V_{2im} D_{i} + \overline{V}_{Bim} D_{yi} \\ hA_{\tau i} N_{2im}^{\mathsf{T}} + hC_{y\tau i}^{\mathsf{T}} \overline{N}_{Bim}^{\mathsf{T}} \end{bmatrix}, \\ \Gamma_{56im} &= \begin{bmatrix} hM_{1im} D_{i} + h \overline{M}_{Bim} D_{yi} \\ hM_{2im} D_{i} + h \overline{M}_{Bim} D_{yi} \\ hN_{2im} D_{i} + h \overline{N}_{Bim} D_{yi} \end{bmatrix}. \end{split}$$

Then, a desired filter can be chosen with parameters as

$$A_{fim} = P_{2im}^{-1} \overline{A}_{fim}, \qquad B_{fim} = P_{2im}^{-1} \overline{B}_{fim},$$

$$C_{fim} = \overline{C}_{fim}.$$
(63)

Proof. We denote that

$$P_{im} = \begin{bmatrix} P_{1im} & P_{2im} \\ P_{2im} & P_{2im} \end{bmatrix}.$$
 (64)

The term $P_{im}\overline{A}_i$ can be rewritten as

$$P_{im}\overline{A}_{i} = \begin{bmatrix} P_{1im}A_{i} + P_{2im}B_{fim}C_{yi} & P_{2im}A_{fim} \\ P_{2im}A_{i} + P_{2im}B_{fim}C_{yi} & P_{2im}A_{fim} \end{bmatrix}.$$
 (65)

Similarly, we have

$$P_{im}\overline{B}_{i} = \begin{bmatrix} P_{1im}A_{\tau i} + P_{2im}B_{fim}C_{y\tau i} \\ P_{2im}A_{\tau i} + P_{2im}B_{fim}C_{y\tau i} \end{bmatrix},$$

$$P_{im}\overline{D}_{i} = \begin{bmatrix} P_{1im}D_{i} + P_{2im}B_{fim}D_{yi} \\ P_{2im}D_{i} + P_{2im}B_{fim}D_{yi} \end{bmatrix}$$
(66)

Define $\overline{A}_{fim} = P_{2im}A_{fim}$, $\overline{B}_{fim} = P_{2im}B_{fim}$, $\overline{C}_{fim} = C_{fim}$, $\overline{M}_{Aim} = M_{2im}A_{fim}$, $\overline{M}_{Bim} = M_{2im}B_{fim}$, $\overline{N}_{Aim} = N_{2im}A_{fim}$, $\overline{N}_{Bim} = N_{2im}B_{fim}$, $\overline{V}_{Aim} = V_{2im}A_{fim}$, and $\overline{V}_{Bim} = V_{2im}B_{fim}$. Therefore, if (59) and (60) hold, system (9) is finite-time bounded with a prescribed H_{∞} performance index γ . The proof is completed.

Remark 16. In many actual applications, the minimum value of γ_{\min}^2 is of interest. In Theorem 12, with a fixed λ , γ_{\min} can be obtained through the following optimization procedure:

min
$$\gamma^2$$
 (67)
s.t. (48)-(50).

In Theorem 15, as for finite-time stability and boundedness, once the state bound c_2 is not ascertained, the minimum value $c_{2\min}$ is of interest. With a fixed λ , define $\lambda_1 = 1$; then the following optimization problem can be formulated to get minimum value $c_{2\min}$:

min
$$\varsigma \gamma^2 + (1 - \varsigma) c_2$$

s.t. (59)-(60) and (50), (68)

where ς is weighted factor, and $\varsigma \in [0, 1]$.

