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Research Article

Passivity and Passification for Delay Fuzzy System Based on Delay Partitioning Approach

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A delay partitioning approach is introduced to solve problems of passivity and passification for continuous T-S fuzzy system with time delay. Our aim is to design a state feedback controller such that the resulting closed system is passive. By constructing a Lyapunov-Krasovskii functional, delay-dependent passivity/passification performance conditions are formulated in terms of linear matrix inequalities (LMIs). Finally, numerical examples are used to illustrate the effectiveness of the proposed approaches which can further reduce conservatism and become more obvious with partitioning getting thinner.

1. Introduction

The passivity concept was introduced by Willems [1] and developed further by Hill and Moylan [2]. Passivity of nonlinear systems has attracted great interest in the control area mainly because of the link between stability and passivity. The passivity theory provides a nice tool for analyzing the stability of systems and has found applications in diverse areas such as stability, signal processing, chaos control, and synchronization.

Since most physical systems in real world are nonlinear, researchers have been devoting their efforts to the control of nonlinear systems. Among the methods, the fuzzy control has been proven to be effective in dealing with the analysis of nonlinear systems. especially the T-S fuzzy control [3, 4]. It is denoted by a group of IF-THEN rules that the conventional linear system theory can be applied to the analysis of the class of nonlinear systems, and numerous nonlinear analysis problems have been studied based on this T-S fuzzy model, such as [5–7] reported the problem of stability analysis, and [8–10] investigated the H_∞ control designs. References [11–13] mentioned the fault detection of the T-S fuzzy systems. H_∞ model reduction is addressed in [14]. The fuzzy controller was carried out via Parallel Distributed Compensation (PDC) technique [15]. Based on the PDC technique, the fuzzy controller can also be designed to guarantee the passivity of T-S fuzzy systems.

On the other hand, the time delay exists naturally in various control systems. Time delays often degrade the system's performance and even cause instability. Therefore, time delays have received great attention in recent years and many researchers have studied various analytical techniques and developed many synthesis methods for time-delay systems. For instance, model reduction is addressed in [16] and filtering problems are investigated in [17, 18]. So, the passivity and passification analysis of nonlinear systems with time delays is worth to be discussed and researched. To date, researches have gained many results in passivity control of T-S fuzzy systems; the passivity of delayed neural networks is considered in [19], passivity of fuzzy time-delay systems is investigated in [20] which adopt delay moom's inequality, and passive controller design for T-S fuzzy systems is addressed in [21]. The passivity of uncertain fuzzy systems is considered in [22]. However, the above methods still have strong conservation, and it is necessary for us to further study.

In this paper, we adopt a delay partitioning approach to study the passivity and passification of T-S fuzzy systems with time delay. Based on this idea, we can further reduce the conservatism, and it becomes even less conservative when the partitioning goes finer. The results of this paper are given in terms of LMIs. The rest of the paper is organized as follows. In Section 2, the problem to be studied is stated and some preliminaries are presented. Passivity analysis results are presented in Section 3. Based on the results obtained in Section 3, we design the controller in Section 4. In Section 5, numerical examples are given to demonstrate the effectiveness of the theoretical results. Finally, conclusions are drawn in Section 6.

Notations. Throughout the paper, A^{-1} and A^T denote the inverse and transpose of a square matrix A. R^n denotes the n-dimensional Euclidean apace and $\|\cdot\|$ refers to the Euclidean vector norm. The notation $A > 0$ is used to define a symmetric positive definite matrix and sym (A) is defined as $A+A^T$. Matrices are assumed to have compatible dimensions.

2. Problem Statement and Preliminaries

Consider the T-S fuzzy system with time delay has the following form.

Plant Rule i:

IF $Z_1(t)$ is M_{i1} and ... and $Z_p(t)$ is M_{ip} THEN

$$\begin{gathered}\dot{x}(t) = A_i x(t) + A_{di} x(t-h) + B_i u(t) + B_{1i}\omega(t), \\ y(t) = C_i x(t) + C_{di} x(t-h) + D_i \omega(t), \\ x(t) = \varphi(t), \quad t \in [-h, 0], \\ i = 1, \dots, r,\end{gathered} \tag{1}$$

where $x(t) \in R^n$ is the state vector; $u(t)$ is the control input vector; h is a time delay; $\varphi(t)$ is the initial condition.

Controller Rule i:

IF $z_1(t)$ is M_{i1} and... and $z_p(t)$ is M_{ip} THEN

$$u(t) = K_i x(t), \quad i = 1, \dots, r, \tag{2}$$

where M_{ij} is the fuzzy set; r is the number of IF-THEN rules; $z(t) = [z_1(t), z_2(t), \dots, z_p(t)]$ is the premise variables vector; K_i, $i = 1, \dots, r$ are constant matrices representing state-feedback control gains. Let $\lambda_i(t)$ be the normalized membership function of the fuzzy set

$$\lambda_i(t) = \frac{\prod_{j=1}^{p} M_{ij}(z_j(t))}{\sum_{i=1}^{r}\left\{\prod_{j=1}^{p} M_{ij}(z_j(t))\right\}}, \tag{3}$$

where $M_{ij}(z_j(t))$ is the grade of membership function of $z_j(t)$ in $M_{ij}(t)$. It is assumed that $\prod_{j=1}^{p} M_{ij}(z_j(t)) \geq 0$, $i = 1, \dots, r$, and $\sum_{i=1}^{r}\{\prod_{j=1}^{p} M_{ij}(z_j(t))\} > 0$ for all t. Therefore, $\lambda_i(t) \geq 0$ and $\sum_{i=1}^{r} \lambda_i(t) = 1$ for all t. By applying (3) into (1), the fuzzy system can be expressed as

$$\begin{aligned}\dot{x}(t) = {} & A(t)x(t) + A_d(t)x(t-h) \\ & + B(t)u(t) + B_1(t)\omega(t), \\ y(t) = {} & C(t)x(t) + C_d(t)x(t-h) + D(t)\omega(t)\end{aligned} \tag{4}$$

with

$$\begin{gathered}A(t) = \sum_{i=1}^{r}\lambda_i(t)A_i, \qquad A_d(t) = \sum_{i=1}^{r}\lambda_i(t)A_{di}, \\ B(t) = \sum_{i=1}^{r}\lambda_i(t)B_i, \qquad B_1(t) = \sum_{i=1}^{r}\lambda_i(t)B_{1i}, \\ C(t) = \sum_{i=1}^{r}\lambda_i(t)C_i, \qquad C_d(t) = \sum_{i=1}^{r}\lambda_i(t)C_{di}, \\ D(t) = \sum_{i=1}^{r}\lambda_i(t)D_i.\end{gathered} \tag{5}$$

