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Research Article

Tradeoff Analysis for Optimal Multiobjective Inventory Model

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Deterministic inventory model, the economic order quantity (EOQ), reveals that carrying inventory or ordering frequency follows a relation of tradeoff. For probabilistic demand, the tradeoff surface among annual order, expected inventory and shortage are useful because they quantify what the firm must pay in terms of ordering workload and inventory investment to meet the customer service desired. Based on a triobjective inventory model, this paper employs the successive approximation to obtain efficient control policies outlining tradeoffs among conflicting objectives. The nondominated solutions obtained by successive approximation are further used to plot a 3D scatterplot for exploring the relationships between objectives. Visualization of the tradeoffs displayed by the scatterplots justifies the computation effort done in the experiment, although several iterations needed to reach a nondominated solution make the solution procedure lengthy and tedious. Information elicited from the inverse relationships may help managers make deliberate inventory decisions. For the future work, developing an efficient and effective solution procedure for tradeoff analysis in multiobjective inventory management seems imperative.

1. Introduction

Inventory control is an important activity that appears in any kind of organization. For this reason, it has been studied extensively in the past several decades. Most inventory models aggregate different cost concepts, such as ordering cost, carrying cost, and shortage cost, into a single-objective formulation. Optimal decisions about when to order and how much to order are then solved by single-objective optimization techniques. However the insight gained from the oldest inventory model, economic order quantity (EOQ), reveals that inventory management should be considered as a biobjective optimization problem to strike a balance between inventory carrying and annual orders. Practically speaking, inventory decisions involve tradeoffs related to operational efficiency and customer service.

Brown [1, 2] first examined the tradeoff between investment in working stock and annual ordering cost. He introduced the concept of exchange curve shown in Figure 1. The curve demonstrates how capital invested in working stocks can be traded for operating expenses of ordering. Points below the curve are infeasible, and decisions located above the curve are suboptimal. Suboptimal policies can be improved by moving back to the curve (i.e., seeking possible improvement from point A to B or C). Starr and Miller [3] determined tradeoffs between two performance measures: (i) number of orders per year (workload) and (ii) average investment in inventory in the case of multiple items. Gardner and Dannenbring [4] introduced customer service as another measure, along with workload and inventory investment, and generalized above exchange curve analysis to the optimal policy surface in case of probabilistic demand.

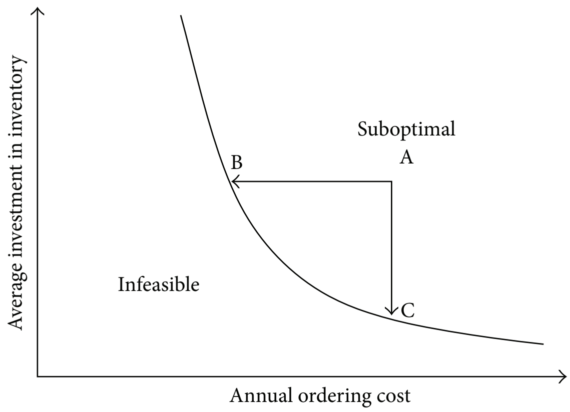

Figure 1: The exchange curve for deterministic inventory model.

The model and solution technique they used is still based on single-objective optimization. Bookbinder and Chen [5] proposed a multiobjective formulation for analyzing multi-echelon inventory and distribution systems. They argued that points on exchange curve or policy surface are equivalent to the nondominated solutions concept of multiobjective optimization. Although it could be the first multiple criteria generalizations of earlier studies, the model was solved by classical optimization techniques.

To a certain extent, the aforementioned tradeoff analysis of inventory management is developed by single-objective optimization. The motivation of this study aims to develop an intrinsically multiobjective approach for building the tradeoff surface of probabilistic inventory systems. Differences between traditional and multiobjective approach are not only in their problem formulations, but also the latter simultaneously treats several objectives analytically or heuristically under certain notion of multiobjective optimality [6, 7].

Gutiérrez et al. [8] considered a dynamic single facility single-item lot size problem. Although the total demand is assumed to be a fixed value, the distribution of this demand among different periods is unknown. They determined all the Pareto-optimal or nondominated production plans that are robust to any possible occurrence of all scenarios. Gutiérrez et al. [9] presented the characterization of the nondominated optimal solution set and use it to correct the solution method previously proposed by Bookbinder and Chen [5].

