Hindawi Publishing Corporation
Journal of Applied Mathematics
Volume 2014, Article ID 842409, 10 pages
http://dx.doi.org/10.1155/2014/842409

*Research Article*

# Differential Game Analyses of Logistics Service Supply Chain Coordination by Cost Sharing Contract

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Received 28 January 2014; Accepted 12 March 2014; Published 13 April 2014

Academic Editor: X. Zhang



Cooperation of all the members in a supply chain plays an important role in logistics service. The service integrator can encourage cooperation from service suppliers by sharing their cost during the service, which we assume can increase the sales by accumulating the reputation of the supply chain. A differential game model is established with the logistics service supply chain that consists of one service integrator and one supplier. And we derive the optimal solutions of the Nash equilibrium without cost sharing contract and the Stackelberg equilibrium with the integrator as the leader who partially shares the cost of the efforts of the supplier. The results make the benefits of the cost sharing contract in increasing the profits of both players as well as the whole supply chain explicit, which means that the cost sharing contract is an effective coordination mechanism in the long-term relationship of the members in a logistics service supply chain.

## 1. Introduction

A logistics service supply chain, abbreviated as LSSC, is a typical service supply chain, which consists of functional service providers, service integrators, and customers [1]. A functional logistics service provider (FLSP) is a traditional logistics enterprise that can provide standard service, such as transportation or warehousing. A logistics service integrator (LSI) can integrate various functional services in different regions to satisfy different needs of customers who may be a manufacture or other enterprise. LSI is the focal company in LSSC which integrates and controls all types of resources by the use of the information technology to meet the needs of customers and enhance the performance while decreasing the total cost of service. Meanwhile, LSI and FLSP are separate and independent economic entities, which results in the conflict and competition in revenue or the risk during the cooperation in service. As a result, the key issue is to develop mechanisms that can incentivize the members to coordinate their activities so as to enhance the performance of the whole supply chain.

The coordination of supply chain is an important area of supply chain management, and a lot of strategies have been proposed so far [2]. Among them, a common mechanism is to develop a set of properly designed contracts [3]. Since the concept of supply chain contract was first introduced by Pasternack [4]; the design of contracts has received a considerable interest. A large number of contracts have been proposed including the wholesale price contract [5–10], the quantity flexibility contract [11–16], the buy-back contract [17–22], the revenue sharing contract [23–30], the option contract [31–35], and the cost sharing contract [36–38].

Nevertheless, most of the previous studies have focused on contracts with respect to the coordination of traditional supply chain, and few studies focus on service supply chain contracts, which will be very important given the rapid development of service industry. To quote a few, Wei and Hu [39] introduce a wholesale price contract into service supply chain and study the ordering, pricing, and allocation strategy. Liu et al. [40] analyze the fairest revenue sharing coefficient in the coordination of a two-stage logistics service supply chain under the stochastic demand condition and expand the result to a three-stage supply chain. Cui and Liu [41] propose a coordination mechanism by the use of the option contract and analyze the allocation of extra profits between the integrator and the provider. Lu [42] makes use of the

cost sharing contract to coordinate the service supply chain under the assumption that the market demand depends on the efforts made by the service provider and the integrator.

On the other hand, the supply chain coordination by the use of contracts is based on one-time transaction between members. But members in a supply chain are often long-term and stable partners in reality. That means that time becomes a critical factor in the game of members because all the members may make different decisions by taking the future into consideration. So, we propose a differential game strategy which has been widely applied in the management of investment, dual-oligopoly market, and cooperative advertising [43–46].

In this study, we consider time to be a critical factor in the coordination of LSSC. The rest of this paper is organized as follows. Section 2 presents the differential model of LSSC under the assumption that reputation contributes to the increase of sales. In Sections 3 and 4, we come up with the optimal decisions of both parties in supply chain under the Nash (without cost sharing contract) feedback equilibrium and Stackelberg (with cost sharing contract) feedback equilibrium. Section 5 compares the results of two different equilibrium strategies and analyzes the coordinating role of the cost sharing contract. Section 6 concludes.

## 2. Model

The logistics service supply chain in the model consists of one service integrator and one professional service supplier, both of which need to endeavor together to satisfy the customer. Corporate reputation, as one of the most important intangible assets, is the comprehensive reflection of the behavior of corporate in the past [47]. The results of empirical research of service industry indicate that reputation is conducive to the creation and maintaining of customer trust resulting in the increase in successive purchase by loyal customers [48]. Hence, the demand of the LSSC can be reflected by the function of the reputation of LSSC, and the accumulation of LSSC reputation depends on the efforts contributed by the LSI and FLSP in their respective service.

It is assumed that the demand of logistics service $S(t)$ adopts the following specification:

$$S(t) = \theta(t)\, R(t) - \gamma(t)\, R^2(t)\,, \tag{1}$$

where $\theta(t)$ and $\gamma(t)$ are positive parameters capturing the effect of the reputation on the demand and $\theta(t) \gg \gamma(t) > 0$.

The growth and accumulation of the reputation $R(t)$ of LSSC depend on the efforts of supply members. The service integrator controls his efforts $E_i(t)$ while the service supplier controls his efforts $E_p(t)$ during the cooperation in service.