By applying (2) into (4), we can get the following closed-loop system:

$$\begin{aligned}\dot{x}(t) = {} & \sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i(z(t))\lambda_j(z(t)) \\ & \times\left[\left(A_i + B_iK_j\right)x(t) + A_{di}x(t-h) + B_{1i}\omega(t)\right],\end{aligned} \tag{6}$$

$$y(t) = \sum_{i=1}^{r}\lambda_i(z(t))\left[C_i x(t) + C_{di}x(t-h) + D_i\omega(t)\right].$$

Before formulating the main problem, we first give the following definition.

Definition 1 (Li et al. [20]). The fuzzy system (1) is called passive if there exists a scalar $\gamma \geq 0$ such that

$$2\int_0^T \omega(s)^T y(s)\,ds \geq -\gamma\int_0^T \omega^T(s)\,\omega(s)\,ds \tag{7}$$

for all $T \geq 0$.

3. Passivity Analysis

The problems to be addressed in this paper can be expressed as follows.

Problem 1 (passivity analysis). Given the feedback controller gain matrices K_i, $i = 1, \dots, r$ in (2), determine under what conditions the closed-loop system (6) is passive for all $T \geq 0$ in the sense of Definition 1.

Problem 2 (passification). Determine the feedback controller gain matrices, K_i, $i = 1, \dots, r$ in (2), such that the closed-loop system (6) is passive for all $T \geq 0$ in the sense of Definition 1.

In this section, we will present a sufficient condition in terms of LMIs, under which the closed-loop system (6) is passive.

Theorem 2. *Given matrices K_i, an integer $m \geq 1$ and a scalar $h > 0$, if there exist symmetric positive definite matrices P, Q_i, Z_i, R_i and matrices S_{1i}, S_{2i} and scalar $\gamma > 0$, satisfying*

$$\begin{bmatrix} \Theta_{iilk} + \sigma W_\sigma^T W_\sigma & S_{1i} \\ * & -\frac{m}{h} Z_i \end{bmatrix} < 0, \quad i, l, k = 1, \ldots, r \tag{8}$$

$$\begin{bmatrix} \Theta_{ijlk} + \sigma W_\sigma^T W_\sigma & S_{1i} \\ * & -\frac{m}{h} Z_i \end{bmatrix} + \begin{bmatrix} \Theta_{jilk} + \sigma W_\sigma^T W_\sigma & S_{1j} \\ * & -\frac{m}{h} Z_j \end{bmatrix} < 0 \tag{9}$$

$$1 \leq r < j \leq r, \quad l, k = 1, \ldots, r,$$

$$Z_i < R_j, \quad i, j = 1, \ldots, r, \tag{10}$$

where

$$\begin{aligned}
\Omega_{ijlk} &= Wp^T \hat{P}\, Wp + \frac{h}{m} Wr^T R_i Wr + Wq_1^T Q_i Wq_1 \\
&\quad - Wq_2^T Q_l Wq_2 + sym\left(S_i Ws_{jk}\right), \\
\Theta_{ijlk} &= \Omega_{ijlk} - \left(W_1 C_j^T W_2 + W_2^T C_j W_1^T \right. \\
&\quad + W_2^T D_j^T W_2 + W_2^T D_j W_2 \\
&\quad \left. + W_2^T C_{dj} W_3 + W_3^T C_{dj}^T W_2 + \gamma W_2^T W_2\right), \\
Wp &= \begin{pmatrix} I_n & O_{n,mn} & O_{n,n} & O_{n,n} \\ O_{n,n} & O_{n,mn} & I_n & O_{n,n} \\ O_{n,n} & O_{n,mn} & O_{n,n} & O_{n.n} \end{pmatrix}, \\
\hat{P} &= \begin{pmatrix} O_{n,n} & P & O_{n,n} \\ P & O_{n,n} & O_{n,n} \\ O_{n,n} & O_{n,n} & O_{n,n} \end{pmatrix}, \qquad W_\sigma = \left(I_n, O_{n,mn+2n}\right), \\
Wr &= \left(O_{n,mn+n}, I_n, O_{n,n}\right), \qquad Wq_1 = \left(I_{mn}, O_{mn,3n}\right), \\
Wq_2 &= \left(O_{mn,n}, I_{mn}, O_{mn,2n}\right), \qquad S_i = \left[S_{1i} \quad S_{2i}\right], \\
W_{Sjk} &= \begin{bmatrix} I_n & & -I_n & & O_{n,mn+n} \\ A_j + B_j K_k & O_{n,mn-n} & A_{dj} & -I_n & B_{1j} \end{bmatrix}, \\
W_1 &= \begin{pmatrix} I_n \\ O_{mn+2n,n} \end{pmatrix}, \qquad W_2 = \left(O_{n,mn+2n}, I_n\right) \\
W_3 &= \left(O_{n,mn}, I_n, O_{n,2n}\right).
\end{aligned} \tag{11}$$

Then the fuzzy system (1) *is passive in the sense of Definition 1 for the time delay* $0 \leq \tau \leq h$.

