

Research Article

Lyapunov-Type Inequality for a Class of Discrete Systems with Antiperiodic Boundary Conditions

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Received 23 June 2013; Accepted 16 August 2013

Academic Editor: XianHua Tang

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A class of higher-order 3-dimensional discrete systems with antiperiodic boundary conditions is investigated. Based on the existence of the positive solution of linear homogeneous system, several new Lyapunov-type inequalities are established.

1. Introduction

Lyapunov-type inequalities have been proved to be very useful in oscillation theory, disconjugacy, eigenvalue problems, and numerous other applications in the theory of differential and difference equations [1–3]. In recent years, there are many literatures which improved and extended the classical Lyapunov inequality including continuous and discrete cases [4–6]. Guseinov and Kaymakçalan [7] considered the following discrete Hamiltonian system:

$$\begin{aligned} \Delta x(t) &= a(t)x(t+1) + b(t)u(t), \\ \Delta u(t) &= -c(t)x(t+1) - a(t)u(t), \end{aligned} \quad (1)$$

where Δ denotes the forward difference operator, with the coefficients $a(t)$ satisfying the condition $1 - a(t) \neq 0$, $t \in Z$. They [7] presented some Lyapunov-type inequalities for discrete linear scalar Hamiltonian systems when the coefficient $c(t)$ is not necessarily nonnegative value. Applying these inequalities, they [7] obtained some stability criteria for discrete Hamiltonian systems.

For simplicity, the following assumptions are introduced:

$$\begin{aligned} 1 - \alpha(n) &> 0, \quad \forall n \in Z, \\ x(a) &= 0, \quad \text{or} \quad x(a)x(a+1) < 0, \\ x(b) &= 0, \quad \text{or} \quad x(b)x(b+1) < 0, \\ \max_{a \leq n \leq b} |x(n)| &> 0, \quad a, b \in Z. \end{aligned} \quad (2) \quad (3)$$

Recently, Zhang and Tang [8] also considered the discrete linear Hamiltonian system:

$$\begin{aligned} \Delta x(n) &= \alpha(n)x(n+1) + \beta(n)y(n), \\ \Delta y(n) &= -\gamma(n)x(n+1) - \alpha(n)y(n), \end{aligned} \quad (4)$$

where $\alpha(n)$, $\beta(n)$, and $\gamma(n)$ are real-valued functions defined on Z and Δ denotes the forward difference operator defined by $\Delta x(n) = x(n+1) - x(n)$, $\beta(n) \geq 0$. They [8] obtained the following interesting Lyapunov-type inequality.

Theorem A. *Suppose that (2) holds, and let $a, b \in Z$ with $a < b - 1$. Assume (4) has a real solution $(x(n), y(n))$ such that (3) holds. Then one has the following inequality:*

$$\sum_{n=a}^{b-1} |\alpha(n)| + \left[\sum_{n=a}^b \beta(n) \sum_{n=a}^{b-1} \gamma^+(n) \right]^{1/2} \geq 2. \quad (5)$$

In 2012, the following assumptions are introduced in [9].

- (H1) $r_1(n), r_2(n), f_1(n)$, and $f_2(n)$ are real-valued functions, and $r_1(n) > 0$, and $r_2(n) > 0$.
- (H2) $1 < p_1, p_2 < \infty, \alpha_1, \alpha_2, \beta_1, \beta_2 > 0$ satisfy $\alpha_1/p_1 + \alpha_2/p_2 = 1$ and $\beta_1/p_1 + \beta_2/p_2 = 1$.
- (H3) $r_i(n)$ and $f_i(n)$ are real-valued functions and $r_i(n) > 0$ for $i = 1, 2, \dots, m$. Furthermore, $1 < p_i < \infty$ and $\alpha_i(n) > 0$ satisfy $\sum_{i=1}^m (\alpha_i/p_i) = 1$.