5. Illustrative Example

Example 17. Consider that the Markovian jump system and the delay mode switching are governed by a Markov process with the following transition rates:

$$\Lambda = \begin{bmatrix} -0.7 & 0.7 \\ 0.9 & -0.9 \end{bmatrix}, \qquad \Pi = \begin{bmatrix} -1 & 1 \\ 1.2 & -1.2 \end{bmatrix}, \tag{69}$$

as well as with the following parameters:

$$A_{1} = \begin{bmatrix} -3.5 & 0.86 \\ -0.64 & -3.25 \end{bmatrix}, \qquad A_{2} = \begin{bmatrix} -2.5 & 0.34 \\ 1.4 & -0.02 \end{bmatrix},$$
$$A_{\tau 1} = \begin{bmatrix} -0.8 & -1.3 \\ -0.7 & -2.2 \end{bmatrix}, \qquad A_{\tau 2} = \begin{bmatrix} -2.8 & 0.5 \\ -0.8 & -1.4 \end{bmatrix},$$
$$D_{1} = D_{2} = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}, \qquad C_{y1} = \begin{bmatrix} 0.4, 0.7 \end{bmatrix},$$
$$C_{y2} = \begin{bmatrix} 0.5, 0.8 \end{bmatrix}, \qquad C_{y\tau 1} = \begin{bmatrix} 0.1, 0.2 \end{bmatrix}, \qquad (70)$$
$$C_{y\tau 2} = \begin{bmatrix} 0.1, 0.1 \end{bmatrix},$$
$$D_{y1} = D_{y2} = 0.1, \qquad C_{z1} = \begin{bmatrix} 0.1, 0.05 \end{bmatrix},$$
$$C_{z2} = \begin{bmatrix} 0.2, 0.09 \end{bmatrix}, \qquad C_{z\tau 1} = \begin{bmatrix} 0.3, 0.6 \end{bmatrix},$$

 $C_{z\tau 2} = [0.4, 0.8], \qquad D_{z1} = D_{z2} = 0.05.$

Then, we choose R = I, T = 2, $c_1 = 1$, and d = 0.01; through Theorem 15, it yields that the mode-dependent filters are as follows:

$$\begin{split} A_{f11} &= \begin{bmatrix} -6.1254 & 1.5565 \\ -0.2347 & -0.3763 \end{bmatrix}, \\ A_{f12} &= \begin{bmatrix} -6.2682 & 1.4338 \\ -0.3552 & -0.5327 \end{bmatrix}, \\ A_{f21} &= \begin{bmatrix} -8.9214 & 2.4223 \\ -1.5476 & -0.6234 \end{bmatrix}, \\ A_{f22} &= \begin{bmatrix} -8.4638 & 2.8237 \\ -1.3545 & -0.4322 \end{bmatrix}, \\ B_{f11} &= \begin{bmatrix} 5.1465 & -2.8516 \\ -9.1216 & 9.5171 \end{bmatrix}, \\ B_{f12} &= \begin{bmatrix} 5.4203 & -2.3121 \\ -9.3156 & 9.6332 \end{bmatrix}, \\ B_{f21} &= \begin{bmatrix} 4.6193 & -1.1984 \\ -16.2397 & 16.3006 \end{bmatrix}, \\ B_{f22} &= \begin{bmatrix} 4.6512 & -1.0311 \\ -16.1132 & 16.2312 \end{bmatrix}, \\ C_{f11} &= \begin{bmatrix} 0.0294, -0.0397 \end{bmatrix}, \\ C_{f12} &= \begin{bmatrix} 0.0271, -0.03368 \end{bmatrix}, \\ C_{f21} &= \begin{bmatrix} 0.1163, -0.1432 \end{bmatrix}, \\ C_{f22} &= \begin{bmatrix} 0.1342, -0.3672 \end{bmatrix}. \end{split}$$

This paper deals with the finite-time filter design problem for a class of Markovian jump systems; particularly, two different Markov processes are considered for modeling the randomness of system matrix and the state delay. Then, through the numerical example, we can see that results in this paper are feasible, which further verified the correctness of our theory. Therefore, the paper shorten this gap.

6. Conclusions

In this paper, we have examined the problems of finite-time H_{∞} filtering for a class of Markovian jump systems with different system modes. Based on a novel approach, a sufficient condition is derived such that the closed-loop Markovian jump system is finite-time bounded and satisfies a prescribed level of H_{∞} disturbance attenuation in a finite time interval. Finally, a numerical example is also given to illustrate the effectiveness of the proposed design approach. It should be noted that one of future research topics would be to investigate the problems of fault detection and fault tolerant control for time-varying Markovian jump systems with incomplete information over a finite-time horizon.

