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Research Article

Multiagent Consensus Control under Network-Induced Constraints

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Mean consensus problem is studied using a class of discrete time multiagent systems in which information exchange is subjected to some network-induced constraints. These constraints include package dropout, time delay, and package disorder. Using Markov jump system method, the necessary and sufficient condition of mean square consensus is obtained and a design procedure is presented such that multiagent systems reach mean square consensus.

1. Introduction

Cooperative control of networked multiagent systems by information exchange has received extensive attention presently, because of their extensive applications in flocking, swarming, distributed sensor fusion, attitude alignment, and so forth (see [1, 2] for surveys). An important problem for cooperative control is to design an appropriate control law such that a multiagent system reaches consensus in the presence of insecure information exchange. Distributed cooperative control of networked multiagent systems has been investigated in various perspectives [3–7]. In [3], the leaderless consensus problem is studied. The problem of consensus with leader node was researched in [4–7]. For networked multiagent systems of linear dynamics, consensus using state feedback or output feedback was analysed in [8, 9].

Unmodelled time delay during the design phase is an important factor that may affect the performance of dynamical systems. It can even, in some situation, cause instability of a system. In these years, consensus in networked multiagent systems with time delay was discussed using linear matrix inequality (LMI) method [10–12]. In [10], the average-consensus problem for continuous-time multiagents with switching topology and time delay was studied. The work of [11] investigated the average consensus problem in undirected networks with fixed and switching topologies under time-varying communication delays. The consensus problem was solved in [12] on directed graphs of the multiagent system with model uncertainty and time delay.

In the information exchange of network, there are not only time-delay but also other network-induced constraints. The other network-induced constraints, which include package dropout and package disorder, also affect the consensus of networked multiagent systems. However, not many works have studied multiagent systems with these network-induced constraints. Based on Markov jump system method [13–15], this paper considers mean square consensus of multiagent systems of first-order integrator under network-induced constraints such as package dropout, time delay, and package disorder. By system transformation, the necessary and sufficient condition of mean square consensus problem is provided and a corresponding design algorithm is given.

The remainder of the paper is organized as follows. Section 2 contains the formulation of the problem and terminology. The main results are presented in Section 3. Section 4 provides the numerical simulation and Section 5 draws conclusions.

2. Problem Formulation and Preliminaries

In this paper, Z^+ is used to denote the set of all nonnegative integers. The $n\times n$ identity matrix is denoted by I_n. The ith row of I_n is denoted by $e(i,n)$. If a matrix P is positive (negative) definite, it is denoted by $P > 0 (< 0)$. The notation # within a matrix represents the symmetric term of the matrix. The expected value is represented by $E[\bullet]$.

Here a discrete-time system is considered that it consists of 2 agents. Each agent is a first-order integrator, which,

$$\begin{aligned} x_1(k+1) &= x_1(k) + bu_1(k), \\ x_2(k+1) &= x_2(k) + bu_2(k) \quad k \in Z^+, \end{aligned} \tag{1}$$

where $x_1(k) \in R$ and $x_2(k) \in R$ are the state at time step k,$u_1(k) \in R$ and $u_2(k) \in R$ are the control at time step k, and $b \in R$ is constant. The two agents exchange their state through two communication channels: channel no. 1 and channel no. 2. At each k, agent 1 transmits $x_1(k)$ to agent 2 through channel no. 1. Agent 2 utilizes $z(k)$ as the information obtained from channel no. 1 at k. Due to random package dropout, time delay, and package disorder in communication, the receiving scenarios in the side of agent 2 at k are various. Agent 2 may receive one package $x_1(k-t)$ from channel no. 1 at k. The package $x_1(k-t)$ is sent by agent 1 at $k-t$ no later than k. After received, $x_1(k-t)$ is examined to see whether it is of disorder (i.e., whether agent 2 has received any packages sent later than $k-t$). If $x_1(k-t)$ is not of disorder, $z(k) \leftarrow x_1(k-t)$. If $x_1(k-t)$ is of disorder, $x_1(k-t)$ is discarded and $z(k) \leftarrow z(k-1)$. Agent 2 may receive severe package $x_1(k-t_1), x_1(k-t_2), \ldots, x_1(k-t_d)$ from channel no. 1 at k. Except the newest $x_1(k-t^*)$ with $t^* = \min(t_1, t_2, \ldots, t_d)$, these packages are discarded. If $x_1(k-t^*)$ is not of disorder, $z(k) \leftarrow x_1(k-t^*)$. If $x_1(k-t^*)$ is of disorder, it is also discarded and $z(k) \leftarrow z(k-1)$. Agent 2 may receive no package from channel no. 1 at k. In this case, $z(k) \leftarrow z(k-1)$.

