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Research Article

Weak Projective Synchronization in Drive-Response Dynamical Networks with Time-Varying Delay and Parameter Mismatch

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This paper investigates the problem of projective synchronization in drive-response dynamical networks (DRDNs) with time-varying delay and parameter mismatch via impulsive control. Owing to projective factor and parameter mismatch, complete projective synchronization cannot be achieved. Therefore, a weak projective synchronization scheme is proposed to ensure that the DRDNs are in a state of synchronization with an error level. Based on the stability analysis of the impulsive functional differential equations, a general method of the weak projective synchronization with the error level is derived in DRDNs. Numerical simulations are provided to verify the correctness and effectiveness of the proposed method and results.

1. Introduction

A complex dynamical network is a set of coupled nodes interconnected by edges, in which each node represents a dynamical system. The structure of many real systems in nature can be described by the complex dynamical networks, such as social relationship networks, metabolic networks, food chain, Internet, the World Wide Web, power grids, and so on [1, 2]. This has led to much interest in the studies of the complex dynamical networks. In particular, the synchronization of complex networks has received much attention, and many interesting results on synchronization were derived for various complex networks such as time invariant, time-varying, discrete, and impulsive network models [3–11].

More recently, projective synchronization on dynamical networks has been reported by Hu et al. in [12], in which the projective synchronization with the desired scaling factor can be realized in drive-response dynamical networks. Projective synchronization has become a hot topic and attracted much attention from authors in many fields, including chaotic systems [13–16] and complex dynamical networks [17–20]. In these papers, the authors just consider the projective synchronization in DRDNs with coupled partially linear chaotic systems. However, there are always some mismatches between drive system and response network systems in the real world. Indeed, almost all complex dynamical networks have different nodes, such as the nodes in network community and the Internet, are in general different. In this case, the DRDNs cannot synchronize completely. Nevertheless, when parameter mismatch is small enough, the synchronization error can converge to a small region containing the origin. In [21], the authors investigated the effect of parameter mismatch on lag synchronization of chaotic systems. In [22], the synchronization of a class of coupled chaotic delayed systems with parameters mismatch and stochastic perturbation was studied. In [23], the weak synchronization criterion of coupled delayed chaotic systems with parameters mismatches was obtained. In [24], the authors studied the synchronization of two coupled identical chaotic systems with parameter mismatch via using periodically intermittent control. In [25], the weak projective synchronization of neural networks with mixed time-varying delays and parameter mismatch was discussed. Unfortunately, there exist few results of a weak projective synchronization method for DRDNs with time-varying delay and parameter mismatch. Therefore, it is worth proposing a weak projective synchronization method in which the problems mentioned above are considered.

Motivated by the above discussions, in this paper, we introduce a drive-response dynamical network with time-varying delay and parameter mismatch. It is known that complete synchronization is destroyed by parameter mismatch and projective factor. We propose the weak projective synchronization properties of this model via impulsive control. Based on the obtained results, one can control the projective synchronization error in a predetermined level. Results of numerical example show the effectiveness of the proposed approach. The rest of this paper is organized as follows. In Section 2, the DRDNs model with parameter mismatch and some preliminaries are given. In Section 3, some criteria for the weak projective synchronization are derived. Numerical simulations are shown in Section 4. The conclusion is finally given in Section 5.

The notation throughout the paper is quite standard. R and R^n denote the real number set and n-dimensional Euclidean space, respectively. $\|\cdot\|$ stands for either the Euclidean vector norm or its induced matrix 2-norm. $\lambda_{\max}(A)(\lambda_{\min}(A))$ represents the maximum (minimum) eigenvalue of the symmetric matrix A. $\mu(A)=\lambda_{\max}(A^T+A)$. sup denotes the upper bound. I_n is the identity matrix with order n. Matrices, if not explicitly stated, are assumed to have compatible dimensions. $\otimes$ is the Kronecker product of two matrices. $PC([-\tau,0],R^n)$ denotes the set of all functions of bounded variation and right-continuous on any compact subinterval of $[-\tau,0]$.

2. Model Description and Preliminaries

2.1. Model Description. In this paper, we consider DRDNs with time-varying coupling delays and parameter mismatch as follows:

$$\dot{x}^d(t)=A^d x^d(t)+B^d f\left(x^d(t)\right)+C^d g\left(x^d(t-\tau(t))\right), \quad (1)$$

$$\begin{aligned}\dot{x}_i^r(t)&=A_i^r x^d(t)+B_i^r f\left(x_i^r(t)\right)\\&\quad+C_i^r g\left(x_i^r(t-\tau(t))\right)+\gamma\sum_{j=1}^{N}c_{ij}\Gamma x_j^r\left(t-\tau_1(t)\right),\end{aligned} \quad (2)$$