Under the boundary value conditions, Zhang and Tang [9] considered the following quasilinear difference systems with hypotheses (H1) and (H2):

$$\begin{aligned}
 & -\Delta \left(r_1(n) |\Delta u(n)|^{p_1-2} \Delta u(n) \right) \\
 & = f_1(n) |u(n+1)|^{\alpha_1-2} |v(n+1)|^{\alpha_2} u(n+1), \\
 & -\Delta \left(r_2(n) |\Delta v(n)|^{p_1-2} \Delta v(n) \right) \\
 & = f_2(n) |u(n+1)|^{\beta_1} |v(n+1)|^{\beta_2-2} v(n+1),
 \end{aligned} \tag{6}$$

and the quasilinear difference systems involving the (p_1, p_2, \dots, p_m) -Laplacian:

$$\begin{aligned}
 & -\Delta \left(r_1(n) |\Delta u_1(n)|^{p_1-2} \Delta u_1(n) \right) \\
 & = f_1(n) |u_1(n+1)|^{\alpha_1-2} \\
 & \quad \times |u_2(n+1)|^{\alpha_2} \cdots |u_m(n+1)|^{\alpha_m} u_1(n+1), \\
 & -\Delta \left(r_2(n) |\Delta u_2(n)|^{p_2-2} \Delta u_2(n) \right) \\
 & = f_2(n) |u_1(n+1)|^{\alpha_1} \\
 & \quad \times |u_2(n+1)|^{\alpha_2-2} \cdots |u_m(n+1)|^{\alpha_m} u_2(n+1), \\
 & \vdots \\
 & -\Delta \left(r_m(n) |\Delta u_m(n)|^{p_m-2} \Delta u_m(n) \right) \\
 & = f_m(n) |u_1(n+1)|^{\alpha_1} \\
 & \quad \times |u_2(n+1)|^{\alpha_2} \cdots |u_m(n+1)|^{\alpha_m-2} u_m(n+1).
 \end{aligned} \tag{7}$$

Some Lyapunov-type inequalities are established in [9].

Recently, antiperiodic problems have received considerable attention as antiperiodic boundary conditions appear in numerous situations [10–12]. For the sake of convenience, in this paper, one will only consider the following higher-order 3-dimensional discrete system:

$$\begin{aligned}
 & |\Delta^m x(n)|^{p_1-2} \Delta^m x(n) \\
 & + f_1(n) \psi_{q_{1,1}}(x(n)) \psi_{q_{1,2}}(y(n)) \psi_{q_{1,3}}(z(n)) = 0, \\
 & |\Delta^m y(n)|^{p_2-2} \Delta^m y(n) \\
 & + f_2(n) \psi_{q_{2,1}}(x(n)) \psi_{q_{2,2}}(y(n)) \psi_{q_{2,3}}(z(n)) = 0, \\
 & |\Delta^m z(n)|^{p_3-2} \Delta^m z(n) \\
 & + f_3(n) \psi_{q_{3,1}}(x(n)) \psi_{q_{3,2}}(y(n)) \psi_{q_{3,3}}(z(n)) = 0,
 \end{aligned} \tag{8}$$

where $1 < p_k < +\infty$ for $k = 1, 2, 3$; $q_{i,j}$ are nonnegative constants for $i, j = 1, 2, 3$; $\psi_q(u) = |u|^{q-1}u$ for $q > 0$ with $\psi_0(u) = \text{sign}(u) = \pm 1$ for $q = 0$.

Obviously, the results obtained in [9] required that $\alpha_1/p_1 + \alpha_2/p_2 = 1$ and $\beta_1/p_1 + \beta_2/p_2 = 1$ or $\sum_{i=1}^m (\alpha_i/p_i) = 1$. The

order of the quasilinear difference systems considered in [9] is less than 3. In this paper, one will remove the unreasonably severe constraints $\alpha_1/p_1 + \alpha_2/p_2 = 1$ and $\beta_1/p_1 + \beta_2/p_2 = 1$ or $\sum_{i=1}^m (\alpha_i/p_i) = 1$ in [9]. one will introduce the antiperiodic boundary conditions instead of boundary conditions in [9]. In this paper, one will establish some new Lyapunov-type inequalities for higher-order 3-dimensional discrete system (8) by a method different from that in [9] under the following antiperiodic boundary conditions:

$$\begin{aligned}
 \Delta^i x(a) + \Delta^i x(b) & = \Delta^i y(a) + \Delta^i y(b) \\
 & = \Delta^i z(a) + \Delta^i z(b) = 0, \\
 i & = 0, 1, \dots, m-1.
 \end{aligned} \tag{9}$$

