# Convergence of Viscosity Iteration Process for a Finite Family of Generalized Asymptotically Quasi-Nonexpansive Mappings 

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#### Abstract

We introduce a general iteration method for a finite family of generalized asymptotically quasi-nonexpansive mappings. The results presented in the paper extend and improve some recent results in the works by Shahzad and Udomene (2006); L. Qihou (2001); Khan et al. (2008).


## 1. Introduction and Preliminaries

Let $C$ be a nonempty subset of a real Banach space $E$ and $T$ a self-mapping of $C$. The set of fixed points of $T$ is denoted by $F(T)$ and we assume that $F(T) \neq \emptyset$. The mapping $T$ is said to be
(i) contractive mapping if there exists a constant $\alpha$ in $(0,1)$ such that $\|f(x)-f(y)\| \leq \alpha\|x-y\|$, for all $x, y \in C$;
(ii) asymptotically nonexpansive mapping if there exists a sequence $\left\{u_{n}\right\}$ in $[0, \infty)$ with $\lim _{n \rightarrow \infty} u_{n}=0$ such that $\left\|T^{n} x-T^{n} y\right\| \leq\left(1+u_{n}\right)\|x-y\|$, for all $x, y \in C$ and $n=1,2,3, \ldots$;
(iii) asymptotically quasi-nonexpansive if there exists a sequence $\left\{u_{n}\right\}$ in $[0, \infty)$ with $\lim _{n \rightarrow \infty} u_{n}=0$ such that $\left\|T^{n} x-p\right\| \leq\left(1+u_{n}\right)\|x-p\|$, for all $x \in C, p \in F(T)$ and $n=1,2,3, \ldots$;
(iv) generalized asymptotically quasi-nonexpansive [1] if there exist two sequences $\left\{u_{n}\right\},\left\{h_{n}\right\}$ in $[0, \infty)$ with $\lim _{n \rightarrow \infty} u_{n}=0$ and $\lim _{n \rightarrow \infty} h_{n}=0$ such that
$\left\|T^{n} x-p\right\| \leq\left(1+u_{n}\right)\|x-p\|+h_{n}, \quad \forall x \in C, p \in F(T)$,
where $n=1,2,3, \ldots$;
(v) uniformly $L$-Lipschitzian if there exists a constant $L>$ 0 such that $\left\|T^{n} x-T^{n} y\right\| \leq L\|x-y\|$, for all $x, y \in C$ and $n=1,2,3, \ldots$;
(vi) $(L-\gamma)$ uniform $L$-Lipschitz if there are constants $L>0$ and $\gamma>0$ such that $\left\|T^{n} x-T^{n} y\right\| \leq L\|x-y\|^{\gamma}$, for all $x, y \in C$ and $n=1,2,3, \ldots$;
(vii) semicompact if for a sequence $\left\{x_{n}\right\}$ in $C$ with $\lim _{n \rightarrow \infty}\left\|x_{n}-T x_{n}\right\|=0$, there exists a subsequence $\left\{x_{n_{i}}\right\}$ of $\left\{x_{n}\right\}$ such that $x_{n_{i}} \rightarrow p \in C$.

In (1), if $h_{n}=0$ for all $n \geq 1$, then $T$ becomes an asymptotically quasi-nonexpansive mapping; if $u_{n}=0$ and $h_{n}=0$ for all $n \geq 1$, then $T$ becomes a quasi-nonexpansive mapping. It is known that an asymptotically nonexpansive mapping is an asymptotically quasi-nonexpansive and a uniformly $L$ Lipschitzian mapping is $(L-1)$ uniform $L$-Lipschitz.

The mapping $T: C \rightarrow E$ is said to be demiclosed at 0 if for each sequence $\left\{x_{n}\right\} \subset C$ converging weakly to $x_{0}$ and $\left\{T x_{n}\right\}$ converging strongly to 0 , we have $T x_{0}=0$.

A Banach space $E$ is said to satisfy Opial's property if for each $x \in E$ and each sequence $\left\{x_{n}\right\}$ weakly convergent to $x$, the following condition holds for all $x \neq y$ :

$$
\begin{equation*}
\liminf _{n \rightarrow \infty}\left\|x_{n}-x\right\|<\liminf _{n \rightarrow \infty}\left\|x_{n}-y\right\| . \tag{2}
\end{equation*}
$$

Let $C$ be a nonempty closed convex subset of a real Banach space $E$ and $\left\{T_{i}: i=1,2, \ldots k\right\}$ a finite family of asymptotically nonexpansive mappings of $C$ into itself. Suppose that
$\alpha_{i n} \in[0,1], n=1,2,3, \ldots$, and $i=1,2, \ldots k$. Then we consider the following mapping of $C$ into itself:

$$
\begin{align*}
& U_{1 n}=\left(1-\alpha_{1 n}\right) I+\alpha_{1 n} T_{1}^{n} U_{0 n}, \\
& U_{2 n}=\left(1-\alpha_{2 n}\right) I+\alpha_{2 n} T_{2}^{n} U_{1 n}, \\
& \quad \vdots  \tag{3}\\
& U_{(k-1) n}=\left(1-\alpha_{(k-1) n}\right) I+\alpha_{(k-1) n} T_{k-1}^{n} U_{(k-2) n}, \\
& W_{n}=U_{k n}=\left(1-\alpha_{k n}\right) I+\alpha_{k n} T_{k}^{n} U_{(k-1) n},
\end{align*}
$$

where $U_{0 n}=I$ (identity mapping). Such a mapping $W_{n}$ is called the modified $W$-mapping generated by $T_{1}, T_{2}, \ldots, T_{k}$ and $\alpha_{1 n}, \alpha_{2 n}, \ldots, \alpha_{k n}$ (see $[2,3]$ ).