where the superscripts d and r stand for the drive system and response networks, respectively. In (1), $x^d(t)=(x_1^d(t),x_2^d(t),\ldots,x_n^d(t))^T\in R^n$ denotes the state vector of the drive system; A^d, B^d, and $C^d$are constant $n\times n$ matrices. $f: R^n\to R^n$ and $g: R^n\to R^n$ are continuously differentiable vector functions. In (2), $x_i^r(t)=(x_{i1}^r(t),x_{i2}^r(t),\ldots,x_{in}^r(t))^T\in R^n, i=1,2,\ldots,N$, denotes the state vector of the ith node; A_i^r, A_i^r, and A_i^r are constant $n\times n$ matrices. $\tau(t),\tau_1(t)$ are the time-varying delays. The constant $\gamma>0$ represents the coupling strength of the network, and $\Gamma\in R^{n\times n}$ is the inner-coupling matrix; $C=(c_{ij})_{N\times N}\in R^{N\times N}$ is the coupling matrix, standing for the coupling configuration of the network. If there is connection between node i and node $(i\neq j)$, $c_{ij}\neq 0$; otherwise, $c_{ij}=0$. The row sum of C is zero; that is, $c_{ii}=-\sum_{j=1,j\neq i}^{N}c_{ij}$.

2.2. Preliminaries. In order to demonstrate this paper clearly, we give some necessary definitions, assumptions, and lemmas, which are useful in deriving projective synchronization criteria.

Definition 1. The drive system (1) and response dynamical networks (2) are said to be weak projective synchronized with an error level $\varsigma>0$, if there exists a $T\geq 0$ such that $\|x_i^r(t)-\lambda x_d(t)\|\leq\varsigma$ for all $t\geq T$, where λ is a desired scaling factor.

Assumption 2. For any $z_1,z_2\in R^n$, there exist constants $l_f>0$, $l_g>0$, $i=1,2,\ldots,N$, such that $\|f(z_1)-f(z_2)\|\leq l_f\|z_1-z_2\|$, $\|g_i(z_1)-g_i(z_2)\|\leq l_g\|z_1-z_2\|$.

Assumption 3. $\tau(t)$ and $\tau_1(t)$ are the time-varying delay satisfying $0\leq\tau(t),\tau_1(t)\leq\tau$, where τ is a positive constant. Clearly, this assumption is certainly ensured if the time-varying delay is a constant.

Remark 4. It should be pointed out that in Assumption 3 we do not require that the time-varying delay is differential function with a bound of its derivative, which means that the time-varying delay satisfying Assumption 3 includes a wide range of functions.

Assumption 5. It is assumed that the trajectory of the drive system (1) is bounded with

$$\Omega=\left\{x^d(t)\mid\left\|x^d(t)\right\|\leq\delta\right\},\quad\forall t\geq-\tau. \quad (3)$$

Remark 6. Assumption 5 is reasonable due to its chaotic feature.

Lemma 7 (see [26]). *Let $P\in R^{n\times n}$ be a symmetric positive definite matrix and $P=Q^TQ$. For any $x,y\in R^n$ and $A\in R^{n\times n}$ such that*

(1) $x^TA^TPAx\leq\|QAQ^{-1}\|^2x^TPx$

(2) $x^T(A^TP+PA)x\leq 2\mu(QAQ^{-1})x^TPx$

(3) $|x^TPy|\leq\sqrt{x^TPx}\sqrt{y^TPy}$.

Lemma 8 (see [26]). *Let $0\leq\tau(t),\tau_1(t),\tau_2(t),\ldots,\tau_m(t)\leq\tilde{\tau}$, $\tilde{\tau}=\max(\tau,\tau_1,\tau_2,\ldots,\tau_m)$, $F(t,u,\overline{u}_1,\ldots,\overline{u}_m):R^+\times\overbrace{R\times\cdots\times R}^{m+1}\to R$ be nondecreasing in $\overline{u}_i$ for each fixed $(t,u,\overline{u}_1,\ldots,\overline{u}_{i-1},\overline{u}_{i+1},\overline{u}_m)$, $i=1,2,\ldots,m$, and let $I_k(u):R\to R$ be nondecreasing in u. Suppose that $u(t)$ and $v(t)$ satisfy*

$$\begin{gathered}D^+u(t)\leq F\left(t,u(t),u_1\left(t-\tau_1(t)\right),\ldots,u_m\left(t-\tau_m(t)\right)\right),\\ t\geq 0,\\ u\left(t_k\right)\leq I_k u\left(t_k\right),\quad k\in N,\\ D^+v(t)>F\left(t,v(t),v_1\left(t-\tau_1(t)\right),\ldots,v_m\left(t-\tau_m(t)\right)\right),\\ t\geq 0,\\ v\left(t_k^+\right)\geq I_k v\left(t_k\right),\quad k\in N;\end{gathered} \quad (4)$$

then $u(t) \le v(t)$ for $-\tau \le t \le 0$ implies that $u(t) \le v(t)$ for $t \ge 0$, where the right and upper Dini's derivative $D^+u(t)$ is defined as $D^+u(t) = \overline{\lim}_{h\to 0^+}((u(t+h)-u(t))/h)$.