For the multiobjective exchange curve, Tsou [7] presented a two-stage framework consisting of multiobjective particle swarm optimization (MOPSO) and technique for order preference by similarity to ideal solution (TOPSIS). At the first stage, MOPSO is used to generate the tradeoff (or nondominated) front of the triobjective inventory model in Agrell [10]. Then, a preferred solution is selected by TOPSIS according to subjective preferences of decision makers. Tsou and Kao [11] also developed a metaheuristic based on electromagnetism-like mechanism (EM) to approximate the Pareto-optimal front without using any prior or interactive preference. They showed that the metaheuristic can find similar Pareto-optimal solutions as the popular interactive procedure Step method (STEM) did [12]. Tsou [13] further showed that evolutionary Pareto optimizers could generate tradeoff solutions potentially ignored by the well-known simultaneous method.

Nevertheless, we recently notice that the tradeoff solutions of the above studies actually laid on an exchange curve, instead of forming a tradeoff surface in the 3D objective space. It apparently indicates that some of the objectives, including minimization of expected annual cost, expected annual number of stockout occasions, and expected annual number of items stocked out, are not conflicting with each other. Among which, the last two objectives are redundant because they relate to the same concept of shortage but different measures. Consequently, such a kind of triobjective models was not properly justified in the above studies.

This paper first presents a triobjective model without redundancy in the next section. Nonredundancy is assured by dropping all the marginal cost parameters out of the classical fixed order model. After that, a successive approximation procedure based on the Lagrange method is utilized to iteratively search for nondominated solutions and efficient control policies. Tradeoffs among workload, inventory, and shortage are visualized by three-dimensional scatterplots. Although it is a time-consuming job to use the successive approximation to find the tradeoff surfaces of multiobjective model, all solutions found are ensured to be Pareto-optimal in comparison with other search methods, such as genetic algorithms. Finally, conclusions and directions for future research are drawn out accordingly.

2. Model Building and Solution Procedures

2.1. A Triobjective Model. The reorder point lot size system, (r, Q), is a popular control method under probabilistic demand. An order of size Q will be triggered immediately whenever the inventory position drops to the reorder point r or lower. Classical (r, Q) model minimizes a lump-sum cost including ordering cost, carrying cost, and stockout cost [14]. The triobjective model below intrinsically restores the nature of conflicts among objectives that are to minimize the workload, inventory, and shortage. Also, multiobjective (r, Q) model does not run into the incommensurate issue while aggregating objectives of different measures into a single one. The notations used here are described as follows.

D is the average annual demand,

L is the lead time,

D_L is the lead time demand. It is normally distributed with mean μ_L and standard deviation σ_L,

SS is the safety stock, which is proportional to the standard deviation of lead time demand. That is, $SS = k\sigma_L$, where k represents the safety factor,

r is the aforementioned reorder point, which equals to the average lead time demand plus the safety stock. That is, $r = \mu_L + k\sigma_L$, and $\varphi(z)$ is the probability density function of standard normal distribution.

A multiobjective (r, Q) model is formulated as follows:

$$\underset{k,Q}{\text{Min}} \quad W = \frac{D}{Q}, \tag{1}$$

$$\underset{k,Q}{\text{Min}} \quad I = \frac{Q}{2} + k\sigma_L, \tag{2}$$

$$\underset{k,Q}{\text{Min}} \quad S = \frac{D\sigma_L}{Q} \int_k^{\infty} (z-k)\,\varphi(z)\,dz \tag{3}$$

subject to the following:

$$0 \le Q \le D, \tag{4}$$

$$0 \le k \le \frac{D}{\sigma}. \tag{5}$$

Equation (1) represents the number of annual order (in cycles per year, also called workload). Equation (2) is the sum of cycle and safety stocks (in units carrying per year). Equation (3) denotes the average number of demand not covered from stock annually (in units short per year). Inequality (4) ensures that the order size (units per order) should be nonnegative and not more than the average annual demand. Inequality (5) guarantees that the safety stock (in units) will not be greater than the average annual demand and should be nonnegative.