The similar results for higher-order m -dimensional discrete system are easy to obtain.

Throughout this paper, $p_i > 1$ and p'_i is a conjugate exponent; that is, $1/p_i + 1/p'_i = 1, i = 1, 2, 3$.

2. Main Results

Theorem 1. Let $a < b$, and assume that there exists a positive solution (e_1, e_2, e_3) of the following linear homogeneous system:

$$\begin{aligned}
 (q_{1,1} + 1 - p_1) e_1 + q_{2,1} e_2 + q_{3,1} e_3 & = 0, \\
 q_{1,2} e_1 + (q_{2,2} + 1 - p_2) e_2 + q_{3,2} e_3 & = 0, \\
 q_{1,3} e_1 + q_{2,3} e_2 + (q_{3,3} + 1 - p_3) e_3 & = 0.
 \end{aligned} \tag{10}$$

If $(x(n), y(n), z(n))$ is a nonzero solution of (8) satisfying the antiperiodic boundary conditions (9), then

$$\begin{aligned}
 & \prod_{k=1}^3 \left(\sum_{n=a}^{b-1} |f_k(n)|^{p_k/(p_k-1)} \right)^{(1-1/p_k)e_k} \\
 & \geq (b-a)^{\sum_{i=1}^3 \sum_{j=1}^3 (q_{i,j}/p_j)e_i} \left(\frac{2}{b-a} \right)^{m \sum_{i=1}^3 (p_i-1)e_i}.
 \end{aligned} \tag{11}$$

Proof. Let $(x(n), y(n), z(n))$ be a nonzero solution of (8). By the antiperiodic boundary conditions (9), $x(a) + x(b) = 0$. For $n \in Z[a, b]$, we have

$$\begin{aligned}
 x(n) & = \frac{1}{2} \sum_{k=a}^{n-1} [x(k+1) - x(k)] - \frac{1}{2} \sum_{k=n}^{b-1} [x(k+1) - x(k)] \\
 & = \frac{1}{2} \sum_{k=a}^{n-1} \Delta x(k) - \frac{1}{2} \sum_{k=n}^{b-1} \Delta x(k).
 \end{aligned} \tag{12}$$

Using discrete Hölder inequality gives

$$\begin{aligned}
 |x(n)| & \leq \frac{1}{2} \sum_{k=a}^{b-1} |\Delta x(k)| \\
 & \leq \frac{1}{2} (b-a)^{1/p'_1} \left(\sum_{k=a}^{b-1} |\Delta x(k)|^{p_1} \right)^{1/p_1}.
 \end{aligned} \tag{13}$$

Similarly,

$$\begin{aligned}
 |\Delta^i x(n)| &\leq \frac{1}{2} \sum_{k=a}^{b-1} |\Delta^{i+1} x(k)| \\
 &\leq \frac{1}{2} (b-a)^{1/p'_1} \left(\sum_{k=a}^{b-1} |\Delta^{i+1} x(k)|^{p_1} \right)^{1/p_1}. \tag{14}
 \end{aligned}$$

Then

$$|\Delta^i x(n)|^{p_1} \leq \left(\frac{1}{2}\right)^{p_1} (b-a)^{p_1/p'_1} \left(\sum_{k=a}^{b-1} |\Delta^{i+1} x(k)|^{p_1} \right). \tag{15}$$