In the sequel, we assume that $F=\bigcap_{i=1}^{k} F\left(T_{i}\right)$.
In 2008, Khan et al. [4] introduced the following iteration process for a family of asymptotically quasi-nonexpansive mappings, for an arbitrary $x_{1} \in C$ :

$$
\begin{align*}
y_{1 n}= & \left(1-\alpha_{1 n}\right) x_{n}+\alpha_{1 n} T_{1}^{n} y_{0 n} \\
y_{2 n}= & \left(1-\alpha_{2 n}\right) x_{n}+\alpha_{2 n} T_{2}^{n} y_{1 n} \\
& \vdots  \tag{4}\\
y_{(k-1) n}= & \left(1-\alpha_{(k-1) n}\right) x_{n}+\alpha_{(k-1) n} T_{k-1}^{n} y_{(k-2) n}, \\
x_{n+1}= & \left(1-\alpha_{k n}\right) x_{n}+\alpha_{k n} T_{k}^{n} y_{(k-1) n},
\end{align*}
$$

where $y_{0 n}=x_{n}, \alpha_{i n} \in[0,1], i=1,2, \ldots, k, n=1,2, \ldots$ and proved that the iterative sequence $\left\{x_{n}\right\}$ defined by (4) converges strongly to a common fixed point of the family of mappings if and only if lim $\inf _{n \rightarrow \infty} d\left(x_{n}, F\right)=0$, where $d(x, F)=$ $\inf _{p \in F}\|x-p\|$. With the help of (3), we write (4) as

$$
\begin{equation*}
x_{n+1}=W_{n} x_{n} . \tag{5}
\end{equation*}
$$

Recently, Chang et al. [5] introduced the following iteration process of asymptotically nonexpansive mappings in Banach space:

$$
\begin{gather*}
x_{n+1}=\lambda_{n} f\left(x_{n}\right)+\left(1-\lambda_{n}\right) T^{n} y_{n},  \tag{6}\\
y_{n}=\beta_{n} x_{n}+\left(1-\beta_{n}\right) T^{n} x_{n},
\end{gather*}
$$

where $\left\{\lambda_{n}\right\},\left\{\beta_{n}\right\} \subset[0,1]$ and $f$ is a fixed contractive mapping, and necessary and sufficient conditions are given for the iterative sequence $\left\{x_{n}\right\}$ to converge to the fixed points of $T$.

For a family of mappings, it is quite significant to devise a general iteration scheme which extends the iteration processes (4) and (6), simultaneously. Thereby, to achieve this goal, we introduce a new iteration process for a family of mappings as follows.

Let $C$ be a nonempty closed convex subset of a real Banach space $E,\left\{T_{i}: C \rightarrow C, i=1,2, \ldots, k\right\}$ a family of generalized asymptotically quasi-nonexpansive mappings, and $f: C \rightarrow$ $C$ a fixed contractive mapping with contractive coefficient $\alpha \in$ $(0,1)$. For a given $x_{1} \in C$, the iteration scheme is defined as follows:

$$
\begin{equation*}
x_{n+1}=\lambda_{n} f\left(x_{n}\right)+\left(1-\lambda_{n}\right) W_{n} x_{n} \tag{7}
\end{equation*}
$$

where $\left\{\lambda_{n}\right\} \in[0,1]$ and $W_{n}$ is the modified $W$-mapping generated by $T_{1}, T_{2}, \ldots, T_{k}$, and $\alpha_{1 n}, \alpha_{2 n}, \ldots, \alpha_{k n}$ for all positive integers $n$.

The purpose of this paper is to study the convergence problem of the iterative sequences $\left\{x_{n}\right\}$ defined by (7). The obtained results extend the corresponding results in [4-8], and Lemma 11 partly improves the method of proof of Lemma 3.1 in [4].

In what follows, we need the following useful known lemmas.

Lemma 1 (see [9]). Let $\left\{a_{n}\right\},\left\{\delta_{n}\right\}$, and $\left\{\gamma_{n}\right\}$ be nonnegative real sequences satisfying the following condition:

$$
\begin{equation*}
a_{n+1} \leq\left(1+\delta_{n}\right) a_{n}+\gamma_{n} \tag{8}
\end{equation*}
$$

where $\sum_{n=1}^{\infty} \delta_{n}<\infty$ and $\sum_{n=1}^{\infty} \gamma_{n}<\infty$; then $\lim _{n \rightarrow \infty} a_{n}$ exists.
Moreover, if in addition, $\lim \inf _{n \rightarrow \infty} a_{n}=0$, then $\lim _{n \rightarrow \infty} a_{n}=0$.

Lemma 2 (see [4]). Let E be a uniformly convex Banach space, $0<b \leq t_{n} \leq c<1$ for all $n \geq 1$, and let $\left\{x_{n}\right\}$ and $\left\{y_{n}\right\}$ be sequences in $E$. Assume that $\lim \sup _{n \rightarrow \infty}\left\|x_{n}\right\| \leq a$, $\lim \sup _{n \rightarrow \infty}\left\|y_{n}\right\| \leq a$, and $\lim _{n \rightarrow \infty}\left\|t_{n} x_{n}+\left(1-t_{n}\right) y_{n}\right\|=a$ for some $a \geq 0$. Then $\lim _{n \rightarrow \infty}\left\|x_{n}-y_{n}\right\|=0$.

## 2. Main Results

Lemma 3. Let $C$ be a nonempty closed convex subset of a real Banach space $E$ and $T$ an asymptotically quasi-nonexpansive self-mapping of $C$ with $\left\{u_{n}\right\} \subset[0, \infty)$ for all $n \geq 1$. Suppose $F(T) \neq \phi$. Then $F(T)$ is a closed subset in $C$.

Proof. Let $\left\{z_{n}\right\}$ be an arbitrary sequence of $F(T)$ and $z_{n} \rightarrow z_{0}$ as $n \rightarrow \infty$. Since $C$ is closed, we have $z_{0} \in C$. For any $\epsilon>0$, there exists a natural number $N$ such that

$$
\begin{equation*}
\left\|z_{n}-z_{0}\right\|<\frac{\epsilon}{2+u_{1}}, \quad \forall n \geq N . \tag{9}
\end{equation*}
$$

Thus, we get

$$
\begin{align*}
\left\|T z_{0}-z_{0}\right\| & \leq\left\|T z_{0}-z_{N}\right\|+\left\|z_{N}-z_{0}\right\| \\
& \leq\left(1+u_{1}\right)\left\|z_{N}-z_{0}\right\|+\left\|z_{N}-z_{0}\right\|  \tag{10}\\
& =\left(2+u_{1}\right)\left\|z_{N}-z_{0}\right\|<\epsilon .
\end{align*}
$$

Since $\epsilon$ is arbitrary, it follows that $\left\|T z_{0}-z_{0}\right\|=0$; that is, $T z_{0}=$ $z_{0}$. Hence $z_{0} \in F(T)$ and $F(T)$ is closed. This completes the proof.

