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Research Article

Exact Solutions of the Space-Time Fractional Bidirectional Wave Equations Using the (G'/G)-Expansion Method

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Based on Jumarie's modified Riemann-Liouville derivative, the fractional complex transformation is used to transform fractional differential equations to ordinary differential equations. Exact solutions including the hyperbolic functions, the trigonometric functions, and the rational functions for the space-time fractional bidirectional wave equations are obtained using the (G'/G)-expansion method. The method provides a promising tool for solving nonlinear fractional differential equations.

1. Introduction

It has recently become more interesting to obtain exact solutions of fractional differential equations (FDEs). These equations have been proved to be an excellent tool in the modelling of many phenomena in various fields such as signal processing, viscoelastic flow, materials and mechanics, biology systems, anomalous diffusion, and medical [1–5]. Such methods as the variational iteration method, the exp-function method, the general Riccati equation, the fractional subequation method, and the first integral method have been proposed to solve the FDEs [6–13]. Here, it is worth to mention the (G'/G)-expansion method [14, 15]. The (G'/G)-expansion method proposed by Wang et al. [16] is one of the most effective direct methods to obtain exact solutions of a large number of nonlinear evolution equations. Based on Jumarie's modified Riemann-Liouville derivative, the (G'/G)-expansion method was further extended [14, 15, 17–20] to find the solutions of fractional differential equations. In this paper, we will apply the (G'/G)-expansion method to obtain more and new exact solutions for the space-time fractional bidirectional wave equations [21]:

$$\begin{gathered} D_t^\alpha v + D_x^\alpha u + uD_x^\alpha v + vD_x^\alpha u + aD_x^\alpha D_x^\alpha D_x^\alpha u \\ - bD_x^\alpha D_x^\alpha D_t^\alpha v = 0, \quad 0 < \alpha \le 1, \\ D_t^\alpha u + D_x^\alpha v + uD_x^\alpha u + cD_x^\alpha D_x^\alpha D_x^\alpha v - dD_x^\alpha D_x^\alpha D_t^\alpha u = 0, \end{gathered} \tag{1}$$

where a, b, c, and d are real constants, t is the elapsed time, x represents the distance along the channel, the variable $u(x,t)$ is the dimensionless horizontal velocity, and $v(x,t)$ is the dimensionless deviation of the water surface from its undisturbed position. When $\alpha = 1$, (1) is the generalization of bidirectional wave equations, which was derived as a model equation describing the propagation of long waves on the surface of water with a small amplitude by Bona and Chen [22]. On the other hand, it is formally equivalent to the classical Boussinesq system and correct through first order with regard to a small parameter characterizing the typical amplitude to depth ratio [23]. Equation (1) for $\alpha = 1$ is studied by many researchers; for instance, Chen [23] used the auxiliary ordinary equation method to obtain some exact solutions of (1) for $\alpha = 1$ and the exact travelling wave solutions by

Lee and Sakthivel [24] by using the modified tanh-coth function method. The exact solutions of the space-time fractional bidirectional wave equations (1) are only reported in [21]. Based on Jumarie's modified Riemann-Liouville derivative and the fractional Riccati equation $D_\xi^\alpha \phi(\xi) = \sigma + \phi^2(\xi)$, Lu [21] obtained the rational formal solutions of (1) by introducing a new general ansätz.

This paper is organized as follows. In Section 2, some basic properties of Jumarie's modified Riemann-Liouville derivative are given. In Section 3, the main steps of the (G'/G)-expansion method are given. In Section 4, we construct the exact solutions of (1) by the present method. Some conclusions are given in Section 5.

2. Preliminaries

In this section, we give some definitions and formulas of Jumarie's modified Riemann-Liouville derivative.

Jumarie [25, 26] defined the fractional derivative in the limit form

$$f^\alpha(x) = \lim_{h\to 0} \frac{\Delta^\alpha [f(x) - f(0)]}{h^\alpha}, \quad 0 < \alpha < 1, \tag{2}$$

where

$$\Delta^\alpha f(x) = \sum_{k=0}^{\infty} (-1)^k \frac{\Gamma(1+\alpha)}{\Gamma(1+k)\Gamma(\alpha-k+1)} f[x + (\alpha - k)h], \tag{3}$$

where $f : R \to R$, $x \to f(x)$ denote a continuous (but not necessarily differentiable) function and h denotes a constant discretization span. An alternative, which is strictly equivalent to (2), is the following expression as

$$f^\alpha(x) = \frac{1}{\Gamma(1-\alpha)} \frac{d}{dx} \int_0^x (x-\xi)^{-\alpha} [f(\xi) - f(0)]\, d\xi, \quad 0 < \alpha < 1, \tag{4}$$

$$f^\alpha(x) = \left(f^{(n)}(x)\right)^{(\alpha-n)}, \quad n \le \alpha \le n+1,\ n \ge 1.$$

Some useful formulas of Jumarie's modified Riemann-Liouville derivative were summarized in [25, 26]; three of them are

$$\begin{gathered} D_x^\alpha x^\gamma = \frac{\Gamma(1+\gamma)}{\Gamma(1+\gamma-\alpha)} x^{\gamma-\alpha}, \quad \gamma > 0, \\ D_x^\alpha (u(x)v(x)) = v(x) D_x^\alpha u(x) + u(x) D_x^\alpha v(x), \\ D_x^\alpha f[u(x)] = f_u'[u(x)] D_x^\alpha u(x) = D_u^\alpha f[u(x)] \left(u'(x)\right)^\alpha. \end{gathered} \tag{5}$$

The previous results are employed in the following sections.