The aim of this paper is to discuss the weak projective synchronization in the DRDNs with time-varying delay and parameters mismatches. We choose the impulsive controller B_{ik} which is a $n\times n$ constant matrix. Thus, the drive system (1) and response networks (2) can be rewritten as the following impulsive differential equations:

$$\begin{aligned}
\dot{x}^d(t) &= A^d x^d(t) + B^d f\left(x^d(t)\right) + C^d g\left(x^d(t-\tau(t))\right),\\
\dot{x}_i^r(t) &= A_i^r x^d(t) + B_i^r f\left(x_i^r(t)\right) + C_i^r g\left(x_i^r(t-\tau(t))\right)\\
&\quad + \gamma\sum_{j=1}^{N} c_{ij}\Gamma x_j^r\left(t-\tau_1(t)\right), \quad t \ne t_k,\\
\Delta x_i^r &= x_i^r\left(t_k^+\right) - x_i^r\left(t_k^-\right) = B_{ik}\left[x_i^r - \lambda x^d\right],\\
&\qquad t = t_k,\ k = 1,2,\ldots,
\end{aligned} \tag{5}$$

where the impulsive time instants t_k satisfy $0 = t_0 < t_1 < t_2 < \cdots < t_k < \cdots$ and $\lim_{k\to\infty} t_k = +\infty$, $x_i^r(t_k^+) = \lim_{t\to t_k^+} x_i^r(t)$, $x_i^r(t_k^-) = \lim_{t\to t_k^-} x_i^r(t)$. Moreover, any solution of (5) is left continuous at each t_k; that is, $x_i^r(t_k^-) = x_i^r(t_k)$.

Letting $e_i(t) = x_i^r(t) - \lambda x^d(t)$, then the synchronization error system between the drive system and the response network can be written as

$$\begin{aligned}
\dot{e}_i(t) &= A_i^r e_i(t) + B_i^r \tilde{f}\left(e_i(t)\right) + C_i^r \tilde{g}\left(e_i(t-\tau(t))\right)\\
&\quad + c\sum_{j=1}^{N} c_{ij}\Gamma e_j\left(t-\tau_1(t)\right)\\
&\quad + \tilde{h}_i\left(x^d(t),\tau(t),\lambda\right), \quad t \ne t_k,\\
\Delta e_i &= B_{ik} e_i, \quad t = t_k,
\end{aligned} \tag{6}$$

where $\tilde{f}(e_i(t)) = f(x_i^r(t)) - f(\lambda x^d(t))$, $\tilde{g}(e_i(t-\tau(t))) = g(x_i^r(t-\tau(t))) - g(\lambda x^d(t-\tau(t)))$ and $\tilde{h}_i(x^d(t),\tau(t),\lambda) = \lambda(A_i^r - A)x^d(t) + B_i^r f(\lambda x^d(t)) - \lambda B^d f(x^d(t)) + C_i^r g(\lambda x^d(t-\tau(t))) - \lambda C^d g(x^d(t-\tau(t)))$.

Let $e(t) = (e_1^T(t), e_2^T(t), \ldots, e_N^T(t))^T$, rewriting (6) in its compact form

$$\begin{aligned}
\dot{e}(t) &= \left(I_N\otimes A_i^r\right)e(t) + \left(I_N\otimes B_i^r\right)F\left(e(t)\right) + \left(I_N\otimes C_i^r\right)\\
&\quad \times G\left(e\left(t-\tau(t)\right)\right) + \gamma\left(C\otimes\Gamma\right)e\left(t-\tau_1(t)\right)\\
&\quad + H\left(x^d(t),\tau(t),\lambda\right), \quad t\ne t_k,\\
e\left(t_k^+\right) &= \left(I_{Nn} + \left(I_N\otimes B_{ik}\right)\right)e\left(t_k\right), \quad t = t_k,\ k = 1,2,\ldots,
\end{aligned} \tag{7}$$

where $F(e(t)) = (\tilde{f}^T(e_1(t)), \tilde{f}^T(e_2(t)), \ldots, \tilde{f}^T(e_N(t)))^T$, $G(e(t-\tau(t))) = (\tilde{g}^T(e_1(t-\tau(t))), \tilde{g}^T(e_2(t-\tau(t))), \ldots, \tilde{g}^T(e_N(t-\tau(t))))^T$ and $H(x^d(t),\tau(t),\lambda) = (\tilde{h}_1^T(x^d(t),\tau(t),\lambda), \ldots, \tilde{h}_N^T(x^d(t),\tau(t),\lambda))^T$.

The initial condition of the error system (7) is defined as $e(s) = \phi(s)$, $-\tau \le s < 0$, where $\phi(\cdot) \in C([-\tau,0],R^n)$. $\|\phi\| = \sup_{-\tau\le s\le 0}\|\phi(s)\|$ is used to denote the norm of a function $\phi \in C([-\tau,0],R^n)$. It is assumed that (7) has a unique solution with respect to initial condition.

3. Main Results

In this section, by combining the stability analysis of impulsive functional differential equations, some sufficient conditions for weak projective synchronization in drive-response dynamical networks with time-varying delay and parameter mismatch under impulsive control are given below.