From the above mechanism, it is seen that $z(k) = x_1(k-\alpha_k)$ with some random $\alpha_k \in Z^+$ constrained by

$$\alpha_{k+1} \le \alpha_k + 1 \quad \forall k \in Z^+. \tag{2}$$

On α_k, we adopt an assumption which is made by some researchers in networked control [16]; that is, α_k is assumed to be a Markov chain taking values in a finite set $\{0, 1, \ldots, m\}$ with transition probabilities:

$$\phi_{s,l} = \Pr(\alpha_{k+1} = l \mid \alpha_k = s) \quad \forall s, l \in \{0, 1, \ldots, m\}, \tag{3}$$

where m is a given nonnegative integer. The transition probability matrix

$$\Phi = \begin{bmatrix} \phi_{0,0} & \phi_{0,1} & 0 & \cdots & 0 \\ \phi_{1,0} & \phi_{1,1} & \phi_{1,2} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \phi_{m-1,0} & \phi_{m-1,1} & \phi_{m-1,2} & \cdots & \phi_{m-1,m} \\ \phi_{m,0} & \phi_{m,1} & \phi_{m,2} & \cdots & \phi_{m,m} \end{bmatrix} \in R^{(m+1)\times(m+1)} \tag{4}$$

is also known. The expression (4) of Φ displays that, for the reason of constraint (2), $\phi_{s,l} = 0$ when $l > s+1$. Thus, we have described the communication in channel no. 1 using Markov chain α_k. The same method is applied to describe the communication in channel no. 2. Agent 1 obtains $x_2(k-\beta_k)$ from channel no. 2 at k, where β_k is a Markov chain taking values in a known set $\{0, 1, \ldots, n\}$ with a known transition probability matrix

$$\Psi = \begin{bmatrix} \varphi_{0,0} & \varphi_{0,1} & 0 & \cdots & 0 \\ \varphi_{1,0} & \varphi_{1,1} & \varphi_{1,2} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \varphi_{n-1,0} & \varphi_{n-1,1} & \varphi_{n-1,2} & \cdots & \varphi_{n-1,n} \\ \varphi_{n,0} & \varphi_{n,1} & \varphi_{n,2} & \cdots & \varphi_{n,n} \end{bmatrix} \in R^{(n+1)\times(n+1)}. \tag{5}$$

The goal of agents 1 and 2 is a prescribed state $x^* \in R$. In this paper, agent 1 is aware of x^* while agent 2 is not. Consequently, agent 1 employs control law

$$u_1(k) = -h\left(\left(x_2(k-\beta_k) - x_1(k)\right) - \left(x_1(k) - x^*\right)\right) \tag{6}$$

but agent 2 employs control law

$$u_2(k) = -h\left(x_1(k-\alpha_k) - x_2(k)\right), \tag{7}$$

where $h \in R$ is the control parameter.

The above multiagent system is said to be mean square consensus if $\forall x_1(0) \in R$, $\forall x_2(0) \in R$, $\forall \alpha_0 \in \{0, 1, \ldots, m\}$, $\forall \beta_0 \in \{0, 1, \ldots, m\}$,

$$\begin{aligned} \lim_{k\to\infty} E\left[\left(x_1(k) - x^*\right)^2\right] &= 0 \\ \lim_{k\to\infty} E\left[\left(x_2(k) - x^*\right)^2\right] &= 0. \end{aligned} \tag{8}$$

Our objective is to design h such that the two agents reach mean square consensus.

3. Main Result

Define

$$\begin{aligned} y_1(k) &= x_1(k) - x^* \\ y_2(k) &= x_2(k) - x^* \quad \forall k \in Z^+. \end{aligned} \tag{9}$$

Then from (1), (6), (7), and (9), we have

$$\begin{aligned} y_1(k+1) &= (1+2bh)\, y_1(k) - bhy_2(k-\beta_k) \\ y_2(k+1) &= -bhy_1(k-\alpha_k) + (1+bh)\, y_2(k). \end{aligned} \tag{10}$$

Further, denote

$$Y(k) = \begin{bmatrix} y_1(k) \\ \vdots \\ y_1(k-m) \\ y_2(k) \\ \vdots \\ y_2(k-n) \end{bmatrix}^T \in R^{m+n+2}. \tag{11}$$