The notion of optimality in single-objective optimization is straightforward, because the optimal solution is the one that realizes the maximum (or the minimum) of the objective function. However, the optimality for a multiobjective optimization problem is not so easy to understand because not all feasible solutions can be compared completely. Generally speaking, multiobjective optimization problems rarely have solutions that simultaneously optimize all of the objectives; as a result we are trying to optimize each objective to the greatest extent possible. There exists a set of solutions, referred as nondominated solutions, which are better than others in the search space when considering all the objectives. For the minimization problem (in Section 2.1), a control parameter $x^1 = (k_1, Q_1)$ is said to strongly dominate $x^2 = (k_2, Q_2)$ (denoted by $x^1 \prec x^2$) if and only if $W(x^1) < W(x^2)$, $I(x^1) < I(x^2)$, and $S(x^1) < S(x^2)$. That is, solution x^1 is strictly better than solution x^2 in all the cost and service objectives ([15, pp. 32]). Less stringently, a decision vector x^1 dominates x^2 (denoted by $x^1 \preceq x^2$) if and only if $W(x^1) \le W(x^2)$, $I(x^1) \le I(x^2)$, $S(x^1) \le S(x^2)$ and at least one of above inequality is strictly held ([15, pp. 28]). For other multiobjective (or multicriteria) concepts, please refer to Ehrgott [16].

Undoubtedly, we are not interested in solutions dominated by other solutions. Solutions that are not dominated by any other solutions are called nondominated in objective space or efficient in decision space. It means that the improvement of some objective could only be achieved at the expense of other objectives. This coincides with the exchange curve concept mentioned earlier. In a multiobjective optimization problem, there are normally a large number of nondominated solutions due to the conflicts among objectives. Hence, it is difficult to find the whole set of nondominated solutions. And because the nondominated set is usually unknown, most optimizers try to find a finite number of nondominated solutions to approximate the set. The successive approximation approach stated below can be used to search for the nondominated solutions of the triobjective inventory model.

2.2. Successive Approximation Based on Lagrange Method. A single objective transformation is first developed as follows. Equation (3) is kept as the objective function and treats (1) and (2) as constraints. That is,

$$\begin{aligned} &\underset{k,Q}{\text{Min}} \quad S \\ &\text{subject to} \quad W = W' \\ &\qquad\qquad\quad\; I = I', \end{aligned} \tag{6}$$

where W' and I' are budgets on workload and inventory.

To solve this equality constrained optimization problem, the Lagrange method is employed here. After introducing the Lagrangian multipliers λ_W and λ_I, the Lagrangian function is as follows:

$$\begin{aligned} L(k, Q, \lambda_W, \lambda_I) = {} & \frac{D\sigma_L}{Q} \int_k^{\infty} (z-k)\,\varphi(z)\,dz \\ & + \lambda_I \left(\frac{Q}{2} + k\sigma_L - I'\right) + \lambda_W \left(\frac{D}{Q} - W'\right). \end{aligned} \tag{7}$$

Some simplifying notations are introduced before presenting the successive approximation algorithm. Let

$$P = \int_k^{\infty} \varphi(z)\,dz, \tag{8}$$

$$E = \sigma_L \int_k^{\infty} (z-k)\,\varphi(z)\,dz, \tag{9}$$

P is the probability of a stockout per order cycle and E is the expected number of shortage per order cycle. Simple algebra provides the following equations used in the successive approximation:

$$\lambda_I = \frac{DP}{2(I' - k\sigma)}, \tag{10}$$

$$P = \frac{\lambda_I Q}{D}, \tag{11}$$

$$\lambda_W = \frac{1}{W'} \left(\frac{\lambda_I Q}{2} - \frac{DE}{Q}\right), \tag{12}$$

$$Q = \sqrt{\frac{2D(E + \lambda_W)}{\lambda_I}}. \tag{13}$$

To search for the efficient (k_i, Q_i) policy, the search steps are proposed as follows.