3. Description of the (G'/G)-Expansion Method

In this section, we give the description of the (G'/G)-expansion method [14, 15] for solving the nonlinear FDE as

$$P\left(u, D_t^\alpha u, D_x^\beta u, D_t^\alpha D_t^\alpha u, D_t^\alpha D_x^\beta u, D_x^\beta D_x^\beta u \ldots\right) = 0, \quad 0 < \alpha,\ \beta \le 1, \tag{6}$$

where u is an unknown function and P is a polynomial of u and its partial fractional derivatives, in which the highest order derivatives and nonlinear terms are involved.

Step 1. Li and He [27] and He and Li [28] proposed a fractional complex transformation to convert fractional differential equations into ordinary differential equations (ODE), so all analytical methods devoted to the advanced calculus can be easily applied to the fractional calculus. The complex wave variable was as follows:

$$u(x,t) = U(\xi), \qquad \xi = \frac{Rx^\beta}{\Gamma(\beta+1)} + \frac{St^\alpha}{\Gamma(\alpha+1)}, \tag{7}$$

where R and S are nonzero arbitrary constants; the nonlinear FDE (6) is reduced to a nonlinear ODE:

$$Q\left(U, U', U'', U''', \ldots\right) = 0, \tag{8}$$

where the prime denotes the derivation with respect to ξ.

Step 2. Suppose that the solution of ODE (8) can be expressed as a polynomial in (G'/G) as follows:

$$U(\xi) = \sum_{i=0}^{n} a_i \left(\frac{G'}{G}\right)^i, \tag{9}$$

where a_i $(i = 0, 1, 2, \ldots, n)$ are constants to be determined later and n is a positive integer that is given by the homogeneous balance principle, and $G = G(\xi)$ satisfies the second order linear differential equation:

$$G'' + \lambda G' + \mu G = 0, \tag{10}$$

where λ and μ are real constants. Using the general solutions of (10), we have

$$\frac{G'}{G} = \begin{cases} -\frac{\lambda}{2} + \frac{\sqrt{\lambda^2 - 4\mu}}{2} \times \left(\left(C_1 \sinh\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}\xi \right) + C_2 \cosh\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}\xi \right) \right) \times \left(C_1 \cosh\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}\xi \right) + C_2 \sinh\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}\xi \right) \right)^{-1} \right), & \lambda^2 - 4\mu > 0, \\ -\frac{\lambda}{2} + \frac{\sqrt{4\mu - \lambda^2}}{2} \times \left(\left(-C_1 \sin\left(\frac{\sqrt{4\mu - \lambda^2}}{2}\xi \right) + C_2 \cos\left(\frac{\sqrt{4\mu - \lambda^2}}{2}\xi \right) \right) \times \left(C_1 \cos\left(\frac{\sqrt{4\mu - \lambda^2}}{2}\xi \right) + C_2 \sin\left(\frac{\sqrt{4\mu - \lambda^2}}{2}\xi \right) \right)^{-1} \right), & \lambda^2 - 4\mu < 0, \\ \left(\frac{C_2}{C_1 + C_2\xi} \right) - \frac{\lambda}{2}, & \lambda^2 - 4\mu = 0, \end{cases} \tag{11}$$

where C_1 and C_2 are arbitrary constants.

Step 3. Substituting (9) along with (10) into (8), collecting all terms with the same order of (G'/G) together, the left-hand side of (8) is converted into a polynomial in (G'/G), and then setting coefficients of the polynomial to zero yields a set of algebraic equations for R, S, λ, and μ and a_i $(i = 0, 1, \ldots, n)$.

Step 4. Solving the algebraic equations obtained in Step 3, the constants R, S, λ, and μ and a_i $(i = 0, 1, \ldots, n)$ can be expressed. Substituting these values into expression (9), we can obtain the general form of the exact solution of (8).

Step 5. Substituting the exact solutions of (10) into the general form of exact solution obtained in Step 4, then we can obtain the exact solutions of (6).

4. Exact Solutions of (1)

In this section, we use above the (G'/G)-expansion method to explore the exact solutions of (1).

Let

$$u(x,t) = U(\xi), \qquad v(x,t) = V(\xi),$$
$$\xi = \frac{Rx^\alpha}{\Gamma(\alpha+1)} + \frac{St^\alpha}{\Gamma(\alpha+1)}, \tag{12}$$

where R and S are nonzero constants, and substituting (12) into (1), we obtain

$$SV' + RU' + RUV' + RVU' + aR^3U''' - bR^2SV''' = 0,$$
$$SU' + RV' + RUU' + cR^3V''' - dR^2SU''' = 0. \tag{13}$$

We suppose that (13) has the solution in the form

$$U(\xi) = \sum_{i=0}^{n} a_i \left(\frac{G'}{G}\right)^i,$$
$$V(\xi) = \sum_{j=0}^{m} b_j \left(\frac{G'}{G}\right)^j. \tag{14}$$

Balancing the highest order derivative terms and nonlinear terms in (13), we get $m = n = 2$. So we can write

$$U(\xi) = a_0 + a_1\left(\frac{G'}{G}\right) + a_2\left(\frac{G'}{G}\right)^2,$$
$$V(\xi) = b_0 + b_1\left(\frac{G'}{G}\right) + b_2\left(\frac{G'}{G}\right)^2, \tag{15}$$

where a_0, a_1, a_2, b_0, b_1, and b_2 are constants to be determined later.

Substituting (15) along with (10) into (13), collecting all the terms of powers of (G'/G), the left-hand side of (13) is converted into a polynomial in (G'/G), and then setting each coefficient to zero yields a set of algebraic equations for R, S, λ, μ, a_0, a_1, a_2, b_0, b_1, b_2. Solving the set of algebraic equations by Maple, we get the following results.

