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Review Article

Review of Input Congestion Estimating Methods in DEA

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Congestion is an economic phenomenon in the production process where excessive amounts of the input cause a reduction of the output. The motivation of this paper is to investigate and compare the methods provided for estimating congestion in the DEA literature.

1. Introduction

Data envelopment analysis (DEA) is one of the very popular operations research methods which is designed for evaluating the efficiency of peer decision making units (DMUs) when multiple inputs and outputs are present. CCR is the first model of this tool which was proposed by Charnes et al. [1]. Then, a variable returns to scale version of the CCR model (BCC) was introduced by Banker et al. [2]. To see the other basic DEA models, refer to [3–6].

DEA can be used not only for estimating the performance of units but also for solving other problems of decision making, management, and economics. One of these problems is to estimate the input congestion of DMUs. If some inputs are used in amounts, they cause output reduction, and then input congestion exists. Such an occurrence may be found in many economic activities. For example, a petroleum industry has a difficulty in transporting oil and natural gas from its production facilities to consumption areas because a pipeline network among them has a capacity limit on transportation. Another example may be found in a transmission line of electricity that connects between generators and end users. The transmission line has a line limit that determines not only the amount of electricity but also the electricity price in a power trading market. Also, an excess of miners bumping into each other in an underground mine is an example, where a reduction in the number of miners can result in an increase in the amount mined. In what follows, the exact definition of congestion in general case is provided.

Definition 1 (input congestion). Input congestion occurs whenever the increasing of one or more inputs decreases some outputs without improving other inputs or outputs. Conversely, congestion occurs when decreasing of some of the inputs increases some outputs without worsening other inputs or outputs.

Definition 2 (weak congestion). A DMU evidences weak congestion if some, rather than all, outputs increase as some inputs decrease, without changing others.

Definition 3 (strong congestion). A DMU evidences strong congestion if all outputs increase as some inputs decrease, without changing others.

Notice that strong congestion implies weak congestion but not vice versa. In addition, in a single input and a single output case, there is no distinction between strong and weak congestions. Weak but not strong congestion occurs only for the case with more than one input or one output. The necessary condition for the presence of congestion is inefficiency [7]. So the efficiency is achieved if and only if it is not possible to improve some inputs or outputs without worsening other inputs or outputs. To clarify the relationship between input congestion and technical inefficiency, we provide the definition of technical inefficiency which is as follows.

Definition 4 (technical inefficiency). Technical inefficiency is present when it is possible to improve some inputs or outputs without worsening other inputs or outputs.

So far several methods for measuring the effect of congestion have been developed under the framework of DEA. The congestion estimating approaches were originated by Färe and Svensson [8]. Färe and Grosskopf [9] and Färe et al. [10] extended and developed a DEA model to calculate the congestion amounts. Hereafter, this procedure is referred to as Färe's approach. This model is a radial approach, in that the congestion effect is measured as the difference between technologies under weak and strong disposability inputs. Based on that model, Byrnes et al. [11, 12] used examples of coal mines to illustrate the decomposition of production efficiency, where the congestion effect is a component. To see the other references of this approach, the reader is referred to [13–16]. A big advantage of this approach is that it is possible to decompose its measure of overall technical efficiency in a straightforward way into pure technical efficiency, scale efficiency, and congestion efficiency. It is the assumption that congestion is the difference between weak disposability and strong disposability. This approach fails to detect congestion in some cases, especially when there is only one input. Also, it does not measure the amount of congestion.

Cooper et al. [17] published an alternative approach for determining input congestion that has some advantages to previous method. This method uses the slacks to determine the congestion amount. In this method, the congestion effect is measured as the difference between the observed amounts and the expected amounts. It was subsequently developed by Brockett et al. [18] and Cooper et al. [19]. For simplicity, this procedure is referred to as Cooper's approach. Unlike the radial approach which requires all inputs to reduce in proportion to eliminate congestion, this method allows each input to reduce in different proportions. This approach has been applied to identify congesting inputs in Chinese industries by Brockett et al. [18] and Cooper et al. [20]. Later on, Cooper et al. [7] introduced a one-model version of Cooper's method. To see the other references of this approach, refer to [21–25].