Lemma 4. Let $C$ be a nonempty closed convex subset of a real Banach space E. Let $\left\{T_{i}: i=1,2, \ldots, k\right\}$ be $k$ generalized asymptotically quasi-nonexpansive self-mappings of $C$ with $\left\{u_{i n}\right\},\left\{h_{i n}\right\} \subset[0, \infty)$ such that $\sum_{n=1}^{\infty} u_{i n}<\infty$ and $\sum_{n=1}^{\infty} h_{i n}<$ $\infty$ for all $i \in\{1,2,3, \ldots, k\}$. Suppose $F \neq \emptyset$ and $\left\{\alpha_{i n}\right\}_{n \geq 1} \subset[0,1]$ for all $i \in\{1,2,3, \ldots, k\}$. Let $W_{n}$ be the modified $W$-mapping generated by $T_{1}, T_{2}, \ldots, T_{k}$ and $\alpha_{1 n}, \alpha_{2 n}, \ldots, \alpha_{k n}$. Let the sequence $\left\{x_{n}\right\}$ be defined by (7) and assuming $\sum_{n=1}^{\infty} \lambda_{n}<\infty$, then
(1) there exist two sequences $\left\{\nu_{n}\right\}$ and $\left\{\xi_{n}\right\}$ in $[0, \infty)$ with $\sum_{n=1}^{\infty} v_{n}<\infty, \sum_{n=1}^{\infty} \xi_{n}<\infty$ such that
$\left\|x_{n+1}-p\right\| \leq\left(1+v_{n}\right)^{k}\left\|x_{n}-p\right\|+\xi_{n}, \quad \forall p \in F, n \geq 1 ;$
(2) there exists a constant $M_{1}>0$, such that

$$
\begin{gather*}
\left\|x_{n+m}-p\right\| \leq M_{1}\left\|x_{n}-p\right\|+M_{1} \sum_{i=n}^{\infty} \xi_{i}  \tag{12}\\
\forall p \in F, n, m=1,2,3, \ldots
\end{gather*}
$$

where $\left\{\xi_{n}\right\} \in[0, \infty)$ and $\Sigma_{n=1}^{\infty} \xi_{n}<\infty$.
Proof. (1) Let $v_{n}=\max _{1 \leq i \leq k} u_{i n}$, for all $n$. Since $\sum_{n=1}^{\infty} u_{i n}<\infty$ for each $i$, we can get $\sum_{n=1}^{\infty} v_{n}<\infty$. For all $p \in F$, it follows from (3) that

$$
\begin{align*}
\left\|U_{1 n} x_{n}-p\right\| \leq & \left(1-\alpha_{1 n}\right)\left\|x_{n}-p\right\|+\alpha_{1 n}\left\|T_{1}^{n} x_{n}-p\right\| \\
\leq & \left(1-\alpha_{1 n}\right)\left\|x_{n}-p\right\| \\
& +\alpha_{1 n}\left[\left(1+u_{1 n}\right)\left\|x_{n}-p\right\|+h_{1 n}\right]  \tag{13}\\
\leq & \left(1+u_{1 n}\right)\left\|x_{n}-p\right\|+h_{1 n} \\
\leq & \left(1+v_{n}\right)\left\|x_{n}-p\right\|+h_{1 n}
\end{align*}
$$

Assume that $\left\|U_{j n} x_{n}-p\right\| \leq\left(1+v_{n}\right)^{j}\left\|x_{n}-p\right\|+\left(1+v_{n}\right)^{j-1} \sum_{i=1}^{j} h_{i n}$ for some $1 \leq j \leq k-1$. Then

$$
\begin{aligned}
&\left\|U_{(j+1) n} x_{n}-p\right\| \\
& \leq\left(1-\alpha_{(j+1) n}\right)\left\|x_{n}-p\right\| \\
&+\alpha_{(j+1) n}\left\|T_{j+1}^{n} U_{j n} x_{n}-p\right\| \\
& \leq\left(1-\alpha_{(j+1) n}\right)\left\|x_{n}-p\right\| \\
&+\alpha_{(j+1) n}\left(\left(1+u_{(j+1) n}\right)\left\|U_{j n} x_{n}-p\right\|+h_{(j+1) n}\right) \\
& \leq\left(1-\alpha_{(j+1) n}\right)\left\|x_{n}-p\right\|+\alpha_{(j+1) n} h_{(j+1) n} \\
&+\alpha_{(j+1) n}\left(1+u_{(j+1) n}\right) \\
& \times\left(\left(1+v_{n}\right)^{j}\left\|x_{n}-p\right\|+\left(1+v_{n}\right)^{j-1} \sum_{i=1}^{j} h_{i n}\right) \\
& \leq\left(\left(1-\alpha_{(j+1) n}\right)+\alpha_{(j+1) n}\left(1+v_{n}\right)^{j+1}\right)\left\|x_{n}-p\right\| \\
& \leq\left(\left(1-\alpha_{(j+1) n}\right)\left(1+v_{n}\right)^{j+1}+\alpha_{(j+1) n}\left(1+v_{n}\right)^{j+1}\right)
\end{aligned}
$$

$$
\begin{align*}
& \times\left\|x_{n}-p\right\|+\left(1+v_{n}\right)^{j} \sum_{i=1}^{j+1} h_{i n} \\
\leq & \left(1+v_{n}\right)^{j+1}\left\|x_{n}-p\right\|+\left(1+v_{n}\right)^{j} \sum_{i=1}^{j+1} h_{i n} . \tag{14}
\end{align*}
$$

Thus, by induction, we have

$$
\begin{equation*}
\left\|U_{j n} x_{n}-p\right\| \leq\left(1+v_{n}\right)^{j}\left\|x_{n}-p\right\|+\left(1+v_{n}\right)^{j-1} \sum_{i=1}^{j} h_{i n} \tag{15}
\end{equation*}
$$

for all $j=1,2, \ldots, k$. Hence,

$$
\begin{align*}
\left\|W_{n} x_{n}-p\right\|= & \left\|U_{k n} x_{n}-p\right\| \leq\left(1+v_{n}\right)^{k}\left\|x_{n}-p\right\| \\
& +\left(1+v_{n}\right)^{k-1} \sum_{i=1}^{k} h_{i n} \tag{16}
\end{align*}
$$

