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Research Article

A Novel Method for Dynamic Multicriteria Decision Making with Hybrid Evaluation Information

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How to select the most desirable pattern(s) is often a crucial step for decision making problem. By taking uncertainty as well as dynamic of database into consideration, in this paper, we construct a dynamic multicriteria decision making procedure, where the evaluation information of criteria is expressed by real number, intuitionistic fuzzy number, and interval-valued intuitionistic fuzzy number. During the process of algorithm construction, the evaluation information at all time episodes is firstly aggregated into one, and then it is transformed into the unified interval-valued intuitionistic fuzzy number representational form. Similar to most multicriteria decision making approaches, the TOPSIS method is applied in the proposed decision making algorithm. In particular, the distance between possible patterns and the ideal solutions is defined in terms of cosine similarity by considering all aspects of the unified evaluation information. Experimental results show that the proposed decision making approach can effectively select desirable pattern(s).

1. Introduction

It is well-known that how to discover useful information from mass data effectively has aroused lots of people's interest in many fields, especially in decision making analysis area. In practical processes of information retrieval, decision makers usually have to face complicated database, such as time series database, hybrid database, incomplete database, and so forth. Decision making is extremely intuitive when considering single criterion problems, since we only need to choose the alternative with the highest preference rating. To choose the most desirable ones, multiple criteria are usually considered by decision makers, which is the so-called multicriteria decision making problem. In view of the potential advantages of multicriteria decision making during the process of decision making, this trade-off method has been combined with many theories, such as rough sets [1, 2], fuzzy set and intuitionistic fuzzy set [3, 4], grey theory [5], Choquet integral [6], soft sets [7], and so forth. Moreover, it has been widely applied to many areas such as layout [8–10], management [11, 12], pattern recognition [13], and others [8, 14, 15].

In order to deal with multicriteria decision making problems, Tzeng and Huang [16] proposed that the first step is to figure out how many criteria exist in the problem and then collect the appropriate information of the possible alternatives; the second step is to select an appropriate method to evaluate and outrank the possible alternatives. For the latter issue, Hwang and Yoon [17] proposed the well-known "technique for order preference by similarity to an ideal solution" method (TOPSIS, in short). Because of the complexity of practical problems, many researchers extended TOPSIS method to fuzzy environment, which can be regarded as a natural generalization of classical TOPSIS method. For example, Shih et al. in [18], Wang and Lee in [19], and Park et al. in [20] discussed extension of TOPSIS method for group decision making problems. Jahanshahloo et al. in [21] constructed an algorithmic method to extend TOPSIS for decision making with interval data. In particular, literature [19] introduced an approach to find the ideal solution and [22] proposed an extension of TOPSIS approach that integrates subjective and objective weight. Abo-Sinna and Amer [23] extended TOPSIS to solve multiobjective large-scale nonlinear programming problems. Moreover, some researchers discussed the extension of TOPSIS to other aspects, such as fuzzy data [21], interval-valued fuzzy data [24–26], interval-valued intuitionistic fuzzy data [20, 27], and others [6, 12, 28–32].

However, detailed investigation of the aforementioned literatures shows us that the extended TOPSIS method for multicriteria decision making under various fuzzy environment has its limitations. The reason is that on the one hand, the context on which the problem is based is static. On the other hand, the style of information expression under all criteria has the same representation format. Yu et al. [33] introduced a preference degree based method for handling hybrid multiple attribute decision making problems but the information is still static. Meanwhile, using score function and accuracy function proposed by [34] to rank alternatives is unreasonable to some extent, from which, in this paper, we will focus on the problem of dynamic multicriteria decision making with hybrid evaluation information. Before using TOPSIS method to select the most desirable ones, we presuppose that the values of criteria weights generally are different at the same time episode, in which case the evaluation information of each alternative under each criterion is a time series. Then, by aggregating all the time series into an overall information database, the extended TOPSIS can be applied to deal with the MCDM problems. In this context, the weights of criteria with respect to all alternatives are determined by a mathematical model based on which the weighted distance between alternative and ideal solutions is calculated by means of cosine similarity measure.

The remainder of this paper is organized as follows. Section 2 presents some basic concepts such as intuitionistic fuzzy sets, interval-valued intuitionistic fuzzy sets, multicriteria decision making, and TOPSIS method. In Section 3, we make a detailed discussion on dynamic multicriteria decision making with hybrid evaluation information, in which case the evaluation information of every alternative is regarded as time series and the weights vary with criteria at each time episode. The issue of how to make logical weights of criteria is also investigated in this section. In Section 4, an illustrative example is applied to show the validity of the proposed approach. Finally, Section 5 concludes this paper.

2. Preliminaries

Throughout this paper, let $X = \{x_1, x_2, \dots, x_m\}$ be a fixed set, the universe of discourse, and let $I([0,1])$ be the collection of all closed intervals belonging to unit interval $[0,1]$; then the intuitionistic fuzzy sets as well as interval-valued intuitionistic fuzzy sets proposed by Atanassov and Gargov [35, 36] can be expressed as follows.

Definition 1. An intuitionistic fuzzy set $\widetilde{A}$ based on X can be expressed as

$$\widetilde{A} = \{(x_i, \mu_{\widetilde{A}}(x_i), \nu_{\widetilde{A}}(x_i)) \mid x_i \in X\}, \tag{1}$$

where $\mu_{\widetilde{A}}(x_i)$ and $\nu_{\widetilde{A}}(x_i)$ are, respectively, the membership degree and nonmembership degree of x_i with the condition $\mu_{\widetilde{A}}(x_i) + \nu_{\widetilde{A}}(x_i) \leqslant 1$ for all $x_i \in X$.

Definition 2. An interval-valued intuitionistic fuzzy set $\mathscr{A}$ based on X can be expressed as

$$\mathscr{A} = \left\{\left(x_i \left[\underline{\mu}_{\mathscr{A}}(x_i), \overline{\mu}_{\mathscr{A}}(x_i)\right], \left[\underline{\nu}_{\mathscr{A}}(x_i), \overline{\nu}_{\mathscr{A}}(x_i)\right]\right) \mid x_i \in X\right\}, \tag{2}$$

where $[\underline{\mu}_{\mathscr{A}}(x_i), \overline{\mu}_{\mathscr{A}}(x_i)] \subseteq I([0,1])$ and $[\underline{\nu}_{\mathscr{A}}(x_i), \overline{\nu}_{\mathscr{A}}(x_i)] \subseteq I([0,1])$ are, respectively, the membership degree interval and nonmembership degree interval with the condition $\overline{\mu}_{\mathscr{A}}(x_i) + \overline{\nu}_{\mathscr{A}}(x_i) \leqslant 1$ for all $x_i \in X$.