Proof. Choose a Lyapunov-Krasovskii functional as $V(t) = V_1(t) + V_2(t) + V_3(t)$ with

$$\begin{aligned}
V_1(t) &= x^T(t) P x(t), \\
V_2(t) &= \int_{-h/m}^{0} \int_{t+\beta}^{t} \dot{x}^T(\alpha) R(\alpha) \dot{x}(\alpha)\, d\alpha\, d\beta, \\
V_3(t) &= \int_{t-h/m}^{t} \gamma^T(\alpha) Q(\alpha) \gamma(\alpha)\, d\alpha,
\end{aligned} \tag{12}$$

where

$$\gamma(t) = \left[x^T(t) \quad x^T\left(t - \frac{h}{m}\right) \quad \cdots \quad x^T\left(t - \frac{m-1}{m} h\right)\right]^T. \tag{13}$$

The time-derivative of $V(t)$ along the trajectory of the system in (6) is given by

$$\begin{aligned}
\dot{V}_1 &= \dot{x}^T(t) P x(t) + x^T(t) P \dot{x}(t), \\
\dot{V}_2 &= \frac{h}{m} \dot{x}^T(t) R(t) \dot{x}(t) - \int_{t-h/m}^{t} \dot{x}^T(\alpha) R(\alpha) \dot{x}(\alpha)\, d\alpha \\
\dot{V}_3 &= \gamma^T(t) Q(t) \gamma(t) - \gamma^T\left(t - \frac{h}{m}\right) Q\left(t - \frac{h}{m}\right) \gamma\left(t - \frac{h}{m}\right).
\end{aligned} \tag{14}$$

Define

$$\begin{aligned}
\xi(t) &= \left[\gamma^T(t) \quad x^T(t-h) \quad \dot{x}^T(t) \quad \omega^T(t)\right]^T, \\
\omega(t) &= \left[\omega_1(t) \quad \cdots \quad \omega_n(t)\right]^T
\end{aligned} \tag{15}$$

and according to the Newton-Leibniz formula and the system in (6), we have

$$\begin{aligned}
\Pi_1 &= 2\xi^T(t) S_1(t) \\
&\quad \times \left[x(t) - x\left(t - \frac{h}{m}\right) - \int_{t-h/m}^{t} \dot{x}(\alpha)\, d\alpha\right] = 0, \\
\Pi_2 &= 2\xi^T(t) S_2(t) \left[\left(A(t) + B(t) K(t)\right) x(t) \right. \\
&\quad + A_d(t) x(t-h) \\
&\quad \left. + B_1(t) \omega(t) - \dot{x}(t)\right] = 0, \\
\Pi_3 &= \frac{h}{m} \xi^T(t) S_1(t) Z^{-1}(t) S_1^T(t) \xi(t) \\
&\quad - \int_{t-h/m}^{t} \xi^T(t) S_1(t) Z^{-1}(t) S_1^T(t) \xi(t)\, d\alpha = 0.
\end{aligned} \tag{16}$$

Therefore

$$\begin{aligned}
\dot{V}(t) \leq{} & \dot{V}_1(t)+\dot{V}_2(t)+\dot{V}_3(t)+\Pi_1+\Pi_2+\Pi_3 \\
={} & \dot{x}^T(t)Px(t)+x^T(t)P\dot{x}(t)+\frac{h}{m}\dot{x}^T(t)R(t)\dot{x}(t) \\
& -\int_{t-h/m}^{t}\dot{x}^T(\alpha)R(\alpha)\dot{x}(\alpha)\,d\alpha+\gamma^T(t)Q(t)\gamma(t) \\
& -\gamma^T\left(t-\frac{h}{m}\right)Q\left(t-\frac{h}{m}\right)\gamma\left(t-\frac{h}{m}\right)+2\xi^T(t)S_1(t) \\
& \times\left[x(t)-x\left(t-\frac{h}{m}\right)-\int_{t-h/m}^{t}\dot{x}(\alpha)\,d\alpha\right] \\
& +2\xi^T(t)S_2(t) \\
& \times\left[(A(t)+B(t)K(t))x(t)+A_d(t)x(t-h)\right. \\
& \quad\left.+B_1(t)\omega(t)-\dot{x}(t)\right] \\
& +\frac{h}{m}\xi^T(t)S_1(t)Z^{-1}(t)S_1^T(t)\xi(t) \\
& -\int_{t-h/m}^{t}\xi^T(t)S_1(t)Z^{-1}(t)S_1^T(t)\xi(t)\,d\alpha \\
={} & \Lambda(t)+\frac{h}{m}\xi^T(t)S_1(t)Z^{-1}(t)S_1^T(t)\xi(t) \\
& -\int_{t-h/m}^{t}\dot{x}^T(\alpha)R(\alpha)\dot{x}(\alpha)\,d\alpha \\
& -2\xi^T(t)S_1(t)\int_{t-h/m}^{t}\dot{x}(\alpha)\,d\alpha \\
& -\int_{t-h/m}^{t}\xi^T(t)S_1(t)Z^{-1}(t)S_1^T(t)\xi(t)\,d\alpha.
\end{aligned} \tag{17}$$

If

$$Z(t)<R(\alpha). \tag{18}$$

Then

$$Z^{-1}(t)>R^{-1}(\alpha), \tag{19}$$

$$\begin{aligned}
&-\int_{t-h/m}^{t}\xi^T(t)S_1(t)Z^{-1}(t)S_1^T(t)\xi(t)\,d\alpha \\
&\quad<-\int_{t-h/m}^{t}\xi^T(t)S_1(t)R^{-1}(\alpha)S_1^T(t)\xi(t)\,d\alpha.
\end{aligned} \tag{20}$$