The efforts of both parties during the service contribute to the accumulation of the reputation of LSSC $R(t)$, which evolves according to the Nerlove and Arrow model [49]; that is,

$$\begin{aligned} R'(t) &= \lambda_i(t)\, E_i(t) + \lambda_p(t)\, E_p(t) - \delta(t)\, R(t)\,, \\ R(0) &= R_0 \geq 0, \end{aligned} \tag{2}$$

where $\lambda_i(t) > 0$ and $\lambda_p(t) > 0$ reflecting the efficiency of the efforts of the integrator and the supplier, respectively. Since the reputation decays by time [50], $\delta(t) > 0$, in fact, is the constant decay rate of LSSC reputation caused by environment disturbance.

Let $C_i(t)$ and $C_p(t)$ be the cost of both parties to maintain the reputation, which are convex and increasing, indicating increasing marginal costs of the efforts and assuming to be quadratic,

$$C_i(t) = \frac{\mu_i(t)}{2} E_i^2(t)\,, \qquad C_p(t) = \frac{\mu_p(t)}{2} E_p^2(t)\,, \tag{3}$$

where $\mu_i(t)$ and $\mu_p(t)$ are the positive coefficient.

Let $D(t)$ be the rate of cost that the integrator will share with the supplier, which ranges from zero (the integrator will not share the cost) to one (the integrator pays the full cost). Let $\rho$ denote the discount rate of both parties. $\pi_i(t)$ and $\pi_p(t)$ represent the profit margin of the integrator and the supplier, respectively. Consider

$$\begin{aligned} J_i &= \int_0^\infty e^{-\rho t} \left\{ \pi_i(t) \left[\theta(t)\, R(t) - \gamma(t)\, R^2(t)\right] - \frac{\mu_i(t)}{2} E_i^2(t) \right. \\ &\qquad \left. - \frac{\mu_p(t)}{2} D(t)\, E_p^2(t) \right\} dt, \\ J_p &= \int_0^\infty e^{-\rho t} \left\{ \pi_p(t) \left[\theta(t)\, R(t) - \gamma(t)\, R^2(t)\right] \right. \\ &\qquad \left. - \frac{\mu_p(t)}{2} \left[1 - D(t)\right] E_p^2(t) \right\} dt. \end{aligned} \tag{4}$$

To recapitulate, (1) to (4) define a differential game with two players, three control variables $E_i(t)$, $E_p(t)$, and $D(t)$, and one state variable $R(t)$. The controls are constrained by $E_i(t) \geq 0$, $E_s(t) \geq 0$, $0 \leq D(t) \leq 1$. The state constraint $R(t) \geq 0$ is automatically satisfied.

The optimal decisions of both parties are decided by a feedback game, so that they are all functions of reputation and time. Let $I(R(t), t)$ and $P(R(t), t)$ be the decisions of LSI and FLSP, respectively. To simplify the model, it is assumed that all the parameters in the model, $\mu_i(t)$, $\mu_p(t)$, $\lambda_i(t)$, $\lambda_p(t)$, $\delta(t)$, $\theta(t)$, $\gamma(t)$, $\pi_i(t)$, and $\pi_p(t)$, are constants and have nothing to do with time, which means the players are in the same game in different time horizons. Thus, the differential games can be changed to static games [51]. The decisions of both parties can be defined as $I(R(t))$ and $P(R(t))$. In the following model, $R(t)$, $E_i(t)$, $E_p(t)$, and $D(t)$ are simplified as $R$, $E_i$, $E_p$, and $D$.

## 3. Nash Equilibrium without Cost Sharing Contract

In this section, we try to analyze the decision strategies of players under the circumstance that LSI does not share the cost of FLSP ($D = 0$). We assume that each member determines his optimal decisions independently at the same time, which means the optimal decisions of both players are the solutions of Nash equilibrium.

**Theorem 1.** *If the feasible solutions of the following constraint equations* $(p_1^\#, p_2^\#, p_3^\#, q_1^\#, q_2^\#, q_3^\#)$ *exist, the optimal decisions of both parties can be derived without the cost sharing contract in LSSC. Consider*

$$\begin{gathered}
\frac{2\lambda_i^2 p_1^2}{\mu_i} + \frac{4\lambda_p^2 p_1 q_1}{\mu_p} - (2\delta + \rho) p_1 - \pi_i \gamma = 0, \\
\frac{2\lambda_i^2 p_1 p_2}{\mu_i} + \frac{2\lambda_p^2 (p_1 q_2 + p_2 q_1)}{\mu_p} - (\delta + \rho) p_2 + \pi_i \theta = 0, \\
\frac{\lambda_i^2 p_2^2}{2\mu_i} + \frac{\lambda_p^2 p_2 q_2}{\mu_p} - \rho p_3 = 0, \\
\frac{4\lambda_i^2 p_1 q_1}{\mu_i} + \frac{2\lambda_p^2 q_1^2}{\mu_p} - (2\delta + \rho) q_1 - \pi_p \gamma = 0, \\
\frac{2\lambda_i^2 (p_1 q_2 + p_2 q_1)}{\mu_i} + \frac{2\lambda_p^2 q_1 q_2}{\mu_p} - (\delta + \rho) q_2 + \pi_p \theta = 0, \\
\frac{\lambda_i^2 p_2 q_2}{\mu_i} + \frac{\lambda_p^2 q_2^2}{2\mu_p} - \rho q_3 = 0.
\end{gathered} \tag{5}$$

*The optimal efforts of LSI and FLSP under Nash equilibrium are*

$$\begin{gathered}
E_i^\# = \frac{\lambda_i \left(2p_1^\# R^\# + p_2^\#\right)}{\mu_i}, \\
E_p^\# = \frac{\lambda_p \left(2q_1^\# R^\# + q_2^\#\right)}{\mu_p},
\end{gathered} \tag{6}$$

*where* $R^\# = (R_0 + s/r)e^{rt} - s/r$, $r = 2\lambda_i^2 p_1^\#/\mu_i + 2\lambda_p^2 q_1^\#/\mu_p - \delta$, *and* $s = \lambda_i^2 p_2^\#/\mu_i + \lambda_p^2 q_2^\#/\mu_p$.