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References

- N. N. Krasovskii and È. A. Lidskii, "Analytical design of controllers in systems with random attributes—I. Statement of the problem, method of solving," *Automation and Remote Control*, vol. 22, pp. 1021–1025, 1961.
- [2] J. Tian, Y. Li, J. Zhao, and S. Zhong, "Delay-dependent stochastic stability criteria for Markovian jumping neural networks with mode-dependent time-varying delays and partially known transition rates," *Applied Mathematics and Computation*, vol. 218, no. 9, pp. 5769–5781, 2012.
- [3] L. Wu, X. Su, and P. Shi, "Sliding mode control with bounded L₂ gain performance of Markovian jump singular time-delay systems," *Automatica*, vol. 48, no. 8, pp. 1929–1933, 2012.
- [4] H. Gao, Z. Fei, J. Lam, and B. Du, "Further results on exponential estimates of Markovian jump systems with mode-dependent time-varying delays," *IEEE Transactions on Automatic Control*, vol. 56, no. 1, pp. 223–229, 2011.
- [5] Z. Wu, H. Su, and J. Chu, "H_{co} filtering for singular Markovian jump systems with time delay," *International Journal of Robust* and Nonlinear Control, vol. 20, no. 8, pp. 939–957, 2010.
- [6] P. Tiňo, M. Čerňanský, and L. Beňušková, "Markovian architectural bias of recurrent neural networks," *IEEE Transactions on Neural Networks*, vol. 15, no. 1, pp. 6–15, 2004.
- [7] H. Dong, Z. Wang, D. W. C. Ho, and H. Gao, "Robust H_{co} filtering for Markovian jump systems with randomly occurring nonlinearities and sensor saturation: the finite-horizon case," *IEEE Transactions on Signal Processing*, vol. 59, no. 7, pp. 3048– 3057, 2011.
- [8] Q. Ma, S. Xu, and Y. Zou, "Stability and synchronization for Markovian jump neural networks with partly unknown transition probabilities," *Neurocomputing*, vol. 74, pp. 3404–3341, 2011.
- [9] H. Zhao, S. Xu, and Y. Zou, "Robust H_∞ filtering for uncertain Markovian jump systems with mode-dependent distributed delays," *International Journal of Adaptive Control and Signal Processing*, vol. 24, no. 1, pp. 83–94, 2010.
- [10] Y. Zhang, S. Xu, and B. Zhang, "Discrete-time fuzzy Markovian jump systems with time-varying delays," *IEEE Transactions on Fuzzy Systems*, vol. 17, no. 2, pp. 411–420, 2009.
- [11] Z. Wu, H. Su, and J. Chu, "State estimation for discrete Markovian jumping neural networks with time delay," *Neurocomputing*, vol. 73, no. 10–12, pp. 2247–2254, 2010.
- [12] Q. Zhu and J. Cao, "Robust exponential stability of markovian jump impulsive stochastic Cohen-Grossberg neural networks with mixed time delays," *IEEE Transactions on Neural Networks*, vol. 21, no. 8, pp. 1314–1325, 2010.
- [13] H. Huang, G. Feng, and X. Chen, "Stability and stabilization of Markovian jump systems with time delay via new Lyapunov functionals," *IEEE Transactions on Circuits and Systems. I*, vol. 59, no. 10, pp. 2413–2421, 2012.