So, we can obtain

$$\begin{aligned}
\dot{V}(t) \leq{} & \Lambda(t)+\frac{h}{m}\xi^T(t)S_1(t)Z^{-1}(t)S_1^T(t)\xi(t) \\
& -\int_{t-h/m}^{t}\left(\dot{x}^T(\alpha)R(\alpha)+\xi^T(t)S_1(t)\right)\times R^{-1}(\alpha) \\
& \quad\times\left(R(\alpha)\dot{x}(\alpha)+S_1^T(t)\xi(t)\right)d\alpha,
\end{aligned} \tag{21}$$

where

$$\begin{aligned}
\Lambda(t)={} & \dot{x}^T(t)Px(t)+x^T(t)P\dot{x}(t)+\frac{h}{m}\dot{x}^T(t)R(t)\dot{x}(t) \\
& +\gamma^T(t)Q(t)\gamma(t)-\gamma^T\left(t-\frac{h}{m}\right)Q\left(t-\frac{h}{m}\right) \\
& \times\gamma\left(t-\frac{h}{m}\right)+2\xi^T(t)S_1(t)\left[x(t)-x\left(t-\frac{h}{m}\right)\right] \\
& +2\xi^T(t)S_2(t) \\
& \times\left[(A(t)+B(t)K(t))x(t)\right. \\
& \quad\left.+A_d(t)x(t-h)+B_1(t)\omega(t)-\dot{x}(t)\right].
\end{aligned} \tag{22}$$

Besides

$$\begin{aligned}
\dot{x}(t) &= \left(O_{n,mn+n}, I_n, O_{n,n}\right)\xi(t), \\
x(t) &= \left(I_n, O_{n,mn+2n}\right)\xi(t), \\
\gamma(t) &= \left(I_{mn}, O_{mn,3n}\right)\xi(t), \\
\gamma\left(t-\frac{h}{m}\right) &= \left(O_{mn,n}, I_{mn}, O_{mn,2n}\right)\xi(t).
\end{aligned} \tag{23}$$

Then

$$\begin{aligned}
\Lambda(t) &= \xi^T(t)\Omega(t)\xi(t), \\
\Omega(t) &= W_p^T\hat{P}W_p+\frac{h}{m}W_r^TR(t)W_r+W_{q1}^TQ(t)W_{q1} \\
&\quad -W_{q2}^TQ\left(t-\frac{h}{m}\right)W_{q2}+\operatorname{sym}\left(S(t)W_S(t)\right), \\
\widehat{P} &= \begin{pmatrix} O_{n,n} & P & O_{n,n} \\ P & O_{n,n} & O_{n,n} \\ O_{n,n} & O_{n,n} & O_{n,n} \end{pmatrix}, \qquad S(t)=\begin{bmatrix} S_1(t) & S_2(t) \end{bmatrix} \\
Wp &= \begin{pmatrix} I_n & O_{n,mn} & O_{n,n} & O_{n,n} \\ O_{n,n} & O_{n,mn} & I_n & O_{n,n} \\ O_{n,n} & O_{n,mn} & O_{n,n} & O_{n.n} \end{pmatrix}, \\
Wr &= \left(O_{n,mn+n}, I_n, O_{n,n}\right), \qquad Wq_1=\left(I_{mn}, O_{mn,3n}\right), \\
Wq_2 &= \left(O_{mn,n}, I_{mn}, O_{mn,2n}\right), \\
W_S(t) &= \begin{bmatrix} I_n & & -I_n & & O_{n,mn+n} \\ A(t)+B(t)K(t) & O_{n,mn-n} & A_d(t) & -I_n & B_1(t) \end{bmatrix}.
\end{aligned} \tag{24}$$

Besides

$$
\begin{aligned}
2\omega^T(t)\,y(t) + \gamma\omega^T(t)\,\omega(t)
&= x^T(t)\,C_i^T\omega(t) + \omega^T(t)\,C_i x(t) \\
&\quad + \omega^T(t)\left(D_i^T + D_i\right)\omega(t) + \omega^T(t)\,C_{di}x(t-h) \\
&\quad + x^T(t-h)\,C_{di}^T\omega(t) + \gamma\omega^T(t)\,\omega(t) \\
&= \xi^T(t)\left[W_1C_i^TW_2 + W_2^TC_iW_1^T + W_2^T\left(D_i^T + D_i\right)W_2\right. \\
&\quad \left. + W_2^TC_{di}W_3 + W_3^TC_{di}^TW_2 + \gamma W_2^TW_2\right]\xi(t),
\end{aligned} \tag{25}
$$

where

$$
W_1 = \begin{pmatrix} I_n \\ O_{mn+2n,n} \end{pmatrix}, \qquad W_2 = \left(O_{n,mn+2n}, I_n\right),
\qquad W_3 = \left(O_{n,mn}, I_n, O_{n,2n}\right). \tag{26}
$$

So, we have

$$
\begin{aligned}
\dot V(t) - 2\omega^T(t)\,y(t) - \lambda\omega^T(t)\,\omega(t)
&\le \xi^T(t)\,\Theta(t)\,\xi(t) + \frac{h}{m}\xi^T(t)\,S_1(t)\,Z^{-1}(t)\,S_1^T(t)\,\xi(t) \\
&\quad - \int_{t-h/m}^{t}\left(\dot x^T(\alpha)\,R(\alpha) + \xi^T(t)\,S_1(t)\right)\times R^{-1}(\alpha) \\
&\quad \times\left(R(\alpha)\,\dot x(\alpha) + S_1^T(t)\,\xi(t)\right)d\alpha
\end{aligned} \tag{27}
$$

with

$$
\begin{aligned}
\Theta(t) = \Omega(t) - \left(W_1C_i^TW_2 + W_2^TC_iW_1^T\right. & \\
+ W_2^T\left(D_i + D_i^T\right)W_2 & \\
\left. + W_2^TC_{di}W_3 + W_3^TC_{di}^TW_2 + \gamma W_2^TW_2\right). &
\end{aligned} \tag{28}
$$

If

$$
\Theta(t) + \sigma W_\sigma^TW_\sigma + \frac{h}{m}S_1(t)\,Z^{-1}(t)\,S_1^T(t) < 0 \tag{29}
$$

then

$$
\begin{aligned}
&\Theta(t) + \frac{h}{m}S_1(t)\,Z^{-1}(t)\,S_1^T(t) < -\sigma W_\sigma^TW_\sigma, \\
&\dot V(t) - 2\omega^T(t)\,y(t) - \gamma\omega^T(t)\,\omega(t) \\
&\quad < \xi^T(t)\,\Theta(t)\,\xi(t) + \frac{h}{m}\xi^T(t)\,S_1(t)\,Z^{-1}(t)\,S_1^T(t)\,\xi(t) \\
&\quad < -\xi^T(t)\,\sigma W_\sigma^TW_\sigma\xi(t) < -\sigma\|x(t)\|^2 < 0.
\end{aligned} \tag{30}
$$