Theorem 9. *Under Assumptions 2, 3, and 5, let a nonsingular matrix $Q \in R^{Nn\times Nn}$, $0 < \rho = \sup\{t_k - t_{k-1}\} < \infty$, and $\sup_{t\ge 0}\|H(t,x^d(t),\lambda)\| \le \omega < \infty$. If the following inequalities hold*

$$\left\|I_{Nn} + Q\left(I_N\otimes B_{ik}\right)Q^{-1}\right\| \le \beta, \quad 0 < \beta < 1, \tag{8}$$

$$\frac{2\ln\beta}{\rho} + a + \beta^{-2}b + \beta^{-2}c < 0, \tag{9}$$

where $\alpha = -a - (2\ln\beta/\rho)$, $a = \max_{1\le i\le N}[(\mu[Q(I_N\otimes A_i^r)Q^{-1}]) + (2l_f\lambda_{\max}(P)\|I_N\otimes B_i^r\| + l_g\lambda_{\max}(P)\|I_N\otimes C_i^r\|\zeta)/\lambda_{\min}(P) + \|P\|^2/\xi\lambda_{\min}(P) + \gamma]$, $b = l_g\lambda_{\max}(P)\|I_N\otimes C_i^r\|/\zeta\lambda_{\min}(P)$, $c = \gamma\|Q(C\otimes\Gamma)Q^{-1}\|^2$, ζ and ξ are positive constants. $\nu > 0$ is a unique solution of $\nu - \alpha + \beta^{-2}be^{\nu\tau} + \beta^{-2}ce^{\nu\tau_1} = 0$. Then, the error system (7) can converge globally exponentially to the small region M containing the origin, where $M = \{e(t) \in R^{Nn} \mid \|e(t)\| \le (\omega/\beta)\sqrt{\xi/(\alpha-\beta^{-2}b-\beta^{-2}c)}\}$, which implies the weak projective synchronization in DRDNs is achieved.

Proof. Consider the following Lyapunov functional:

$$V(t) = e^T(t)Pe(t), \tag{10}$$

where P is a symmetric matrix and $P = Q^TQ$.

For $t \ne t_k$, the time derivative of $V(t)$ along the trajectories of (7) is

$$\begin{aligned}
\dot{V}(t) &= 2e^T(t)P\dot{e}(t)\\
&= 2e^T(t)P\Big[\left(I_N\otimes A_i^r\right)e(t) + \left(I_N\otimes B_i^r\right)F\left(e(t)\right)\\
&\quad + \left(I_N\otimes C_i^r\right)G\left(e\left(t-\tau(t)\right)\right)\\
&\quad + \gamma\left(C\otimes\Gamma\right)e\left(t-\tau_1(t)\right)\\
&\quad + H\left(x^d(t),\tau(t),\lambda\right)\Big]
\end{aligned}$$

$$\begin{aligned}
&= e^T(t)\left[\left(I_N\otimes A_i^r\right)^T P + P\left(I_N\otimes A_i^r\right)\right]e(t)\\
&\quad + 2\gamma e^T(t)P(C\otimes\Gamma)e\left(t-\tau_1(t)\right)\\
&\quad + 2e^T(t)P\left(I_N\otimes B_i^r\right)F(e(t))\\
&\quad + 2e^T(t)P\left(I_N\otimes C_i^r\right)G(e(t-\tau(t)))\\
&\quad + 2e^T(t)PH\left(x^d(t),\tau(t),\lambda\right).
\end{aligned}\tag{11}$$

From Lemmas 7-8 and Assumption 2, it is clear that

$$\begin{aligned}
&\dot V(t)\\
&\le \mu\left(Q\left(I_N\otimes A_i\right)Q^{-1}\right)e^T(t)Pe(t)+\frac{\|P\|^2}{\xi}\|e(t)\|^2\\
&\quad + \xi\left\|H\left(x^d(t),\tau(t),\lambda\right)\right\|^2\\
&\quad + 2l_f\lambda_{\max}(P)\left\|I_N\otimes B_i^r\right\|\|e(t)\|^2\\
&\quad + 2l_g\lambda_{\max}(P)\left\|I_N\otimes C_i^r\right\|\|e(t)\|\|e(t-\tau(t))\|\\
&\quad + 2\gamma\sqrt{e^T(t)Pe(t)}\\
&\quad \times\sqrt{e^T\left(t-\tau_1(t)\right)(C\otimes\Gamma)^T P(C\otimes\Gamma)e\left(t-\tau_1(t)\right)}\\
&\le \max_{1\le i\le N}\Bigg[\left(\mu\left[Q\left(I_N\otimes A_i^r\right)Q^{-1}\right]\right)\\
&\qquad + \frac{2l_f\lambda_{\max}(P)\left\|I_N\otimes B_i^r\right\| + l_g\lambda_{\max}(P)\left\|I_N\otimes C_i^r\right\|\zeta}{\lambda_{\min}(P)}\\
&\qquad + \frac{\|P\|^2}{\xi\lambda_{\min}(P)}+\gamma\Bigg]e^T(t)Pe(t)\\
&\quad + \frac{l_g\lambda_{\max}(P)\left\|I_N\otimes C_i^r\right\|}{\zeta\lambda_{\min}(P)}e^T(t-\tau(t))Pe(t-\tau(t))\\
&\quad + \gamma\left\|Q(C\otimes\Gamma)Q^{-1}\right\|^2 e^T\left(t-\tau_1(t)\right)\\
&\quad \times Pe\left(t-\tau_1(t)\right)+\xi\omega^2\\
&= aV(t)+bV(t-\tau(t))+cV\left(t-\tau_1(t)\right)+\xi\omega^2.
\end{aligned}\tag{12}$$