- [14] Q. Zhu and J. Cao, "Stability of Markovian jump neural networks with impulse control and time varying delays," *Nonlinear Analysis. Real World Applications*, vol. 13, no. 5, pp. 2259–2270, 2012.
- [15] M. Sun, J. Lam, S. Xu, and Y. Zou, "Robust exponential stabilization for Markovian jump systems with mode-dependent input delay," *Automatica*, vol. 43, no. 10, pp. 1799–1807, 2007.
- [16] M. Mohammadian, A. Abolmasoumi, and H. Momeni, "H_{co} mode-independent filter design for Markovian jump genetic regulatory networks with time-varying delays," *Neurocomputing*, vol. 87, pp. 10–18, 2012.
- [17] S. Boyd, L. El Ghaoui, E. Feron, and V. Balakrishnan, *Linear Matrix Inequalities in System and Control Theory*, vol. 15 of *SIAM Studies in Applied Mathematics*, SIAM, Philadelphia, Pa, USA, 1994.
- [18] S. Mou, H. Gao, W. Qiang, and K. Chen, "New delay-dependent exponential stability for neural networks with time delay," *IEEE Transactions on Systems, Man, and Cybernetics Part B*, vol. 38, no. 2, pp. 571–576, 2008.
- [19] O. M. Kwon, J. Park, S. Lee, and E. Cha, "Analysis on delaydependent stability for neural networks with time-varying delays," *Neurocomputing*, vol. 103, pp. 114–120, 2013.
- [20] L. Hu, H. Gao, and W. X. Zheng, "Novel stability of cellular neural networks with interval time-varying delay," *Neural Networks*, vol. 21, no. 10, pp. 1458–1463, 2008.
- [21] H. Zhang, Z. Liu, G. B. Huang, and Z. Wang, "Novel weightingdelay-based stability criteria for recurrent neural networks with time-varying delay," *IEEE Transactions on Neural Networks*, vol. 21, no. 1, pp. 91–106, 2010.
- [22] Z. Zuo, C. Yang, and Y. Wang, "A new method for stability analysis of recurrent neural networks with interval time-varying delay," *IEEE Transactions on Neural Networks*, vol. 21, no. 2, pp. 339–344, 2010.
- [23] P. Park, J. W. Ko, and C. Jeong, "Reciprocally convex approach to stability of systems with time-varying delays," *Automatica*, vol. 47, no. 1, pp. 235–238, 2011.
- [24] D. Zhang, L. Yu, and W. Zhang, "Delay-dependent fault detection for switched linear systems with time-varying delays-the average dwell time approach," *Signal Processing*, vol. 91, pp. 832– 840, 2011.
- [25] Z. Fei, H. Gao, and P. Shi, "New results on stabilization of Markovian jump systems with time delay," *Automatica*, vol. 45, no. 10, pp. 2300–2306, 2009.
- [26] H. R. Karimi, "Robust delay-dependent H_{∞} control of uncertain time-delay systems with mixed neutral, discrete, and distributed time-delays and Markovian switching parameters," *IEEE Transactions on Circuits and Systems I*, vol. 58, no. 8, pp. 1910–1923, 2011.
- [27] M. Luo, G. Liu, and S. Zhong, "Robust fault detection of Markovian jump systems with different system modes," *Applied Mathematical Modelling*, vol. 37, no. 7, pp. 5001–5012, 2013.
- [28] B. Zhang and Y. Li, "Exponential L_2-L_{∞} filtering for distributed delay systems with Markovian jumping parameters," *Signal Processing*, vol. 93, no. 1, pp. 206–216, 2013.
- [29] A. Abolmasoumi and H. Momeni, "Robust observer-based H_{∞} control of a Markovian jump system with different delay and system modes," *International Journal of Control, Automation, and Systems*, vol. 9, no. 4, pp. 768–776, 2011.
- [30] L. Zhang, E.-K. Boukas, and J. Lam, "Analysis and synthesis of Markov jump linear systems with time-varying delays and partially known transition probabilities," *IEEE Transactions on Automatic Control*, vol. 53, no. 10, pp. 2458–2464, 2008.