By (7) and (16), we obtain

$$
\begin{align*}
\left\|x_{n+1}-p\right\| \leq & \lambda_{n}\left\|f\left(x_{n}\right)-p\right\|+\left(1-\lambda_{n}\right)\left\|W_{n} x_{n}-p\right\| \\
\leq & \lambda_{n}\left\|f\left(x_{n}\right)-f(p)\right\|+\lambda_{n}\|f(p)-p\| \\
& +\left(1-\lambda_{n}\right)\left\|W_{n} x_{n}-p\right\| \\
\leq & \lambda_{n} \alpha\left\|x_{n}-p\right\|+\lambda_{n}\|f(p)-p\|+\left(1-\lambda_{n}\right) \\
& \times\left[\left(1+v_{n}\right)^{k}\left\|x_{n}-p\right\|+\left(1+v_{n}\right)^{k-1} \sum_{i=1}^{k} h_{i n}\right] \\
\leq & \left(1+v_{n}\right)^{k}\left\|x_{n}-p\right\| \\
& +\left(1-\lambda_{n}\right)\left(1+v_{n}\right)^{k-1} \sum_{i=1}^{k} h_{i n} \\
& +\lambda_{n}\|f(p)-p\| \tag{17}
\end{align*}
$$

Since $\sum_{n=1}^{\infty} v_{n}<\infty,\left\{v_{n}\right\}_{n=1}^{\infty}$ is bounded. Setting $M=$ $\max \left\{\sup _{n}\left(1+v_{n}\right)^{k-1},\|f(p)-p\|\right\}$, we get that

$$
\begin{equation*}
\left\|x_{n+1}-p\right\| \leq\left(1+v_{n}\right)^{k}\left\|x_{n}-p\right\|+\xi_{n}, \quad \forall p \in F, n \geq 1 \tag{18}
\end{equation*}
$$

where $\xi_{n}=M\left(\sum_{i=1}^{k} h_{i n}+\lambda_{n}\right)$ and $\sum_{n=1}^{\infty} \xi_{n}<\infty$. This completes the proof of $(1)$.
(2) If $t \geq 0$, then $1+t \leq e^{t}$ and consequently, $(1+t)^{k} \leq e^{k t}$, $k=1,2, \ldots$ Thus, from part (1), we get

$$
\begin{aligned}
& \left\|x_{n+m}-p\right\| \leq\left(1+v_{n+m-1}\right)^{k}\left\|x_{n+m-1}-p\right\|+\xi_{n+m-1} \\
& \leq \exp \left\{k v_{n+m-1}\right\}\left\|x_{n+m-1}-p\right\|+\xi_{n+m-1} \\
& \leq \\
& \quad \exp \left\{k v_{n+m-1}\right\} \\
& \quad \times\left(\exp \left\{k v_{n+m-2}\right\}\left\|x_{n+m-2}-p\right\|+\xi_{n+m-2}\right) \\
& \quad+\xi_{n+m-1}
\end{aligned}
$$

$$
\begin{align*}
\leq & \exp \left\{k\left(v_{n+m-1}+v_{n+m-2}\right)\right\}\left\|x_{n+m-2}-p\right\| \\
& +\exp \left\{k v_{n+m-1}\right\}\left(\xi_{n+m-2}+\xi_{n+m-1}\right) \\
\vdots & \\
\leq & \exp \left\{k \sum_{i=n}^{n+m-1} v_{i}\right\}\left\|x_{n}-p\right\| \\
& +\exp \left\{k \sum_{i=n+1}^{n+m-1} v_{i}\right\} \sum_{i=n}^{n+m-1} \xi_{i} \\
\leq & M_{1}\left\|x_{n}-p\right\|+M_{1} \sum_{i=n}^{\infty} \xi_{i} \tag{19}
\end{align*}
$$

for any positive integers $m, n$, where $M_{1}=\exp \left\{k \sum_{i=1}^{\infty} v_{i}\right\}$, $\sum_{i=1}^{\infty} \xi_{i}<\infty$. This completes the proof of (2).

Remark 5. Lemma 4 generalizes Lemma 2.1 in [4].
Theorem 6. Let C be a nonempty closed convex subset of a real Banach space E. Let $\left\{T_{i}: \quad i=1,2, \ldots, k\right\}$ be $k$ generalized asymptotically quasi-nonexpansive self-mappings of $C$ with $\left\{u_{i n}\right\},\left\{h_{i n}\right\} \subset[0, \infty)$ such that $\sum_{n=1}^{\infty} u_{i n}<\infty$ and $\sum_{n=1}^{\infty} h_{i n}<$ $\infty$ for all $i \in\{1,2,3, \ldots, k\}$. Let $\left\{\alpha_{i n}\right\}_{n \geq 1} \subset[0,1]$ for all $i \in$ $\{1,2,3, \ldots, k\}$ and let $W_{n}$ be a modified $W$-mapping generated by $T_{1}, T_{2}, \ldots, T_{k}$ and $\alpha_{1 n}, \alpha_{2 n}, \ldots, \alpha_{k n}$. Suppose that $F \neq \emptyset$ is closed and $\sum_{n=1}^{\infty} \lambda_{n}<\infty$. Starting from arbitrary $x_{1} \in C$, define the sequence $\left\{x_{n}\right\}$ by the recursion (7); then the sequence $\left\{x_{n}\right\}$ converges strongly to $p \in F$ if and only if $\lim \inf _{n \rightarrow \infty} d\left(x_{n}, F\right)=0$.