For an intuitionistic fuzzy set $\widetilde{A}$, the pair $(\mu_{\widetilde{A}}(x_i), \nu_{\widetilde{A}}(x_i))$ is called an intuitionistic fuzzy number and is denoted by $\widetilde{\alpha}_i = (a_i, b_i)$ for convenience, the same as that of interval-valued intuitionistic fuzzy number. In what follows we present a brief review of the mathematical description of multicriteria decision making and TOPSIS method [17, 37].

The procedure of multicriteria decision making can be summarized in three steps, which are evaluating, prioritizing, and selecting. For the first process, which is evaluating, decision makers usually invite experts to provide some evaluation information for some alternatives under certain criteria, and prioritizing is a trade-off process in its nature; the selecting phase is to rank all alternatives with corresponding values obtained from second stage and select the most desirable one(s).

Mathematically speaking, the multicriteria decision making problem of m alternatives with n criteria can be expressed as

$$\begin{array}{c} \\ x_1 \\ x_2 \\ \vdots \\ x_m \end{array} \begin{array}{c} \begin{array}{cccc} c_1 & c_2 & \cdots & c_n \end{array} \\ \begin{pmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ r_{m1} & r_{m2} & \cdots & r_{mn} \end{pmatrix} \end{array}, \tag{3}$$

where r_{ij} for $i = 1, 2, \dots, m$ and $j = 1, 2, \dots, n$ is the evaluation information of alternative x_i under criterion c_j provided by experts, and what follows is to assign an overall evaluation information to each alternative by trading off techniques. Based on foregoing information, the decision maker can obtain a linear order of all alternatives; take $x_{i_1} \succcurlyeq x_{i_2} \succcurlyeq \cdots x_{i_m}$, for example, and then the alternative x_{i_1} is the best choice under existing criteria.

Just like what [38] claimed, how to aggregate the evaluation information of each alternative to a unique value plays a crucial role in the final selection of the best alternative, which in turn means that suitable techniques need to be selected carefully. For this reason, in what follows we present a brief review of the famous trade-off method, which is TOPSIS method. Its main idea came from the concept of compromise solution in order to choose the best alternative nearest to the positive ideal solution (PIS) and farthest from the negative ideal solution (NIS). The general process of TOPSIS method can be summarized as follows [37].

(1) Choose PIS and NIS as

$$\text{PIS} = \{r_1^+, r_2^+, \ldots, r_n^+\}, \qquad \text{NIS} = \{r_1^-, r_2^-, \ldots, r_n^-\}, \tag{4}$$

where r_j^+ represents the obtainable maximum value under c_j if c_j is beneficial criterion (larger is better); otherwise r_j^+ is the minimum value; r_j^- represents the obtainable minimum value under c_j if c_j is costly criterion (smaller is better); otherwise r_j^- is the maximum value.

(2) Calculate the separation from the PIS and NIS between alternatives by

$$D_i^+ = \sqrt{\sum_{j=1}^{n}\left(r_{ij} - r_j^+\right)^2}, \qquad D_i^- = \sqrt{\sum_{j=1}^{n}\left(r_{ij} - r_j^-\right)^2} \tag{5}$$

for $i = 1, 2, \ldots, m$.

(3) Calculate the overall score of each alternative by

$$D_i = \frac{D_i^-}{D_i^+ + D_i^-} \tag{6}$$

and make a choice, where $i = 1, 2, \ldots, m$.

It deserves to be pointed out that, (1) in practical application, the criteria are usually given weights by various means; (2) many measuring distances are applied to measure the distance between alternative and PIS as well as NIS, such as [39–41]. For detailed description of this trade-off method, the interested readers can refer to many literatures such as [16, 37, 42].

3. Multicriteria Decision Making with Hybrid Evaluation Information

In this section, our work can be divided into three components, which are normalization of hybrid evaluation information, determination of weighted vector of criteria as well as time episodes, and construction of the detailed procedure for dynamic multicriteria decision making.

3.1. Normalization of Hybrid Evaluation Information. Due to the complexity of the real word, database with hybrid types of information is unavoidable. For example, datum $x = \langle 2, (0.3, 0.5), ([0.7, 0.8], [0, 0.1])\rangle$ is a hybrid datum with respect to criteria (c_1, c_2, c_3) because each component of x has different types. Hereinto, by taking uncertainty of information into consideration, next we mainly discuss three types of evaluation information, which are real number, intuitionistic fuzzy number, and interval-valued intuitionistic fuzzy number.

It is well-known that during the process of decision making, the ultimate goal is to obtain the whole evaluation of each alternative under many criteria. And if the evaluation information of an alternative contains different types of values, the first choice for many decision makers is to transform them into single type. In this light, next we make a detailed discussion on how an datum that components have different types is changed into a datum that components have same type.

Given that only three types of evaluation information appeared in datum x_i:

$$x_i = \begin{array}{c} \begin{matrix} c_1 & \cdots & c_l & \cdots & c_k & \cdots & c_q & \cdots & c_n \end{matrix} \\ \left(r_{i1} \;\cdots\; (\mu_{il}, \nu_{il}) \;\cdots\; r_{ik} \;\cdots\; \left(\left[\underline{\mu}_{iq}, \overline{\mu}_{iq}\right], \left[\underline{\nu}_{iq}, \overline{\nu}_{iq}\right]\right) \;\cdots\; r_{in} \right) \end{array}, \tag{7}$$

where (μ_{il}, ν_{il}) is an intuitionistic fuzzy number and $([\underline{\mu}_{iq}, \overline{\mu}_{iq}], [\underline{\nu}_{iq}, \overline{\nu}_{iq}])$ is an interval-valued intuitionistic fuzzy number, then we have the following.

