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Research Article

Cooperative Game Study of Airlines Based on Flight Frequency Optimization

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By applying the game theory, the relationship between airline ticket price and optimal flight frequency is analyzed. The paper establishes the payoff matrix of the flight frequency in noncooperation scenario and flight frequency optimization model in cooperation scenario. The airline alliance profit distribution is converted into profit distribution game based on the cooperation game theory. The profit distribution game is proved to be convex, and there exists an optimal distribution strategy. The results show that joining the airline alliance can increase airline whole profit, the change of negotiated prices and cost is beneficial to profit distribution of large airlines, and the distribution result is in accordance with aviation development.

1. Introduction

With the increasing demand of air travel, not a single airline is able to achieve global seamless service. Thus, a new form of cooperation emerges in the aviation field, the airline alliance. The airline alliance [1] is a partnership formed by two or more airlines with a cooperation agreement, to establish a global flight system, a market integrated by code sharing, and so forth. The airlines in an alliance are independent from each other, while their rights and obligations are clarified in the agreement. In this paper we develop a methodology to analyze cooperative game among airline alliance market. The methodology may be used to explain airline performance and to predict the profit before and after the alliance. Game Theory is more reasonable and accurate for alliance profit distribution more than other methodologies.

The burgeoning alliance makes the distribution of profit particularly important. Literature [2–7] analyzed the development and constraints of the airline alliance and the multilateral relationships in it. Literature [8–11] discussed the general code sharing and the code sharing in the airline alliance and its influence on travelers. The airline alliance is an organization driven by market opportunity. Its fundamental purpose is to obtain economic benefits for the airlines. This is crucial not only for the stability and improvement of the alliance, but also for the participating motivation of the members. Most of the above literatures studied the profit distribution in the alliance based on assumed flight frequency. However, it is more accurate to firstly determine the flight frequency. In the paper, we analyze the relationship between the ticket price and the flight frequency under competition and cooperation circumstances and develop a profit distribution model for the airline alliance.

2. Optimal Flight Frequency in an Airline Alliance

2.1. Flight Frequency. For route i, if it is a monopoly route, use plane model k; then the flight frequency for demand D_i is [5]

$$F_i = \frac{D_i}{l_i s_k}, \tag{1}$$

where l_i is the load factor and s_k is the number of available seats (aircraft capacity) of model k.

If route i is not a monopoly route, both airline A and airline B compete for it and market share of the airline A is MS_{1i}; then

$$\mathrm{MS}_{1i} = \frac{F_{1i}s_{1k}}{F_{1i}s_{1k} + F_{2i}s_{2k}},$$
$$F_{1i}l_{1i}s_{1k} + F_{2i}l_{2i}s_{2k} = D_i, \tag{2}$$

where F_{1i}, s_{1k}, l_{1i} and F_{2i}, s_{2k}, l_{2i} represent the flight frequency, aircraft capacity, and load factor of A and B, respectively; then the flight frequency for route i is

$$F_{1i} = \frac{D_i}{s_{1k}\left[l_{1i} + l_{2i}\left(1/\mathrm{MS}_{1i} - 1\right)\right]},$$
$$F_{2i} = \frac{D_i}{s_{2k}\left[l_{1i}MS_{1i}/\left(1 - \mathrm{MS}_{1i}\right) + l_{2i}\right]}. \tag{3}$$

2.2. Flight Frequency in Competition. In business competition, airline A and airline B usually compete on price to attract visitors by providing more discounts or increasing the number of flights. Table 1 shows the payoff matrix.

Table 1: Payoff matrix of two airlines in competition.

		Airline A	
		a	b
Airline B	c	$a\overline{p}l_{1i}s_{1k}F_{1i}^1 - CF_{1i}^1$ $c\overline{p}l_{2i}s_{2k'}F_{2i}^1 - CF_{2i}^1$	$b\overline{p}l_{1i}s_{1k}F_{1i}^2 - CF_{1i}^2$ $c\overline{p}l_{2i}s_{2k'}F_{2i}^2 - CF_{2i}^2$
	d	$a\overline{p}l_{1i}s_{1k}F_{1i}^3 - CF_{1i}^3$ $d\overline{p}l_{2i}s_{2k'}F_{2i}^3 - CF_{2i}^3$	$b\overline{p}l_{1i}s_{1k}F_{1i}^4 - CF_{1i}^4$ $d\overline{p}l_{2i}s_{2k'}F_{2i}^4 - CF_{2i}^4$