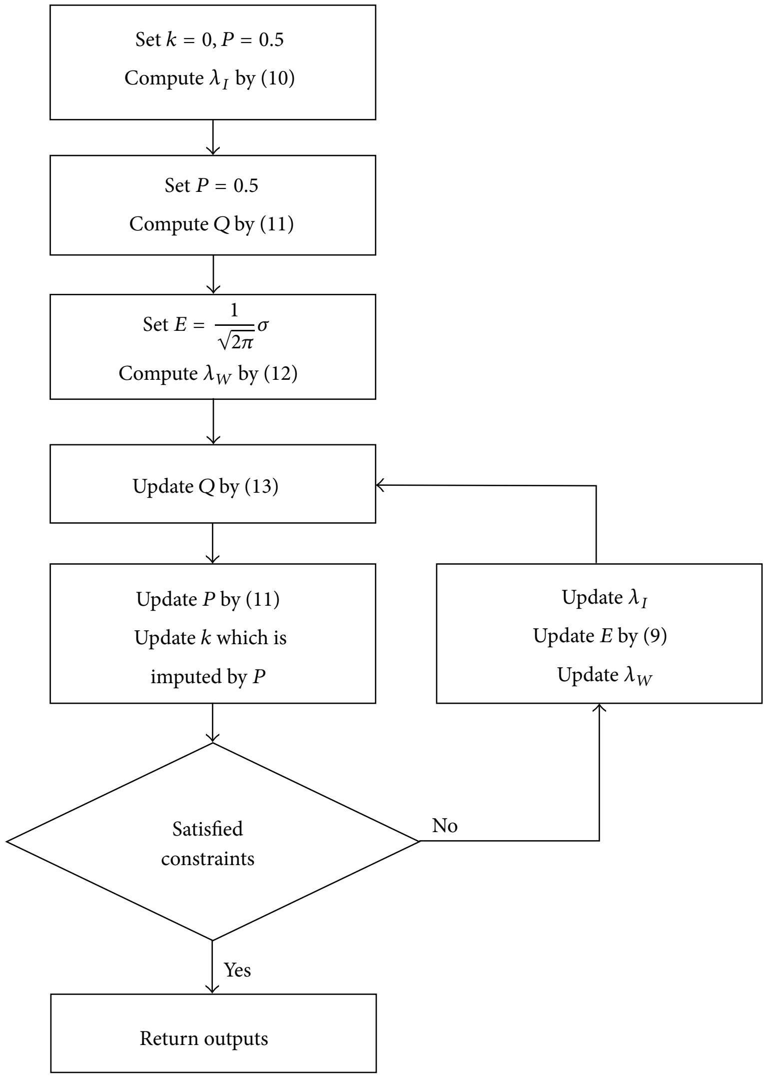

Figure 2: Flow chart of the search algorithm.

Step 1. Initialize λ_I: Compute λ_I using (10) with $k = 0$ and $P = 0.5$.

Step 2. Initialize Q: Compute Q using (11) with $P = 0.5$.

Step 3. Initialize λ_W: Compute λ_W using (12) with $E = (1/\sqrt{2\pi})\sigma$ corresponds to zero safety stock.

Step 4. Update Q: Compute Q using (13).

Step 5. Update P and k: Compute P with (11) and look up k imputed by P.

Step 6. Check constraints: If both workload and investment constraints are satisfied, then output the results. Otherwise, update λ_I, E (using (9)), and λ_W, then go back to Step 4.

The search begins with an initial guess of zero safety stock. This allows us to use (10) to derive an initial λ_I, which in turn, determines the initial Q using (11). However, (12)-(13) for λ_W and Q are interdependent, preventing their use in the initialization phase. Rearranging (11), however, we can derive an equation for Q which does not require an estimate of λ_W. The Q based on (11) is then used to provide an initial estimate of λ_W from (13). Thereafter the search progresses by iteratively updating values for Q, P (and correspondingly k), λ_I, and λ_W, using (13), (11), (10), and (12), respectively, until both the workload and investment constraints are satisfied. The flow chart of the above search algorithm is shown in Figure 2.

3. Numerical Results

Pharmaceutical inventory data with four items (Table 1) were fed into the triobjective model. The successive approximation was coded in R [17], and all computation was executed on a laptop computer. Ten representative solutions for each

Table 1: The pharmaceutical data.

Item	D	σ_L
1	3412	53.354
2	490	5.027
3	4736	57.911
4	200	2.969

Table 2: Tradeoff solutions of Item 1 generated by successive approximation.

Sol. no.	Efficient solution		Nondominated solution			Iter
	Q	k	W	I	S	
1	94.804	0.049	35.990	50.000	720.207	81*
2	162.532	0.351	20.993	99.992	277.522	20†
3	98.804	0.986	35.990	99.999	164.354	34
4	83.238	1.094	40.991	99.999	151.799	41
5	162.51	1.287	20.996	149.931	52.407	20†
6	94.798	1.923	35.992	149.993	19.999	27
7	131.234	2.517	25.999	199.92	2.635	26
8	213.250	2.686	16.000	249.915	0.948	41
9	110.065	3.653	31.000	249.922	0.052	32
10	110.065	4.590	31.000	299.911	0.001	40
Min.	83.238	0.049	16.000	50.000	0.001	20
Max.	213.250	4.590	40.991	299.911	720.207	81

† and * represent the minimum and the maximum numbers of iterations, respectively.

Table 3: Tradeoff solutions of Item 2 generated by successive approximation.