Summing (15) from a to $b-1$, we have

$$\begin{aligned}
 \sum_{n=a}^{b-1} |\Delta^i x(n)|^{p_1} \\
 \leq (b-a) \left(\frac{1}{2}\right)^{p_1} (b-a)^{p'_1} \left(\sum_{k=a}^{b-1} |\Delta^{i+1} x(k)|^{p_1} \right); \tag{16}
 \end{aligned}$$

that is,

$$\left(\sum_{k=a}^{b-1} |\Delta^i x(k)|^{p_1} \right)^{1/p_1} \leq \frac{b-a}{2} \left(\sum_{k=a}^{b-1} |\Delta^{i+1} x(k)|^{p_1} \right)^{1/p_1}. \tag{17}$$

So

$$\begin{aligned}
 |x(n)| &\leq \frac{1}{2} (b-a)^{1/p'_1} \left(\sum_{k=a}^{b-1} |\Delta x(k)|^{p_1} \right)^{1/p_1} \\
 &\leq \frac{1}{2} (b-a)^{1/p'_1} \left(\frac{b-a}{2}\right)^{m-1} \left(\sum_{k=a}^{b-1} |\Delta^m x(k)|^{p_1} \right)^{1/p_1}. \tag{18}
 \end{aligned}$$

Similarly,

$$|y(n)| \leq \frac{1}{2} (b-a)^{1/p'_2} \left(\frac{b-a}{2}\right)^{m-1} \left(\sum_{k=a}^{b-1} |\Delta^m y(k)|^{p_2} \right)^{1/p_2}, \tag{19}$$

$$|z(n)| \leq \frac{1}{2} (b-a)^{1/p'_3} \left(\frac{b-a}{2}\right)^{m-1} \left(\sum_{k=a}^{b-1} |\Delta^m z(k)|^{p_3} \right)^{1/p_3}. \tag{20}$$

Multiplying the first equation of (8) by $\Delta^m x(n)$ and using inequalities (18)–(20), we have

$$\begin{aligned}
 &|\Delta^m x(n)|^{p_1} \\
 &= |-f_1(n) \psi_{q_{1,1}}(x(n)) \psi_{q_{1,2}}(y(n)) \psi_{q_{1,3}}(z(n)) \Delta^m x(n)| \\
 &= |f_1(n)| |x(n)|^{q_{1,1}} |y(n)|^{q_{1,2}} |z(n)|^{q_{1,3}} |\Delta^m x(n)| \\
 &\leq \left[\frac{1}{2} (b-a)^{1/p'_1} \left(\frac{b-a}{2}\right)^{m-1} \right. \\
 &\quad \times \left. \left(\sum_{k=a}^{b-1} |\Delta^m x(k)|^{p_1} \right)^{1/p_1} \right]^{q_{1,1}} \\
 &\quad \times \left[\frac{1}{2} (b-a)^{1/p'_2} \left(\frac{b-a}{2}\right)^{m-1} \right. \\
 &\quad \times \left. \left(\sum_{k=a}^{b-1} |\Delta^m y(k)|^{p_2} \right)^{1/p_2} \right]^{q_{1,2}} \\
 &\quad \times \left[\frac{1}{2} (b-a)^{1/p'_3} \left(\frac{b-a}{2}\right)^{m-1} \right. \\
 &\quad \times \left. \left(\sum_{k=a}^{b-1} |\Delta^m z(k)|^{p_3} \right)^{1/p_3} \right]^{q_{1,3}} \\
 &\quad \times |f_1(n)| |\Delta^m x(n)|. \tag{21}
 \end{aligned}$$