*Proof.* We apply a standard sufficient condition for a stationary Markov perfect Nash equilibrium and wish to find bounded and continuously differentiable functions $V_n(R)$, $n \in \{i, p\}$, which satisfy for all $R \geq 0$ the Hamilton-Jacobi-Bellman (HJB) equations [52]:

$$\begin{aligned}
\rho V_i(R) = \operatorname*{Max}_{E_i \geq 0} \Big\{ & \pi_i\left(\theta R - \gamma R^2\right) - \frac{\mu_i}{2} E_i^2 \\
& + V_i'(R) \left[\lambda_i E_i + \lambda_p E_p - \delta R\right] \Big\},
\end{aligned} \tag{7}$$

$$\begin{aligned}
\rho V_p(R) = \operatorname*{Max}_{E_p \geq 0} \Big\{ & \pi_p\left(\theta R - \gamma R^2\right) - \frac{\mu_p}{2} E_p^2 \\
& + V_p'(R) \left[\lambda_i E_i + \lambda_p E_p - \delta R\right] \Big\}.
\end{aligned} \tag{8}$$

Equations (7) and (8) are both concave in $E_i$ and $E_p$, yielding the unique effort level of both parties

$$E_i = \frac{\lambda_i V_i'(R)}{\mu_i}, \qquad E_p = \frac{\lambda_p V_p'(R)}{\mu_p}. \tag{9}$$

Substitute $E_i$ and $E_p$ into (7) to obtain

$$\begin{aligned}
\rho V_i(R) &= \left[\pi_i(\theta - \gamma R) - \delta V_i'(R)\right] R \\
&\quad + \frac{[\lambda_i V_i'(R)]^2}{2\mu_i} + \frac{\lambda_p^2 V_i'(R) V_p'(R)}{\mu_p}, \\
\rho V_p(R) &= \left[\pi_p(\theta - \gamma R) - \delta V_p'(R)\right] R \\
&\quad + \frac{\lambda_i^2 V_i'(R) V_p'(R)}{\mu_i} + \frac{[\lambda_p V_p'(R)]^2}{2\mu_p}.
\end{aligned} \tag{10}$$

We conjecture that the solutions to (10) will be quadratic

$$\begin{aligned}
V_i(R) &= p_1 R^2 + p_2 R + p_3, \\
V_p(R) &= q_1 R^2 + q_2 R + q_3,
\end{aligned} \tag{11}$$

in which $p_1$, $p_2$, $p_3$ and $q_1$, $q_2$, $q_3$ are constants. We substitute $V_i(R)$, $V_p(R)$ and their derivatives from (11) into (10) to obtain

$$\begin{aligned}
\rho\left(p_1 R^2 + p_2 R + p_3\right) &= \left[\left(\pi_i \theta - \delta p_2\right) R - \left(\pi_i \gamma + 2\delta p_1\right) R^2\right] \\
&\quad + \frac{\lambda_i^2 (2p_1 R + p_2)^2}{2\mu_i} \\
&\quad + \frac{\lambda_p^2 (2p_1 R + p_2)(2q_1 R + q_2)}{\mu_p},
\end{aligned} \tag{12}$$

$$\begin{aligned}
\rho\left(q_1 R^2 + q_2 R + q_3\right) &= \left[\left(\pi_p \theta - \delta q_2\right) R - \left(\pi_p \gamma + 2\delta q_1\right) R^2\right] \\
&\quad + \frac{\lambda_i^2 (2p_1 R + p_2)(2q_1 R + q_2)}{\mu_i} \\
&\quad + \frac{\lambda_p^2 (2q_1 R + q_2)^2}{2\mu_p}.
\end{aligned} \tag{13}$$

Equations (12) and (13) both satisfy for all $R \geq 0$. Thus, we can find out that a set of values for $(p_1^\#, p_2^\#, p_3^\#, q_1^\#, q_2^\#, q_3^\#)$,

which can result in the maximization of the profits, is the solution for the following equations:

$$\begin{aligned}
\rho p_1 &= \frac{2\lambda_i^2 p_1^2}{\mu_i} + \frac{4\lambda_p^2 p_1 q_1}{\mu_p} - (\pi_i\gamma + 2\delta p_1),\\
\rho p_2 &= \frac{2\lambda_i^2 p_1 p_2}{\mu_i} + \frac{2\lambda_p^2 (p_1 q_2 + p_2 q_1)}{\mu_p} + (\pi_i\theta - \delta p_2),\\
\rho p_3 &= \frac{\lambda_i^2 p_2^2}{2\mu_i} + \frac{\lambda_p^2 p_2 q_2}{\mu_p},\\
\rho q_1 &= \frac{4\lambda_i^2 p_1 q_1}{\mu_i} + \frac{2\lambda_p^2 q_1^2}{\mu_p} - (\pi_p\gamma + 2\delta q_1),\\
\rho q_2 &= \frac{2\lambda_i^2 (p_1 q_2 + p_2 q_1)}{\mu_i} + \frac{2\lambda_p^2 q_1 q_2}{\mu_p} + (\pi_p\theta - \delta q_2),\\
\rho q_3 &= \frac{\lambda_i^2 p_2 q_2}{\mu_i} + \frac{\lambda_p^2 q_2^2}{2\mu_p}.
\end{aligned} \tag{14}$$