When $t=t_k$, one gets

$$\begin{aligned}
V\left(t_k^+\right) &= e^T\left(t_k\right)\left(I_{Nn}+\left(I_N\otimes B_{ik}\right)\right)^T\\
&\quad \times P\left(I_{Nn}+\left(I_N\otimes B_{ik}\right)\right)e\left(t_k\right)\\
&\le \left\|I_{Nn}+Q\left(I_N\otimes B_{ik}\right)Q^{-1}\right\|^2 e^T\left(t_k\right)Pe\left(t_k\right)\\
&= \beta^2 V\left(t_k\right),\quad k=1,2,\ldots.
\end{aligned}\tag{13}$$

For any $\varepsilon>0$, let $v(t)$ be a unique solution of the following impulsive delayed system:

$$\begin{aligned}
\dot v(t) &= av(t)+bv(t-\tau(t))+cv\left(t-\tau_1(t)\right)+\xi\omega^2+\varepsilon,\\
&\qquad t\ne t_k,\\
v\left(t_k^+\right) &= \beta^2 v\left(t_k\right),\quad k\in N,\\
v(s) &= \lambda_{\max}(P)\|\phi(s)\|^2,\quad -\tau\le s\le 0.
\end{aligned}\tag{14}$$

From Lemma 8 and $V(t)\le\|\phi(t)\|^2$ for $-\tau\le t\le 0$, we conclude that $V(t)\le v(t)$, for $t>0$.

The trivial solution of the comparison system is

$$\begin{aligned}
v(t) &= w(t,0)v(0)\\
&\quad + \int_0^t w(t,s)\left(bv(s-\tau(s))+cv\left(s-\tau_1(s)\right)\right.\\
&\qquad \left.+\xi\omega^2+\varepsilon\right)ds,
\end{aligned}\tag{15}$$

where $w(t,s)$, $0\le s\le t$ is Cauchy matrix of the linear impulsive system.

Since $0<\beta<1$, $t_k-t_{k-1}\le\rho$, one has

$$\begin{aligned}
w(t,s) &= e^{a(t-s)}\prod_{s<t_k\le t}\beta^2\\
&\le e^{(-\alpha-2\ln\beta/\rho)(t-s)}\beta^{2(((t-s)/\rho)-1)}\\
&\le \beta^{-2}e^{-\alpha(t-s)},\quad 0\le s\le t.
\end{aligned}\tag{16}$$

Let $\sigma=\beta^{-2}\lambda_{\max}(P)\sup_{-\tau\le s\le 0}\|\phi(s)\|^2$, from (15) and (16), one has

$$\begin{aligned}
v(t) &\le \beta^{-2}e^{-\alpha t}v(0)\\
&\quad + \int_0^t e^{-\alpha(t-s)}\beta^{-2}\\
&\qquad \times\left[bv(s-\tau(s))+cv\left(s-\tau_1(s)\right)+\xi\omega^2+\varepsilon\right]ds\\
&\le \sigma e^{-\alpha t}\\
&\quad + \int_0^t e^{-\alpha(t-s)}\beta^{-2}\\
&\qquad \times\left[bv(s-\tau(s))+cv\left(s-\tau_1(s)\right)+\xi\omega^2+\varepsilon\right]ds.
\end{aligned}\tag{17}$$

Denote $\varphi(\nu)=\nu-\alpha+\beta^{-2}be^{\nu\tau}+\beta^{-2}ce^{\nu\tau_1}$; from (9), one has $\alpha>0$, $b>0$, $c>0$, $\alpha-\beta^{-2}b-\beta^{-2}c>0$, $\varphi(0)<0$, $\varphi(+\infty)>0$, $\varphi'(\nu)=1+\beta^{-2}b\tau e^{\nu\tau}+\beta^{-2}c\tau_1 e^{\nu\tau_1}>0$; then $\varphi(\nu)=0$ has a unique solution $\nu>0$. Since $\beta^{-2}(\xi\omega^2+\varepsilon)>0$, $\nu>0$, $\alpha-\beta^{-2}b-\beta^{-2}c>0$, and $\beta^{-2}>1$; we derive that

$$v(t)\le\beta^{-2}\sup_{-\tau\le s\le 0}v(s)<\sigma e^{-\nu t}+\frac{\xi\omega^2+\varepsilon}{\beta^2\alpha-b-c},\quad -\tau\le t\le 0.\tag{18}$$

In the following, we will prove that the following inequality holds:

$$v(t) < \sigma e^{-\nu t} + \frac{\xi\omega^2 + \varepsilon}{\beta^2\alpha - b - c}, \quad t > 0. \tag{19}$$