Sol. no.	Efficient solution		Nondominated solution			Iter
	Q	k	W	I	S	
1	18.8519	0.114	25.9921	9.9989	45.0181	15†
2	23.3406	0.6575	20.9935	14.9757	16.193	15†
3	30.6264	0.9159	15.9992	19.9176	7.8443	17
4	13.6139	1.6284	35.9927	14.9929	3.9316	21
5	11.9541	1.794	40.9901	14.9954	2.9865	22
6	18.8469	2.0863	25.999	19.9113	0.8779	21
7	15.8077	2.3897	30.9976	19.917	0.4372	21
8	11.9532	2.781	40.9934	19.9568	0.1671	22
9	18.8462	3.0794	26	24.9033	0.0376	30
10	30.625	3.8973	16	34.9042	0.0009	60*
Min.	11.9532	0.114	15.9992	9.9989	0.0009	15
Max.	30.6264	3.8973	40.9934	34.9042	45.0181	60

† and * represent the minimum and the maximum numbers of iterations, respectively.

Table 4: Tradeoff solutions of Item 3 generated by successive approximation.

Sol. no.	Efficient solution		Nondominated solution			Iter
	Q	k	W	I	S	
1	296.134	0.032	15.993	149.924	354.824	8†
2	430.546	0.598	11	249.913	107.77	25
3	789.333	0.954	6	449.91	31.58	59*
4	152.82	1.27	30.991	149.964	87.038	21
5	225.525	1.505	21	199.918	35.237	25
6	152.781	2.133	30.999	199.911	10.597	24
7	115.534	2.455	40.992	199.945	5.463	24
8	296	2.623	16	299.901	1.26	50
9	131.556	3.18	36	249.913	0.416	30
10	152.774	3.86	31	299.91	0.024	41
Min.	115.512	0.032	6	149.924	0.024	8
Max.	789.333	3.86	40.992	499.91	354.824	59

† and * represent the minimum and the maximum numbers of iterations, respectively.

item generated by successive approximation are shown in Tables 2, 3, 4, and 5. The columns of nondominated solutions demonstrate that the improvement of some objective(s) could only be achieved at the expense of other objectives. For example, solution 3, compared to solution 4, in Table 2 gets better in workload at the expense of shortage.

Three-dimensional scatterplots for each item are illustrated in Figures 3, 4, 5, and 6. Any one can visually check the tradeoffs displayed in scatterplots by adding planes parallel to I-S or W-S plane. With a fixed workload, expected shortage decreases as expected inventory increases. At a fixed inventory level, increases in workload lead to a reduction of expected shortage. All these findings are intuitive and straightforward.

For the computation effort, we notice that only one solution in Table 2 can be obtained after at least twenty iterations. And the largest iterations to reach a nondominated solution is eighty-one. Ranges of other items are between eight to sixty-six iterations. Hence, creating the scatterplot of workload, inventory, and shortage by successive approximation is lengthy and tedious.

The quality of a set of tradeoff solutions is evaluated quantitatively in terms of accuracy and diversity. A metric called hypervolume (H) is used to demonstrate the accuracy of the nondominated solutions. It calculates the size of the area that is dominated by a nondominated set and is defined as follows. The idea is that the larger the area the solutions can dominate in the objective space, the better it is [18]:

$$\prod_{i=1}^{M} \left(f_i^{\max} - f_i^{\min} \right), \tag{14}$$

where M is the number of objectives. Figure 7 shows the pictorial explanation of H in which O' represents the reference point and S is the nondominated set.

Keeping the nondominated set as diverse as possible is very important. Here spacing (S) and maximum spread (D) are used to evaluate the distribution and spread of

Table 5: Tradeoff solutions of Item 4 generated by successive approximation.

Sol. no.	Efficient solution		Nondominated solution			Iter
	Q	k	W	I	S	
1	18.1965	0.2896	10.9911	9.9582	8.8351	10†
2	5.5568	0.7482	35.9916	5	14.059	36
3	33.3333	1.0901	6	19.9032	1.2465	28
4	12.5064	1.2415	15.9918	9.9393	2.4446	15
5	9.527	1.7531	20.993	9.9686	1.0003	18
6	7.6947	2.0674	25.9918	9.9854	0.546	20
7	5.5571	2.4315	35.9902	9.9978	0.2642	24
8	33.3333	2.7747	6	24.9047	0.0148	66*
9	12.5	2.9139	16	14.9015	0.0245	28
10	7.6925	3.7241	25.9993	14.9032	0.0018	25
Min.	5.5568	0.2896	6	5	0.0018	10
Max.	33.3333	3.7241	35.9916	24.9047	14.059	66

† and * represent the minimum and the maximum numbers of iterations, respectively.