Then, the optimal profits of LSI and FLSP can be represented as

$$\begin{aligned}
V_i^{\#}(R) &= p_1^{\#} R^2 + p_2^{\#} R + p_3^{\#},\\
V_p^{\#}(R) &= q_1^{\#} R^2 + q_2^{\#} R + q_3^{\#}.
\end{aligned} \tag{15}$$

Substituting the derivatives of $V_i^{\#}(R)$, $V_p^{\#}(R)$ obtained from (15) into (9), the optimal effort level of both parties can be represented as

$$E_i^{\#} = \frac{\lambda_i(2p_1^{\#}R + p_2^{\#})}{\mu_i}, \qquad E_p^{\#} = \frac{\lambda_p(2q_1^{\#}R + q_2^{\#})}{\mu_p}. \tag{16}$$

Substitute the results into (2) to obtain

$$R' = rR + s, \tag{17}$$

in which $r = 2\lambda_i^2 p_1^{\#}/\mu_i + 2\lambda_p^2 q_1^{\#}/\mu_p - \delta$, $s = \lambda_i^2 p_2^{\#}/\mu_i + \lambda_p^2 q_2^{\#}/\mu_p$.

The general solution to (17) is

$$R = we^{kt} + w_1. \tag{18}$$

By substituting $R$ and its derivative, for $t$, (19) can be obtained

$$w_1 = -\frac{s}{r}. \tag{19}$$

Hence, the general solution can be represented as

$$R = we^{rt} - \frac{s}{r}, \tag{20}$$

in which $w$ is a constant. Given $R|_{t=0} = R_0 \geq 0$, a particular solution can be obtained

$$R^{\#} = \left(R_0 + \frac{s}{r}\right)e^{rt} - \frac{s}{r}. \tag{21}$$

□

## 4. Stackelberg Equilibrium with Cost Sharing Contract

In this section, we apply a cost sharing contract to realize the coordination of LSSC in a Stackelberg game with the LSI as the leader of the game and the provider as the follower. Firstly, the integrator decides the optimal efforts $E_i$ and the sharing rate $D$. Then, the functional service provider determines his optimal efforts after observing the strategy of the integrator. The optimal decisions of both parties are the results of Stackelberg equilibrium.

**Theorem 2.** *If the feasible solutions to the following constraint equations $(f_1^*, f_2^*, f_3^*, g_1^*, g_2^*, g_3^*)$ exist, the optimal decisions of both parties can be derived after adopting the cost sharing contract in LSSC:*

$$\begin{aligned}
&\frac{2\lambda_i^2 f_1^2}{\mu_i} + \frac{\lambda_p^2(2f_1 + g_1)^2}{2\mu_p} - (2\delta + \rho) f_1 - \pi_i\gamma = 0,\\
&\frac{2\lambda_i^2 f_1 f_2}{\mu_i} + \frac{\lambda_p^2 (2f_1 + g_1)(2f_2 + g_2)}{2\mu_p} - (\delta + \rho) f_2 + \pi_i\theta = 0,\\
&\frac{\lambda_i^2 f_2^2}{2\mu_i} + \frac{\lambda_p^2(2f_2 + g_2)^2}{8\mu_p} - \rho f_3 = 0,\\
&\frac{2\lambda_i^2 f_1 g_1}{\mu_i} + \frac{\lambda_p^2 g_1(2f_1 + g_1)}{\mu_p} - (2\delta + \rho) g_1 - \pi_p\gamma = 0,\\
&\frac{\lambda_i^2(f_1 g_2 + f_2 g_1)}{\mu_i} + \frac{\lambda_p^2(f_1 g_2 + f_2 g_1 + g_1 g_2)}{\mu_p}\\
&\quad - (\delta + \rho) g_2 + \pi_p\theta = 0,\\
&\frac{\lambda_i^2 f_2 g_2}{2\mu_i} + \frac{\lambda_p^2 g_2(2f_2 + g_2)}{4\mu_p} - \rho g_3 = 0.
\end{aligned} \tag{22}$$

*The optimal decisions of LSI and FLS in the Stackelberg equilibrium with LSI as the leader can be represented as*

$$\begin{aligned}
E_i^* &= \frac{\lambda_i(2f_1^* R^* + f_2^*)}{\mu_i},\\
E_p^* &= \frac{\lambda_p[(4f_1^* + 2g_1^*)R^* + (2f_2^* + g_2^*)]}{2\mu_p},\\
D^* &= \frac{(4f_1^* - 2g_1^*)R^* + (2f_2^* - g_2^*)}{(4f_1^* + 2g_1^*)R^* + (2f_2^* + g_2^*)},
\end{aligned} \tag{23}$$

*in which $R^* = (R_0 + d/k)e^{kt} - d/k$, $k = 2\lambda_i^2 f_1^*/\mu_i + \lambda_p^2(2f_1^* + g_1^*)/\mu_p - \delta$, and $d = \lambda_i^2 f_2^*/\mu_i + \lambda_p^2(2f_2^* + g_2^*)/2\mu_p$.*