We can obtain

$$
\dot V(t) < 2y^T(t)\,\omega(t) + \gamma\omega^T(t)\,\omega(t). \tag{31}
$$

It follows by integrating (31) with respect to t over the time period $0 \sim T$ that (7) holds, and hence the delayed fuzzy system (1) is passive in the sense of Definition 1. Our next objective is to convert the inequalities in (18) and (29) to some finite LMIs, then (29) can be rewritten as

$$
\begin{aligned}
\eta_{ijlk} &= \eta_{iilk} + \left(\eta_{ijlk(i<j)} + \eta_{jilk(i<j)}\right) < 0, \\
\eta_{ijlk} &= \sum_{i=1}^{r}\lambda_i(\theta(t))\sum_{j=1}^{r}\lambda_j(\theta(t)) \\
&\quad \times\sum_{l=1}^{r}\lambda_l\left(\theta\left(t-\frac{h}{m}\right)\right)\sum_{k=1}^{r}\lambda_k(\theta(t)) \\
&\quad \times\left[\Omega_{ijlk} - \left(W_1C_j^TW_2 + W_2^TC_jW_1^T\right.\right. \\
&\qquad + W_2^T\left(D_j + D_j^T\right)W_2 \\
&\qquad \left. + W_2^TC_{dj}W_3 + W_3^TC_{dj}^TW_2 + \gamma W_2^TW_2\right) \\
&\qquad + \sigma W_\sigma^TW_\sigma \\
&\qquad \left. + \frac{h}{m}S_{1i}Z_i^{-1}S_{1i}^T\right].
\end{aligned} \tag{32}
$$

Equation (18) can be rewritten as

$$
Z_i < R_j. \tag{33}
$$

From the Schur complement, we can get the inequalities (8), (9), and (10), and the proof is completed. □

4. Controller Design

In this section, fuzzy state feedback controllers will be designed based on the result developed in the previous section.

Theorem 3. *Given an integer $m > 1$ and scalars $h > 0$, τ_1, $\tau_2, \ldots, \tau_{(m+3)}$, there exists a fuzzy state feedback controller such that the closed-loop system* (4) *is passive if there exist symmetric* *positive definite matrices $\overline{P}$, $\overline{Q_i}$, $\overline{Z_i}$, $\overline{R_i}$, and matrices X, $\overline{S_{1i}}$, M_i and scalars $\gamma > 0$, $\sigma > 0$, satisfying*

$$\begin{bmatrix} \varphi_{iil} & \overline{S_{1i}} & W_\sigma^T X^T & W_2^T X^T & W_2^T X^T \\ * & -\frac{m}{h}\overline{Z_i} & 0 & 0 & 0 \\ * & * & -\sigma^{-1} I_n & 0 & 0 \\ * & * & * & D_i^{-T} + D_i^{-1} & 0 \\ * & * & * & * & \gamma^{-1} I_n \end{bmatrix} < 0 \quad (34)$$

$$i, l = 1, \ldots, r,$$

$$\begin{bmatrix} \phi_{ijl} & \overline{S_{1i}} & W_\sigma^T X^T & W_2^T X^T & W_2^T X^T \\ * & -\frac{m}{h}\overline{Z_i} & 0 & 0 & 0 \\ * & * & -\sigma^{-1} I_n & 0 & 0 \\ * & * & * & D_i^{-T} + D_i^{-1} & 0 \\ * & * & * & * & \gamma^{-1} I_n \end{bmatrix} + \begin{bmatrix} \phi_{jil} & \overline{S_{1j}} & W_\sigma^T X^T & W_2^T X^T & W_2^T X^T \\ * & -\frac{m}{h}\overline{Z_j} & 0 & 0 & 0 \\ * & * & -\sigma^{-1} I_n & 0 & 0 \\ * & * & * & D_j^{-T} + D_j^{-1} & 0 \\ * & * & * & * & \gamma^{-1} I_n \end{bmatrix} < 0$$

$$1 \le i < j \le r, \quad l = 1, \ldots, r, \quad (35)$$

$$\overline{Z_i} < \overline{R_j}, \quad i, j = 1, \ldots, r, \quad (36)$$

where

$$\varphi_{ijl} = W_p^T \hat{P} W_p + \frac{h}{m} W_r^T \overline{R_i} W_r + W_{q1}^T \overline{Q_i} W_{q1} - W_{q2}^T \overline{Q_l} W_{q2} + sym\left(V_i \overline{W_{Sij}} - W_1 X^T C_i^T W_2 - W_2^T C_{di} X W_3\right),$$

$$\hat{P} = \begin{pmatrix} O_{n,n} & \overline{P} & O_{n,n} \\ \overline{P} & O_{n,n} & O_{n,n} \\ O_{n,n} & O_{n,n} & O_{n,n} \end{pmatrix}, \quad V_i = \begin{bmatrix} \overline{S_{1i}} & U \end{bmatrix},$$

$$U = \begin{bmatrix} \tau_1 I_n & \tau_2 I_n & \cdots & \tau_{(m+3)} I_n \end{bmatrix}^T,$$

$$\overline{W_{Sij}} = \left[\begin{array}{ccccc} & I_n & -I_n & O_{n,mn+n} & \\ \hline A_i X + B_i M_j & O_{n,mn-n} & A_{di} X & -X & B_{1i} \end{array}\right]. \quad (37)$$