The previous approaches express the congestion effect in terms of excessive inputs. According to the definition, congestion occurs when increases in some inputs results in decreases in some outputs. Hence, congestion can also be measured as shortfalls in outputs. Tone and Sahoo [26] and Wei and Yan [27] developed a method from the output point of view. This model, hereafter, is referred to as Tone's approach. Sueyoshi and Sekitani [28] proposed a modified approach which is able to measure the degree of congestion under the occurrence of multiple solutions. This method is also a radial approach, so a product form of efficiency decomposition is possible. Flegg and Allen [29] applied these three approaches to examine whether the rapid growth in the number of students in British universities has led to congestion. One of the shortcomings of Tone's method is its inability to identify the excessive amount of each input. Also, in the presence of alternative optimal solutions, this approach is incapable of detecting congestion (strong and weak). Furthermore, in this approach all inputs and outputs of DMUs have been assumed positive, while, in real applications, data is often nonnegative. Khoveyni et al. [30] removed this drawback of Tone's approach.

In DEA, the inputs proportions changes are often based on inputs reduction. Apparently this subject is a reasonable justification economically because of reducing the costs of the reduced inputs. But, in some cases, inputs reduction such as labor may be faced with social tensions, as it happened in China. Jahanshahloo and Khodabakhshi [31, 32] proposed a method to determine the sources and amounts of input congestion which is based on the relaxed combination of inputs. The framework of this method is similar to that of Cooper et al. [7]. Later on, Khodabakhshi [33] provided a one-model approach of input congestion based on input relaxation model. This approach reduces solving three problems with the two-model approach introduced by Jahanshahloo and Khodabakhshi [31], which is certainly important from the computational point of view. To see more references about this approach, the readers are referred to [34, 35, 37]. Hereafter, this procedure is referred to as Khodabakhshi's approach. Using this approach in textile industry of China, show that flexibility in using inputs causes output augmentation because of new inputs combination. So it has more benefits than the probable increasing cost of some inputs. Also, Khodabakhshi [36] and Asgharian et al. [38] proposed some methods for determining the input congestion in the stochastic conditions (See also [37]). Jahanshahloo et al. [39] and Khodabakhshi et al. [40] proposed some methods for computing the congestion in DEA models with productions trade-offs and weight restrictions.

Noura et al. [41] proposed another algorithm which is equivalent to Cooper's approach. Their method requires considerably less computation than Cooper's approach. This method is called Noura's approach.

The above approaches only consider the desirable output. But, in the production process, undesirable outputs like smoke pollution or waste are usually produced with desirable outputs. The desirable outputs are good outputs for consumers and undesirable outputs are bad outputs for consumers, such as pollution. Therefore, congestion with undesirable outputs is different from that in the traditional scenario. Wu et al. [42] combined the methods of Seiford and Zhu [43] and Wei and Yan [27] to propose a method for determining input congestion in the presence of undesirable outputs, which is named Wu's approach.

To see more references about input congestion measurement subject, the readers can refer to [44–49].

The rest of the paper is structured as follows. In Sections 2–7 the approaches of Färe, Cooper, Tone, Khodabakhshi, Noura, and Wu are briefly introduced. The final section concludes the paper.

2. Färe's Approach

Throughout this paper, we deal with n decision making units DMU_j ($j = 1,\ldots,n$), each having m inputs x_{ij} ($i = 1,\ldots,m$) for producing s outputs y_{rj} ($r = 1,\ldots,s$). The underevaluation of DMU is denoted by DMU_o. Also, suppose that all inputs and outputs are nonnegative deterministic numbers. The efficiency score of DMU_o under the strong disposal technology is determined as follows:

$$\begin{aligned} \theta^* = \min \quad & \theta \\ \text{s.t.} \quad & \sum_{j=1}^{n} x_{ij}\lambda_j \le x_{io}\theta, \quad i = 1,\ldots,m \\ & \sum_{j=1}^{n} y_{rj}\lambda_j \ge y_{ro}, \quad r = 1,\ldots,s \\ & \sum_{j=1}^{n} \lambda_j = 1 \\ & \lambda_j \ge 0, \quad j = 1,\ldots,n. \end{aligned} \tag{1}$$

Also, the efficiency score of DMU_o under the weak disposal technology is determined as follows:

$$\begin{aligned} \beta^* = \min \quad & \beta \\ \text{s.t.} \quad & \sum_{j=1}^{n} x_{ij}\lambda_j = x_{io}\beta, \quad i = 1,\ldots,m \\ & \sum_{j=1}^{n} y_{rj}\lambda_j \ge y_{ro}, \quad r = 1,\ldots,s \\ & \sum_{j=1}^{n} \lambda_j = 1 \\ & \lambda_j \ge 0, \quad j = 1,\ldots,n. \end{aligned} \tag{2}$$

This measure identifies the presence of congestion provided $\theta^*/\beta^* < 1$ and no congestion is present if $\theta^*/\beta^* = 1$. When there is only one input, strong disposability and weak disposability in input are the same; therefore, Färe's approach-type congestion does not exist.