*Proof.* Given that the integrator announced decisions $E_i$ and $D$, the provider faces a maximization problem where the profit function must satisfy the HJB equation

$$\rho V_p(R) = \underset{E_p \geq 0}{\text{Max}} \left\{ \pi_p\left(\theta R - \gamma R^2\right) - \frac{\mu_p}{2}(1-D)\, E_p^2 + V_p'(R)\left[\lambda_i E_i + \lambda_p E_p - \delta R\right] \right\}. \tag{24}$$

Maximization on the right side of (24) yields

$$E_p = \frac{\lambda_p V_p'(R)}{\mu_p(1-D)}. \tag{25}$$

The integrator, as the leader of game, can forecast the decision of the provider, and then the leader HJB equation is

$$\rho V_i(R) = \underset{E_i \geq 0}{\text{Max}} \left\{ \pi_i\left(\theta R - \gamma R^2\right) - \frac{\mu_i}{2}E_i^2 - \frac{\mu_p}{2}DE_p^2 + V_i'(R)\left[\lambda_i E_i + \lambda_p E_p - \delta R\right] \right\}. \tag{26}$$

Insert (25) into (26) and obtain the efforts and sharing rate as

$$E_i = \frac{\lambda_i V_i'(R)}{\mu_i}, \tag{27}$$

$$D = \frac{2V_i'(R) - V_p'(R)}{2V_i'(R) + V_p'(R)}. \tag{28}$$

Insert (25), (27), and (28) into (24) and (26) to obtain

$$\begin{aligned} \rho V_i(R) &= \left[\pi_i\left(\theta R - \gamma R^2\right) - \delta V_i'(R)\right] R + \frac{\left[\lambda_i V_i'(R)\right]^2}{2\mu_i} + \frac{\lambda_p^2\left[2V_i'(R) + V_p'(R)\right]^2}{8\mu_p}, \\ \rho V_p(R) &= \left[\pi_p\left(\theta R - \gamma R^2\right) - \delta V_p'(R)\right] R + \frac{\lambda_i^2 V_i'(R)\, V_p'(R)}{2\mu_i} + \frac{\lambda_p^2 V_p'(R)\left[2V_i'(R) + V_p'(R)\right]}{4\mu_p}. \end{aligned} \tag{29}$$

We conjecture that the solutions of (29) will be quadratic

$$V_i(R) = f_1 R^2 + f_2 R + f_3, \qquad V_p(R) = g_1 R^2 + g_2 R + g_3, \tag{30}$$

in which $f_1$, $f_2$, and $f_3$ and $g_1$, $g_2$, and $g_3$ are all constants. We substitute $V_i(R)$, $V_p(R)$ and their derivatives from (30) into (29) to obtain

$$\begin{aligned} &\rho\left(f_1 R^2 + f_2 R + f_3\right) \\ &= \left[\left(\pi_i\theta - \delta f_2\right) R - \left(\pi_i\gamma + 2\delta f_1\right) R^2\right] + \frac{\lambda_i^2(2f_1 R + f_2)^2}{2\mu_i} \\ &\quad + \frac{\lambda_p^2\left[2\left(2f_1 R + f_2\right) + \left(2g_1 R + g_2\right)\right]^2}{8\mu_p}, \end{aligned} \tag{31}$$

$$\begin{aligned} &\rho\left(g_1 R^2 + g_2 R + g_3\right) \\ &= \left[\left(\pi_p\theta - \delta g_2\right) R - \left(\pi_p\gamma + 2\delta g_1\right) R^2\right] \\ &\quad + \frac{\lambda_i^2\left(2f_1 R + f_2\right)\left(2g_1 R + g_2\right)}{2\mu_i} \\ &\quad + \frac{\lambda_p^2\left(2g_1 R + g_2\right)\left[2\left(2f_1 R + f_2\right) + \left(2g_1 R + g_2\right)\right]}{4\mu_p}. \end{aligned} \tag{32}$$

Equations (31) and (32) both satisfy for all $R \geq 0$. Thus, we can find out that a set of values for $(f_1^*, f_2^*, f_3^*, g_1^*, g_2^*, g_3^*)$, which can result in the maximization of the profits, is the solution for the following equations:

$$\begin{aligned} \rho f_1 &= \frac{2\lambda_i^2 f_1^2}{\mu_i} + \frac{\lambda_p^2(2f_1 + g_1)^2}{2\mu_p} - \left(\pi_i\gamma + 2\delta f_1\right), \\ \rho f_2 &= \frac{2\lambda_i^2 f_1 f_2}{\mu_i} + \frac{\lambda_p^2\left(2f_1 + g_1\right)\left(2f_2 + g_2\right)}{2\mu_p} + \left(\pi_i\theta - \delta f_2\right), \\ \rho f_3 &= \frac{\lambda_i^2 f_2^2}{2\mu_i} + \frac{\lambda_p^2(2f_2 + g_2)^2}{8\mu_p}, \\ \rho g_1 &= \frac{2\lambda_i^2 f_1 g_1}{\mu_i} + \frac{\lambda_p^2 g_1\left(2f_1 + g_1\right)}{\mu_p} - \left(\pi_p\gamma + 2\delta g_1\right), \\ \rho g_2 &= \frac{\lambda_i^2\left(f_1 g_2 + f_2 g_1\right)}{\mu_i} + \frac{\lambda_p^2\left(f_1 g_2 + f_2 g_1 + g_1 g_2\right)}{\mu_p}, \\ &\quad + \left(\pi_p\theta - \delta g_2\right) \\ \rho g_3 &= \frac{\lambda_i^2 f_2 g_2}{2\mu_i} + \frac{\lambda_p^2 g_2\left(2f_2 + g_2\right)}{4\mu_p}. \end{aligned} \tag{33}$$