If the above conditions are feasible, the gains of the controller are given by

$$K_i = M_i X^{-1}, \quad i = 1, \ldots, r. \quad (38)$$

Proof. Assume that X is invertible, define $S = X^{-T}$, and

$$G1 = \operatorname{diag}\{S, \ldots, S, I_n, S, S, S, S\} \in R^{(mn+7n)\times(mn+7n)},$$
$$G2 = \operatorname{diag}\{S, \ldots, S, I_n\} \in R^{(mn+3n)\times(mn+3n)}, \quad (39)$$
$$G3 = \operatorname{diag}\{S, \ldots, S\} \in R^{mn\times mn}.$$

Premultiplying and postmultiplying (34) with G_1 and G_1^T, then we obtain

$$\begin{bmatrix} \Delta & G_2\overline{S_{1i}}S^T & G_2 W_\sigma^T X^T S^T & G_2 W_2^T X^T S^T & G_2 W_2^T X^T S^T \\ * & -\frac{m}{h} S\overline{Z_i} S^T & 0 & 0 & 0 \\ * & * & -\sigma^{-1} S S^T & 0 & 0 \\ * & * & * & S\left(D_i^{-T} + D_i^{-1}\right) S^T & 0 \\ * & * & * & * & \gamma^{-1} S S^T \end{bmatrix} < 0, \quad (40)$$

where

$$\Delta = G_2 \phi_{iil} G_2^T,$$

$$G_2 \phi_{iil} G_2^T = W_p^T \begin{bmatrix} O_{n,n} & S\overline{P}S^T & O_{n,n} \\ S\overline{P}S^T & O_{n,n} & O_{n,n} \\ O_{n,n} & O_{n,n} & O_{n,n} \end{bmatrix} W_p + \frac{h}{m} W_r^T S\overline{R_i} S^T W_r + W_{q1}^T G_3 \overline{Q_i} G_3^T W_{q1} - W_{q2}^T G_3 \overline{Q_l} G_3^T W_{q2} + \operatorname{sym}\left(G_2 V_i \overline{W_{Sii}} G_2^T\right) - W_1 C_i^T W_2 - W_2^T C_i W_1^T - W_3^T C_{di}^T W_2 - W_2^T C_{di} W_3, \quad (41)$$

$$G_2 W_\sigma^T X^T S^T = W_\sigma^T S^T, \qquad G_2 W_2^T X^T S^T = W_2^T S^T.$$

We defining

$$P = S\overline{P}S^T, \qquad R_i = S\overline{R_i}S^T, \qquad Q_i = G_3 \overline{Q_i} G_3^T,$$
$$S_{1i} = G_2 \overline{S_{1i}} S^T, \qquad Z_i = S\overline{Z_i} S^T, \quad (42)$$
$$S_{2i} = \begin{bmatrix} \tau_1 S^T & \tau_2 S^T & \cdots & \tau_{(m+2)} S^T & \tau_{(m+3)} I_n \end{bmatrix}^T.$$

Then

$$\begin{aligned}
G_2V_iW_{Sii}G_2^T &= G_2\overline{S_{1i}}\left[I_n \quad -I_n \quad O_{n,mn+n}\right]G_2^T \\
&\quad + G_2U\left[A_iX+B_iM_i \quad O_{n,mn-n} \quad A_{di}X \quad -X \quad B_{1i}\right]G_2^T \\
&= G_2\overline{S_{1i}}S^T\left[I_n \quad -I_n \quad O_{n,mn+n}\right] \\
&\quad + S_{2i}\left[A_i+B_iK_k \quad O_{n,mn-n} \quad A_{di} \quad -I_n \quad B_{1i}\right] \\
&= \left[S_{1i} \quad S_{2i}\right]\left[\frac{I_n \quad -I_n \quad O_{n,mn+n}}{A_i+B_iK_k \quad O_{n,mn-n} \quad A_{di} \quad -I_n \quad B_{1i}}\right]
\end{aligned} \tag{43}$$

then

$$\begin{aligned}
G_2\varphi_{iil}G_2^T &= W_p^T\begin{bmatrix} O_{n,n} & P & O_{n,n} \\ P & O_{n,n} & O_{n,n} \\ O_{n,n} & O_{n,n} & O_{n,n}\end{bmatrix}W_p + \frac{h}{m}W_r^TR_iW_r \\
&\quad + W_{q1}^TQ_iW_{q1} - W_{q2}^TQ_lW_{q2} + \operatorname{sym}\left(G_2V_i\overline{W_{Sii}}G_2^T\right) \\
&\quad - W_1C_i^TW_2 - W_2^TC_iW_1^T \\
&\quad - W_3^TC_{di}^TW_2 - W_2^TC_{di}W_3.
\end{aligned} \tag{44}$$

Thus, by the Schur complement, we can obtain (40) is equivalent to (8). Pre- and postmultiplying (35) with G_1 and G_1^T, we obtain (9), pre- and postmultiplying (36) with S and S^T, we obtain (10). The proof is completed. □

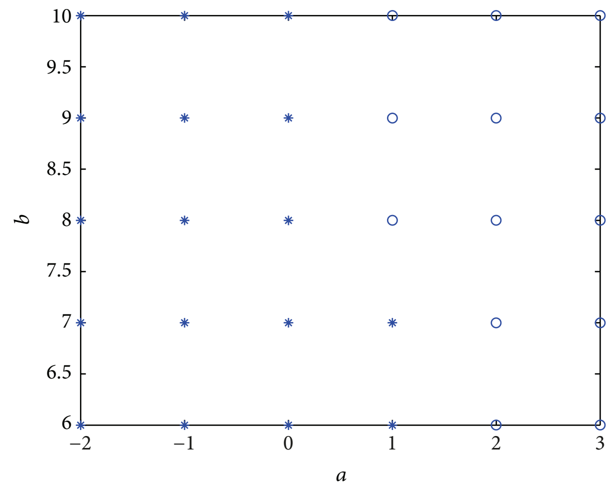

Figure 1: The feasible region based on Theorem 2.