Proof. We will only prove the sufficiency; the necessity is obvious. From Lemma 4(1), we have

$$
\begin{equation*}
\left\|x_{n+1}-p\right\| \leq\left(1+v_{n}\right)^{k}\left\|x_{n}-p\right\|+\xi_{n} \tag{20}
\end{equation*}
$$

for all $p \in F$ and all $n$. Therefore,

$$
\begin{align*}
d\left(x_{n+1}, F\right) \leq & \left(1+v_{n}\right)^{k} d\left(x_{n}, F\right)+\xi_{n} \\
= & \left(1+\sum_{r=1}^{k} \frac{k(k-1) \cdots(k-r+1)}{r!} v_{n}^{r}\right)  \tag{21}\\
& \times d\left(x_{n}, F\right)+\xi_{n} .
\end{align*}
$$

As $\sum_{n=1}^{\infty} v_{n}<\infty$, so $\sum_{r=1}^{k}(k(k-1) \cdots(k-r+1) / r!) v_{n}^{r}<\infty$. By Lemma 1 and $\lim _{\inf _{n \rightarrow \infty}} d\left(x_{n}, F\right)=0$, we get that $\lim _{n \rightarrow \infty} d\left(x_{n}, F\right)=0$. Next, we prove that $\left\{x_{n}\right\}$ is a Cauchy sequence. From Lemma 4(2), we have

$$
\begin{equation*}
\left\|x_{n+m}-p\right\| \leq M_{1}\left\|x_{n}-p\right\|+M_{1} \sum_{i=n}^{\infty} \xi_{i} \tag{22}
\end{equation*}
$$

$\forall p \in F, n, m \geq 1$.

Hence, for all integers $m \geq 1$ and all $p \in F$,

$$
\begin{align*}
\left\|x_{n+m}-x_{n}\right\| & \leq\left\|x_{n+m}-p\right\|+\left\|x_{n}-p\right\| \\
& \leq\left(M_{1}+1\right)\left\|x_{n}-p\right\|+M_{1} \sum_{j=n}^{\infty} \xi_{j} . \tag{23}
\end{align*}
$$

Taking infimum over $p \in F$ in (23) gives

$$
\begin{equation*}
\left\|x_{n+m}-x_{n}\right\| \leq\left(M_{1}+1\right) d\left(x_{n}, F\right)+M_{1} \sum_{j=n}^{\infty} \xi_{j} \tag{24}
\end{equation*}
$$

Now, since $\lim _{n \rightarrow \infty} d\left(x_{n}, F\right)=0$ and $\sum_{j=1}^{\infty} \xi_{j}<\infty$, given $\epsilon>$ 0 , there exists an integer $N_{1}>0$ such that for all $n \geq N_{1}$, $d\left(x_{n}, F\right)<\epsilon /\left(2\left(M_{1}+2\right)\right)$ and $\sum_{j=n}^{\infty} \xi_{n}<\epsilon /\left(2\left(M_{1}+1\right)\right)$. So for all integers $n \geq N_{1}, m \geq 1$, we obtain from (24) that

$$
\begin{equation*}
\left\|x_{n+m}-x_{n}\right\|<\epsilon, \quad \forall n \geq N_{1}, \quad m \geq 1 . \tag{25}
\end{equation*}
$$

Hence, $\left\{x_{n}\right\}$ is a Cauchy sequence in $E$. Since $E$ is complete, there exists $q \in E$ such that $\lim _{n \rightarrow \infty} x_{n}=q$. We now show that $q \in F$. Since $d\left(x_{n}, F\right) \rightarrow 0$ and $x_{n} \rightarrow q$ as $n \rightarrow \infty$, for each $\bar{\epsilon}>0$, there exists an integer $N_{2}>0$ such that, $d\left(x_{n}, F\right)=$ $\inf _{p \in F}\left\|x_{n}-p\right\|<\bar{\epsilon} / 3$ and $\left\|x_{n}-q\right\|<\bar{\epsilon} / 2$ for all $n \geq N_{2}$. In particular, we have $d\left(x_{N_{2}}, F\right)=\inf _{p \in F}\left\|x_{N_{2}}-p\right\|<\bar{\epsilon} / 3$; that is, there exists a $\bar{p} \in F$ such that $\left\|x_{N_{2}}-\bar{p}\right\|<\bar{\epsilon} / 2$; hence

$$
\begin{equation*}
\|q-\bar{p}\| \leq\left\|x_{N_{2}}-q\right\|+\left\|x_{N_{2}}-\bar{p}\right\|<\bar{\epsilon} \tag{26}
\end{equation*}
$$

Since $F$ is a closed subset of $E$, we obtain $q \in F$. This completes the proof.

Remark 7. Theorem 6 generalizes and extends Theorem 2.2 of Khan et al. [4], Theorem 3.1 of Ghosh and Debnath [8], Theorem 3.2 of Shahzad and Udomene [6], and Theorem 1 of Qihou [7] together with its Corollaries 1 and 2.

Asymptotically nonexpansive mappings and asymptotically quasi-nonexpansive mappings are all generalized asymptotically quasi-nonexpansive, by Theorem 6 and Lemma 3, so we have

Corollary 8. Let $C$ be a nonempty closed convex subset of a real Banach space E. Let $\left\{T_{i}: i=1,2, \ldots, k\right\}$ be $k$ asymptotically quasi-nonexpansive self-mappings of $C$ with $\left\{u_{i n}\right\} \subset$ $[0, \infty)$ such that $\sum_{n=1}^{\infty} u_{i n}<\infty$ for all $i \in\{1,2,3, \ldots, k\}$. Let $\left\{\alpha_{i n}\right\}_{n \geq 1} \subset[0,1]$ for all $i \in\{1,2,3, \ldots, k\}$ and let $W_{n}$ be a modified $W$-mapping generated by $T_{1}, T_{2}, \ldots, T_{k}$ and $\alpha_{1 n}, \alpha_{2 n}, \ldots, \alpha_{k n}$. Suppose $F \neq \emptyset$ and $\sum_{n=1}^{\infty} \lambda_{n}<\infty$. Starting from arbitrary $x_{1} \in C$, define the sequence $\left\{x_{n}\right\}$ by the recursion (7). Then the sequence $\left\{x_{n}\right\}$ converges strongly to $p \in F$ if and only if $\lim \inf _{n \rightarrow \infty} d\left(x_{n}, F\right)=0$.