The optimal profits of LSI and FLSP can be represented as

$$V_i^*(R) = f_1^* R^2 + f_2^* R + f_3^*, \tag{34}$$

$$V_p^*(R) = g_1^* R^2 + g_2^* R + g_3^*. \tag{35}$$

Substituting the derivative of $V_i^*(R)$ from (34) into (27), the optimal efforts of LSI can be represented as

$$E_i^* = \frac{\lambda_i(2f_1^* R + f_2^*)}{\mu_i}. \tag{36}$$

Substituting the derivatives of $V_i^*(R)$ and $V_p^*(R)$ from (34) and (35) into (28), the optimal sharing rate can be represented as

$$D^* = \frac{(4f_1^* - 2g_1^*)R + (2f_2^* - g_2^*)}{(4f_1^* + 2g_1^*)R + (2f_2^* + g_2^*)}. \tag{37}$$

Also, we can obtain the optimal efforts of FLSP by substituting $V_p^*(R)$'s derivative and $D^*$ from (35) and (37) into (25)

$$E_p^* = \frac{\lambda_p[(4f_1^* + 2g_1^*)R + (2f_2^* + g_2^*)]}{2\mu_p}. \tag{38}$$

Substitute (36) and (38) into differential game equation (2) to obtain

$$R' = kR + d, \tag{39}$$

in which $k = 2\lambda_i^2 f_1^*/\mu_i + \lambda_p^2(2f_1^* + g_1^*)/\mu_p - \delta$, $d = \lambda_i^2 f_2^*/\mu_i + \lambda_p^2(2f_2^* + g_2^*)/2\mu_p$.

Same as the proof of Theorem 2, the general solutions can represented as

$$R = ce^{kt} - \frac{d}{k}, \tag{40}$$

in which $c$ is a constant. Given $R|_{t=0} = R_0 \geq 0$, the particular solution is

$$R^* = \left(R_0 + \frac{d}{k}\right)e^{kt} - \frac{d}{k}. \tag{41}$$

□

## 5. Numerical Analysis

Consider a case with the following set of parameters:

$$\begin{aligned} &\mu_i = 1.2, \quad \mu_p = 1, \quad \lambda_i = 0.6, \\ &\lambda_p = 0.5, \quad \delta = 0.01, \quad \theta = 0.5, \\ &\gamma = 0.05, \quad \pi_i = 0.6, \quad \pi_p = 0.4, \\ &\rho = 0.9, \quad R_0 = 0.5. \end{aligned} \tag{42}$$

Then, we can get the results of (5) and (22)

$$\begin{aligned} &(p_1^\#, p_2^\#, p_3^\#, q_1^\#, q_2^\#, q_3^\#) \\ &\quad = (-0.031269, 0.315981, 0.034968, \\ &\qquad - 0.020664, 0.208800, 0.028047), \\ &(f_1^*, f_2^*, f_3^*, g_1^*, g_2^*, g_3^*) \\ &\quad = (-0.031046, 0.313701, 0.040789, \\ &\qquad - 0.020847, 0.210668, 0.023275), \\ &r = \frac{2\lambda_i^2 p_1^\#}{\mu_i} + \frac{2\lambda_p^2 q_1^\#}{\mu_p} - \delta = -0.039093, \\ &s = \frac{\lambda_i^2 p_2^\#}{\mu_i} + \frac{\lambda_p^2 q_2^\#}{\mu_p} = 0.146994, \\ &k = \frac{2\lambda_i^2 f_1^*}{\mu_i} + \frac{\lambda_p^2(2f_1^* + g_1^*)}{\mu_p} - \delta = -0.049362, \\ &d = \frac{\lambda_i^2 f_2^*}{\mu_i} + \frac{\lambda_p^2(2f_2^* + g_2^*)}{2\mu_p} = 0.198869. \end{aligned} \tag{43}$$

As a result, we can obtain the optimal solutions of the differential game.

*(1) Nash Equilibrium without Cost Sharing Contract.* The optimal efforts of LSI and FLSP are

$$\begin{aligned} E_i^\# &= \frac{\lambda_i(2p_1^\# R^\# + p_2^\#)}{\mu_i} = 0.1020e^{-0.04t} + 0.0404, \\ E_p^\# &= \frac{\lambda_p(2q_1^\# R^\# + q_2^\#)}{\mu_p} = 0.0673e^{-0.04t} + 0.0267. \end{aligned} \tag{44}$$

Under the circumstances, the accumulation of the reputation of LSSC is

$$R^\#(t) = \left(R_0 + \frac{s}{r}\right)e^{rt} - \frac{s}{r} = -3.26e^{-0.04t} + 3.76. \tag{45}$$