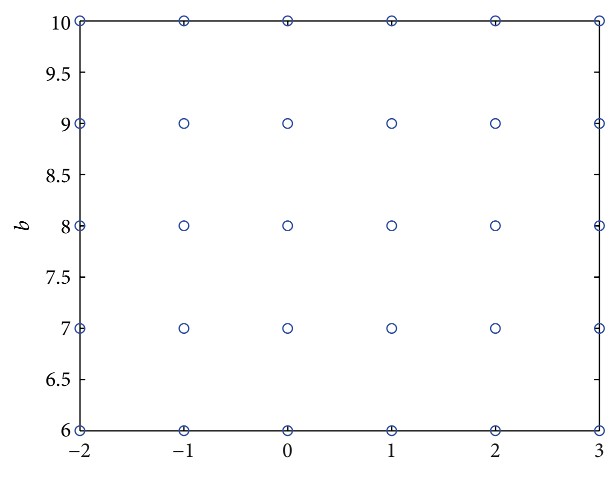

Figure 2: The feasible region based on [22].

5. Numerical Example

Without delay and uncertainty, Example 1 designs different passive controllers by applying the theorem of our paper and the literature [22], respectively. We can compare the region of feasible solution.

Example 1. Consider a fuzzy system of the following form.

Plant Rules:

Rule 1: IF $x_1(t)$ is M_1, THEN

$$\begin{aligned}\dot{x}(t) &= A_1x(t) + B_1u(t) + B_{11}\omega(t), \\ y(t) &= C_1x(t) + D_1\omega(t).\end{aligned} \tag{45}$$

Rule 2: IF $x_1(t)$ is M_2, THEN

$$\begin{aligned}\dot{x}(t) &= A_2x(t) + B_2u(t) + B_{12}\omega(t), \\ y(t) &= C_2x(t) + D_2\omega(t)\end{aligned} \tag{46}$$

with

$$\begin{gathered}
A_1 = \begin{bmatrix} a & -0.02 & 1 \\ 1 & 0 & 5 \\ 1 & 2 & 3\end{bmatrix}, \qquad A_2 = \begin{bmatrix} -0.225 & -0.02 & 0 \\ 1 & 0 & 3 \\ 1 & 1 & 1\end{bmatrix}, \\
B_1 = \begin{bmatrix} 1 \\ 0 \\ b\end{bmatrix}, \qquad B_2 = \begin{bmatrix} 1 \\ 0 \\ 2\end{bmatrix}, \qquad B_{11} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 0 & 1 & 0\end{bmatrix}, \\
B_{12} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 3 \\ 0 & 1 & 2\end{bmatrix}, \qquad C_1 = C_2 = \begin{bmatrix} 2.5 & 0 & 0 \\ 0 & 2.5 & 0 \\ 1 & 2 & 1\end{bmatrix}, \\
D_1 = D_2 = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 1 & 1 & 1\end{bmatrix}, \quad a \in [-2, 3], \ b \in [6, 10].
\end{gathered} \tag{47}$$

We can obtain Figure 1 by applying our method and obtain Figure 2 by applying method in [22]. Symbol "*" shows that feasible solution exists at that point; symbol "o" shows that feasible solution does not exist at that point.

It is obvious that Theorem 3 can relax the conditions in [22], and our method futher reduced the conservatism.

Example 2 (Li et al. [20]). Consider a delayed fuzzy system of the following form.

Plant Rules:

Rule 1: IF $x_1(t)$ is M_1, THEN

$$\begin{aligned} \dot{x}(t) &= A_1 x(t) + A_{d1} x(t-h) + B_1 u(t) + B_{11}\omega(t), \\ y(t) &= C_1 x(t) + C_{d1} x(t-h) + D_1 \omega(t). \end{aligned} \tag{48}$$

Rule 2: IF $x_1(t)$ is M_2, THEN

$$\begin{aligned} \dot{x}(t) &= A_2 x(t) + A_{d2} x(t-h) + B_2 u(t) + B_{12}\omega(t), \\ y(t) &= C_2 x(t) + C_{d2} x(t-h) + D_2 \omega(t) \end{aligned} \tag{49}$$

with

$$\begin{gathered} A_1 = \begin{bmatrix} -1 & 0.2 \\ 0 & -0.1 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} -1 & 0 \\ 0.1 & -0.5 \end{bmatrix}, \\ A_{d1} = \begin{bmatrix} 0.1 & 0.6 \\ 0.2 & -0.1 \end{bmatrix}, \qquad A_{d2} = \begin{bmatrix} -0.2 & 0.1 \\ 0.1 & -0.2 \end{bmatrix}, \\ B_1 = \begin{bmatrix} 0.2 \\ -0.5 \end{bmatrix}, \qquad B_2 = \begin{bmatrix} 0.3 \\ 0.3 \end{bmatrix}, \qquad B_{11} = \begin{bmatrix} 1 & 0 \\ 0.2 & 0 \end{bmatrix}, \\ B_{12} = \begin{bmatrix} -0.2 & 0 \\ 0 & 0 \end{bmatrix}, \qquad C_1 = \begin{bmatrix} 1 & 0.2 \\ 0 & 0 \end{bmatrix}, \\ C_2 = \begin{bmatrix} 0.1 & 1 \\ 0 & 0 \end{bmatrix}, \qquad C_{d1} = \begin{bmatrix} -0.1 & 0.2 \\ 0 & 0 \end{bmatrix}, \\ C_{d2} = \begin{bmatrix} 0.1 & -0.1 \\ 0 & 0 \end{bmatrix}, \qquad D_1 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0 \end{bmatrix}, \\ D_2 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0 \end{bmatrix} \end{gathered} \tag{50}$$

we let $h = 7.5$, $m = 2$, $\tau_1 = 10$, $\tau_2 = 0$, $\tau_3 = 5$, $\tau_4 = 10$, $\tau_5 = 10$ and we can obtain

$$\begin{gathered} P = \begin{bmatrix} 29.0804 & 1.8737 \\ 1.8737 & 3.7272 \end{bmatrix}, \qquad X = \begin{bmatrix} 1.8020 & 0.0328 \\ 0.1385 & 0.4057 \end{bmatrix}, \\ \gamma = 237.5809. \end{gathered} \tag{51}$$

By using the delay-dependent criterion Theorem 3 of [20], we can obtain $\gamma = 604.9523$ when $\tau = 7.5$. And applying our result, $\gamma = 237.5809$ is obtained. We can see that our results is not conservative since γ is much smaller than the result of [20].