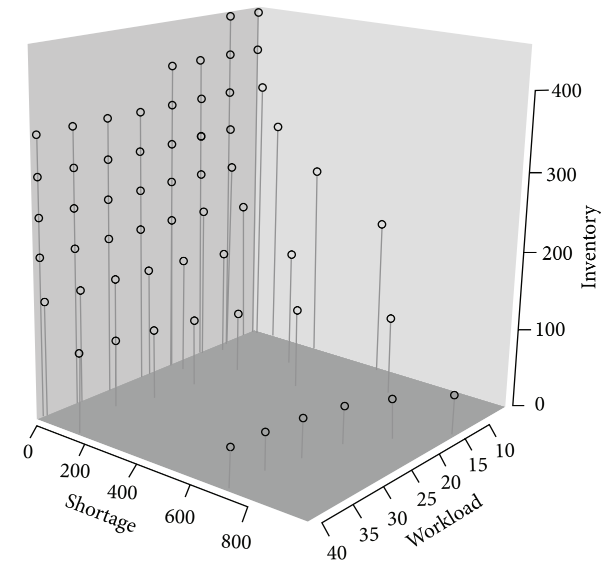

Figure 3: Scatterplot of Item 1 nondominated solutions from successive approximation.

Table 6: Performance measures of the successive approximation approach.

ID	Accuracy	Distribution	
	H	Spacing	Spread
1	42,694,515	0.091	1.532
2	7,215,908	0.097	1.232
3	68,798,878	0.121	1.637
4	476,041	0.105	1.362

the nondominated fronts generated by successive approximation:

$$S = \sqrt{\frac{1}{|\widetilde{A}|}\sum_{i=1}^{|\widetilde{A}|}\left(d_i - \overline{d}\right)^2},$$
$$D = \sqrt{\sum_{k=1}^{3}\left(\max_{i=1}^{|\widetilde{A}|} f_k^i - \min_{i=1}^{|\widetilde{A}|} f_k^i\right)^2}, \quad (15)$$

where $d_i = \min_{j\in\widetilde{A}\wedge j\neq i}\sum_{k=1}^{3}|f_k^i - f_k^j|$, f_k^i represents the kth criterion function value of the nondominated solution i, and $\overline{d}$ is the mean value of the absolute distance measure where $\overline{d} = \sum_{i=1}^{|\widetilde{A}|}(d_i/|\widetilde{A}|)$.

Larger above measures are better except for the spacing. Table 6 shows the results of the successive approximation method. If there is a reference solutions set known to decision makers or generated by other solution procedures, one can use the figures in Table 6 to compare successive approximation with the benchmark that he/she is interested in. After verifying the validity of nondominated (W, I, S) solutions generated by successive approximation, they can be used to construct a tradeoff surface for inventory control. It helps managers choose an appropriate control policy for probabilistic demand, such as the fund level tied in inventory versus the service level under fixed workload.

4. Conclusions and Suggestions

Tradeoff analysis in inventory management is useful in quantifying what the firm must pay in terms of workload and investment to meet the desired customer service. A triobjective model is presented first to generate the efficient (k, Q) policies in decision space that correspond to the nondominated (W, I, S) solutions in objective space. Nonredundancy is assured by dropping all the marginal cost parameters out of the classical fixed order model. Such that leads to a triobjective model which intends to minimize the workload, inventory, and shortage that are all conflicting with each other.

To solve the triobjective model, successive approximation approach is employed in this paper. The successive approximation is in an attempt to build the optimal tradeoff surface under probabilistic demand. Successive approximation can be used to obtain the nondominated solutions of workload, inventory, and shortage. Specifically, lots of trial and error involve in deriving the edge of each objective with a Lagrangian model. Several iterations needed to reach a nondominated solution for all items make the creating of scatterplot of workload, inventory, and shortage by successive approximation lengthy and tedious. However, visualization of the tradeoffs displayed by the scatterplots of Figures 3, 4, 5, and 6 justifies the computation effort done in the experiment. The inverse relationship among workload, inventory, and shortage conforms to our intuition.