Corollary 9. Let C be a nonempty closed convex subset of a real Banach space E. Let $\left\{T_{i}: i=1,2, \ldots, k\right\}$ be $k$ asymptotically nonexpansive self-mappings of $C$ with $\left\{u_{i n}\right\} \subset[0, \infty)$ such that $\sum_{n=1}^{\infty} u_{i n}<\infty$ for all $i \in\{1,2,3, \ldots, k\}$. Let $\left\{\alpha_{i n}\right\}_{n \geq 1} \subset$ $[0,1]$ for all $i \in\{1,2,3, \ldots, k\}$ and let $W_{n}$ be a modified $W$-mapping generated by $T_{1}, T_{2}, \ldots, T_{k}$ and $\alpha_{1 n}, \alpha_{2 n}, \ldots, \alpha_{k n}$.

Suppose $F \neq \emptyset$ and $\sum_{n=1}^{\infty} \lambda_{n}<\infty$. Starting from arbitrary $x_{1} \in$ $C$, define the sequence $\left\{x_{n}\right\}$ by the recursion (7). Then the sequence $\left\{x_{n}\right\}$ converges strongly to $p \in F$ if and only if $\lim \inf _{n \rightarrow \infty} d\left(x_{n}, F\right)=0$.

Corollary 10. Let $C$ be a nonempty closed convex subset of a real Banach space E. Let $\left\{T_{i}: i=1,2, \ldots, k\right\}$ be $k$ generalized asymptotically quasi-nonexpansive self-mappings of $C$ with $\left\{u_{i n}\right\},\left\{h_{i n}\right\} \subset[0, \infty)$ such that $\sum_{n=1}^{\infty} u_{i n}<\infty$ and $\sum_{n=1}^{\infty} h_{i n}<$ $\infty$ for all $i \in\{1,2,3, \ldots, k\}$. Let $\left\{\alpha_{i n}\right\}_{n \geq 1} \subset[0,1]$ for all $i \in$ $\{1,2,3, \ldots, k\}$ and let $W_{n}$ be a modified $W$-mapping generated by $T_{1}, T_{2}, \ldots, T_{k}$ and $\alpha_{1 n}, \alpha_{2 n}, \ldots, \alpha_{k n}$. Suppose that $F \neq \emptyset$ is closed and $\sum_{n=1}^{\infty} \lambda_{n}<\infty$. Starting from arbitrary $x_{1} \in C$, define the sequence $\left\{x_{n}\right\}$ by the recursion (7). Then the sequence $\left\{x_{n}\right\}$ converges strongly to $p \in F$ if and only if there exists a subsequence $\left\{x_{n_{j}}\right\}$ of $\left\{x_{n}\right\}$ which converges to $p$.

## 3. Results in Uniformly Convex Banach Spaces

Lemma 11. Let $C$ be a nonempty closed convex subset of a uniformly convex Banach space E. Let $\left\{T_{i}: i=1,2, \ldots, k\right\}$ be $k(L-\gamma)$ uniform Lipschitz and generalized asymptotically quasi-nonexpansive self-mappings of $C$ with $\left\{u_{i n}\right\},\left\{h_{i n}\right\} \subset$ $[0, \infty)$ such that $\sum_{n=1}^{\infty} u_{i n}<\infty$ and $\sum_{n=1}^{\infty} h_{i n}<\infty$ for all $i \in$ $\{1,2,3, \ldots, k\}$. Let $\alpha_{i n} \in[\delta, 1-\delta]$ for some $\delta \in(0,1 / 2)$ and let $W_{n}$ be a modified $W$-mapping generated by $T_{1}, T_{2}, \ldots, T_{k}$ and $\alpha_{1 n}, \alpha_{2 n}, \ldots, \alpha_{k n}$. Suppose $F=\bigcap_{i=1}^{k} F\left(T_{i}\right) \neq \emptyset$ and $\sum_{n=1}^{\infty} \lambda_{n}<$ $\infty$. Starting from arbitrary $x_{1} \in C$, define the sequence $\left\{x_{n}\right\}$ by the recursion (7). Then $\lim _{n \rightarrow \infty}\left\|x_{n}-T_{j} x_{n}\right\|=0$ for each $j \in$ $\{1,2,3, \ldots k\}$.

Proof. Let $p \in F$ and $v_{n}=\max _{1 \leq i \leq k} u_{i n}$, for all $n$. By Lemma 1 and Lemma 4(1), it follows that $\lim _{n \rightarrow \infty}\left\|x_{n}-p\right\|$ exists for all $p \in F$. Assume that

$$
\begin{equation*}
\lim _{n \rightarrow \infty}\left\|x_{n}-p\right\|=c \tag{27}
\end{equation*}
$$

From (2) and (27) we obtain that

$$
\begin{equation*}
\limsup _{n \rightarrow \infty}\left\|U_{j n} x_{n}-p\right\| \leq c, \quad \forall 1 \leq j \leq k \tag{28}
\end{equation*}
$$

From (7), we have

$$
\begin{align*}
\left\|x_{n+1}-p\right\|= & \left\|\lambda_{n} f\left(x_{n}\right)+\left(1-\lambda_{n}\right) W_{n} x_{n}-p\right\| \\
\leq & \lambda_{n} \alpha\left\|x_{n}-p\right\|+\lambda_{n}\|f(p)-p\|  \tag{29}\\
& +\left(1-\lambda_{n}\right)\left\|U_{k n} x_{n}-p\right\|
\end{align*}
$$

therefore,

$$
\begin{equation*}
\liminf _{n \rightarrow \infty}\left\|U_{k n} x_{n}-p\right\| \geq c \tag{30}
\end{equation*}
$$

From (28) and (30) we can obtain that

$$
\begin{equation*}
\lim _{n \rightarrow \infty}\left\|U_{k n} x_{n}-p\right\|=c \tag{31}
\end{equation*}
$$

Suppose that $\lim _{n \rightarrow \infty}\left\|U_{(j+1) n} x_{n}-p\right\|=c$ for some $1 \leq j \leq$ $k-1$. Since

$$
\begin{align*}
\left\|U_{(j+1) n} x_{n}-p\right\| \leq & \left(1-\alpha_{(j+1) n}\right)\left\|x_{n}-p\right\| \\
& +\alpha_{(j+1) n}\left\|T_{j+1}^{n} U_{j n} x_{n}-p\right\| \\
\leq & \left(1-\alpha_{(j+1) n}\right)\left\|x_{n}-p\right\|+\alpha_{(j+1) n} \\
& \times\left[\left(1+u_{(j+1) n}\right)\left\|U_{j n} x_{n}-p\right\|+h_{(j+1) n}\right] \tag{32}
\end{align*}
$$

so we obtain that

$$
\begin{equation*}
\lim _{n \rightarrow \infty} \inf _{\|}\left\|U_{j n} x_{n}-p\right\| \geq c \tag{33}
\end{equation*}
$$