(1) If r_{ij} is an intuitionistic fuzzy number for some $j \in \{1, 2, \ldots, n\}$, then replace r_{ij} with $([\mu_{ij}, \mu_{ij}], [\nu_{ij}, \nu_{ij}])$; that is,

$$r_{ij} = \left(\mu_{ij}, \nu_{ij}\right) = \left(\left[\mu_{ij}, \mu_{ij}\right], \left[\nu_{ij}, \nu_{ij}\right]\right). \tag{8}$$

(2) If r_{ij} is a real number belonging to unit interval $[0, 1]$, then replace it with $([r_{ij}, r_{ij}], [1 - r_{ij}, 1 - r_{ij}])$; that is,

$$r_{ij} = \left(\left[r_{ij}, r_{ij}\right], \left[1 - r_{ij}, 1 - r_{ij}\right]\right). \tag{9}$$

Otherwise, at first we normalize r_{ij} by

$$r'_{ij} = \frac{r_{ij}}{\max r_{lj}}, \tag{10}$$

where $l \in \{1, 2, \ldots, m\}$ and then do as (8) does.

(3) Do nothing if r_{ij} is an interval-valued intuitionistic fuzzy number.

Using above transformation, datum x_i then changes into an interval-valued intuitionistic fuzzy vector as follows:

$$x_i = \begin{matrix} & c_1 & c_2 & \cdots & c_n \\ (& ([\underline{\mu}_{i1}, \overline{\mu}_{i1}], [\underline{\nu}_{i1}, \overline{\nu}_{i1}]) & ([\underline{\mu}_{i2}, \overline{\mu}_{i2}], [\underline{\nu}_{i2}, \overline{\nu}_{i2}]) & \cdots & ([\underline{\mu}_{in}, \overline{\mu}_{in}], [\underline{\nu}_{in}, \overline{\nu}_{in}]) &). \end{matrix} \tag{11}$$

Example 1. Given that for a dynamic multicriteria decision making with alternatives x_1, x_2, x_3 and criteria c_1, c_2, c_3, c_4, c_5, the evaluation information at some episode is

$$\begin{matrix} & c_1 & c_2 & c_3 & c_4 & c_5 \\ x_1 & 1 & (0.8, 0.1) & 0.4 & (0.3, 0.6) & ([0.5, 0.7], [0.1, 0.2]) \\ x_2 & 3 & (0.7, 0.1) & 0.8 & (0.2, 0.1) & ([0.2, 0.4], [0.3, 0.5]) \\ x_3 & 4 & (0.3, 0.4) & 0.6 & (0.5, 0.0) & ([0.0, 1.0], [0.6, 0.7]) \end{matrix}. \tag{12}$$

Obviously, the information with respect to criteria c_1 and c_3 is real number, and the information with respect to criteria c_2 and c_4 is intuitionistic fuzzy number. By (11) the normalization information of criteria c_1 and c_3 can be expressed as

$$\begin{aligned} &\begin{matrix} & c_1 & c_3 \\ x_1 & 0.25 & 0.50 \\ x_2 & 0.75 & 1.00 \\ x_3 & 1.00 & 0.75 \end{matrix} \\ &= \begin{pmatrix} c_1 & c_3 \\ ([0.25, 0.25], [0.75, 0.75]) & ([0.50, 0.50], [0.50, 0.50]) \\ ([0.75, 0.75], [0.25, 0.25]) & ([1.00, 1.00], [0.00, 0.00]) \\ ([1.00, 1.00], [0.00, 0.00]) & ([0.75, 0.75], [0.25, 0.25]) \end{pmatrix}. \end{aligned} \tag{13}$$

For criteria c_2 and c_4, we have that

$$\begin{matrix} & x_1 & x_2 & x_3 \\ c_2 & ([0.8, 0.8], [0.1, 0.1]) & ([0.7, 0.7], [0.1, 0.1]) & ([0.3, 0.3], [0.4, 0.4]) \\ c_4 & ([0.3, 0.3], [0.6, 0.6]) & ([0.2, 0.2], [0.1, 0.1]) & ([0.5, 0.5], [0.0, 0.0]) \end{matrix}. \tag{14}$$

Therefore, we have

$$\begin{matrix} & x_1 & x_2 & x_3 \\ c_1 & ([0.25, 0.25], [0.75, 0.75]) & ([0.75, 0.75], [0.25, 0.25]) & ([0.50, 0.50], [0.50, 0.50]) \\ c_2 & ([0.80, 0.80], [0.10, 0.10]) & ([0.70, 0.70], [0.10, 0.10]) & ([0.30, 0.30], [0.40, 0.40]) \\ c_3 & ([0.50, 0.50], [0.50, 0.50]) & ([1.00, 1.00], [0.00, 0.00]) & ([0.75, 0.75], [0.25, 0.25]) \\ c_4 & ([0.30, 0.30], [0.60, 0.60]) & ([0.20, 0.20], [0.10, 0.10]) & ([0.50, 0.50], [0.00, 0.00]) \\ c_5 & ([0.50, 0.70], [0.10, 0.20]) & ([0.20, 0.40], [0.30, 0.50]) & ([0.00, 0.10], [0.60, 0.70]) \end{matrix}. \tag{15}$$

3.2. Determination of Weights. To make a reasonable decision for certain problem, how to determine the weights of criteria has been discussed broadly, and many methods are also being used to calculate the corresponding weights, such as maximizing deviation method [43, 44], entropy method [45, 46], and others [7, 47–49].

Due to the increasing complexity of practical socioeconomic development, uncertainty and diversification have become the normal situation for information obtained from flow process line, especially for the information changing with time, that is, time series database. In this paper, from the view point of uncertainty of dynamic evaluation information, at first we construct an approach to determine the weighted vector of episodes, in which different criteria are assigned different weights at the same episode. The reason for that is that if we pay equal weights to each criterion at the same time episode, it may be illogic. For example, a wise teacher ought to know that, for the purpose of estimating a student's performance, different subjects should not be paid same importance at the end of one semester.