If it is not true, there exists a $t^* > 0$ such that

$$v(t^*) \geq \sigma e^{-\nu t^*} + \frac{\xi\omega^2 + \varepsilon}{\beta^2\alpha - b - c}, \tag{20}$$

$$v(t) < \sigma e^{-\nu t} + \frac{\xi\omega^2 + \varepsilon}{\beta^2\alpha - b - c}, \quad t < t^*. \tag{21}$$

From Assumption 3, (17), and (21), we obtain

$$\begin{aligned}
& v(t^*) \\
& \leq \sigma e^{-\alpha t^*} + \int_0^{t^*} e^{-\alpha(t^*-s)}\beta^{-2} \\
& \qquad \times \left[bv(s-\tau(s)) + cv(s-\tau_1(s)) + \xi\omega^2 + \varepsilon\right] ds \\
& < e^{-\alpha t^*}\left(\sigma + \frac{\xi N\omega^2 + \varepsilon}{\beta^2\alpha - b - c}\right) \\
& \quad + e^{-\alpha t^*}\int_0^{t^*} e^{\alpha s}\beta^{-2} b\left(\sigma e^{-\nu(s-\tau(s))} + \frac{\xi\omega^2 + \varepsilon}{\beta^2\alpha - b - c}\right) ds \\
& \quad + e^{-\alpha t^*}\int_0^{t^*} e^{\alpha s}\beta^{-2} c\left(\sigma e^{-\nu(s-\tau_1(s))} + \frac{\xi\omega^2 + \varepsilon}{\beta^2\alpha - b - c}\right) ds \\
& \quad + e^{-\alpha t^*}\int_0^{t^*} e^{\alpha s}\beta^{-2}\left(\xi\omega^2 + \varepsilon\right) ds \\
& \leq e^{-\alpha t^*}\left\{\sigma + \frac{\xi\omega^2 + \varepsilon}{\beta^2\alpha - b - c} + \beta^{-2}\sigma\left(be^{\nu\tau} + ce^{\nu\tau_1}\right)e^{\nu\tau}\right. \\
& \qquad \times \int_0^{t^*} e^{(a-\nu)s} ds \\
& \qquad + \left(\frac{\beta^{-2}(b+c)\left(\xi\omega^2 + \varepsilon\right)}{\beta^2\alpha - b - c} + \frac{\xi\omega^2 + \varepsilon}{\beta^2}\right) \\
& \qquad \left. \times \int_0^{t^*} e^{as} ds\right\} \\
& = \sigma e^{-\lambda t^*} + \frac{\xi\omega^2 + \varepsilon}{\beta^2\alpha - b - c}
\end{aligned} \tag{22}$$

which contradicts with (20). Consequently, (19) holds.

Let $\varepsilon \to 0$; then one obtains

$$V(t) = \|e(t)\|^2 \leq v(t) \leq \sigma e^{-\lambda t} + \frac{\xi\omega^2}{\beta^2\alpha - b - c}. \tag{23}$$

Thus, one has

$$\|e(t)\| \leq \sqrt{\sigma}e^{-(\nu/2)t} + \frac{\omega}{\beta}\sqrt{\frac{\xi}{\alpha - \beta^{-2}b - \beta^{-2}c}}. \tag{24}$$

When $t \to \infty$, the synchronization error system (7) converges exponentially to a small region M containing the origin: $M = \{e(t) \in R^{Nn} \mid \|e(t)\| \leq (\omega/\beta)\sqrt{\xi/(\alpha - \beta^{-2}b - \beta^{-2}c)}\}$, which implies that the DRDNs achieve the weak projective synchronization. The proof is completed. □

By further estimating the value of $H(x^d(t), \tau(t), \lambda)$ and selecting ξ appropriately, we have Corollary 10.

Corollary 10. *Under Assumptions 2, 3, and 5, suppose a nonsingular matrix* $Q \in R^{Nn\times Nn}$, $0 < \rho = \sup\{t_k - t_{k-1}\} < \infty$, *and* $\delta|\lambda|[\|A_i^r - A\| + (\|B_i^r\| + \|B^d\|)l_f + (\|C_i^r\| + \|C^d\|)l_g] \leq \omega_i$. *For given synchronization scaling factor* λ, *if the following inequalities hold*

$$\left\|I_{Nn} + Q\left(I_N \otimes B_{ik}\right)Q^{-1}\right\| \leq \beta, \quad 0 < \beta < 1, \tag{25}$$