And the profits of both parties and the whole supply chain are

$$\begin{aligned} V_i^\#(R) &= p_1^\# R^2 + p_2^\# R + p_3^\# \\ &= -0.3323e^{-0.08t} - 0.2635e^{-0.04t} + 0.7810, \\ V_p^\#(R) &= q_1^\# R^2 + q_2^\# R + q_3^\# \\ &= -0.2196e^{-0.08t} - 0.1741e^{-0.04t} + 0.5210, \\ V_{\text{LSSC}}^\# &= V_i^\# + V_p^\# = -0.5519e^{-0.08t} \\ &\quad - 0.4376e^{-0.04t} + 1.3020. \end{aligned} \tag{46}$$

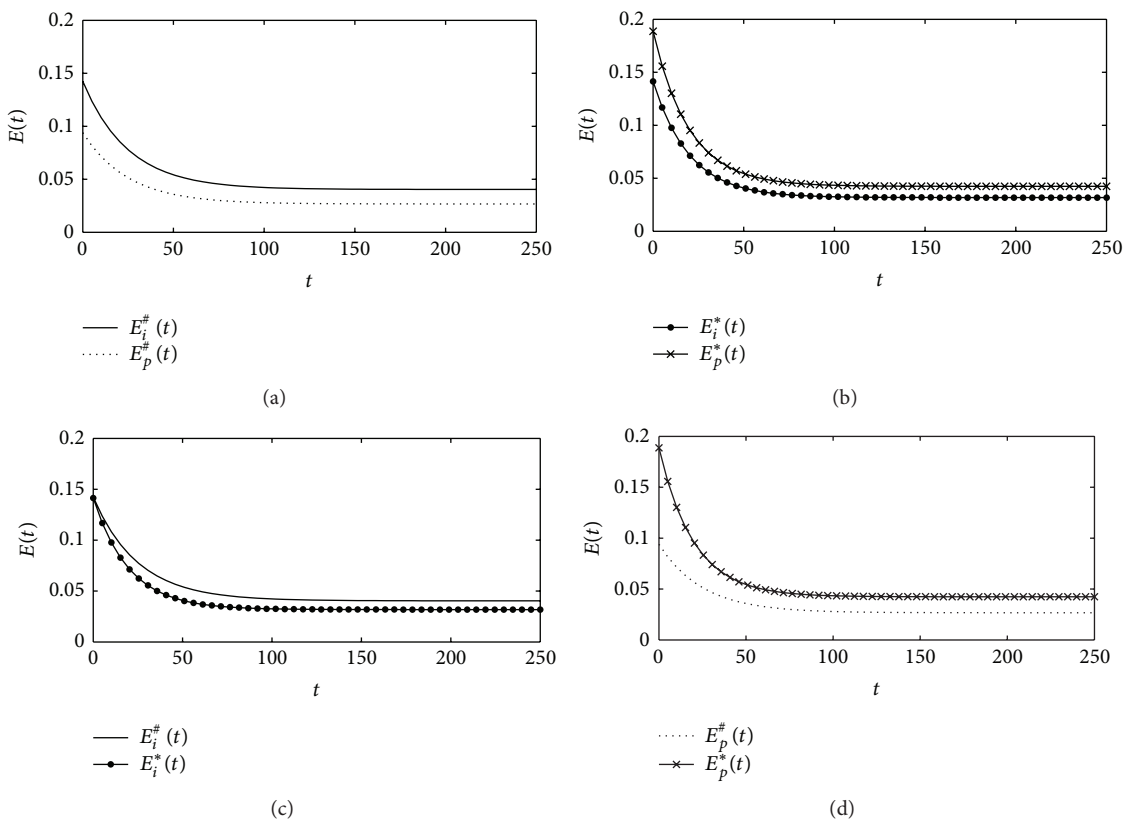


Figure 1: Difference of optimal efforts between Nash equilibrium and Stackelberg equilibrium.

*(2) Stackelberg Equilibrium with Cost Sharing Contract.* The optimal efforts of LSI and FLSP are

$$E_i^* = \frac{\lambda_i(2f_1^*R^* + f_2^*)}{\mu_i} = 0.1096e^{-0.05t} + 0.0318,$$
$$E_p^* = \frac{\lambda_p[(4f_1^* + 2g_1^*)R^* + (2f_2^* + g_2^*)]}{2\mu_p} = 0.1463e^{-0.05t} + 0.0424. \quad (47)$$

The optimal sharing rate of LSI is

$$D^* = \frac{(4f_1^* - 2g_1^*)R^* + (2f_2^* - g_2^*)}{(4f_1^* + 2g_1^*)R^* + (2f_2^* + g_2^*)} = \frac{0.2911e^{-0.05t} + 0.0844}{0.5853e^{-0.05t} + 0.1698}. \quad (48)$$

Under the circumstances, the accumulation of the reputation of LSSC is

$$R^*(t) = \left(R_0 + \frac{d}{k}\right)e^{kt} - \frac{d}{k} = -3.53e^{-0.05t} + 4.03. \quad (49)$$

And the profits of both parties and the whole supply chain are

$$\begin{aligned} V_i^*(R) &= f_1^*R^2 + f_2^*R + f_3^* \\ &= -0.3866e^{-0.10t} - 0.2242e^{-0.05t} + 0.8007, \\ V_p^*(R) &= g_1^*R^2 + g_2^*R + g_3^* \\ &= -0.2596e^{-0.10t} - 0.1507e^{-0.05t} + 0.5336, \\ V_{\text{LSSC}}^* &= V_i^* + V_p^* \\ &= -0.6462e^{-0.10t} - 0.3749e^{-0.05t} + 1.3343. \end{aligned} \quad (50)$$

In the long-term relationship among members in LSSC, LSI and FLSP are faced with dynamic games, which means that the optimal strategies of LSI and FLSP will change with time. As a result, both players will adjust their decisions to maximize their profits in an infinite-time horizon. However, the optimal decisions will become stable at last, suggesting the autonomy of the LSSC.