Case 1. Consider

$$a_0 = \frac{8a_2^2\mu + a_2^2\lambda^2 - 12}{12a_2} + \frac{2c - aa_2^2}{a_2(b - 2d)}, \qquad a_1 = a_2\lambda,$$
$$b_0 = \frac{8a_2^2\mu - 12a_2^2 + a_2^2\lambda^2 + 12}{12a_2^2}, \qquad b_1 = \lambda, \qquad b_2 = 1,$$

$$R = \frac{a_2}{2}\sqrt{\frac{b-2d}{3daa_2^2-3cb}},$$
$$S = \frac{aa_2^2-2c}{6\left(daa_2^2-cb\right)\sqrt{(b-2d)/\left(3daa_2^2-3cb\right)}}, \tag{16}$$

where a_2 is nonzero arbitrary constant.

Substituting (16) into (15), we obtain the following formal solution of (13):

$$U(\xi) = \frac{8a_2^2\mu + a_2^2\lambda^2 - 12}{12a_2} + \frac{2c - aa_2^2}{a_2(b-2d)} + a_2\lambda\left(\frac{G'}{G}\right) + a_2\left(\frac{G'}{G}\right)^2,$$
$$V(\xi) = \frac{8a_2^2\mu - 12a_2^2 + a_2^2\lambda^2 + 12}{12a_2^2} + \lambda\left(\frac{G'}{G}\right) + \left(\frac{G'}{G}\right)^2, \tag{17}$$

where $\xi = (a_2/2)\sqrt{(b-2d)/(3daa_2^2-3cb)}(x^\alpha/\Gamma(\alpha+1)) + ((aa_2^2-2c)/6(daa_2^2-cb)\sqrt{(b-2d)/(3daa_2^2-3cb)})(t^\alpha/\Gamma(\alpha+1))$.

Substituting the general solution of (10) into (17), we obtain the three types of traveling wave solutions depending on the sign of $\Delta = \lambda^2 - 4\mu$.

If $\Delta = \lambda^2 - 4\mu > 0$, we have the following general hyperbolic traveling wave solutions of (1):

$$\begin{aligned} u_1(x,t) &= \frac{8a_2^2\mu + a_2^2\lambda^2 - 12}{12a_2} + \frac{2c - aa_2^2}{a_2(b-2d)} + a_2\lambda \\ &\quad\times\left[-\frac{\lambda}{2} + \frac{\sqrt{\Delta}}{2}\times\left(\frac{C_1\sinh\left(\left(\sqrt{\Delta}/2\right)\xi\right) + C_2\cosh\left(\left(\sqrt{\Delta}/2\right)\xi\right)}{C_1\cosh\left(\left(\sqrt{\Delta}/2\right)\xi\right) + C_2\sinh\left(\left(\sqrt{\Delta}/2\right)\xi\right)}\right)\right] + a_2 \\ &\quad\times\left[-\frac{\lambda}{2} + \frac{\sqrt{\Delta}}{2}\times\left(\frac{C_1\sinh\left(\left(\sqrt{\Delta}/2\right)\xi\right) + C_2\cosh\left(\left(\sqrt{\Delta}/2\right)\xi\right)}{C_1\cosh\left(\left(\sqrt{\Delta}/2\right)\xi\right) + C_2\sinh\left(\left(\sqrt{\Delta}/2\right)\xi\right)}\right)\right]^2, \\ v_1(x,t) &= \frac{8a_2^2\mu - 12a_2^2 + a_2^2\lambda^2 + 12}{12a_2^2} + \lambda \\ &\quad\times\left[-\frac{\lambda}{2} + \frac{\sqrt{\Delta}}{2}\times\left(\frac{C_1\sinh\left(\left(\sqrt{\Delta}/2\right)\xi\right) + C_2\cosh\left(\left(\sqrt{\Delta}/2\right)\xi\right)}{C_1\cosh\left(\left(\sqrt{\Delta}/2\right)\xi\right) + C_2\sinh\left(\left(\sqrt{\Delta}/2\right)\xi\right)}\right)\right] \\ &\quad+\left[-\frac{\lambda}{2} + \frac{\sqrt{\Delta}}{2}\times\left(\frac{C_1\sinh\left(\left(\sqrt{\Delta}/2\right)\xi\right) + C_2\cosh\left(\left(\sqrt{\Delta}/2\right)\xi\right)}{C_1\cosh\left(\left(\sqrt{\Delta}/2\right)\xi\right) + C_2\sinh\left(\left(\sqrt{\Delta}/2\right)\xi\right)}\right)\right]^2, \end{aligned} \tag{18}$$

where $\xi = (a_2/2)\sqrt{(b-2d)/(3daa_2^2-3cb)}(x^\alpha/\Gamma(\alpha+1)) + ((aa_2^2-2c)/6(daa_2^2-cb)\sqrt{(b-2d)/(3daa_2^2-3cb)})(t^\alpha/\Gamma(\alpha+1))$.

In particular, setting $C_2 = 0$ and $C_1 \neq 0$, then (18) can be written as

$$\begin{aligned} u_{1(1)}(x,t) &= \frac{8a_2^2\mu + a_2^2\lambda^2 - 12}{12a_2} + \frac{2c - aa_2^2}{a_2(b-2d)} + a_2\lambda \\ &\quad\times\left[-\frac{\lambda}{2} + \frac{\sqrt{\Delta}}{2}\tanh\left(\frac{\sqrt{\Delta}}{2}\xi\right)\right] \\ &\quad+ a_2\left[-\frac{\lambda}{2} + \frac{\sqrt{\Delta}}{2}\tanh\left(\frac{\sqrt{\Delta}}{2}\xi\right)\right]^2, \\ v_{1(1)}(x,t) &= \frac{8a_2^2\mu - 12a_2^2 + a_2^2\lambda^2 + 12}{12a_2^2} + \lambda \\ &\quad\times\left[-\frac{\lambda}{2} + \frac{\sqrt{\Delta}}{2}\tanh\left(\frac{\sqrt{\Delta}}{2}\xi\right)\right] \\ &\quad+\left[-\frac{\lambda}{2} + \frac{\sqrt{\Delta}}{2}\tanh\left(\frac{\sqrt{\Delta}}{2}\xi\right)\right]^2, \end{aligned} \tag{19}$$

where $\xi = (a_2/2)\sqrt{(b-2d)/(3daa_2^2-3cb)}(x^\alpha/\Gamma(\alpha+1)) + ((aa_2^2-2c)/6(daa_2^2-cb)\sqrt{(b-2d)/(3daa_2^2-3cb)})(t^\alpha/\Gamma(\alpha+1))$.