Generally speaking, (3) can be regarded as one time episode of the dynamic multicriteria decision making, and the whole process of it can be depicted as

$$\underbrace{\begin{matrix} & c_1 & \cdots & c_n \\ x_1 & r_{11}^{(1)} & \cdots & r_{1n}^{(1)} \\ \vdots & \vdots & \ddots & \vdots \\ x_m & r_{m1}^{(1)} & \cdots & r_{mn}^{(1)} \end{matrix}}_{t_1}, \quad \underbrace{\begin{pmatrix} r_{11}^{(2)} & \cdots & r_{1n}^{(2)} \\ \vdots & \ddots & \vdots \\ r_{m1}^{(2)} & \cdots & r_{mn}^{(2)} \end{pmatrix}}_{t_2}, \ldots, \quad \underbrace{\begin{pmatrix} r_{11}^{(p)} & \cdots & r_{1n}^{(p)} \\ \vdots & \ddots & \vdots \\ r_{m1}^{(p)} & \cdots & r_{mn}^{(p)} \end{pmatrix}}_{t_p}, \tag{16}$$

where $r_{ij}^{(k)}$ represents the evaluation information of alternative x_i under criterion c_j at time episode t_k. Next, we make a detailed description of how to calculate the weights of each criterion at all time episodes. From above flow chart (i.e., (16)) we apply

$$\begin{matrix} & t_1 & t_2 & \cdots & t_p \\ x_1 & r_{1j}^{(1)} & r_{1j}^{(2)} & \cdots & r_{1j}^{(p)} \\ x_2 & r_{2j}^{(1)} & r_{2j}^{(2)} & \cdots & r_{2j}^{(p)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_m & r_{mj}^{(1)} & r_{mj}^{(2)} & \cdots & r_{mj}^{(p)} \end{matrix} \tag{17}$$

to depict the evaluation information of criterion c_j for all time episodes. Because the value style of $r_{ij}^{(k)}$ varies with criteria, in what follows we propose separately the approaches to determining weighted vector of time episodes in detail.

(1) $r_{ij}^{(k)}$ is a positive real number for $i = 1, 2, \ldots, m$, in which case $\omega_j^{(k)}$ for $k = 1, 2, \ldots, p$ can be calculated by

$$\omega_j^{(k)} = \frac{r_{*j}^{(k)}}{\sum_{k=1}^{p} r_{*j}^{(k)}}, \tag{18}$$

where $r_{*j}^{(k)} = \sum_{i=1}^{m} \eta_i r_{ij}^{(k)}$ if c_j is a beneficial criterion; otherwise $r_{*j}^{(k)} = \sum_{i=1}^{m} \eta_i (1/r_{ij}^{(k)})$. Here, η_i represents the alternatives' weight for the ith pattern.

(2) $r_{ij}^{(k)}$ is an intuitionistic fuzzy number for $k = 1, 2, \ldots, p$, $i = 1, 2, \ldots, m$; that is, $r_{ij}^{(k)} = (\mu_{ij}^{(k)}, \nu_{ij}^{(k)})$ with condition $\mu_{ij}^{(k)} + \nu_{ij}^{(k)} \leqslant 1$, in which case we first aggregate $r_{ij}^{(k)}$, for $i = 1, 2, \ldots, m$, at time episode t_k into $r_{*j}^{(k)} = (\mu_{*j}^{(k)}, \nu_{*j}^{(k)})$ by IFWA$_\eta$ operator or IFWG$_\eta$ operator provided in [50, 51]. After that, compute the weighted vector of time episodes by

$$\omega_j^{(k)} = \frac{S_j^{(k)}}{\sum_{k=1}^{p} S_j^{(k)}}, \tag{19}$$

where $S_j^{(k)} = \mu_{*j}^{(k)}$ if c_j is a beneficial criterion; otherwise $S_j^{(k)} = \nu_{*j}^{(k)}$.

(3) $r_{ij}^{(k)}$ is an interval-valued intuitionistic fuzzy number for $t = 1, 2, \ldots, p$ and $i = 1, 2, \ldots, m$; that is, $r_{ij}^{(k)} = ([\underline{\mu}_{ij}^{(k)}, \overline{\mu}_{ij}^{(k)}], [\underline{\nu}_{ij}^{(k)}, \overline{\nu}_{ij}^{(k)}])$ with condition $\overline{\mu}_{ij}^{(k)} + \overline{\nu}_{ij}^{(k)} \leqslant 1$, in which case we first aggregate $r_{ij}^{(k)}$, for $i = 1, 2, \ldots, m$, at time episode t_k into $r_{*j}^{(k)} = ([\underline{\mu}_{*j}^{(k)}, \overline{\mu}_{*j}^{(k)}], [\underline{\nu}_{*j}^{(k)}, \overline{\nu}_{*j}^{(k)}])$ by IIFWA$_\eta$ operator or IIFWG$_\eta$ operator provided in [34]. After that, compute the weighted vector of time episodes by

$$\omega_j^{(k)} = \frac{H_j^{(k)}}{\sum_{k=1}^{p} H_j^{(k)}}, \tag{20}$$

where $H_j^{(k)} = (1/2)(\underline{\mu}_{*j}^{(k)} + \overline{\mu}_{*j}^{(k)})$ if c_j is beneficial criterion; otherwise $H_j^{(k)} = (1/2)(\underline{\nu}_{*j}^{(k)} + \overline{\nu}_{*j}^{(k)})$.

Notice that if the decision makers treat all alternative without distinction, then during the calculation of $r_{*j}^{(k)}$, the weighted vector of alternatives is $\eta = \{1/m, 1/m, \ldots, 1/m\}$; otherwise $\eta = \{\eta_1, \eta_2, \ldots, \eta_m\}$ with the condition $\eta_1 + \eta_2 + \cdots + \eta_m = 1$.

For the rest of this subsection, we present a discussion of how to determine the weighted vector on criteria, at each time episode, take time episode t_k for example. Given that there is a dynamic multicriteria decision making problem with alternatives $x_1, x_2, \ldots, x_m$ and criteria $c_1, c_2, \ldots, c_n$, the evaluation information $r_{ij}^{(k)}$ for $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$ can be expressed as

$$\begin{array}{c} \\ x_1 \\ x_2 \\ \vdots \\ x_m \end{array} \begin{array}{c} \begin{array}{cccc} c_1 & c_2 & \cdots & c_n \end{array} \\ \left(\begin{array}{cccc} r_{11}^{(k)} & r_{12}^{(k)} & \cdots & r_{1n}^{(k)} \\ r_{21}^{(k)} & r_{22}^{(k)} & \cdots & r_{2n}^{(k)} \\ \vdots & \vdots & \ddots & \vdots \\ r_{m1}^{(k)} & r_{m2}^{(k)} & \cdots & r_{mn}^{(k)} \end{array} \right) \end{array}. \quad (21)$$

Generally, the starting point of assigning weights to criteria is to make the alternative have the best performance with respect to other alternatives as much as possible under provided criteria. Therefore, the mathematical model can be constructed as follows.