Figure 1 illustrates the difference between efforts of both parties obtained from Nash equilibrium and Stackelberg equilibrium. The efforts will decline with time until they reach an appropriate level. This is because the marginal utility of reputation for revenue is diminishing while the cost to

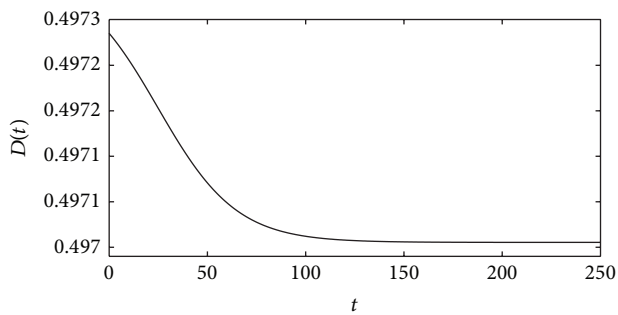


Figure 2: The optimal cost sharing rate of the LSI.

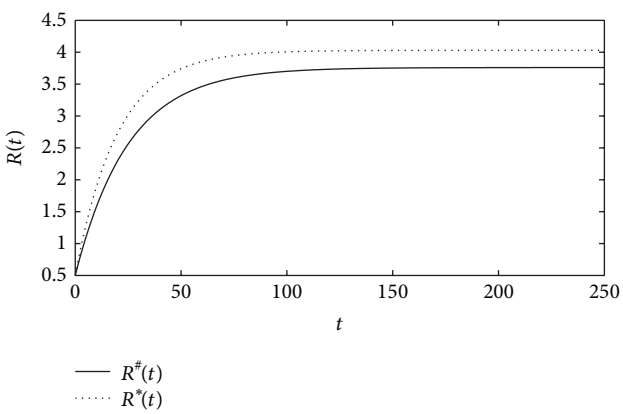


Figure 3: Change of the reputation after adopting cost sharing contract.

maintain reputation remains the same. By comparing the results from the two equilibria, we can see that efforts of FLSP enhance remarkably under the Stackelberg equilibrium, which means that the cost sharing contract can help LSI to encourage FLSP to endeavor in service.

The optimal sharing rate of the LSI can be observed in Figure 2, declining until it reaches an appropriate level which is about 0.5. Thus, LSI should share about half of the cost of FLSP to realize the coordination of the supply chain.

Figure 3 illustrates the whole process of the building reputation. The reputation of LSSC will increase because of accumulation. But the rate of growth will decrease with the decline of the efforts of both parties, resulting in a stable level at last. The stable state of reputation gets higher in the Stackelberg equilibrium with the cost sharing contract, which suggests that the cost sharing contract contributes to the growth of reputation. Same as reputation, the profits of LSSC as well as members can be enhanced by the use of the cost sharing contract, as seen in Figure 4.

To sum up, the long-term relationship among the members of the supply chain contributes to the stability of the service supply chain. In the long-term relationship between LSI and FLSP, the cost sharing contract is helpful in increasing the reputation and profits of the supply chain by encouraging a higher level of the efforts by FLSP, which implies that cost sharing contract is an effective coordination mechanism.

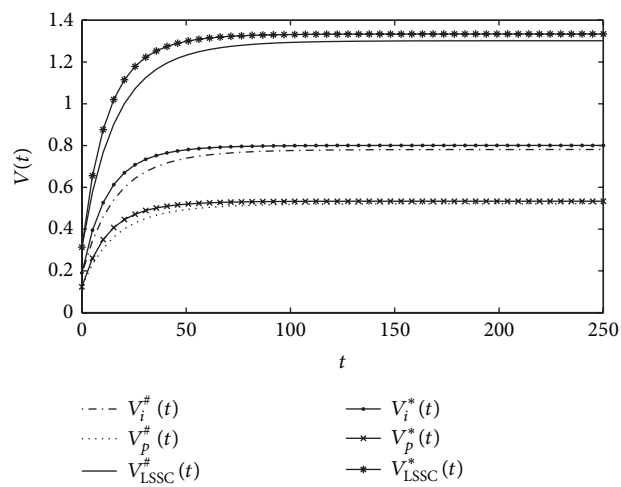


Figure 4: Change of profits of LSSC and the members after adopting cost sharing contract.

## 6. Conclusions

Given the long-term relationship among members in a supply chain, we take time and reputation into consideration as the main factors in a dynamic differential model in LSSC consisting of one LSI and one FLSP. The results indicate the autonomy of the logistics service system and the effectiveness of cost sharing contract in the coordination of the supply chain.

However, we simplify the parameters, the expression form of which may be various in reality resulting in different forms of solutions, in the model to obtain the optimal decisions of both parties, which is an interesting subject for future work. Meanwhile, further studies in the coordination of the supply chain that consists of multi-integrators and multiproviders and the empirical analysis of the mathematical models will play an important role in the practice of supply chain management.

## Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

## Acknowledgments

This research was supported by the NSFC Major Program (Grant no. 71090404/71090400) and NSFC General Program (Grant nos. 71272045 and 71101107).

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