From (28) and (33), we have that

$$
\begin{equation*}
\lim _{n \rightarrow \infty}\left\|U_{j n} x_{n}-p\right\|=c \tag{34}
\end{equation*}
$$

Thus, by induction, we have

$$
\begin{equation*}
\lim _{n \rightarrow \infty}\left\|U_{j n} x_{n}-p\right\|=c \tag{35}
\end{equation*}
$$

for each $j=1,2,3, \ldots, k$. That is,

$$
\begin{equation*}
\lim _{n \rightarrow \infty}\left\|\left(1-\alpha_{j n}\right)\left(x_{n}-p\right)+\alpha_{j n}\left(T_{j}^{n} U_{(j-1) n} x_{n}-p\right)\right\|=c, \tag{36}
\end{equation*}
$$

for each $j=1,2,3, \ldots, k$. From (28), we obtain

$$
\begin{equation*}
\lim \sup _{n \rightarrow \infty}\left\|T_{j}^{n} U_{(j-1) n} x_{n}-p\right\| \leq c \tag{37}
\end{equation*}
$$

for each $j=1,2,3, \ldots, k$. By Lemma 2 , we get

$$
\begin{equation*}
\lim _{n \rightarrow \infty}\left\|T_{j}^{n} U_{(j-1) n} x_{n}-x_{n}\right\|=0, \quad \forall 1 \leq j \leq k \tag{38}
\end{equation*}
$$

If $j=1$, from (38), we have

$$
\begin{equation*}
\lim _{n \rightarrow \infty}\left\|T_{1}^{n} x_{n}-x_{n}\right\|=0 \tag{39}
\end{equation*}
$$

If $j=2,3, \ldots, k$, then we have

$$
\begin{align*}
\left\|T_{j}^{n} x_{n}-x_{n}\right\| \leq & \left\|T_{j}^{n} x_{n}-T_{j}^{n} U_{(j-1) n} x_{n}\right\|+\left\|T_{j}^{n} U_{(j-1) n} x_{n}-x_{n}\right\| \\
\leq & L\left\|x_{n}-U_{(j-1) n} x_{n}\right\|^{\gamma} \\
& +\left\|T_{j}^{n} U_{(j-1) n} x_{n}-x_{n}\right\| \\
= & L\left(\alpha_{(j-1) n}\left\|x_{n}-T_{j-1}^{n} U_{(j-2) n} x_{n}\right\|\right)^{\gamma} \\
& +\left\|T_{j}^{n} U_{(j-1) n} x_{n}-x_{n}\right\| . \tag{40}
\end{align*}
$$

Hence,

$$
\begin{equation*}
\lim _{n \rightarrow \infty}\left\|T_{j}^{n} x_{n}-x_{n}\right\|=0, \quad \forall 1 \leq j \leq k \tag{41}
\end{equation*}
$$

Note that

$$
\begin{aligned}
\left\|x_{n+1}-x_{n}\right\| & =\left\|\lambda_{n} f\left(x_{n}\right)+\left(1-\lambda_{n}\right) W_{n} x_{n}-x_{n}\right\| \\
& \leq \lambda_{n}\left(\alpha\left\|x_{n}-p\right\|+\|f(p)-p\|+\left\|x_{n}-p\right\|\right)
\end{aligned}
$$

$$
\begin{align*}
& +\left(1-\lambda_{n}\right)\left\|W_{n} x_{n}-x_{n}\right\| \\
= & \lambda_{n}\left(\alpha\left\|x_{n}-p\right\|+\|f(p)-p\|+\left\|x_{n}-p\right\|\right) \\
& +\left(1-\lambda_{n}\right) \alpha_{k n}\left\|T_{k}^{n} U_{(k-1) n} x_{n}-x_{n}\right\| \tag{42}
\end{align*}
$$

therefore, we have

$$
\begin{equation*}
\lim _{n \rightarrow \infty}\left\|x_{n+1}-x_{n}\right\|=0 \tag{43}
\end{equation*}
$$

Now, we observe that

$$
\begin{align*}
\left\|x_{n}-T_{j} x_{n}\right\| \leq & \left\|x_{n}-x_{n+1}\right\|+\left\|x_{n+1}-T_{j}^{n+1} x_{n+1}\right\| \\
& +\left\|T_{j}^{n+1} x_{n+1}-T_{j}^{n+1} x_{n}\right\|+\left\|T_{j}^{n+1} x_{n}-T_{j} x_{n}\right\| \\
\leq & \left\|x_{n}-x_{n+1}\right\|+\left\|x_{n+1}-T_{j}^{n+1} x_{n+1}\right\| \\
& +L\left\|x_{n+1}-x_{n}\right\|^{\gamma}+L\left\|T_{j}^{n} x_{n}-x_{n}\right\|^{\gamma} . \tag{44}
\end{align*}
$$

By (41) and (43), we have

$$
\begin{equation*}
\lim _{n \rightarrow \infty}\left\|x_{n}-T_{j} x_{n}\right\|=0 \tag{45}
\end{equation*}
$$

for $j=1,2,3, \ldots, k$. This completes the proof.
Theorem 12. Let $C$ be a nonempty closed convex subset of a uniformly convex Banach space $E$. Let $\left\{T_{i}: i=1,2, \ldots, k\right\}$ be $k(L-\gamma)$ uniform Lipschitz and generalized asymptotically quasi-nonexpansive self-mappings of $C$ with $\left\{u_{i n}\right\},\left\{h_{i n}\right\} \quad \subset$ $[0, \infty)$ such that $\sum_{n=1}^{\infty} u_{i n}<\infty$ and $\sum_{n=1}^{\infty} h_{i n}<\infty$ for all $i \in$ $\{1,2,3, \ldots, k\}$. Let $\alpha_{i n} \in[\delta, 1-\delta]$ for some $\delta \in(0,1 / 2)$ and let $W_{n}$ be a modified $W$-mapping generated by $T_{1}, T_{2}, \ldots, T_{k}$ and $\alpha_{1 n}, \alpha_{2 n}, \ldots, \alpha_{k n}$. Suppose $F=\bigcap_{i=1}^{k} F\left(T_{i}\right) \neq \emptyset, \sum_{n=1}^{\infty} \lambda_{n}<\infty$ and there exists one member in $\left\{T_{i}^{m}: i=1,2, \ldots, k\right\}$ which is semicompact for some positive integer $m$. Starting from arbitrary $x_{1} \in C$, define the sequence $\left\{x_{n}\right\}$ by the recursion (7). Then $\left\{x_{n}\right\}$ converges strongly to some common fixed point of the family $\left\{T_{i}: i=1,2, \ldots, k\right\}$.