The quality of a set of tradeoff solutions has to be evaluated quantitatively when comparing to other benchmarks, although developing an efficient and effective solution procedure for tradeoff analysis of multiobjective inventory management will be our future work. After verifying the validity of nondominated (W, I, S) solutions, they can be used to construct a tradeoff surface that helps managers choose

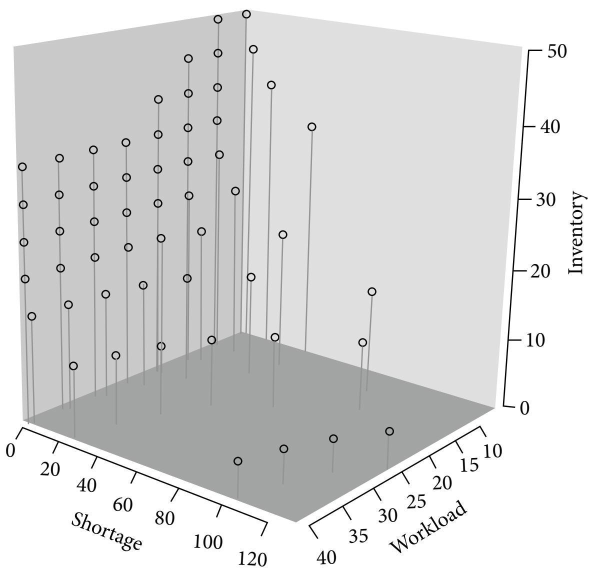

Figure 4: Scatterplot of Item 2 nondominated solutions from successive approximation.

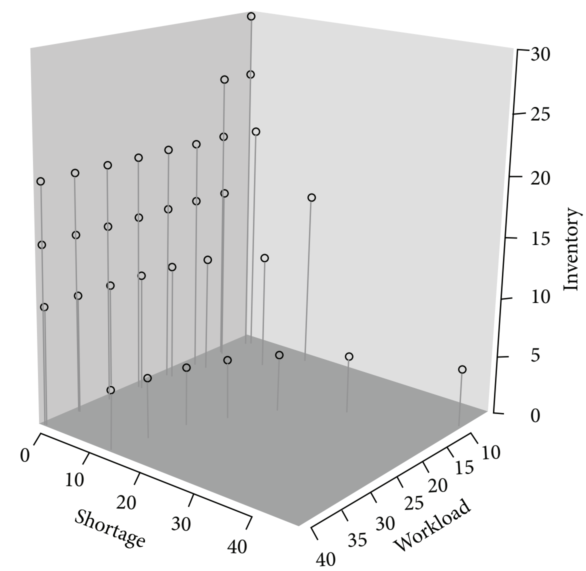

Figure 6: Scatterplot of Item 4 nondominated solutions from successive approximation.

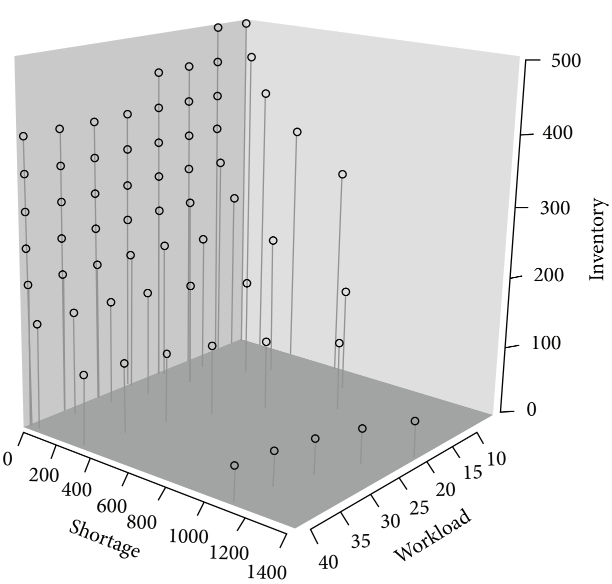

Figure 5: Scatterplot of Item 3 nondominated solutions from successive approximation.

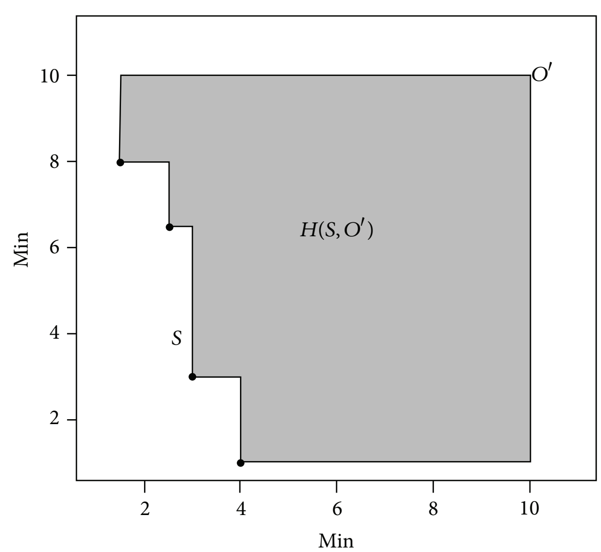

Figure 7: The hypervolume index in the minimization problem.

an appropriate control policy for probabilistic demand. For example, a common debate on the fund level tied in inventory versus the service level under fixed workload usually arises between warehousing and sales departments. By utilizing the information coming from the tradeoff surface, deliberate decisions among conflicting objectives can be easily made.