C is the cost of flights, a, b, c, and d represent different discounts, and $a > c > b > d$. F_{1i}^1, F_{1i}^2, F_{1i}^3, and F_{1i}^4, respectively, denote the flight frequency in the four cases for airline A, and $F_{1i}^2 > F_{1i}^4 > F_{1i}^1 > F_{1i}^3$ according to market competition rules. While for airline B, $F_{2i}^3 > F_{2i}^4 > F_{2i}^1 > F_{2i}^2$. Based on the above relations, the matrix is going to be $\begin{bmatrix} \begin{bmatrix} (ac, ca) & (bc, cb) \\ (ad, da) & (bd, db) \end{bmatrix} \end{bmatrix}$, and $bc > bd > ac > ad$. Similarly, $db > da > ca > cb$, and it can be concluded that (bd, db) is the Nash equilibrium in a noncooperative game. Thus, b and d, respectively, are the lowest fares provided by airlines A and B, and the corresponding flight frequency is at the medium high level.

2.3. Flight Frequency after the Airline Alliance. After joining airline alliance, the number of passengers is greatly increased due to code sharing, as well as the profit. It can be concluded that the airline alliance is one of the best ways for airlines' benefits. The airlines of alliance are considered as one company. So the flight frequency can be simplified as follows.

The function between the profit and flight frequency is

$$P_r = \left(\overline{p} - c_p\right)D - \left(c_r + c_l\right)F, \tag{4}$$

where P_r is the airline profit; $\overline{p}$ is the average fare; c_p is per passenger cost; and c_r and c_l are the route running cost and the landing fee. Considering the passenger demand D is a function of the flight frequency F, then the derivation of flight frequency is

$$\frac{dP_r}{dF} = \left(\overline{p} - c_p\right)\frac{dD}{dF} - \left(c_r + c_l\right) = 0. \tag{5}$$

Introducing the demand elasticity for flight frequency $e = (dD/D)/(dF/F)$, the optimal frequency of flights is

$$F^* = \frac{\left(\overline{p} - c_p\right)eD}{c_r + c_l}. \tag{6}$$

It can be seen from (6) that the optimal flight frequency is proportional to the flight fare after the alliance.

3. Profit Distribution in an Airline Alliance

3.1. The Profit Distribution Model. In an airline alliance, code sharing is a very effective marketing strategy, which decides whether consumers are willing to pay higher fares to increase their market power and space. Not only can code sharing expand the network coverage and the market presence of the alliance members, but also can strengthen the network with lower costs by reducing input of airplanes [6]. For travelers, they are provided with more choices and also enjoy a seamless transit service, because the interconnecting flights by alliance partners are strengthened by the network, where the connectivity and service quality are significantly improved.

There are three main forms of code sharing: the first is the parallel operation on the backbone routes; that is, two airlines operate their routes together and help each other sell seats; the second is the unilateral operation; that is, airline X sells the seats for airline Y, and airline Y only operates flights; the third is that airline X operates route A-B and airline Y operate route B-C while sharing codes with each other and selling seats of routes operated by Y [7]. In this paper, we are focusing on the first form, where the airlines operate the routes and sell seats mutually.

Airlines share codes on the same route in agreement that sets an agreement price P at first. And if there are n airlines, the profit distribution function for airline i after code sharing is

$$\Pi_i = PQ\frac{\exp\left(UF_i\right)}{\sum_{i=1}^{n}\exp\left(UF_i\right)} - C_i\frac{\exp\left(UF_i\right)}{\sum_{i=1}^{n}\exp\left(UF_i\right)}F^*$$
$$i = 1, 2, 3, \ldots, n, \tag{7}$$

$$UF_i = \frac{\left(\theta_{0i} + \theta_{mi} + \theta_{fi} + \theta_{xi} + \theta_{ui}\right)}{5}, \tag{8}$$

where P is the agreement price and Q is the passenger volume on the route, $(\exp(UF_i))/\sum_{i=1}^{n}\exp(UF_i)$ is airline i's market share on the route using the Logit model [8], C_i is the flight operating cost, and F^* is the number of flights after code sharing from (6). Equation (8) is the utility function of civil aviation passengers. Although travelers consider many factors when choosing airline, the service is always the first. According to the service evaluation indexes, we select the flight attendant service, the flight broadcast, the flight meals, cabin facilities, and flight entertainment as parameters.