Model-1:

$$\text{Max:} \quad x_i(\omega) = \sum_{j=1}^{n} \left(L\left(r_{ij}^{(k)}\right) \omega_{ij}^{(k)} \right)^2, \quad i = 1, 2, \ldots, m$$

$$\text{s.t.} \quad \omega^{(k)} \in \mathscr{H}, \quad \sum_{j=1}^{n} \omega_{ij}^{(k)} = 1, \quad \omega_{ij}^{(k)} \geqslant 0, \ j = 1, 2, \ldots, n, \quad (22)$$

where

$$L\left(r_{ij}^{(k)}\right) = \frac{\underline{\mu}_{ij}^{(k)} + \overline{\mu}_{ij}^{(k)} - \overline{\nu}_{ij}^{(k)}\left(1 - \overline{\mu}_{ij}^{(k)}\right) - \underline{\nu}_{ij}^{(k)}\left(1 - \underline{\mu}_{ij}^{(k)}\right)}{2}, \quad (23)$$

is the score of interval-valued intuitionistic fuzzy number $r_{ij}^{(k)}$ given by [52]. Solve mathematical model (Model-1) by means of Lagrange multiplier method.

Let λ be Lagrange multiplier and construct Lagrange function as

$$\begin{aligned} F(x_i, \lambda) &= x_i(\omega) + \lambda \left(\sum_{j=1}^{n} \omega_{ij}^{(k)} - 1 \right) \\ &= \sum_{j=1}^{n} \left(L\left(r_{ij}^{(k)}\right) \omega_{ij}^{(k)} \right)^2 + \lambda \left(\sum_{j=1}^{n} \omega_{ij}^{(k)} - 1 \right). \end{aligned} \quad (24)$$

Differentiating (24) we have that

$$\frac{\partial F(x_i, \lambda)}{\partial \omega_{ij}^{(k)}} = 2\left(L\left(r_{ij}^{(k)}\right)\right)^2 \omega_{ij}^{(k)} + \lambda = 0. \quad (25)$$

With constraint condition $\sum_{j=1}^{n} \omega_{ij}^{(k)} = 1$, we have

$$\lambda = -\frac{1}{\sum_{j=1}^{n} \left(1/2\left(L\left(r_{ij}^{(k)}\right)\right)^2 \right)}. \quad (26)$$

Taking (26) into (25), we can get that

$$\omega_{ij}^{(k)} = \frac{1}{\left(L\left(r_{ij}^{(k)}\right)\right)^2} \frac{1}{\sum_{j=1}^{n} \left(1/\left(L\left(r_{ij}^{(k)}\right)\right)^2 \right)} \quad (27)$$

for $i = 1, 2, \ldots, m$. From above, we obtain the weighted vector of criteria at each time episode with respect to every alternative.

3.3. Procedures for Multicriteria Decision Making. Based on aforementioned analysis, in what follows we propose an approach to dynamic multicriteria decision making for hybrid evaluation information. Given that the dynamic multicriteria decision making problem about m alternatives with n criteria at p time episodes is depicted as flow chart (i.e., (16)), then the procedure for decision making can be constructed as follows.

Step 1. Utilize (18)–(20) to compute $\omega_j^{(k)}$, where $k = 1, 2, \ldots, p$ and $j = 1, 2, \ldots, n$.

Step 2. Aggregate $r_{ij}^{(k)}$ into r_{ij} for $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$, where

$$r_{ij} = \sum_{k=1}^{p} \omega_j^{(k)} r_{ij}^{(k)}, \quad (28)$$

if $r_{ij}^{(k)}$ is a real number; and

$$\begin{aligned} r_{ij} &= \text{IFWA}_{\omega_j}\left(r_{ij}^{(1)}, r_{ij}^{(2)}, \ldots, r_{ij}^{(p)}\right) \\ &= \left(1 - \prod_{k=1}^{p} \left(1 - \mu_{ij}^{(k)}\right)^{\omega_j^{(k)}}, \prod_{k=1}^{p} \left(\nu_{ij}^{(k)}\right)^{\omega_j^{(k)}} \right) \end{aligned} \quad (29)$$

or

$$\begin{aligned} r_{ij} &= \text{IFWG}_{\omega_j}\left(r_{ij}^{(1)}, r_{ij}^{(2)}, \ldots, r_{ij}^{(p)}\right) \\ &= \left(\prod_{k=1}^{p} \left(\mu_{ij}^{(k)}\right)^{\omega_j^{(k)}}, 1 - \prod_{k=1}^{p} \left(1 - \nu_{ij}^{(k)}\right)^{\omega_j^{(k)}} \right) \end{aligned} \quad (30)$$

if $r_{ij}^{(k)}$ is an intuitionistic fuzzy number; and

$$\begin{aligned} r_{ij} &= \text{IIFWA}_{\omega_j}\left(r_{ij}^{(1)}, r_{ij}^{(2)}, \ldots, r_{ij}^{(p)}\right) \\ &= \left(\left[1 - \prod_{k=1}^{p} \left(1 - \underline{\mu}_{ij}^{(k)}\right)^{\omega_j^{(k)}}, 1 - \prod_{k=1}^{p} \left(1 - \overline{\mu}_{ij}^{(k)}\right)^{\omega_j^{(k)}} \right], \right. \\ &\quad \left. \left[\prod_{k=1}^{p} \left(\underline{\nu}_{ij}^{(k)}\right)^{\omega_j^{(k)}}, \prod_{k=1}^{p} \left(\overline{\nu}_{ij}^{(k)}\right)^{\omega_j^{(k)}} \right] \right) \end{aligned} \quad (31)$$