Obviously, mean square consensus (8) is equivalent to $\lim_{k\to\infty}E[Y^{\mathrm{T}}(k)Y(k)] = 0$. Using (11), system (10) is transformed into

$$Y(k+1) = G_h(\alpha_k, \beta_k)\, Y(k), \tag{12}$$

where

$$\begin{aligned}
G_h(\alpha_k, \beta_k) &= \begin{bmatrix} G_{h11} & G_{h12}(\beta_k) \\ G_{h21}(\alpha_k) & G_{h22} \end{bmatrix} \in R^{(m+n+2)\times(m+n+2)}, \\
G_{h11} &= \begin{bmatrix} 1+2bh & 0 & \cdots & 0 & 0 \\ 1 & & & & 0 \\ & 1 & & & 0 \\ & & \ddots & & \vdots \\ & & & 1 & 0 \end{bmatrix} \in R^{(m+1)\times(m+1)}, \\
G_{h12}(\beta_k) &= bh \begin{bmatrix} -e(\beta_k, n+1) \\ 0 \end{bmatrix} \in R^{(m+1)\times(n+1)}, \\
G_{h21}(\beta_k) &= bh \begin{bmatrix} -e(\alpha_k, m+1) \\ 0 \end{bmatrix} \in R^{(n+1)\times(m+1)}, \\
G_{h22} &= \begin{bmatrix} 1+bh & 0 & \cdots & 0 & 0 \\ 1 & & & & 0 \\ & 1 & & & 0 \\ & & \ddots & & \vdots \\ & & & 1 & 0 \end{bmatrix} \in R^{(n+1)\times(n+1)}.
\end{aligned} \tag{13}$$

On system (12), [16] presented the following.

Lemma 1. *Suppose that Markov chains α_k and β_k are independent. System* (12) *achieves* $\lim_{k\to\infty}E[Y^T(k)Y(k)] = 0$ *if and only if there exist positive definite matrices $P(\alpha,\beta) \in R^{(m+n+2)\times(m+n+2)}$, $\alpha \in \{0,1,\ldots,m\}$, $\beta \in \{0,1,\ldots,n\}$ such that*

$$P(\alpha,\beta) - G_h^T(\alpha,\beta)\left(\sum_{i=0}^{m}\sum_{j=0}^{n}\phi_{\alpha,i}\varphi_{\beta,j}P(i,j)\right)G_h(\alpha,\beta) > 0. \tag{14}$$

Actually, the condition in Lemma 1 can be converted using Schur complement.

Lemma 2 (see [17]). *Let $S = \left[\begin{smallmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{smallmatrix}\right]$ be given partitioned matrix. Then $S > 0$ if and only if $S_{22} > 0$ and $S_{11} - S_{12}S_{22}^{-1}S_{21} > 0$.*

Through the above converting, the necessary and sufficient condition is obtained from mean square consensus of the multiagent system.

Theorem 3. *Suppose that Markov chains α_k and β_k are independent. The multiagent system in Section 2 achieves mean square consensus if and only if there exist $h \in R$ and positive definite matrices $P(\alpha,\beta) \in R^{(m+n+2)\times(m+n+2)}$, $Q(\alpha,\beta) \in R^{(m+n+2)\times(m+n+2)}$, $\alpha \in \{0,1,\ldots,m\}$, $\beta \in \{0,1,\ldots,n\}$ such that*

$$\begin{bmatrix} P(\alpha,\beta) & & & \# \\ \sqrt{\phi_{\alpha,0}\varphi_{\beta,0}}G_h(\alpha,\beta) & Q(0,0) & & \\ \vdots & & \ddots & \\ \sqrt{\phi_{\alpha,m}\varphi_{\beta,n}}G_h(\alpha,\beta) & & & Q(m,n) \end{bmatrix} > 0 \tag{15}$$

$$P(\alpha,\beta) - Q^{-1}(\alpha,\beta) = 0.$$

In order to deal with the condition in Theorem 3 using Cone Complementarity Linearisation algorithm [18], for $r \in Z^+$, we construct LMI

$$\begin{bmatrix} P_r(\alpha,\beta) & & & \# \\ \sqrt{\phi_{\alpha,0}\varphi_{\beta,0}}G_h(\alpha,\beta) & Q(0,0) & & \\ \vdots & & \ddots & \\ \sqrt{\phi_{\alpha,m}\varphi_{\beta,n}}G_h(\alpha,\beta) & & & Q(m,n) \end{bmatrix} > 0 \tag{16}$$

$$\begin{aligned}
&\begin{bmatrix} P_r(\alpha,\beta) & I \\ I & Q_r(\alpha,\beta) \end{bmatrix} > 0 \\
&P_r(\alpha,\beta) \in R^{(m+n+2)\times(m+n+2)}, \\
&Q_r(\alpha,\beta) \in R^{(m+n+2)\times(m+n+2)}, \\
&\alpha \in \{0,1,\ldots,m\},\ \beta \in \{0,1,\ldots,n\}
\end{aligned} \tag{17}$$

which is denoted by $L(P_r(\alpha,\beta), Q_r(\alpha,\beta), h) > 0$. The following is our design steps:

Step 1. Specify an enough small real number $\varepsilon > 0$ and an enough large integer T. Set $r = 0$. Find feasible $P_0(\alpha,\beta)$, $Q_0(\alpha,\beta)$, and h_0, $\forall\alpha \in \{0,1,\ldots,m\}$ and $\forall\beta \in \{0,1,\ldots,n\}$ satisfy $L(P_0(\alpha,\beta), Q_0(\alpha,\beta), h_0) > 0$. If there is none, exit.

Step 2. Solve the LMI problem

$$\begin{aligned}
\mu_{r+1} = \min\ \operatorname{trace} &\sum_{\alpha=0}^{m}\sum_{\beta=0}^{n} P_{r+1}(\alpha,\beta)\, Q_r(\alpha,\beta) \\
&+ P_r(\alpha,\beta)\, Q_{r+1}(\alpha,\beta) \\
\text{s.t.}\quad & L(P_{r+1}(\alpha,\beta), Q_{r+1}(\alpha,\beta), h_{r+1}) > 0,
\end{aligned} \tag{18}$$

and obtain $P_{r+1}(\alpha,\beta)$, $Q_{r+1}(\alpha,\beta)$ and h_{r+1}.

Step 3. If $|\mu_{r+1} - 2(m+1)(n+1)(m+n+2)| < \varepsilon$, let $h = h_{r+1}$ and terminate. Otherwise, $r \leftarrow r+1$ and go to Step 4.

Step 4. If $r > T$, exist. Otherwise, go to Step 2

It should be pointed out that the above method is easy to be extended to q agents when $q > 2$. Among q agents, there are $q(q-1)$ communication channels. We utilize $q(q-1)$ independent Markov chains to describe communication in these channels and can arrive at a similar result as Theorem 3.

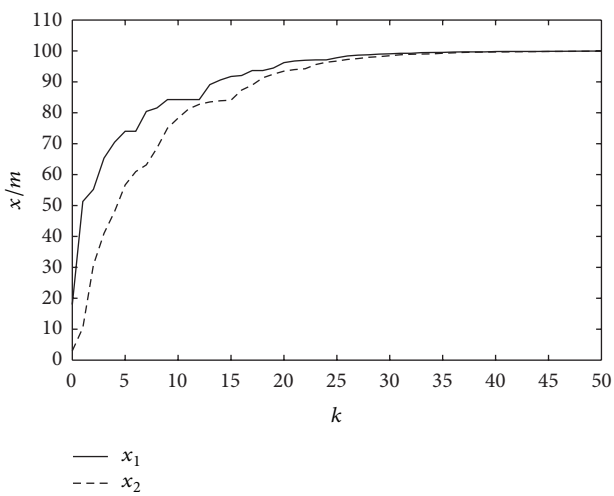

Figure 1: State response of two agents.

4. Numerical Example

In the numerical example, we give $m = 3$, $n = 3$, $b = 0.8$, and transition probability of α_k and β_k is given as

$$\Phi = \begin{bmatrix} 0.6 & 0.4 & 0 & 0 \\ 0.25 & 0.3 & 0.45 & 0 \\ 0.2 & 0.2 & 0.1 & 0.5 \\ 0.1 & 0.55 & 0.1 & 0.25 \end{bmatrix},$$
$$\Psi = \begin{bmatrix} 0.7 & 0.3 & 0 & 0 \\ 0.55 & 0.35 & 0.1 & 0 \\ 0.2 & 0.4 & 0.25 & 0.15 \\ 0.3 & 0.35 & 0.15 & 0.2 \end{bmatrix}. \tag{19}$$

Using the design steps in Section 3, we get $h = -0.6215$. Figure 1 shows the state response of two agents with

$$x_1(0) = 18, \qquad x_2(0) = 3,$$
$$x^* = 100, \quad \alpha_0 = 0, \quad \beta_0 = 0. \tag{20}$$

It can be seen that x_1 and x_2 converge at x^*.

5. Conclusion

The consensus control problem of multiagent systems of first-order integrator is studied under network-induced constraints. A new model is presented to describe the network communication with package dropout, time delay, and package disorder. For the new model, the definition of mean square consensus is given multiagent systems. Further, the necessary and sufficient condition of mean square consensus is proposed in the form of matrix inequalities. Based on this condition and Cone Complementarity Linearisation algorithm, a consensus control law can be designed to make systems reach mean square consensus.

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