Proof. By Lemma 11, we have

$$
\begin{equation*}
\lim _{n \rightarrow \infty}\left\|x_{n}-T_{j} x_{n}\right\|=0 \tag{46}
\end{equation*}
$$

for each $j=1,2,3, \ldots k$. Without loss of generality, we may assume that $T_{1}^{m}$ is semicompact for some $m \geq 1$; then we have

$$
\begin{align*}
\left\|T_{1}^{m} x_{n}-x_{n}\right\| \leq & \left\|T_{1}^{m} x_{n}-T_{1}^{m-1} x_{n}\right\|+\left\|T_{1}^{m-1} x_{n}-T_{1}^{m-2} x_{n}\right\| \\
& +\cdots+\left\|T_{1} x_{n}-x_{n}\right\| \\
& \leq\left\|T_{1} x_{n}-x_{n}\right\|+(m-1) L\left\|T_{1} x_{n}-x_{n}\right\|^{\gamma} \longrightarrow 0 . \tag{47}
\end{align*}
$$

Since $T_{1}^{m}$ is semicompact, then there exists a subsequence $\left\{x_{n_{j}}\right\}$ of $\left\{x_{n}\right\}$ such that $x_{n_{j}} \rightarrow q \in C$. Hence, we have

$$
\begin{equation*}
\left\|q-T_{i} q\right\|=\lim _{j \rightarrow \infty}\left\|x_{n_{j}}-T_{j} x_{n_{j}}\right\|=0 \tag{48}
\end{equation*}
$$

for each $i=1,2,3, \ldots, k$. This implies that $q \in F$. By Corollary 10, $\left\{x_{n}\right\}$ converges strongly to some common fixed point of the family $\left\{T_{i}: i=1,2, \ldots, k\right\}$.

Theorem 13. Let $C$ be a nonempty closed convex subset of a uniformly convex Banach space E. Let $\left\{T_{i}: i=1,2, \ldots, k\right\}$ be $k(L-\gamma)$ uniform Lipschitz and generalized asymptotically quasi-nonexpansive self-mappings of $C$ with $\left\{u_{i n}\right\},\left\{h_{i n}\right\} \subset$ $[0, \infty)$ such that $\sum_{n=1}^{\infty} u_{i n}<\infty$ and $\sum_{n=1}^{\infty} h_{i n}<\infty$ for all $i \epsilon$ $\{1,2,3, \ldots, k\}$. Let $\alpha_{i n} \in[\delta, 1-\delta]$ for some $\delta \in(0,1 / 2)$ and let $W_{n}$ be a modified $W$-mapping generated by $T_{1}, T_{2}, \ldots, T_{k}$ and $\alpha_{1 n}, \alpha_{2 n}, \ldots, \alpha_{k n}$. Suppose $F=\bigcap_{i=1}^{k} F\left(T_{i}\right) \neq \emptyset, \sum_{n=1}^{\infty} \lambda_{n}<\infty$ and each $I-T_{i}, i=1,2, \ldots, k$, is demiclosed at 0 . If $E$ satisfies Opial's condition, then the sequence $\left\{x_{n}\right\}$ defined by (7) converges weakly to a common fixed point of the family $\left\{T_{i}: i=\right.$ $1,2, \ldots, k\}$.

Proof. From the proof of Lemma 11, we know that $\left\{x_{n}\right\}$ is a bounded sequence in $C$. Since $E$ is uniformly convex, it must be reflexive. Therefore, there exists a subsequence $\left\{x_{n_{j}}\right\}$ in $\left\{x_{n}\right\}$ converging weakly to $u \in C$. By Lemma $11, \lim _{j \rightarrow \infty} \| x_{n_{j}}-$ $T_{i} x_{n_{j}} \|=0$ and $I-T_{i}$ is demiclosed at 0 for $i=1,2, \ldots k$, so we obtain $T_{i} u=u$. That is, $u \in F$. Suppose that there exists another subsequence $\left\{x_{n_{k}}\right\}$ of $\left\{x_{n}\right\}$ converging weakly to $v \in$ C. As above, we can prove $v \in F$. By (27) we know that $\lim _{n \rightarrow \infty}\left\|x_{n}-u\right\|$ and $\lim _{n \rightarrow \infty}\left\|x_{n}-v\right\|$ exist. Assume $v \neq u$. Then by the Opial's condition, we have

$$
\begin{align*}
\lim _{n \rightarrow \infty}\left\|x_{n}-u\right\| & =\lim _{n_{j} \rightarrow \infty}\left\|x_{n_{j}}-u\right\|<\lim _{n_{j} \rightarrow \infty}\left\|x_{n_{j}}-v\right\| \\
& =\lim _{n \rightarrow \infty}\left\|x_{n}-v\right\|=\lim _{n_{k} \rightarrow \infty}\left\|x_{n_{k}}-v\right\|  \tag{49}\\
& <\lim _{n_{k} \rightarrow \infty}\left\|x_{n_{k}}-u\right\|=\lim _{n \rightarrow \infty}\left\|x_{n}-u\right\|,
\end{align*}
$$

which is a contradiction. Hence $u=v$. This implies that $\left\{x_{n}\right\}$ converges weakly to a common fixed point of the family $\left\{T_{i}\right.$ : $i=1,2, \ldots, k\}$.

Remark 14. Lemma 11, Theorem 12, and Theorem 13 extend Lemma 3.1, Theorem 3.3, and Theorem 3.2 of Khan et al. [4], respectively.

## Conflict of Interests

The author declares that there is no conflict of interests.

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