or

$$\begin{aligned} r_{ij} &= \text{IIFWG}_{\omega_j}\left(r_{ij}^{(1)}, r_{ij}^{(2)}, \ldots, r_{ij}^{(p)}\right) \\ &= \left(\left[\prod_{k=1}^{p} \left(\underline{\mu}_{ij}^{(k)}\right)^{\omega_j^{(k)}}, \prod_{k=1}^{p} \left(\overline{\mu}_{ij}^{(k)}\right)^{\omega_j^{(k)}} \right], \right. \\ &\quad \left. \left[1 - \prod_{k=1}^{p} \left(1 - \underline{\nu}_{ij}^{(k)}\right)^{\omega_j^{(k)}}, 1 - \prod_{k=1}^{p} \left(1 - \overline{\nu}_{ij}^{(k)}\right)^{\omega_j^{(k)}} \right] \right) \end{aligned} \quad (32)$$

if $r_{ij}^{(k)}$ is an interval-valued intuitionistic fuzzy number. And denote the final evaluation information by $R = (r_{ij})_{m\times n}$.

Table 1: Evaluation information at t_1 time episode.

	c_1	c_2	c_3	c_4
x_1	([0.5, 0.7], [0.2, 0.3])	0.7	(0.8, 0.1)	(0.4, 0.4)
x_2	([0.6, 0.8], [0.1, 0.2])	0.6	(0.7, 0.0)	(0.5, 0.2)
x_3	([0.3, 0.5], [0.1, 0.2])	0.6	(0.6, 0.4)	(0.4, 0.5)
x_4	([0.5, 0.6], [0.1, 0.2])	0.5	(0.3, 0.5)	(0.7, 0.1)
x_5	([0.4, 0.5], [0.0, 0.1])	0.8	(0.5, 0.2)	(0.8, 0.1)

Table 2: Evaluation information at t_2 time episode.

	c_1	c_2	c_3	c_4
x_1	([0.6, 0.7], [0.1, 0.2])	0.8	(0.6, 0.0)	(0.5, 0.4)
x_2	([0.6, 0.7], [0.2, 0.3])	0.5	(0.8, 0.1)	(0.5, 0.3)
x_3	([0.4, 0.5], [0.2, 0.3])	0.6	(0.5, 0.4)	(0.4, 0.6)
x_4	([0.5, 0.5], [0.0, 0.2])	0.6	(0.4, 0.6)	(0.8, 0.1)
x_5	([0.4, 0.6], [0.2, 0.3])	0.7	(0.6, 0.4)	(0.7, 0.2)

Step 3. Transform r_{ij} for $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$ obtained from (28)–(32) into interval-valued intuitionistic fuzzy matrix $\widetilde{R} = (\widetilde{r}_{ij})_{m\times n}$, where $\widetilde{r}_{ij}$ is the interval-valued information fuzzy number of r_{ij} as (8)–(10) depicted.

Step 4. Calculate the score matrix of $\widetilde{R}$ by (23) and it is denoted by $S(\widetilde{R})$.

Step 5. Calculate weights of criteria for each alternative by (27), with respect to the final evaluation information matrix $\widetilde{R}$ and it is denoted by $\Lambda = (\lambda_{ij})_{m\times n}$.

Step 6. Calculate PIS and NIS of $\widetilde{R}$ according to (4) and it is denoted by

$$\mathrm{PIS} = \left(r_1^+, r_2^+, \ldots, r_n^+\right), \qquad \mathrm{NIS} = \left(r_1^-, r_2^-, \ldots, r_n^-\right), \tag{33}$$

where $r_j^+ = ([a_j^+, b_j^+], [c_j^+, d_j^+])$ and $r_j^- = ([a_j^-, b_j^-], [c_j^-, d_j^-])$ for $j = 1, 2, \ldots, n$.

Step 7. Calculate weighted evaluation information $\widetilde{R}_\Lambda = (\widetilde{r}^*_{ij})_{m\times n}$, where

$$\begin{aligned}\widetilde{r}^*_{ij} &= \lambda_{ij}\widetilde{r}_{ij}\\ &= \lambda_{ij}\left(\left[\underline{\mu}_{ij}, \overline{\mu}_{ij}\right], \left[\underline{\nu}_{ij}, \overline{\nu}_{ij}\right]\right)\\ &= \left(\left[1-\left(1-\underline{\mu}_{ij}\right)^{\lambda_{ij}}, 1-\left(1-\overline{\mu}_{ij}\right)^{\lambda_{ij}}\right], \left[\underline{\nu}_{ij}^{\lambda_{ij}}, \overline{\nu}_{ij}^{\lambda_{ij}}\right]\right)\\ &= \left(\left[\underline{\mu}^*_{ij}, \overline{\mu}^*_{ij}\right], \left[\underline{\nu}^*_{ij}, \overline{\nu}^*_{ij}\right]\right).\end{aligned} \tag{34}$$

Step 8. Calculate the distances D_i^+ and D_i^- for $i = 1, 2, \ldots, m$ by

$$D_i^+ = \frac{C_i\left(x_i, \mathrm{PIS}\right)}{\sqrt{T_i\left(x_i\right) T_i\left(\mathrm{PIS}\right)}},$$

Table 3: Evaluation information at t_3 time episode.

	c_1	c_2	c_3	c_4
x_1	([0.5, 0.7], [0.2, 0.3])	0.7	(0.7, 0.1)	(0.4, 0.4)
x_2	([0.6, 0.7], [0.1, 0.2])	0.6	(0.9, 0.1)	(0.5, 0.4)
x_3	([0.5, 0.6], [0.2, 0.3])	0.7	(0.4, 0.5)	(0.4, 0.5)
x_4	([0.5, 0.6], [0.1, 0.2])	0.8	(0.5, 0.4)	(0.6, 0.2)
x_5	([0.5, 0.6], [0.1, 0.3])	0.6	(0.5, 0.3)	(0.8, 0.1)

Table 4: Evaluation information at t_4 time episode.

	c_1	c_2	c_3	c_4
x_1	([0.7, 0.7], [0.2, 0.3])	0.6	(0.8, 0.1)	(0.5, 0.4)
x_2	([0.6, 0.8], [0.1, 0.2])	0.7	(0.8, 0.0)	(0.5, 0.3)
x_3	([0.4, 0.6], [0.1, 0.3])	0.8	(0.6, 0.3)	(0.5, 0.4)
x_4	([0.4, 0.7], [0.1, 0.3])	0.5	(0.6, 0.2)	(0.7, 0.1)
x_5	([0.4, 0.5], [0.1, 0.2])	0.7	(0.6, 0.3)	(0.7, 0.2)