$$\frac{2\ln\beta}{\rho} + a + \beta^{-2}b + \beta^{-2}c < 0, \tag{26}$$

where $\alpha = -a - (2\ln\beta/\rho)$, $b = l_g\lambda_{\max}(P)\|I_N \otimes C_i^r\|/\zeta\lambda_{\min}(P)$, $a = \max_{1\leq i\leq N}[(\mu[Q(I_N \otimes A_i^r)Q^{-1}]) + \gamma + (\|P\|^2/\xi\lambda_{\min}(P)) + ((2l_f\lambda_{\max}(P)\|I_N \otimes B_i^r\| + l_g\lambda_{\max}(P)\|I_N \otimes C_i^r\|\zeta)/\lambda_{\min}(P))]$, $c = \gamma\|Q(C \otimes \Gamma)Q^{-1}\|^2$ *and* $\nu > 0$ *is an unique solution of* $\nu - \alpha + \beta^{-2}be^{\nu\tau} + \beta^{-2}ce^{\nu\tau_1} = 0$, *then, the error system* (7) *can converge globally exponentially to the small region* M *containing the origin, where* $M = \{e(t) \in R^{Nn} \mid \|e(t)\| \leq (\omega/\beta)\sqrt{(\xi/(\alpha - \beta^{-2}b - \beta^{-2}c))}\}$, *which implies that the weak projective synchronization in DRDNs is achieved.*

Remark 11. For simplicity, we consider the equidistant impulsive interval $t_k - t_{k-1} = \Delta$, and the impulsive control gain matrix $B_k = b_0 I_n$, $k = 1, 2, \ldots$, in Theorem 9. If the following condition holds $\Delta < -2(1+b_0)^2\ln|1+b_0|/((1+b_0)^2 a + b + c)$, $-2 < b_0 < 0$, then the DRDNs achieve weak projective synchronization.

4. Numerical Simulation

In this section, an example is presented to show the effectiveness of the proposed scheme. To show the advantage of the criteria based on matrix measure, a scalar Ikeda oscillator is investigated in the context of weak projective synchronization in the following example.

The dynamics of Ikeda oscillator is described by

$$\dot{x} = -dx + e\sin(x(t - \tau(t))). \tag{27}$$

System (26) exhibits chaotic behavior when $d = 1$, $e = 4$ and $\tau(t) = 2$, as shown in Figure 1. It is known that the chaotic attractor of system (27) is contained in the set $\Omega = \{x \in R \mid |x| \leq 4\}$.

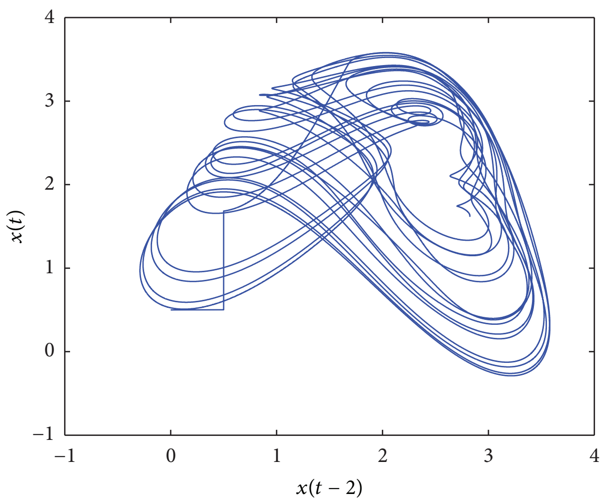

Figure 1: Chaotic attractor of system (27).

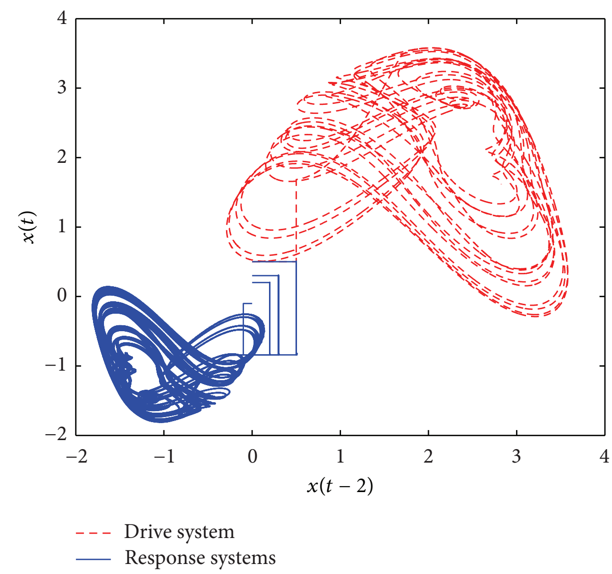

Figure 2: Chaotic attractors of systems (27) and (28) with $\lambda = -0.5$.

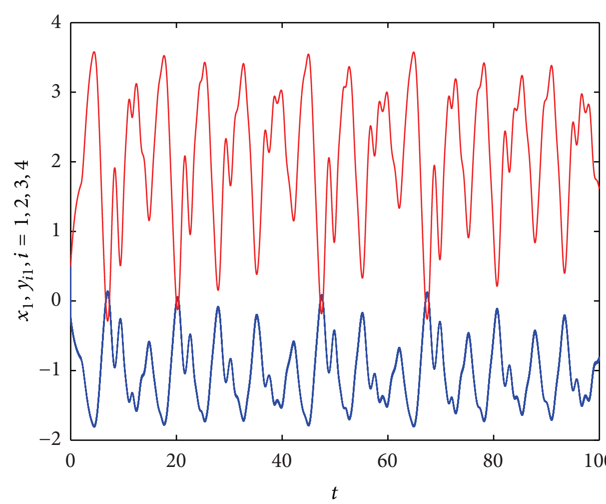

Figure 3: The trajectories of state variables with $\lambda = -0.5$.