Moreover, the tradeoff surface under multi-item context deserves closer attention to bridge the gap between inventory theory and managerial practice, because too much attention was focused on the single-item problem in view of the past literature. Finally, multiechelon inventory and/or distribution systems are very common in business logistics. It is worthwhile to study the multiobjective inventory policies of different parties in a supply chain.

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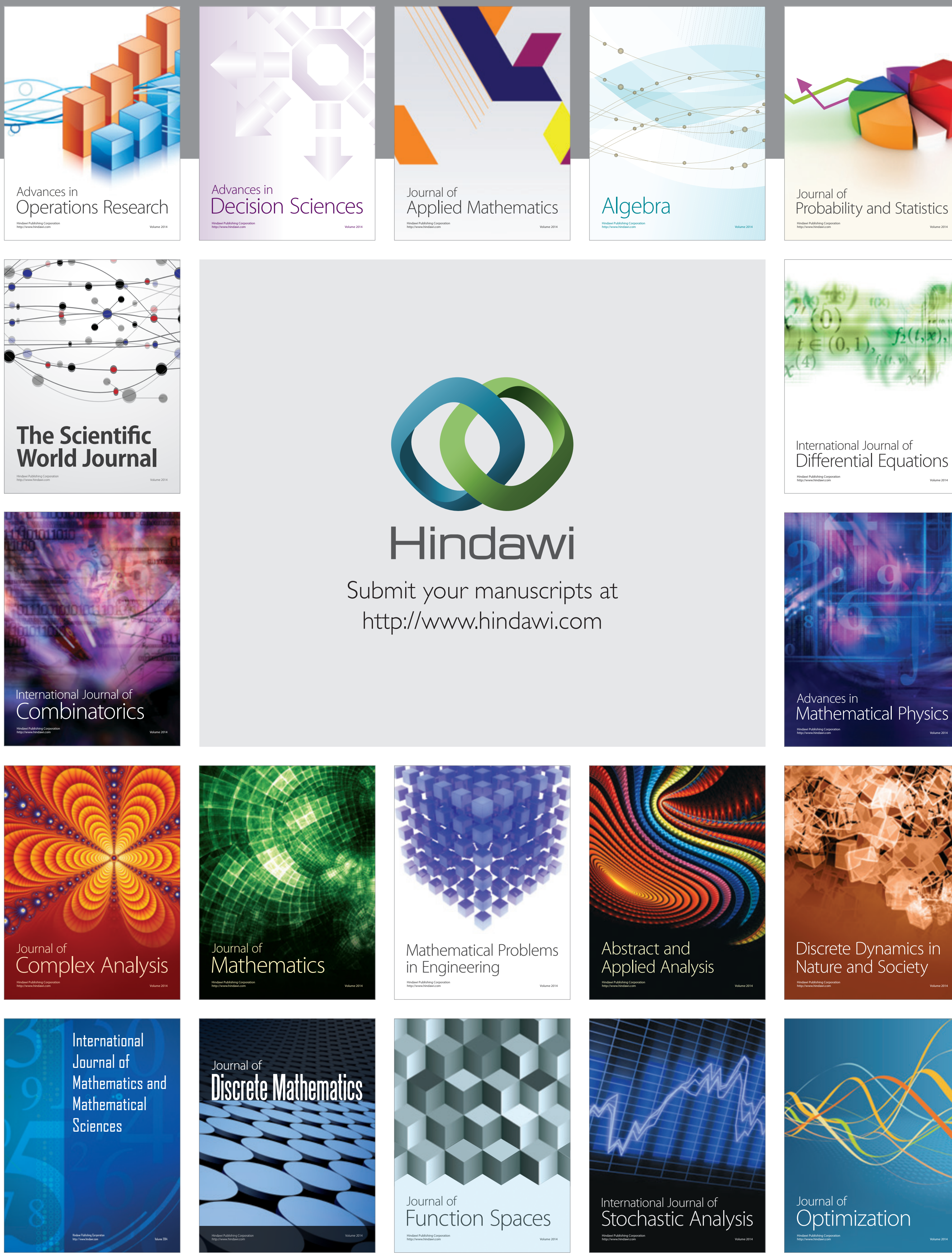
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