Table 5: Evaluation information at t_5 time episode.

	c_1	c_2	c_3	c_4
x_1	([0.6, 0.7], [0.1, 0.3])	0.8	(0.7, 0.0)	(0.6, 0.3)
x_2	([0.6, 0.7], [0.2, 0.2])	0.9	(0.7, 0.1)	(0.6, 0.2)
x_3	([0.5, 0.5], [0.4, 0.5])	0.7	(0.6, 0.4)	(0.5, 0.5)
x_4	([0.6, 0.8], [0.1, 0.2])	0.7	(0.5, 0.3)	(0.7, 0.2)
x_5	([0.6, 0.7], [0.1, 0.2])	0.6	(0.5, 0.3)	(0.8, 0.1)

Table 6: Weights of criteria at each time episode.

$\omega_j^{(k)}$	t_1	t_2	t_3	t_4	c_5
c_1	0.2026	0.1834	0.1967	0.2017	0.2156
c_2	0.1905	0.1905	0.2024	0.1964	0.2202
c_3	0.1847	0.1847	0.2166	0.2166	0.1974
c_4	0.1806	0.2222	0.2222	0.1944	0.1806

$$D_i^- = \frac{C_i\left(x_i, \mathrm{NIS}\right)}{\sqrt{T_i\left(x_i\right) T_i\left(\mathrm{NIS}\right)}}, \tag{35}$$

where

$$\begin{aligned}C_i\left(x_i, \mathrm{PIS}\right) = \sum_{j=1}^{n} \Big\{ &\underline{\mu}^*_{ij} a_j^+ + \overline{\mu}^*_{ij} b_j^+ + \underline{\nu}^*_{ij} c_j^+ + \overline{\nu}^*_{ij} d_j^+\\ &+ \left(1-\underline{\mu}^*_{ij}-\underline{\nu}^*_{ij}\right)\left(1-a_j^+-c_j^+\right)\\ &+ \left(1-\overline{\mu}^*_{ij}-\overline{\nu}^*_{ij}\right)\left(1-b_j^+-d_j^+\right)\Big\},\\ C_i\left(x_i, \mathrm{NIS}\right) = \sum_{j=1}^{n} \Big\{ &\underline{\mu}^*_{ij} a_j^- + \overline{\mu}^*_{ij} b_j^- + \underline{\nu}^*_{ij} c_j^- + \overline{\nu}^*_{ij} d_j^-\\ &+ \left(1-\underline{\mu}^*_{ij}-\underline{\nu}^*_{ij}\right)\left(1-a_j^--c_j^-\right)\\ &+ \left(1-\overline{\mu}^*_{ij}-\overline{\nu}^*_{ij}\right)\left(1-b_j^--d_j^-\right)\Big\}\end{aligned} \tag{36}$$

Table 7: Final evaluation information R.

	c_1	c_2	c_3	c_4
x_1	([0.5918, 0.7047], [0.1479, 0.2720])	0.7214	(0.7014, 0.0000)	(0.4514, 0.4150)
x_2	([0.5684, 0.7143], [0.1516, 0.2462])	0.6666	(0.7688, 0.0000)	(0.5594, 0.2337)
x_3	([0.4195, 0.5368], [0.1792, 0.3124])	0.6815	(0.5752, 0.3656)	(0.4772, 0.4588)
x_4	([0.5070, 0.6594], [0.0000, 0.2141])	0.6238	(0.4967, 0.3435)	(0.6967, 0.1396)
x_5	([0.4921, 0.6164], [0.0000, 0.1809])	0.6768	(0.5380, 0.2977)	(0.7203, 0.1606)

Table 8: Interval-valued intuitionistic fuzzy evaluation information $\widetilde{R}$.

	c_1	c_2
x_1	([0.5918, 0.7047], [0.1479, 0.2720])	([0.7214, 0.7214], [0.2786, 0.2786])
x_2	([0.5684, 0.7143], [0.1516, 0.2462])	([0.6666, 0.6666], [0.3334, 0.3334])
x_3	([0.4195, 0.5368], [0.1792, 0.3124])	([0.6815, 0.6815], [0.3185, 0.3185])
x_4	([0.5070, 0.6594], [0.0000, 0.2141])	([0.6238, 0.6238], [0.3762, 0.3762])
x_5	([0.4921, 0.6164], [0.0000, 0.1809])	([0.6768, 0.6768], [0.3232, 0.3232])
	c_3	c_4
x_1	([0.5918, 0.7047], [0.1479, 0.2720])	([0.7214, 0.7214], [0.2786, 0.2786])
x_2	([0.5684, 0.7143], [0.1516, 0.2462])	([0.6666, 0.6666], [0.3334, 0.3334])
x_3	([0.4195, 0.5368], [0.1792, 0.3124])	([0.6815, 0.6815], [0.3185, 0.3185])
x_4	([0.5070, 0.6594], [0.0000, 0.2141])	([0.6238, 0.6238], [0.3762, 0.3762])
x_5	([0.4921, 0.6164], [0.0000, 0.1809])	([0.6768, 0.6768], [0.3232, 0.3232])

Table 9: $S(\widetilde{R})$: score of $\widetilde{R}$.

$L(\widetilde{r}_{ij})$	c_1	c_2	c_3	c_4
x_1	0.5779	0.6438	0.7014	0.2237
x_2	0.5735	0.5554	0.7688	0.4564
x_3	0.3538	0.5801	0.4199	0.2373
x_4	0.5467	0.4823	0.3238	0.6544
x_5	0.5196	0.5723	0.4005	0.6754

Table 10: $S(\widetilde{R})$: score of $\widetilde{R}$.

λ_{ij}	c_1	c_2	c_3	c_4
x_1	0.1092	0.0880	0.0741	0.7287
x_2	0.2380	0.2538	0.1324	0.3758
x_3	0.2323	0.0864	0.1649	0.5164
x_4	0.1714	0.2203	0.4887	0.1196
x_5	0.2439	0.2011	0.4106	0.1444

and $T_i(x_i) = C_i(x_i, x_i)$, $T_i(\text{PIS}) = C_i(\text{PIS}, \text{PIS})$, and $T_i(\text{NIS}) = C_i(\text{NIS}, \text{NIS})$.