In particular, setting $C_1 = 0$ and $C_2 \neq 0$, then (18) can be written as

$$u_{1(2)}(x,t) = \frac{8a_2^2\mu + a_2^2\lambda^2 - 12}{12a_2} + \frac{2c - aa_2^2}{a_2(b-2d)} + a_2\lambda$$

$$\times\left[-\frac{\lambda}{2}+\frac{\sqrt{\Delta}}{2}\coth\left(\frac{\sqrt{\Delta}}{2}\xi\right)\right]$$
$$+a_2\left[-\frac{\lambda}{2}+\frac{\sqrt{\Delta}}{2}\coth\left(\frac{\sqrt{\Delta}}{2}\xi\right)\right]^2,$$
$$v_{1(2)}(x,t)$$
$$=\frac{8a_2^2\mu-12a_2^2+a_2^2\lambda^2+12}{12a_2^2}+\lambda$$
$$\times\left[-\frac{\lambda}{2}+\frac{\sqrt{\Delta}}{2}\coth\left(\frac{\sqrt{\Delta}}{2}\xi\right)\right]$$
$$+\left[-\frac{\lambda}{2}+\frac{\sqrt{\Delta}}{2}\coth\left(\frac{\sqrt{\Delta}}{2}\xi\right)\right]^2, \tag{20}$$

where $\xi = (a_2/2)\sqrt{(b-2d)/(3daa_2^2-3cb)}(x^\alpha/\Gamma(\alpha+1)) + ((aa_2^2-2c)/6(daa_2^2-cb)\sqrt{(b-2d)/(3daa_2^2-3cb)})(t^\alpha/\Gamma(\alpha+1))$.

If $\Delta = \lambda^2 - 4\mu < 0$, we have the following general trigonometric function solutions of (1):

$$u_2(x,t)$$
$$=\frac{8a_2^2\mu+a_2^2\lambda^2-12}{12a_2}+\frac{2c-aa_2^2}{a_2(b-2d)}+a_2\lambda$$
$$\times\left[-\frac{\lambda}{2}+\frac{\sqrt{-\Delta}}{2}\right.$$
$$\left.\times\left(\frac{-C_1\sin\left(\left(\sqrt{-\Delta}/2\right)\xi\right)+C_2\cos\left(\left(\sqrt{-\Delta}/2\right)\xi\right)}{C_1\cos\left(\left(\sqrt{-\Delta}/2\right)\xi\right)+C_2\sin\left(\left(\sqrt{-\Delta}/2\right)\xi\right)}\right)\right]$$
$$+a_2\left[-\frac{\lambda}{2}+\frac{\sqrt{-\Delta}}{2}\right.$$
$$\left.\times\left(\frac{-C_1\sin\left(\left(\sqrt{-\Delta}/2\right)\xi\right)+C_2\cos\left(\left(\sqrt{-\Delta}/2\right)\xi\right)}{C_1\cos\left(\left(\sqrt{-\Delta}/2\right)\xi\right)+C_2\sin\left(\left(\sqrt{-\Delta}/2\right)\xi\right)}\right)\right]^2,$$
$$v_2(x,t)$$
$$=\frac{8a_2^2\mu-12a_2^2+a_2^2\lambda^2+12}{12a_2^2}+\lambda$$
$$\times\left[-\frac{\lambda}{2}+\frac{\sqrt{-\Delta}}{2}\right.$$
$$\left.\times\left(\frac{-C_1\sin\left(\left(\sqrt{-\Delta}/2\right)\xi\right)+C_2\cos\left(\left(\sqrt{-\Delta}/2\right)\xi\right)}{C_1\cos\left(\left(\sqrt{-\Delta}/2\right)\xi\right)+C_2\sin\left(\left(\sqrt{-\Delta}/2\right)\xi\right)}\right)\right]$$
$$+\left[-\frac{\lambda}{2}+\frac{\sqrt{-\Delta}}{2}\right.$$
$$\left.\times\left(\frac{-C_1\sin\left(\left(\sqrt{-\Delta}/2\right)\xi\right)+C_2\cos\left(\left(\sqrt{-\Delta}/2\right)\xi\right)}{C_1\cos\left(\left(\sqrt{-\Delta}/2\right)\xi\right)+C_2\sin\left(\left(\sqrt{-\Delta}/2\right)\xi\right)}\right)\right]^2, \tag{21}$$

where $\xi = (a_2/2)\sqrt{(b-2d)/(3daa_2^2-3cb)}(x^\alpha/\Gamma(\alpha+1)) + ((aa_2^2-2c)/6(daa_2^2-cb)\sqrt{(b-2d)/(3daa_2^2-3cb)})(t^\alpha/\Gamma(\alpha+1))$.