Then

$$\begin{aligned}
 &\sum_{n=a}^{b-1} |\Delta^m x(n)|^{p_1} \\
 &\leq (b-a)^{-\sum_{j=1}^3 (q_{1,j}/p_j)} \left(\frac{b-a}{2}\right)^{m(\sum_{j=1}^3 q_{1,j})} \\
 &\quad \times \left(\sum_{k=a}^{b-1} |\Delta^m x(k)|^{p_1} \right)^{q_{1,1}/p_1} \left(\sum_{k=a}^{b-1} |\Delta^m y(k)|^{p_2} \right)^{q_{1,2}/p_2} \\
 &\quad \times \left(\sum_{k=a}^{b-1} |\Delta^m z(k)|^{p_3} \right)^{q_{1,3}/p_3} \sum_{n=a}^{b-1} |f_1(n)| |\Delta^m x(n)| \\
 &\leq (b-a)^{-\sum_{j=1}^3 (q_{1,j}/p_j)} \left(\frac{b-a}{2}\right)^{m(\sum_{j=1}^3 q_{1,j})} \\
 &\quad \times \left(\sum_{k=a}^{b-1} |\Delta^m x(k)|^{p_1} \right)^{q_{1,1}/p_1} \left(\sum_{k=a}^{b-1} |\Delta^m y(k)|^{p_2} \right)^{q_{1,2}/p_2}
 \end{aligned}$$

$$\begin{aligned} & \times \left(\sum_{k=a}^{b-1} |\Delta^m z(k)|^{p_3} \right)^{q_{1,3}/p_3} \left(\sum_{n=a}^{b-1} |f_1(n)|^{p'_1} \right)^{1/p'_1} \\ & \times \left(\sum_{n=a}^{b-1} |\Delta^m x(n)|^{p_1} \right)^{1/p_1}. \end{aligned} \tag{22}$$

So

$$\begin{aligned} & \left(\sum_{k=a}^{b-1} |\Delta^m x(k)|^{p_1} \right)^{(q_{1,1}+1)/p_1-1} \left(\sum_{k=a}^{b-1} |\Delta^m y(k)|^{p_2} \right)^{q_{1,2}/p_2} \\ & \times \left(\sum_{k=a}^{b-1} |\Delta^m z(k)|^{p_3} \right)^{q_{1,3}/p_3} \left(\sum_{n=a}^{b-1} |f_1(n)|^{p'_1} \right)^{1/p'_1} \\ & \geq (b-a)^{\sum_{j=1}^3 (q_{1,j}/p_j)} \left(\frac{2}{b-a} \right)^{m(\sum_{j=1}^3 q_{1,j})}. \end{aligned} \tag{23}$$

For the second and third equations of (8), we also have

$$\begin{aligned} & \left(\sum_{k=a}^{b-1} |\Delta^m x(k)|^{p_1} \right)^{q_{2,1}/p_1} \left(\sum_{k=a}^{b-1} |\Delta^m y(k)|^{p_2} \right)^{(q_{2,2}+1)/p_2-1} \\ & \times \left(\sum_{k=a}^{b-1} |\Delta^m z(k)|^{p_3} \right)^{q_{2,3}/p_3} \left(\sum_{n=a}^{b-1} |f_2(n)|^{p'_2} \right)^{1/p'_2} \\ & \geq (b-a)^{\sum_{j=1}^3 (q_{2,j}/p_j)} \left(\frac{2}{b-a} \right)^{m(\sum_{j=1}^3 q_{2,j})}, \end{aligned} \tag{24}$$

$$\begin{aligned} & \left(\sum_{k=a}^{b-1} |\Delta^m x(k)|^{p_1} \right)^{q_{3,1}/p_1} \left(\sum_{k=a}^{b-1} |\Delta^m y(k)|^{p_2} \right)^{q_{3,2}/p_2} \\ & \times \left(\sum_{k=a}^{b-1} |\Delta^m z(k)|^{p_3} \right)^{(q_{3,3}+1)/p_3-1} \left(\sum_{n=a}^{b-1} |f_3(n)|^{p'_3} \right)^{1/p'_3} \\ & \geq (b-a)^{\sum_{j=1}^3 (q_{3,j}/p_j)} \left(\frac{2}{b-a} \right)^{m(\sum_{j=1}^3 q_{3,j})}. \end{aligned} \tag{25}$$