Step 9. Utilize (6) to calculate the trade-off performance of each alternative x_i for $i = 1, 2, \ldots, m$.

Step 10. Rank D_i and select the best alternative.

4. Illustrative Example

In this section, we use a synthetic dynamic database with hybrid evaluation information to illustrate the proposed multicriteria decision making method.

Given that a university wants to select the most desirable candidate from candidates $x_1, x_2, \ldots, x_5$ to attend a special meeting, one of the problems facing the president of the university is to determine how to make a reasonable decision making analysis. The candidates are evaluated by experts from four aspects: level of their scientific research (c_1), social resource (c_2), teaching performance (c_3), and level of subhealth (c_4), where c_1, c_2, and c_3 are beneficial criteria and c_4 is the cost criterion, where (1) the evaluation information of c_1 is expressed as interval-valued intuitionistic fuzzy numbers; (2) the evaluation information of c_2 is expressed as real numbers; and (3) the evaluation information of c_3 and c_4 is expressed as intuitionistic fuzzy numbers. All of these evaluation information for each candidate are gathered up separately from five periods and shown in Tables 1, 2, 3, 4, and 5.

In what follows we make a detailed description of the dynamic multicriteria decision making with above database.

Step 1. Compute $\omega_j^{(k)}$ for $k = 1, 2, \ldots, 5$ and $j = 1, 2, 3, 4$ by the aggregation operators IFWA_η and IIFWA_η. Here we treat all alternatives without distinction, and the corresponding computing results can be found in Table 6.

Step 2. Aggregate $r_{ij}^{(k)}$ into r_{ij}, and Table 7 illustrates the corresponding aggregation results.

Step 3. Change R into interval-valued intuitionistic fuzzy matrix and show it in Table 8.

Table 11: Weighted evaluation information of $\widetilde{R}$.

	c_1	c_2
x_1	([0.0932, 0.1247], [0.8116, 0.8675])	([0.1064, 0.1064], [0.8936, 0.8936])
x_2	([0.1813, 0.2578], [0.6383, 0.7164])	([0.2433, 0.2433], [0.7567, 0.7567])
x_3	([0.1187, 0.1637], [0.6707, 0.7632])	([0.0941, 0.0941], [0.9059, 0.9059])
x_4	([0.1142, 0.1686], [0.0000, 0.7678])	([0.1938, 0.1938], [0.8062, 0.8062])
x_5	([0.1523, 0.2084], [0.0000, 0.6590])	([0.2032, 0.2032], [0.7968, 0.7968])
	c_3	c_4
x_1	([0.0857, 0.0857], [0.0000, 0.0000])	([0.3544, 0.3544], [0.5268, 0.5268])
x_2	([0.1763, 0.1763], [0.0000, 0.0000])	([0.2651, 0.2651], [0.5791, 0.5791])
x_3	([0.1317, 0.1317], [0.8471, 0.8471])	([0.2846, 0.2846], [0.6687, 0.6687])
x_4	([0.2850, 0.2850], [0.5932, 0.5932])	([0.1330, 0.1330], [0.7902, 0.7902])
x_5	([0.2717, 0.2717], [0.6080, 0.6080])	([0.1680, 0.1680], [0.7679, 0.7679])

Step 4. Calculate the score matrix of $\widetilde{R}$ which is illustrated in Table 9.

Step 5. Calculate $\Lambda = (\lambda_{ij})_{m\times n}$ by (27) and Table 10 shows the corresponding results.

Step 6. Calculate PIS and NIS of $\widetilde{R}$:

$$\begin{aligned} \text{PIS} &= (r_1^+, r_2^+, r_3^+, r_4^+) \\ &= (([1,1],[0,0]),([1,1],[0,0]), \\ &\qquad ([1,1],[0,0]),([0,0],[1,1])), \\ \text{NIS} &= (r_1^-, r_2^-, r_3^-, r_4^-) \\ &= (([0,0],[1,1]),([0,0],[1,1]), \\ &\qquad ([0,0],[1,1]),([1,1],[0,0])). \end{aligned} \tag{37}$$

Step 7. Calculate weighted evaluation information $\widetilde{R}_\Lambda$ which is illustrated in Table 11.

Step 8. Calculate the distances from each alternative to PIS as well as NIS:

$$\begin{aligned} D^+ &= (D_1^+, D_2^+, D_3^+, D_4^+, D_5^+) \\ &= (0.2973, 0.4848, 0.3194, 0.4892, 0.5047), \\ D^- &= (D_1^-, D_2^-, D_3^-, D_4^-, D_5^-) \\ &= (0.7497, 0.6762, 0.8494, 0.6646, 0.6746). \end{aligned} \tag{38}$$

Step 9. Calculate D_i for each alternative:

$$\begin{aligned} &(D_1, D_2, D_3, D_2, D_4, D_5) \\ &\quad = (0.7160, 0.5824, 0.7267, 0.5761, 0.5721). \end{aligned} \tag{39}$$

Step 10. Rank the performance order of alternative as $D_5 \prec D_4 \prec D_2 \prec D_1 \prec D_3$ and we have that the most desirable alternative is x_3.

5. Conclusions

In this paper, a multicriteria decision making algorithm with respect to hybrid evaluation information has been proposed. This decision making approach aims at selecting the most desirable patterns(s) from a group of evaluation information, where the evaluation information are gathered from different time episodes and different criteria usually have different representations, such as real number, intuitionistic fuzzy number, interval-valued intuitionistic fuzzy number, and so forth. The experimental results show that the proposed decision making approach is feasible and effective. Since, for decision making problem, the concrete data representation of evaluation information, to some extent, can directly determine the decision approach selection, our proposed approach can enrich the study in the area of diversifying patterns' data representation. However, our proposed algorithm is incapable of handling the decision making problems with missing evaluation information, especially for the case that patterns in different time episode are depicted by different amount of criteria. Bearing these facts in mind, it deserves further investigation for the dynamic multicriteria decision making problem with hybrid evaluation information.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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