The airline alliance is a more in-depth and comprehensive cooperation based on code sharing. The primary considerations for airlines to join the alliance are the expected benefits from the alliance. The benefits mainly include having access to market and complementary resources, obtaining institutional legitimacy and capacity of new markets, and reducing risks and environmental uncertainties [9]. However, it requires plenty of cost to join the alliance, like the accession fees and annual fees when joining the international coalition. Many other costs also exist to maintain the alliance, such as management fees and coordination cost. Then the profit function of airline i in code sharing period is

$$v(u) = \sum_{i=1}^{u} \Pi_i + PQ'_u - C'_u, \quad u = 1, 2, 3, 4, \ldots, n, \tag{9}$$

where $N = \{i \mid i = 1, 2, 3, \ldots, n\}$ represents all airlines alliances, Q'_u denotes the increased passenger volume due to the advantage of the alliance for u airlines, and C'_u is the increased cost after alliance.

3.2. Game Model of the Profit Distribution in an Airline Alliance. The Game Theory can be applied to many questions of airline alliance. It is very effective on consultation, coordination, and achievement of strong constraint agreements. The airlines can share the benefits from the alliance through various ways of cooperation.

There are two major elements in the cooperative game: the participants and the characteristic function. The participants are the airlines involved in an alliance. If there are n airlines, the set is $N = \{i \mid i = 1, 2, 3, \ldots, n\}$ and u is a subset of N, $u \subset N$. $v(u)$ is the corresponding characteristic function, denoting the greatest value that all u members of the alliance can create, while $v(\phi) = 0$ indicates no value created if the subset is empty.

Definition 1. $x = (x_1, x_2, \ldots, x_n)$ is an n-dimensional vector, satisfying the following two conditions:

$$x_i \geq v(\{i\}), \quad i = 1, 2, \ldots, n; \tag{10}$$

$$\sum_{i=1}^{n} x_i = v(N), \tag{11}$$

x is called as an assignment.

Condition (10) is called the individual rational condition, indicating that for the airlines to participate in the alliance is at least better than operating by itself or it would not join the alliance. Condition (11) is called the group rational condition, which means that the sum of the profits allocated to each is the total revenue of the alliance; if less, there must be some undistributed parts of the profit, which apparently the alliance airlines will not agree with, and if greater, the total allocation exceeds the total profit of the alliance, which is impossible.

For the game of the profit distribution, (N, v), if $v(u) \leq v(w)$, $\forall i \in N$, and $\forall u \subset w \subset N$, then the game is linear; if $u \cap w = \phi$, $\forall u, w$, then

$$v(u \cup w) \leq v(u) + v(w). \tag{12}$$

The game also meets an additive condition, $u \subset w \subset N \setminus \{k\}$, $\forall i \in N$; then

$$v(u \cup \{k\}) - v(u) \leq v(w \cup \{k\}) - v(w). \tag{13}$$

The game is a convex game.

Definition 2. For the same route, the larger the alliance, the greater the chance for the travelers to reschedule or cancel the flights, and the better the growth of travel volume is stimulated. However, $Q'_u \leq Q'_v$, $\forall u \subset w \subset N$, but marketing cost can be reduced even if the coordination cost is increased; thus $C'_u \geq C'_v$.

Proposition 3. *Game (N, V) is a convex game.*

Proof. It is true, $\forall u \subset w \subset N \setminus \{k\}$. Consider the following:

$$\begin{aligned} &PQ'_{u\cup k} - C'_{u\cup k} - \left(PQ'_u - C'_u\right) \\ &\quad = P\left(Q'_{u\cup k} - Q'_u\right) + \left(C'_u - C'_{u\cup k}\right), \\ &PQ'_{w\cup k} - C'_{w\cup k} - \left(PQ'_w - C'_w\right) \\ &\quad = P\left(Q'_{w\cup k} - Q'_w\right) + \left(C'_w - C'_{w\cup k}\right). \end{aligned} \tag{14}$$

By Definition 2,

$$\begin{aligned} &PQ'_{u\cup k} - C'_{u\cup k} - \left(PQ'_u - C'_u\right) \\ &\quad \leq PQ'_{w\cup k} - C'_{w\cup k} - \left(PQ'_w - C'_w\right) \\ &\qquad \forall u \subset w \subset N \setminus \{k\}, \\ &\left\{\left(\sum_{i=1}^{w\cup k} \Pi_i + PQ'_{w\cup k} - C'_{w\cup k}\right) - \left(\sum_{i=1}^{w} \Pi_i + PQ'_w - C'_w\right)\right\} \\ &\quad - \left\{\left(\sum_{i=1}^{u\cup k} \Pi_i + PQ'_{u\cup k} - C'_{u\cup k}\right) \right. \\ &\qquad \left. - \left(\sum_{i=1}^{u} \Pi_i + PQ'_u - C'_u\right)\right\} \\ &\quad = \left\{PQ'_{w\cup k} - C'_{w\cup k} - \left(PQ'_w - C'_w\right)\right\} \\ &\qquad - \left\{PQ'_{u\cup k} - C'_{u\cup k} - \left(PQ'_u - C'_u\right)\right\} \geq 0. \end{aligned} \tag{15}$$

Then $v(u \cup \{k\}) - v(u) \leq v(w \cup \{k\}) - v(w)$.