- [31] S. Ma and E. K. Boukas, "Robust H_{∞} filtering for uncertain discrete Markov jump singular systems with mode-dependent time delay," *IET Control Theory & Applications*, vol. 3, no. 3, pp. 351–361, 2009.
- [32] Y. Chen, W. Bi, and W. Li, "Stability analysis for neural networks with time-varying delay: a more general delay decomposition approach," *Neurocomputing*, vol. 73, no. 4–6, pp. 853–857, 2010.
- [33] Z. Shu, J. Lam, and S. Xu, "Robust stabilization of Markovian delay systems with delay-dependent exponential estimates," *Automatica*, vol. 42, no. 11, pp. 2001–2008, 2006.
- [34] X. Huang, W. Lin, and B. Yang, "Global finite-time stabilization of a class of uncertain nonlinear systems," *Automatica*, vol. 41, no. 5, pp. 881–888, 2005.
- [35] C. Qian and J. Li, "Global finite-time stabilization by output feedback for planar systems without observable linearization," *IEEE Transactions on Automatic Control*, vol. 50, no. 6, pp. 885– 890, 2005.
- [36] F. Amato, R. Ambrosino, C. Cosentino, and G. De Tommasi, "Input-output finite time stabilization of linear systems," *Auto-matica*, vol. 46, no. 9, pp. 1558–1562, 2010.
- [37] Y. Hong, "Finite-time stabilization and stabilizability of a class of controllable systems," *Systems & Control Letters*, vol. 46, no. 4, pp. 231–236, 2002.
- [38] F. Amato and M. Ariola, "Finite-time control of discrete-time linear systems," *IEEE Transactions on Automatic Control*, vol. 50, no. 5, pp. 724–729, 2005.
- [39] Z. Zuo, Y. Liu, Y. Wang, and H. Li, "Finite-time stochastic stability and stabilisation of linear discrete-time Markovian jump systems with partly unknown transition probabilities," *IET Control Theory & Applications*, vol. 6, no. 10, pp. 1522–1526, 2012.
- [40] X. Luan, F. Liu, and P. Shi, "Finite-time filtering for non-linear stochastic systems with partially known transition jump rates," *IET Control Theory & Applications*, vol. 4, no. 5, pp. 735–745, 2010.
- [41] F. Amato, M. Ariola, and C. Cosentino, "Finite-time control of discrete-time linear systems: analysis and design conditions," *Automatica*, vol. 46, no. 5, pp. 919–924, 2010.
- [42] X. Lin, H. Du, and S. Li, "Finite-time boundedness and L₂gain analysis for switched delay systems with norm-bounded disturbance," *Applied Mathematics and Computation*, vol. 217, no. 12, pp. 5982–5993, 2011.
- [43] J. Lin, S. Fei, and Z. Gao, "Stabilization of discrete-time switched singular time-delay systems under asynchronous switching," *Journal of the Franklin Institute*, vol. 349, no. 5, pp. 1808–1827, 2012.
- [44] H. Song, L. Yu, D. Zhang, and W.-A. Zhang, "Finite-time H_{co} control for a class of discrete-time switched time-delay systems with quantized feedback," *Communications in Nonlinear Science* and Numerical Simulation, vol. 17, no. 12, pp. 4802–4814, 2012.
- [45] Z. Zuo, H. Li, Y. Liu, and Y. Wang, "On finite-time stochastic stability and stabilization of Markovian jump systems subject to partial information on transition probabilities," *Circuits Systems* and Signal Processing, 2012.
- [46] H. Liu, Y. Shen, and X. Zhao, "Delay-dependent observer-based H_∞ finite-time control for switched systems with time-varying delay," *Nonlinear Analysis. Hybrid Systems*, vol. 6, no. 3, pp. 885– 898, 2012.
- [47] Y. Zhang, C. Liu, and X. Mu, "Robust finite-time H_{∞} control of singular stochastic systems via static output feedback," *Applied Mathematics and Computation*, vol. 218, no. 9, pp. 5629–5640, 2012.

- [48] J. Cheng, H. Zhu, S. Zhong, Y. Zhang, and Y. Zeng, "Finite-time Stabilization of H_{∞} filtering for switched stochastic systems," *Circuits, Systems and Signal Processing*, 2012.
- [49] J. Cheng, H. Zhu, S. Zhong, and Y. Zhang, "Finite-time boundness of H_{∞} filtering for switching discrete-time systems," *International Journal of Control, Automation, and Systems*, vol. 10, pp. 1129–1135, 2012.
- [50] Z. Xiang, C. Qiao, and M. S. Mahmoud, "Finite-time analysis and H_{∞} control for switched stochastic systems," *Journal of the Franklin Institute*, vol. 349, no. 3, pp. 915–927, 2012.



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