In particular, setting $C_2 = 0$ and $C_1 \neq 0$, then (21) can be written as

$$u_{2(1)}(x,t)$$
$$=\frac{8a_2^2\mu+a_2^2\lambda^2-12}{12a_2}+\frac{2c-aa_2^2}{a_2(b-2d)}+a_2\lambda$$
$$\times\left[-\frac{\lambda}{2}-\frac{\sqrt{-\Delta}}{2}\tan\left(\frac{\sqrt{-\Delta}}{2}\xi\right)\right]$$
$$+a_2\left[-\frac{\lambda}{2}-\frac{\sqrt{-\Delta}}{2}\tan\left(\frac{\sqrt{-\Delta}}{2}\xi\right)\right]^2, \tag{22}$$
$$v_{2(1)}(x,t)$$
$$=\frac{8a_2^2\mu-12a_2^2+a_2^2\lambda^2+12}{12a_2^2}+\lambda$$
$$\times\left[-\frac{\lambda}{2}-\frac{\sqrt{-\Delta}}{2}\tan\left(\frac{\sqrt{-\Delta}}{2}\xi\right)\right]$$
$$+\left[-\frac{\lambda}{2}-\frac{\sqrt{-\Delta}}{2}\tan\left(\frac{\sqrt{-\Delta}}{2}\xi\right)\right]^2,$$

where $\xi = (a_2/2)\sqrt{(b-2d)/(3daa_2^2-3cb)}(x^\alpha/\Gamma(\alpha+1)) + ((aa_2^2-2c)/6(daa_2^2-cb)\sqrt{(b-2d)/(3daa_2^2-3cb)})(t^\alpha/\Gamma(\alpha+1))$.

In particular, setting $C_1 = 0$ and $C_2 \neq 0$, then (21) can be written as

$$u_{2(2)}(x,t)$$
$$=\frac{8a_2^2\mu+a_2^2\lambda^2-12}{12a_2}+\frac{2c-aa_2^2}{a_2(b-2d)}+a_2\lambda$$
$$\times\left[-\frac{\lambda}{2}+\frac{\sqrt{-\Delta}}{2}\cot\left(\frac{\sqrt{-\Delta}}{2}\xi\right)\right]$$
$$+a_2\left[-\frac{\lambda}{2}+\frac{\sqrt{-\Delta}}{2}\cot\left(\frac{\sqrt{-\Delta}}{2}\xi\right)\right]^2,$$

$$v_{2(2)}(x,t) = \frac{8a_2^2\mu - 12a_2^2 + a_2^2\lambda^2 + 12}{12a_2^2} + \lambda \times \left[-\frac{\lambda}{2} + \frac{\sqrt{-\Delta}}{2}\cot\left(\frac{\sqrt{-\Delta}}{2}\xi\right)\right] + \left[-\frac{\lambda}{2} + \frac{\sqrt{-\Delta}}{2}\cot\left(\frac{\sqrt{-\Delta}}{2}\xi\right)\right]^2, \tag{23}$$

where $\xi = (a_2/2)\sqrt{(b-2d)/(3daa_2^2-3cb)}(x^\alpha/\Gamma(\alpha+1)) + ((aa_2^2-2c)/6(daa_2^2-cb)\sqrt{(b-2d)/(3daa_2^2-3cb)})(t^\alpha/\Gamma(\alpha+1))$.

If $\Delta = \lambda^2 - 4\mu = 0$, we have the following general rational function solutions of (1):

$$u_3(x,t) = \frac{8a_2^2\mu + a_2^2\lambda^2 - 12}{12a_2} + \frac{2c - aa_2^2}{a_2(b-2d)} + a_2\lambda \times \left[\left(\frac{C_2}{C_1 + C_2\xi}\right) - \frac{\lambda}{2}\right] + a_2\left[\left(\frac{C_2}{C_1 + C_2\xi}\right) - \frac{\lambda}{2}\right]^2,$$
$$v_3(x,t) = \frac{8a_2^2\mu - 12a_2^2 + a_2^2\lambda^2 + 12}{12a_2^2} + \lambda \times \left[\left(\frac{C_2}{C_1 + C_2\xi}\right) - \frac{\lambda}{2}\right] + \left[\left(\frac{C_2}{C_1 + C_2\xi}\right) - \frac{\lambda}{2}\right]^2, \tag{24}$$

where $\xi = (a_2/2)\sqrt{(b-2d)/(3daa_2^2-3cb)}(x^\alpha/\Gamma(\alpha+1)) + ((aa_2^2-2c)/6(daa_2^2-cb)\sqrt{(b-2d)/(3daa_2^2-3cb)})(t^\alpha/\Gamma(\alpha+1))$.

Case 2. Consider

$$a_0 = \left(\frac{\lambda^2}{12} + \frac{2\mu}{3} + \frac{a(b-8c-6d)}{2c(3b-2d)}\right)\sqrt{\frac{4dc-6cb}{ab-6da}},$$
$$a_1 = \sqrt{\frac{4dc-6cb}{ab-6da}}\lambda, \qquad a_2 = \sqrt{\frac{4dc-6cb}{ab-6da}},$$
$$b_0 = \frac{\lambda^2}{12} + \frac{2\mu}{3} - 1 + \frac{a(b-6d)}{2c(2d-3b)}, \qquad b_1 = \lambda, \tag{25}$$
$$b_2 = 1, \qquad R = \sqrt{\frac{3b-2d}{6ab+12da}},$$
$$S = \frac{2\sqrt{(4dc-6cb)/(ab-6da)}}{3(b+2d)\sqrt{(3b-2d)/(6ab+12da)}}.$$

Substituting (25) into (15), we obtain the following formal solution of (13):

$$U(\xi) = \left(\frac{\lambda^2}{12} + \frac{2\mu}{3} + \frac{a(b-8c-6d)}{2c(3b-2d)}\right)\sqrt{\frac{4dc-6cb}{ab-6da}} + \sqrt{\frac{4dc-6cb}{ab-6da}}\lambda\left(\frac{G'}{G}\right) + \sqrt{\frac{4dc-6cb}{ab-6da}}\left(\frac{G'}{G}\right)^2,$$
$$V(\xi) = \frac{\lambda^2}{12} + \frac{2\mu}{3} - 1 + \frac{a(b-6d)}{2c(2d-3b)} + \lambda\left(\frac{G'}{G}\right) + \left(\frac{G'}{G}\right)^2, \tag{26}$$

where $\xi = \sqrt{(3b-2d)/(6ab+12da)}(x^\alpha/\Gamma(\alpha+1)) + (2\sqrt{(4dc-6cb)/(ab-6da)}/3(b+2d)\sqrt{(3b-2d)/(6ab+12da)})(t^\alpha/\Gamma(\alpha+1))$.