Raising both sides of inequalities (23)–(25) to the powers e_1, e_2 , and e_3 , respectively, and multiplying the resulting inequalities give

$$\begin{aligned} & \left(\sum_{k=a}^{b-1} |\Delta^m x(k)|^{p_1} \right)^{(\sum_{i=1}^3 q_{i,1}e_i)/p_1+(1-p_1)e_1/p_1} \\ & \times \left(\sum_{k=a}^{b-1} |\Delta^m y(k)|^{p_2} \right)^{(\sum_{i=1}^3 q_{i,2}e_i)/p_2+(1-p_2)e_2/p_2} \end{aligned}$$

$$\begin{aligned} & \times \left(\sum_{k=a}^{b-1} |\Delta^m z(k)|^{p_3} \right)^{(\sum_{i=1}^3 q_{i,3}e_i)/p_3+(1-p_3)e_3/p_3} \\ & \times \prod_{k=1}^3 \left(\sum_{n=a}^{b-1} |f_k(n)|^{p'_k} \right)^{e_k/p'_k} \\ & \geq (b-a)^{\sum_{i=1}^3 \sum_{j=1}^3 ((q_{i,j}/p_j)e_i)} \left(\frac{2}{b-a} \right)^{m(\sum_{i=1}^3 \sum_{j=1}^3 q_{i,j}e_i)}. \end{aligned} \tag{26}$$

Since (e_1, e_2, e_3) is a positive solution of the linear homogeneous system (10), then

$$\begin{aligned} & \prod_{k=1}^3 \left(\sum_{n=a}^{b-1} |f_k(n)|^{p'_k} \right)^{e_k/p'_k} \\ & \geq (b-a)^{\sum_{i=1}^3 \sum_{j=1}^3 ((q_{i,j}/p_j)e_i)} \left(\frac{2}{b-a} \right)^{m(\sum_{i=1}^3 \sum_{j=1}^3 q_{i,j}e_i)}. \end{aligned} \tag{27}$$

Summing both sides of linear homogeneous system (10) yields

$$\sum_{i=1}^3 \sum_{j=1}^3 q_{i,j}e_i = \sum_{i=1}^3 (p_i - 1)e_i. \tag{28}$$

Noting that $1/p_k + 1/p'_k = 1, k = 1, 2, 3$, we have

$$\begin{aligned} & \prod_{k=1}^3 \left(\sum_{n=a}^{b-1} |f_k(n)|^{p_k/(p_k-1)} \right)^{(1-1/p_k)e_k} \\ & \geq (b-a)^{\sum_{i=1}^3 \sum_{j=1}^3 ((q_{i,j}/p_j)e_i)} \left(\frac{2}{b-a} \right)^{m \sum_{i=1}^3 ((p_i-1)e_i)}. \end{aligned} \tag{29}$$

□

Corollary 2. Let $a < b$ and assume

$$\begin{aligned} & (q_{1,1} + 1 - p_1) + q_{2,1} + q_{3,1} = 0, \\ & q_{1,2} + (q_{2,2} + 1 - p_2) + q_{3,2} = 0, \\ & q_{1,3} + q_{2,3} + (q_{3,3} + 1 - p_3) = 0. \end{aligned} \tag{30}$$

If $(x(n), y(n), z(n))$ is a nonzero solution of (8) satisfying the antiperiodic boundary conditions (9), then

$$\begin{aligned} & \prod_{k=1}^3 \left(\sum_{n=a}^{b-1} |f_k(n)|^{p_k/(p_k-1)} \right)^{(1-1/p_k)} \\ & \geq (b-a)^{\sum_{i=1}^3 \sum_{j=1}^3 (q_{i,j}/p_j)} \left(\frac{2}{b-a} \right)^{m \sum_{i=1}^3 (p_i-1)}. \end{aligned} \tag{31}$$

Acknowledgments

This work is partly supported by NSFC under Grant nos. 61271355 and 61070190, the ZNDXQYYJH under Grant no. 2010QZZD015, and NFSS under Grant no. 10BJL020.

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