3. Cooper's Approach

The BCC model for measuring the efficiency has two variations, input and output orientation. Since the input-oriented model may produce erroneous results in measuring congestion, this method will concentrate on the output-orientation model [19]. Using the output-oriented BCC model, the efficiency of DMU_o is evaluated as follows:

$$\begin{aligned} \max \quad & \phi_o + \varepsilon\left(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+\right) \\ \text{s.t.} \quad & x_{io} = \sum_{j=1}^{n} x_{ij}\lambda_j + s_i^-, \quad i = 1,\ldots,m \\ & 0 = \sum_{j=1}^{n} y_{rj}\lambda_j - y_{ro}\phi_o - s_r^+, \quad r = 1,\ldots,s \\ & 1 = \sum_{j=1}^{n} \lambda_j \\ & \lambda_j, s_i^-, s_r^+ \ge 0. \end{aligned} \tag{3}$$

Consider, in model (3), that ε is a non-Archimedean element; namely, it is not a real number and defined to be smaller than any positive real number. To avoid assigning a value to ε, this model can be solved via a two-stage procedure [32]. In this procedure, first, ϕ_o is maximized while ignoring the slacks in the objective. Then, ϕ_o is replaced with ϕ_o^* (optimal value of first stage), and the sum of the slacks is maximized. Using the optimal solution of the above model, we have the following definitions.

Definition 5 (efficiency). DMU_o is efficient if $\phi_o^* = 1$ and $s_i^{-*} = s_r^{+*} = 0$, for all i, r.

Inefficiency is a necessary condition for the presence of congestion. Therefore, if DMU_o is found to be inefficient, the optimal solution of model (3), $(\phi_o^*, \lambda_j^*, s_i^{-*}, s_r^{+*})$, is used to determine the amount of technical inefficiency in inputs as follows:

$$\begin{aligned} \max \quad & \sum_{i=1}^{m} \delta_i^- \\ \text{s.t.} \quad & x_{io} - s_i^{-*} = \sum_{j=1}^{n} x_{ij}\lambda_j - \delta_i^-, \quad i = 1,\ldots,m \\ & \phi_o^* y_{ro} + s_r^{+*} = \sum_{j=1}^{n} y_{rj}\lambda_j, \quad r = 1,\ldots,s \\ & 1 = \sum_{j=1}^{n} \lambda_j \\ & \delta_i^- \le s_i^{-*}, \quad i = 1,\ldots,m \\ & \delta_i^-, \lambda_j \ge 0. \end{aligned} \tag{4}$$

Finally, the optimum value of model (4) (δ_i^{+*}) is used to determine congestion amounts as follows:

$$s_i^{-c*} = s_i^{-*} - \delta_i^{-*}, \quad i = 1,\ldots,m. \tag{5}$$

Cooper et al. [7] introduced the one-model version of pervious method to deal with the congestion subject. Substituting (5) into (4), we can rewrite the latter as follows:

$$\begin{aligned} \min \quad & \sum_{i=1}^{m} s_i^{-c} \\ \text{s.t.} \quad & x_{io} - s_i^{-c} = \sum_{j=1}^{n} x_{ij}\lambda_j, \quad i = 1,\ldots,m \end{aligned}$$

$$\begin{aligned}
&\phi_o^* y_{ro} + s_r^{+*} = \sum_{j=1}^{n} y_{rj}\lambda_j, \quad r = 1,\dots,s \\
&1 = \sum_{j=1}^{n}\lambda_j \\
&\lambda_j, s_i^{-c} \geq 0, \quad i = 1,\dots,m.
\end{aligned} \tag{6}$$