Example 3. Consider a delayed fuzzy system of the following form.

Plant Rules:

Rule 1: IF $x_2(t)$ is M_1, THEN

$$\begin{aligned} \dot{x}(t) &= A_1 x(t) + A_{d1} x(t-h) + B_1 u(t) + B_{11}\omega(t), \\ y(t) &= C_1 x(t) + D_1 \omega(t). \end{aligned} \tag{52}$$

Table 1: Allowable maximum time delay h.

Methods	The upper bound of h
[21]	1.5
Theorem 3, $m = 1$	2
Theorem 3, $m = 4$	3.16
Theorem 3, $m = 5$	8.8

Rule 2: IF $x_2(t)$ is M_2, THEN

$$\begin{aligned} \dot{x}(t) &= A_2 x(t) + A_{d2} x(t-h) + B_2 u(t) + B_{12}\omega(t), \\ y(t) &= C_2 x(t) + D_2 \omega(t) \end{aligned} \tag{53}$$

with

$$\begin{gathered} A_1 = \begin{bmatrix} -0.1125 & -0.02 \\ 1 & 0 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} -0.1125 & -1.527 \\ 1 & 0 \end{bmatrix}, \\ A_{d1} = \begin{bmatrix} -0.0125 & -0.005 \\ 0 & 0 \end{bmatrix}, \qquad A_{d2} = \begin{bmatrix} -0.0125 & -0.23 \\ 0 & 0 \end{bmatrix}, \\ B_1 = B_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \qquad B_{11} = B_{12} = I_2, \\ C_1 = C_2 = D_1 = D_2 = I_2. \end{gathered} \tag{54}$$

The membership function is

$$M_{11}(x_2(t)) = 1 - \frac{x_2^2(t)}{2.25}, \qquad M_{21}(x_2(t)) = \frac{x_2^2(t)}{2.25}. \tag{55}$$

The initial state is

$$x(0) = [-1 \quad -1.2]^T, \qquad \omega(t) = \begin{bmatrix} \dfrac{\sin(3t)}{\exp(t)} & \dfrac{\cos(3t)}{\exp(t)} \end{bmatrix}^T. \tag{56}$$

First, we should find the allowable maximum time delay h which let the fuzzy system passive. Table 1 shows the maximum time delay obtained by [21] and our method.

It clearly shows that the method in our paper can get larger upper bound than before. It also shows that conservatism is further reduced when m increases.

Then we let $h = 1.5$, $m = 4$, and we can obtain

$$P = \begin{bmatrix} 9.4980 & 9.1547 \\ 9.1547 & 8.8849 \end{bmatrix}, \qquad X = \begin{bmatrix} 0.6351 & 0.4997 \\ 0.5807 & 0.4826 \end{bmatrix}. \tag{57}$$

The fuzzy controller gains by our method are given by

$$\begin{aligned} K1 &= [150.7358 \quad -218.3595], \\ K2 &= [150.9502 \quad -219.0519]. \end{aligned} \tag{58}$$

Figure 3 shows the state response $x_1(t)$ of the closed-loop system with the controller gains in (58), and Figure 4 shows the state response $x_2(t)$ of the closed-loop system.

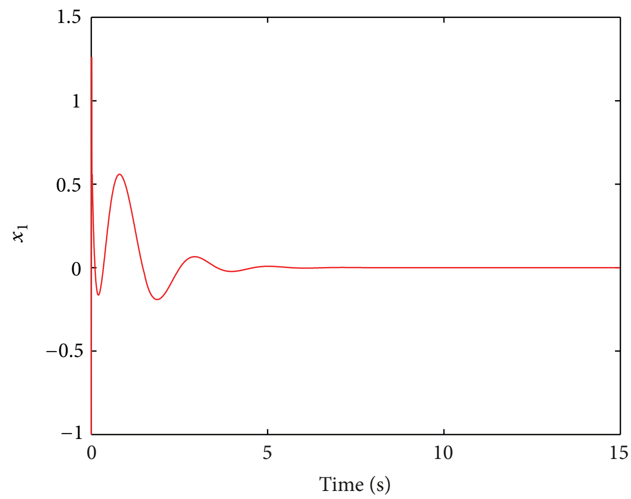

FIGURE 3: State response ×1 of the system.

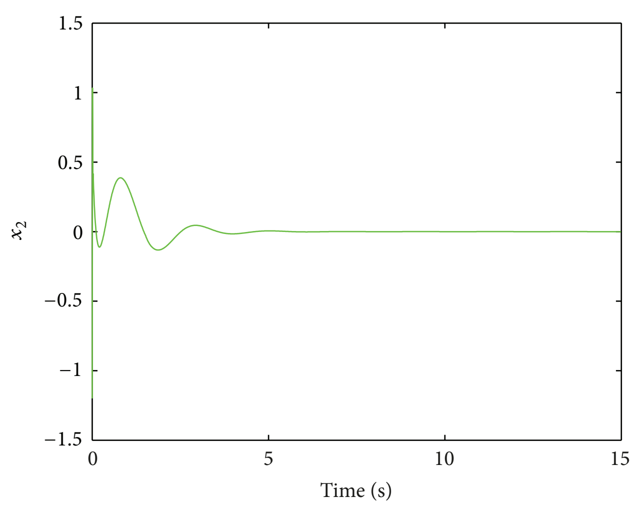

FIGURE 4: State response ×2 of the system.

6. Conclusion

This paper has adopted the delay partitioning approach to analyse the passivity and passification of delay fuzzy system based on T-S model. The theorems given in this paper are all in terms of LMIs. Examples have illustrated the effectiveness of our results. The method in our paper has further reduced the conservatism and the effect has been more apparent when m increases. In addition, the results obtained in this paper also can be extended to the fuzzy system with time-varying delay and uncertainties.

Acknowledgments

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