Substituting the general solution of (10) into (26), we obtain the three types of traveling wave solutions depending on the sign of $\Delta = \lambda^2 - 4\mu$.

If $\Delta = \lambda^2 - 4\mu > 0$, we have the following general hyperbolic traveling wave solutions of (1):

$$u_4(x,t) = \left(\frac{\lambda^2}{12} + \frac{2\mu}{3} + \frac{a(b-8c-6d)}{2c(3b-2d)}\right) \times \sqrt{\frac{4dc-6cb}{ab-6da}} + \sqrt{\frac{4dc-6cb}{ab-6da}}\lambda \times \left[-\frac{\lambda}{2} + \frac{\sqrt{\Delta}}{2} \times \left(\frac{C_1\sinh\left(\left(\sqrt{\Delta}/2\right)\xi\right) + C_2\cosh\left(\left(\sqrt{\Delta}/2\right)\xi\right)}{C_1\cosh\left(\left(\sqrt{\Delta}/2\right)\xi\right) + C_2\sinh\left(\left(\sqrt{\Delta}/2\right)\xi\right)}\right)\right] + \sqrt{\frac{4dc-6cb}{ab-6da}} \times \left[-\frac{\lambda}{2} + \frac{\sqrt{\Delta}}{2} \times \left(\frac{C_1\sinh\left(\left(\sqrt{\Delta}/2\right)\xi\right) + C_2\cosh\left(\left(\sqrt{\Delta}/2\right)\xi\right)}{C_1\cosh\left(\left(\sqrt{\Delta}/2\right)\xi\right) + C_2\sinh\left(\left(\sqrt{\Delta}/2\right)\xi\right)}\right)\right]^2,$$

$$v_4(x,t) = \frac{\lambda^2}{12} + \frac{2\mu}{3} - 1 + \frac{a(b-6d)}{2c(2d-3b)} + \lambda \times \left[-\frac{\lambda}{2} + \frac{\sqrt{\Delta}}{2} \times \left(\frac{C_1\sinh\left(\left(\sqrt{\Delta}/2\right)\xi\right) + C_2\cosh\left(\left(\sqrt{\Delta}/2\right)\xi\right)}{C_1\cosh\left(\left(\sqrt{\Delta}/2\right)\xi\right) + C_2\sinh\left(\left(\sqrt{\Delta}/2\right)\xi\right)}\right)\right]$$

$$+\left[-\frac{\lambda}{2}+\frac{\sqrt{\Delta}}{2}\times\left(\frac{C_1\sinh\left(\left(\sqrt{\Delta}/2\right)\xi\right)+C_2\cosh\left(\left(\sqrt{\Delta}/2\right)\xi\right)}{C_1\cosh\left(\left(\sqrt{\Delta}/2\right)\xi\right)+C_2\sinh\left(\left(\sqrt{\Delta}/2\right)\xi\right)}\right)\right]^2, \tag{27}$$

where $\xi = \sqrt{(3b-2d)/(6ab+12da)}(x^\alpha/\Gamma(\alpha+1)) + (2\sqrt{(4dc-6cb)/(ab-6da)}/3(b+2d)\sqrt{(3b-2d)/(6ab+12da)})(t^\alpha/\Gamma(\alpha+1))$.

In particular, setting $C_2 = 0$ and $C_1 \neq 0$, then (27) can be written as

$$\begin{aligned} u_{4(1)}(x,t) &= \left(\frac{\lambda^2}{12}+\frac{2\mu}{3}+\frac{a(b-8c-6d)}{2c(3b-2d)}\right)\sqrt{\frac{4dc-6cb}{ab-6da}} \\ &\quad+\sqrt{\frac{4dc-6cb}{ab-6da}}\lambda\times\left[-\frac{\lambda}{2}+\frac{\sqrt{\Delta}}{2}\tanh\left(\frac{\sqrt{\Delta}}{2}\xi\right)\right] \\ &\quad+\sqrt{\frac{4dc-6cb}{ab-6da}}\left[-\frac{\lambda}{2}+\frac{\sqrt{\Delta}}{2}\tanh\left(\frac{\sqrt{\Delta}}{2}\xi\right)\right]^2, \\ v_{4(1)}(x,t) &= \frac{\lambda^2}{12}+\frac{2\mu}{3}-1+\frac{a(b-6d)}{2c(2d-3b)}+\lambda \\ &\quad\times\left[-\frac{\lambda}{2}+\frac{\sqrt{\Delta}}{2}\tanh\left(\frac{\sqrt{\Delta}}{2}\xi\right)\right] \\ &\quad+\left[-\frac{\lambda}{2}+\frac{\sqrt{\Delta}}{2}\tanh\left(\frac{\sqrt{\Delta}}{2}\xi\right)\right]^2, \end{aligned} \tag{28}$$

where $\xi = \sqrt{(3b-2d)/(6ab+12da)}(x^\alpha/\Gamma(\alpha+1)) + (2\sqrt{(4dc-6cb)/(ab-6da)}/3(b+2d)\sqrt{(3b-2d)/(6ab+12da)})(t^\alpha/\Gamma(\alpha+1))$.