Using ε, the models ((3), (6)) are combined into the following single model:

$$\begin{aligned}
\max \quad & \phi_o + \varepsilon\left(-\sum_{i=1}^{m} s_i^{-c} + \sum_{r=1}^{s} s_r^{+}\right) \\
\text{s.t.} \quad & x_{io} = \sum_{j=1}^{n} x_{ij}\lambda_j + s_i^{-c}, \quad i = 1,\dots,m \\
& 0 = \sum_{j=1}^{n} y_{rj}\lambda_j - y_{ro}\phi_o - s_r^{+}, \quad r = 1,\dots,s \\
& 1 = \sum_{j=1}^{n}\lambda_j \\
& \lambda_j, s_i^{-c}, s_r^{+} \geq 0.
\end{aligned} \tag{7}$$

Theorem 6. *Congestion is present if and only if, in an optimal solution* $\phi_o^*, \lambda^*, S^{+*}, S^{-c*}$ *of* (7), *at least one of the following two conditions is satisfied.*

(i) $\phi_o^* > 1$ *and there is at least one* $S^{-c*} > 0$ $(1 \leq i \leq m)$.

(ii) *There exists at least one* $S_r^{+*} > 0$ $(1 \leq r \leq s)$ *and at least one* $S^{-c*} > 0$ $(1 \leq i \leq m)$.

Proof. See Cooper et al. [7]. □

4. Tone's Approach

In the approach, which was presented by Tone and Sahoo [26], it is assumed that all inputs and outputs are positive. This approach is based on the following production possibility set (PPS):

$$P_{\text{Convex}} = \left\{(x, y) \mid \sum_{j=1}^{n} x_j\lambda_j = x, \sum_{j=1}^{n} y_j\lambda_j \geq y, \sum_{j=1}^{n}\lambda_j = 1, \lambda_j \geq 0\right\}. \tag{8}$$

In this approach for measuring the congestion effect, the pure technical efficiency is firstly determined using the following model:

$$\begin{aligned}
\max \quad & \mu \\
\text{s.t.} \quad & \sum_{j=1}^{n} x_j\lambda_j = x_o \\
& \sum_{j=1}^{n} y_j\lambda_j - q^{+} = y_o\mu \\
& \sum_{j=1}^{n}\lambda_j = 1 \\
& \lambda_j, q^{+} \geq 0.
\end{aligned} \tag{9}$$

Suppose that $(\overline{x}_0, \overline{y}_o)$ is the projection of DMU_o which is obtained as

$$\begin{aligned}
\overline{x}_0 &= x_0, \\
\overline{y}_o &= \mu^* y_o + q^{+*}.
\end{aligned} \tag{10}$$

In this case, $(\overline{x}_0, \overline{y}_o)$ is on the strongly efficient frontier of P_{Convex}. Here, the definitions of congestion (strong and weak) from Tone's approach are assumed as follows:

Definition 7 (strong congestion). A DMU_o is strongly congested if there exists an activity $(x, y) \in P_{\text{Convex}}$ such that $x = \alpha\tilde{x}_o$ (with $0 < \alpha < 1$) and $\tilde{y}_o \geq \beta y_o$ (with $\beta > 1$).

Definition 8 (weak congestion). A DMU is (weakly) congested if it is strongly efficient and there exists an activity in P_{Convex} that uses less resources in one or more inputs for making more products in one or more outputs.

First, we consider DMU_o which is strongly P_{Convex} efficient. Thus, in each optimal solution of model (9), $\mu^* = 1$ and $q^{+*} = 0$. Then the following mathematical program is solved for a specific DMU_o. Consider the following:

$$\begin{aligned}
\max \quad & \frac{1}{s}\sum_{r=1}^{s}\frac{t_r^{+}}{y_{ro}} + \varepsilon\left(\frac{1}{m}\sum_{i=1}^{m}\frac{t_i^{-}}{x_{io}}\right) \\
\text{s.t.} \quad & \sum_{j=1}^{n} x_j\lambda_j + t_i^{-} = x_o \\
& \sum_{j=1}^{n} y_j\lambda_j - t^{+} = y_o \\
& \sum_{j=1}^{n}\lambda_j = 1 \\
& \lambda_j, t^{+}, t^{-} \geq 0,
\end{aligned} \tag{11}$$

where ε is a non-Archimedean small positive number.

Next, let $(\lambda^*, t^{-*}, t^{+*})$ be an optimal solution of model (11). Then, we will have the following cases.

(i) If $t^{+*} = 0$, DMU_o has no congestion, because the decreasing in inputs cannot increase any outputs.

(ii) If $t^{+*} \neq 0$, t^{-*} is also not zero, since DMU_o is strongly P_{Convex} efficient. Consequently, DMU_o has (weak) congestion.