The corresponding response network systems with parameter mismatch are given by

$$\dot{x}_i^r(t) = -(d - 0.001i)\,x_i^r(t) + (e - 0.001i)\sin\left(x_i^r(t - \tau(t))\right) + \gamma\sum_{j=1}^{4} c_{ij}\Gamma x_j^r(t - \tau_1(t)). \tag{28}$$

Then, the controlled DRDNs are described as follows:

$$\begin{aligned}
\dot{x}^d &= -x^d + 4\sin\left(x^d(t - \tau(t))\right),\\
\dot{x}_i^r(t) &= -(1 - 0.001i)\,x_i^r(t) + (4 - 0.001i)\sin\left(x_i^r(t - \tau(t))\right)\\
&\quad + \gamma\sum_{j=1}^{4} c_{ij}\Gamma x_j^r(t - \tau_1(t)) \qquad t \neq t_k,\ i = 1, 2, 3, 4,\\
\Delta e_i &= b_0 e_i(t_k^-), \qquad t = t_k,\ k = 1, 2, \ldots,
\end{aligned} \tag{29}$$

choosing the coupling configuration matrix

$$C = \begin{pmatrix} -2 & 1 & 1 & 0 \\ 1 & -1 & 0 & 0 \\ 2 & 1 & -3 & 0 \\ 0 & 0 & 1 & -1 \end{pmatrix}. \tag{30}$$

In the numerical simulations, we assume that $\xi = 1$, $\zeta = \gamma = 0.1$, $b_0 = -0.9$, $(1 + b_0)^2 = 0.01 > 0$. $\Gamma = P = 1$, $Q = I_{Nn}$. The two coupling delays are $\tau(t) = 2$ and $\tau_1(t) = 2 + 0.02\sin t$, respectively. After calculations, getting $a = 38.092$, $b = 0.3999$, $c = 1.814$, one has $\Delta < 0.0177$. Taking the impulsive interval $\Delta = t_{k+1} - t_k = 0.01$, then, it is easy to verify that all conditions in Corollary 10 are satisfied. The projective synchronization error is defined by $\|e(t)\| = \sqrt{(x_{i1} - \lambda x_1)^2 + (y_{i2} - \lambda y_2)^2}$, $i = 1, 2, 3, 4$. When $\lambda = -0.5$, as shown in Figures 2–4. Figure 2 shows attractors of the DRDNs network model. Figure 3 displays time evolutions of state trajectories of the controlled DRDNs (29). The evolution process of the error does not converge to zero as shown in Figure 4; from Figure 4, it is easy to see that the projective synchronization is not achieved. The numerical results show that the impulsive controlling scheme for the drive-response coupled dynamical network model with time-varying delays is effective.

5. Conclusion

In this paper, the problem of weak projective synchronization in DRDNs with time-varying coupling delay and parameter mismatch has been investigated by employing impulsive control scheme. Some criteria for realizing the weak projective synchronization are established based on the stability analysis

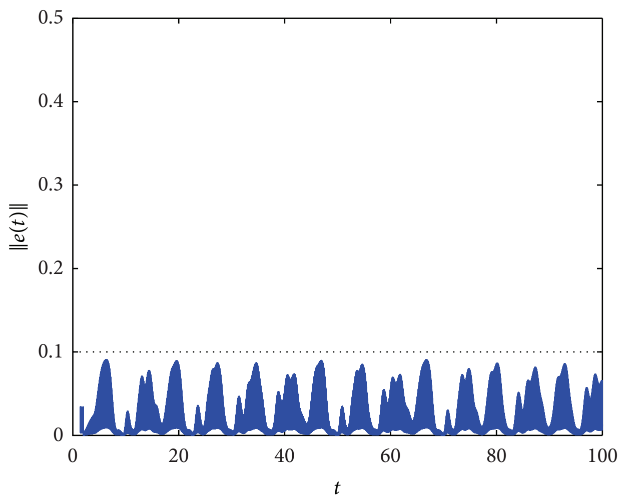

FIGURE 4: Time evolution of $\|e(t)\|$ of coupled networks (29).

of impulsive functional differential equations. Moreover, the DRDNs can be synchronized exponentially within a small error; the error upper bound of weak projective synchronization is estimated easily by the theoretical criteria. Finally, the numerical examples show the effectiveness of the proposed results. However, the results of theoretical analysis in this paper are still conservative. Meanwhile, since the surrounding environment is complex variable, it is desirable to investigate weak projective synchronization problem for complex dynamical networks with noise, stochastic disturbances, and so on, so we will further investigate these problems in the future.

Conflict of Interests

The authors declare that they have no conflict of interests.

Authors' Contributions

Jiang Xu carried out the main part of this paper. Song Zheng participated in the discussion and corrected the main theorem. All authors read and approved the final paper.

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