In particular, setting $C_1 = 0$ and $C_2 \neq 0$, then (27) can be written as

$$\begin{aligned} u_{4(2)}(x,t) &= \left(\frac{\lambda^2}{12}+\frac{2\mu}{3}+\frac{a(b-8c-6d)}{2c(3b-2d)}\right)\sqrt{\frac{4dc-6cb}{ab-6da}} \\ &\quad+\sqrt{\frac{4dc-6cb}{ab-6da}}\lambda\times\left[-\frac{\lambda}{2}+\frac{\sqrt{\Delta}}{2}\coth\left(\frac{\sqrt{\Delta}}{2}\xi\right)\right] \\ &\quad+\sqrt{\frac{4dc-6cb}{ab-6da}}\left[-\frac{\lambda}{2}+\frac{\sqrt{\Delta}}{2}\coth\left(\frac{\sqrt{\Delta}}{2}\xi\right)\right]^2, \\ v_{4(2)}(x,t) &= \frac{\lambda^2}{12}+\frac{2\mu}{3}-1+\frac{a(b-6d)}{2c(2d-3b)}+\lambda \\ &\quad\times\left[-\frac{\lambda}{2}+\frac{\sqrt{\Delta}}{2}\coth\left(\frac{\sqrt{\Delta}}{2}\xi\right)\right] \\ &\quad+\left[-\frac{\lambda}{2}+\frac{\sqrt{\Delta}}{2}\coth\left(\frac{\sqrt{\Delta}}{2}\xi\right)\right]^2, \end{aligned} \tag{29}$$

where $\xi = \sqrt{(3b-2d)/(6ab+12da)}(x^\alpha/\Gamma(\alpha+1)) + (2\sqrt{(4dc-6cb)/(ab-6da)}/3(b+2d)\sqrt{(3b-2d)/(6ab+12da)})(t^\alpha/\Gamma(\alpha+1))$.

If $\Delta = \lambda^2 - 4\mu < 0$, we have the following general trigonometric function solutions of (1):

$$\begin{aligned} u_5(x,t) &= \left(\frac{\lambda^2}{12}+\frac{2\mu}{3}+\frac{a(b-8c-6d)}{2c(3b-2d)}\right) \\ &\quad\times\sqrt{\frac{4dc-6cb}{ab-6da}}+\sqrt{\frac{4dc-6cb}{ab-6da}}\lambda \\ &\quad\times\left[-\frac{\lambda}{2}+\frac{\sqrt{-\Delta}}{2}\times\left(\frac{-C_1\sin\left(\left(\sqrt{-\Delta}/2\right)\xi\right)+C_2\cos\left(\left(\sqrt{-\Delta}/2\right)\xi\right)}{C_1\cos\left(\left(\sqrt{-\Delta}/2\right)\xi\right)+C_2\sin\left(\left(\sqrt{-\Delta}/2\right)\xi\right)}\right)\right] \\ &\quad+\sqrt{\frac{4dc-6cb}{ab-6da}} \\ &\quad\times\left[-\frac{\lambda}{2}+\frac{\sqrt{-\Delta}}{2}\times\left(\frac{-C_1\sin\left(\left(\sqrt{-\Delta}/2\right)\xi\right)+C_2\cos\left(\left(\sqrt{-\Delta}/2\right)\xi\right)}{C_1\cos\left(\left(\sqrt{-\Delta}/2\right)\xi\right)+C_2\sin\left(\left(\sqrt{-\Delta}/2\right)\xi\right)}\right)\right]^2, \\ v_5(x,t) &= \frac{\lambda^2}{12}+\frac{2\mu}{3}-1+\frac{a(b-6d)}{2c(2d-3b)}+\lambda \\ &\quad\times\left[-\frac{\lambda}{2}+\frac{\sqrt{-\Delta}}{2}\times\left(\frac{-C_1\sin\left(\left(\sqrt{-\Delta}/2\right)\xi\right)+C_2\cos\left(\left(\sqrt{-\Delta}/2\right)\xi\right)}{C_1\cos\left(\left(\sqrt{-\Delta}/2\right)\xi\right)+C_2\sin\left(\left(\sqrt{-\Delta}/2\right)\xi\right)}\right)\right] \end{aligned}$$

$$+\left[-\frac{\lambda}{2}+\frac{\sqrt{-\Delta}}{2}\times\left(\frac{-C_1\sin\left(\left(\sqrt{-\Delta}/2\right)\xi\right)+C_2\cos\left(\left(\sqrt{-\Delta}/2\right)\xi\right)}{C_1\cos\left(\left(\sqrt{-\Delta}/2\right)\xi\right)+C_2\sin\left(\left(\sqrt{-\Delta}/2\right)\xi\right)}\right)\right]^2, \tag{30}$$

where $\xi = \sqrt{(3b-2d)/(6ab+12da)}(x^\alpha/\Gamma(\alpha+1)) + (2\sqrt{(4dc-6cb)/(ab-6da)}/3(b+2d)\sqrt{(3b-2d)/(6ab+12da)})(t^\alpha/\Gamma(\alpha+1))$.