5. Khodabakhshi's Approach

Jahanshahloo and Khodabakhshi [31, 32] and Khodabakhshi [33] introduced the following model for improving output:

$$\begin{aligned} \max \quad & \phi_o + \varepsilon \left(\sum_{i=1}^{m} s_{i1}^{-} - \sum_{i=1}^{m} s_{i2}^{+} + \sum_{r=1}^{s} s_r^{+} \right) \\ \text{s.t.} \quad & x_{io} = \sum_{j=1}^{n} x_{ij}\lambda_j + s_{i1}^{-} - s_{i2}^{+}, \quad i = 1, \ldots, m \\ & 0 = \sum_{j=1}^{n} y_{rj}\lambda_j - y_{ro}\phi_o - s_r^{+}, \quad r = 1, \ldots, s \\ & 1 = \sum_{j=1}^{n} \lambda_j \\ & \lambda_j, s_{i1}^{-}, s_{i2}^{+}, s_r^{+} \geq 0. \end{aligned} \tag{12}$$

This model is a version of output-oriented BCC model which is not limited in using observed sources of evaluating DMU. While output-oriented BCC model is limited to the using of observed sources of evaluating DMU, input relaxation model by imposing a few changes on some inputs can produce outputs more than observed output or output suggested by BCC model. Using the above model will be very useful when attracting some inputs such as labor which is necessary to solve employment problem in a society. The conditions of efficiency under the above model for DMU_o being evaluated become as follows:

Definition 9 (efficiency). DMU_o is efficient under model (12) if the following two conditions are satisfied.

(i) $\phi_o^* = 1$.

(ii) The optimal amounts of all slacks are zero.

Using the optimal solution of model (12), $(\phi_o^*, \lambda_j^*, s_{i1}^{-*}, s_{i2}^{+*}, s_r^{+*})$, the following model is solved for determining technical inefficiency of inputs:

$$\begin{aligned} \max \quad & \sum_{i=1}^{m} \delta_i^{+} \\ \text{s.t.} \quad & x_{io} - s_{i1}^{-*} + s_{i2}^{+*} = \sum_{j=1}^{n} x_{ij}\lambda_j - \delta_i^{+}, \quad i = 1, \ldots, m \\ & \phi_o^* y_{ro} + s_r^{+*} = \sum_{j=1}^{n} y_{rj}\lambda_j, \quad r = 1, \ldots, s \\ & 1 = \sum_{j=1}^{n} \lambda_j \\ & \delta_i^{+} \leq s_i^{-*}, \quad i = 1, \ldots, m \\ & \lambda_j \delta_i^{+} \geq 0. \end{aligned} \tag{13}$$

Finally, the amount of congestion for ith input is defined as follows:

$$s_i^{-c} = s_{i1}^{-*} - \delta_i^{+*}, \quad i = 1, \ldots, m, \tag{14}$$

where δ_i^{+*} $(i = 1, \ldots, m)$ is obtained by solving model (13). If the amount of total slacks (total inefficiencies), s_{i1}^{-*}, and noncongesting input are the same, then we do not have any input congestion. In other words, in this case, the amount of input congestion is zero, while, if the amount of total slacks is more than noncongesting input, then the input congestion will exist for ith input. It is obvious that, if s_{i2}^{+*} is positive for ith input, there is no congestion for this input.

Khodabakhshi [33] presented the one-model version of this approach. The two models ((12), (13)) can be replaced by a single model, which provides an alternative procedure to determine input congestion. Using relation (14), model (13) can be rewritten as follows:

$$\begin{aligned} \max \quad & \sum_{i=1}^{m} - s_i^{-c} \\ \text{s.t.} \quad & x_{io} - s_i^{-c} + s_{i2}^{+*} = \sum_{j=1}^{n} x_{ij}\lambda_j, \quad i = 1, \ldots, m \\ & \phi_o^* y_{ro} + s_r^{+*} = \sum_{j=1}^{n} y_{rj}\lambda_j, \quad r = 1, \ldots, s \\ & 1 = \sum_{j=1}^{n} \lambda_j \\ & \lambda_j, s_i^{-c} \geq 0. \end{aligned} \tag{15}$$

If we consider the following model,

$$\begin{aligned} \max \quad & \phi_o + \varepsilon \left(\sum_{i=1}^{m} - s_i^{-c} - \sum_{i=1}^{m} s_{i2}^{+} + \sum_{r=1}^{s} s_r^{+} \right) \\ \text{s.t.} \quad & x_{io} = \sum_{j=1}^{n} x_{ij}\lambda_j + s_i^{-c} - s_{i2}^{+}, \quad i = 1, \ldots, m \\ & 0 = \sum_{j=1}^{n} y_{rj}\lambda_j - \phi_o y_{ro} - s_r^{+}, \quad r = 1, \ldots, s \\ & 1 = \sum_{j=1}^{n} \lambda_j \\ & \lambda_j, s_{i2}^{+}, s_i^{-c}, s_r^{+} \geq 0, \end{aligned} \tag{16}$$

it is obvious that, if $(\phi_o^*, \lambda^*, s_1^{-c*}, s_2^{+*}, s^{+*})$ is an optimal solution of (16), $(\phi_o^*, s_2^{+*}, s^{+*})$ are part of an optimal solution of (12) and (λ^*, s^{+*}) is an optimal solution of (15). In other words, we can regard model (15) as part of a two-stage procedure for solving model (16). Using one-model approach, we reduced the solving of three problems with two-model approach to the solving of two problems with one-model approach. This is certainly important from computational point of view.

Theorem 10. *Congestion is present if and only if optimal solution* $(\phi_o^*, \lambda^*, s_1^{-c*}, s_2^{+*}, s^{+*})$ *is an optimal solution of* (16) *and at least one of the following conditions is satisfied*

(i) $\phi_o^* > 1$ *and there exists at least one* i $(1 \le i \le m)$ *such that* $s_i^{-c*} > 0$.

(ii) *There exists at least one* r $(1 \le r \le s)$ *for which* $s_r^{+*} > 0$ *and also one* i $(1 \le i \le m)$ *such that* $s_i^{-c*} > 0$.

Proof. See Khodabakhshi [33]. □

Theorem 11. *Congestion is present if and only if, for an optimal solution* $(\phi_o^*, \lambda^*, s_1^{-c*}, s_2^{+*}, s^{+*})$ *of* (16), *there is at least one* i $(1 \le i \le m)$ *such that* $s_i^{-c*} > 0$.

Proof. See Khodabakhshi [33]. □

6. Noura's Approach

In this section, the approach of Noura et al. [41] is briefly reviewed. In this method, first, model (3) is solved and the optimal solution $\phi_o^*, \lambda^*, S^{+*}, S^{-*}$ is obtained. Then, the set E is defined as follows:

$$E = \left\{ j \mid \phi_j^* = 1 \right\}. \quad (17)$$

Among the DMUs in set E, there exists at least one, say DMU_l, that has the highest consumption in its first input component compared with the first input component of the remaining DMUs of set E. x_{1l} is denoted by x_1^*. Then, among the DMUs in E, a DMU is found, say DMU_t, that has the highest consumption in its second input component compared to the remaining DMUs in E. x_{2t} is denoted by x_2^*. In a similar manner, $x_1^*, x_2^*, \ldots, x_m^*$ can be identified.

Definition 12. Congestion is present if and only if, in an optimal solution $\phi_o^*, \lambda^*, S^{+*}, S^{-*}$ of (3) for DMU_o, at least one of the following two conditions is satisfied

(i) $\phi_o^* > 1$ and there is at least one $x_{io} > x_i^*$ $(i = 1, \ldots, m)$.

(ii) There exists at least one $S_r^{+*} > 0$ $(r = 1, \ldots, s)$ and at least one $x_{io} > x_i^*$ $(i = 1, \ldots, m)$.

They denote the amount of congestion in the ith input of DMU_o by $s_i^{c'}$ where $x_{io} > x_i^*$ and define it as

$$s_i^{c'} = x_{io} - x_i^*. \quad (18)$$

Congestion is considered not present when $x_{io} \le x_i^*$ and $s_i^{c'} = 0$. The sum of all $s_i^{c'}$ is the amount of congestion in DMU_o.

Noura et al. [41], to establish general validity of their method, prove three theorems, which show that Definition 12 is equivalent to Cooper et al.'s Theorem 6, where $x_{io} \ge x_i^*$. Also, they prove another theorem, which shows that there exists no congestion in DMU_o when $x_{io} \le x_i^*$. The computational effort of this method for calculating congestion is less than Cooper's approach.