In particular, setting $C_2 = 0$ and $C_1 \neq 0$, then (30) can be written as

$$\begin{aligned} u_{5(1)}(x,t) &= \left(\frac{\lambda^2}{12}+\frac{2\mu}{3}+\frac{a(b-8c-6d)}{2c(3b-2d)}\right)\times\sqrt{\frac{4dc-6cb}{ab-6da}}+\sqrt{\frac{4dc-6cb}{ab-6da}}\lambda\\ &\quad\times\left[-\frac{\lambda}{2}-\frac{\sqrt{-\Delta}}{2}\tan\left(\frac{\sqrt{-\Delta}}{2}\xi\right)\right]\\ &\quad+\sqrt{\frac{4dc-6cb}{ab-6da}}\left[-\frac{\lambda}{2}-\frac{\sqrt{-\Delta}}{2}\tan\left(\frac{\sqrt{-\Delta}}{2}\xi\right)\right]^2, \end{aligned} \tag{31}$$

$$\begin{aligned} v_{5(1)}(x,t) &= \frac{\lambda^2}{12}+\frac{2\mu}{3}-1+\frac{a(b-6d)}{2c(2d-3b)}+\lambda\\ &\quad\times\left[-\frac{\lambda}{2}-\frac{\sqrt{-\Delta}}{2}\tan\left(\frac{\sqrt{-\Delta}}{2}\xi\right)\right]\\ &\quad+\left[-\frac{\lambda}{2}-\frac{\sqrt{-\Delta}}{2}\tan\left(\frac{\sqrt{-\Delta}}{2}\xi\right)\right]^2, \end{aligned}$$

where $\xi = \sqrt{(3b-2d)/(6ab+12da)}(x^\alpha/\Gamma(\alpha+1)) + (2\sqrt{(4dc-6cb)/(ab-6da)}/3(b+2d)\sqrt{(3b-2d)/(6ab+12da)})(t^\alpha/\Gamma(\alpha+1))$.

In particular, setting $C_1 = 0$ and $C_2 \neq 0$, then (30) can be written as

$$\begin{aligned} u_{5(2)}(x,t) &= \left(\frac{\lambda^2}{12}+\frac{2\mu}{3}+\frac{a(b-8c-6d)}{2c(3b-2d)}\right)\times\sqrt{\frac{4dc-6cb}{ab-6da}}+\sqrt{\frac{4dc-6cb}{ab-6da}}\lambda\\ &\quad\times\left[-\frac{\lambda}{2}+\frac{\sqrt{-\Delta}}{2}\cot\left(\frac{\sqrt{-\Delta}}{2}\xi\right)\right]\\ &\quad+\sqrt{\frac{4dc-6cb}{ab-6da}}\left[-\frac{\lambda}{2}+\frac{\sqrt{-\Delta}}{2}\cot\left(\frac{\sqrt{-\Delta}}{2}\xi\right)\right]^2, \end{aligned}$$

$$\begin{aligned} v_{5(2)}(x,t) &= \frac{\lambda^2}{12}+\frac{2\mu}{3}-1+\frac{a(b-6d)}{2c(2d-3b)}+\lambda\\ &\quad\times\left[-\frac{\lambda}{2}+\frac{\sqrt{-\Delta}}{2}\cot\left(\frac{\sqrt{-\Delta}}{2}\xi\right)\right]\\ &\quad+\left[-\frac{\lambda}{2}+\frac{\sqrt{-\Delta}}{2}\cot\left(\frac{\sqrt{-\Delta}}{2}\xi\right)\right]^2, \end{aligned} \tag{32}$$

where $\xi = \sqrt{(3b-2d)/(6ab+12da)}(x^\alpha/\Gamma(\alpha+1)) + (2\sqrt{(4dc-6cb)/(ab-6da)}/3(b+2d)\sqrt{(3b-2d)/(6ab+12da)})(t^\alpha/\Gamma(\alpha+1))$.

If $\Delta = \lambda^2 - 4\mu = 0$, we have the following general rational function solutions of (1):

$$\begin{aligned} u_6(x,t) &= \left(\frac{\lambda^2}{12}+\frac{2\mu}{3}+\frac{a(b-8c-6d)}{2c(3b-2d)}\right)\times\sqrt{\frac{4dc-6cb}{ab-6da}}+\sqrt{\frac{4dc-6cb}{ab-6da}}\lambda\\ &\quad\times\left[\left(\frac{C_2}{C_1+C_2\xi}\right)-\frac{\lambda}{2}\right]\\ &\quad+\sqrt{\frac{4dc-6cb}{ab-6da}}\left[\left(\frac{C_2}{C_1+C_2\xi}\right)-\frac{\lambda}{2}\right]^2, \end{aligned} \tag{33}$$

$$\begin{aligned} v_6(x,t) &= \frac{\lambda^2}{12}+\frac{2\mu}{3}-1+\frac{a(b-6d)}{2c(2d-3b)}+\lambda\\ &\quad\times\left[\left(\frac{C_2}{C_1+C_2\xi}\right)-\frac{\lambda}{2}\right]+\left[\left(\frac{C_2}{C_1+C_2\xi}\right)-\frac{\lambda}{2}\right]^2, \end{aligned}$$

where $\xi = \sqrt{(3b-2d)/(6ab+12da)}(x^\alpha/\Gamma(\alpha+1)) + (2\sqrt{(4dc-6cb)/(ab-6da)}/3(b+2d)\sqrt{(3b-2d)/(6ab+12da)})(t^\alpha/\Gamma(\alpha+1))$.

5. Conclusions

The exact solutions of (1) are only reported in [21]. In this work, based on the fractional complex transformation and Jumarie's modified Riemann-Liouville derivative, we successfully obtained some new exact solutions of the space-time fractional bidirectional wave equations using the (G'/G)-expansion method. These solutions are expressed by the hyperbolic functions, the trigonometric functions, and the rational functions. If we set the parameters in the obtained wider set of solutions as special values, a variety of special solutions like kink shaped, antikink shaped, and bell type solitary solutions are obtained. Though the exact solutions of (1) have been obtained via the fractional Riccati equation

method [21], they are different from the solutions obtained in this paper. This method is very efficient and simple in finding the exact solutions for the nonlinear fractional differential equations.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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