7. Wu's Approach

In this section, the input congestion measurement subject in presence of undesirable outputs is considered using the approach of Wu et al. [42]. Assume that x, y, u represent the inputs, desirable outputs, and undesirable outputs, respectively. Evidence of congestion in this method occurs whenever reducing some inputs x can increase desirable outputs y and decrease undesirable outputs u simultaneously. Before measuring this kind of congestion, we should develop the approach solving the problem of undesirable outputs. So far there have been several approaches for addressing the undesirable output problem. Wu et al. [42] choose the approach of Seiford and Zhu [43] to deal with undesirable outputs. The model is shown as follows:

$$\begin{aligned}
\max \quad & \delta \\
\text{s.t.} \quad & \sum_{j=1}^{n} x_{ij}\lambda_j \le x_{i0}, \quad i = 1, \ldots, m \\
& \sum_{j=1}^{n} y_{rj}\lambda_j \ge y_{r0}\delta, \quad r = 1, \ldots, s \\
& \sum_{j=1}^{n} o_{tj}\lambda_j \ge o_{t0}\delta, \quad t = 1, \ldots, k \\
& o_{tj} = -u_{tj} + \alpha_t, \quad j = 1, \ldots, n \\
& \sum_{j=1}^{n} \lambda_j = 1 \\
& \lambda_j \ge 0,
\end{aligned} \quad (19)$$

where α_t is a big enough positive number that can make every o_{tj} positive. The fourth constraint is used to transform the undesirable output to a new variable. The third constraint is used to constrain the new variable in the possible production set. Wu et al. [42] proposed the following model for determining the input congestion of DMU_0 in the presence of undesirable outputs:

$$\begin{aligned}
\max \quad & z \\
\text{s.t.} \quad & \sum_{j=1}^{n} x_{ij}\lambda_j = x_{i0}, \quad i = 1, \ldots, m
\end{aligned}$$

$$\begin{aligned}
&\sum_{j=1}^{n} y_{rj}\lambda_j \geq y_{r0}\delta, \quad r = 1,\ldots,s \\
&\sum_{j=1}^{n} o_{tj}\lambda_j \geq o_{t0}\delta, \quad t = 1,\ldots,k \\
&o_{tj} = -u_{tj} + \alpha_t, \quad j = 1,\ldots,n \\
&\sum_{j=1}^{n} \lambda_j = 1 \\
&\lambda_j \geq 0.
\end{aligned} \tag{20}$$

The production possibility set corresponding to these models is as follows:

$$T = \left\{ (x, y, u) \mid \sum_{j=1}^{n} x_j\lambda_j = x, \sum_{j=1}^{n} y_j\lambda_j \geq y, \sum_{j=1}^{n} u_j\lambda_j \leq u, \sum_{j=1}^{n} \lambda_j = 1, \lambda_j \geq 0 \right\}. \tag{21}$$

Definition 13. If the optimal value of model (20) satisfies $z^* = 1$, then we call DMU_0 weakly efficient.

Definition 14. Assume that DMU_0 is defined as weakly efficient by model (20). If it has $(\hat{x}, \hat{y}, \hat{u}) \in T$, $\hat{x} \leq x_0$ and $\hat{x} \neq x_0$, $\hat{y} > y_0$, $\hat{u} > u_0$, then the DMU evidences congestion.

Definition 15. Assume that DMU_0 is defined as weakly efficient by model (20). If it has $(\hat{x}, \hat{y}, \hat{u}) \in T$, $\hat{x} \leq x_0$ and $\hat{x} \neq x_0$, $\hat{y} \geq y_0$, $\hat{u} \geq u_0$, then the DMU evidences congestion.

Theorem 16. *A weakly efficient DMU_0 of model (20) evidences congestion if and only if it is not weakly efficient in model (19).*

Proof. See Wu et al. [42]. □

8. Conclusion

There are several approaches for dealing with input congestion measurement in DEA. In this review, six important methods are briefly considered and compared. Whilst each technique is useful in a specific area, none of the methodologies can be prescribed as the complete solution to the question of input congestion. However, there is need to design a complete method which can cover all of the following properties:

(1) to determine both sources and amounts of input congestion,

(2) to deal with congestion problem under the condition that all inputs and outputs are nonnegative values,

(3) to measure the input congestion amounts in the presence of the multiple optimal solutions,

(4) to decompose the measure of overall technical efficiency into pure technical efficiency, scale efficiency, and congestion efficiency,

(5) to deal with the congestion problem in the presence of the undesirable outputs.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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