Therefore, game (N, V) is a convex one. When an airline joins a larger alliance, its profit is higher than participating in a smaller one. $\varphi_i(V)$ denotes the profit assigned to airline i in the alliance, where the coallocation of profit for each airline

Table 2: Evaluation values of airline service.

Airlines	Flight attendant service	Flight broadcast	Flight meals	Cabin facilities	Flight entertainment
A	4.09	3.9	3.72	4.07	3.41
B	3.9	3.79	3.08	3.84	2.94
C	3.38	3.4	2.06	3.23	2

has nothing to do with its label, and the sum equals the total profit, $\varphi(V) = (\varphi_1(V), \ldots, \varphi_N(V))$. If n airlines participate in two alliances without interference with each other, the profit distributions should also be independent from each other, and the total profit of all the airlines should equal the sum of the profit from both alliances. From Proposition 3, we know that game (N, V) has a nonempty core, and the Shapley value is the core of the game and the gravity center of the collection. □

$\varphi(V)$ can be calculated by the following formula:

$$\varphi_i(V) = \sum_{S \in S_i} W(|S|)\left[V(S) - V(S - \{i\})\right], \tag{16}$$

$$W(|S|) = \frac{(|S| - 1)!\,(n - |S|)!}{n!}, \tag{17}$$

where S is a subset of N, $S \subseteq N$. Equation (17) is an "impact program" under different circumstances, reflecting the power of each participated airline. A rational distribution can stimulate enthusiasm of the airlines to join the alliance.

3.3. Numerical Example. Assume that airlines A, B, and C form an alliance by cooperation of code sharing on routes. The profits of the three airlines are 61, 49, and 35 million when operating independently. The average number of daily passengers is 3500 before the alliance. The daily flight cost of A, B, and C are 6,000 yuan, 40,000 yuan, and 35,000 yuan, respectively, and the optimal flight frequency is 28 per day. Q_2' is 180 passengers. Q_3' is 110 passengers. C_2' is 30,000 yuan, and C_3' is 29,000 yuan.

The values of service parameters of utility function are shown in Table 2. The profit distributions under different agreement prices calculated by (7), (9), and (16) are shown in Figure 1. The absolute values of each airline's profit allocation increase with the increase of the price P, while, notably, the proportions of airline A and airline B in the total profit also tend to increase. As shown, for airline A, its proportion increases from 39.2% to 39.77% with price P rising from 650 yuan to 1000 yuan, while it increases from 33.7% to 33.8% for airline B, which is less than the increase for A. However, the proportion of airline C tends to decline. It decreases from 27.1% to 26.4% when the price rises from 650 yuan to 1,000 yuan. So the higher the agreement price is, the worse it is for airline C. It becomes apparent that the airline owning more passengers and flights obtains more profits in alliance; composing alliance is especially important to such airlines.

With the same agreement price, the profit distribution of each participated airline at different alliance cost combinations can be seen in Figure 2. With the cost increasing, the profit of each airline is decreasing, and the change is

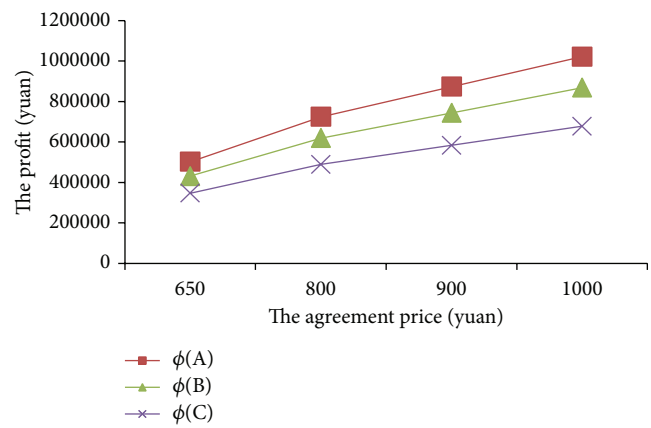

Figure 1: The influence of agreement price on the profit distribution.

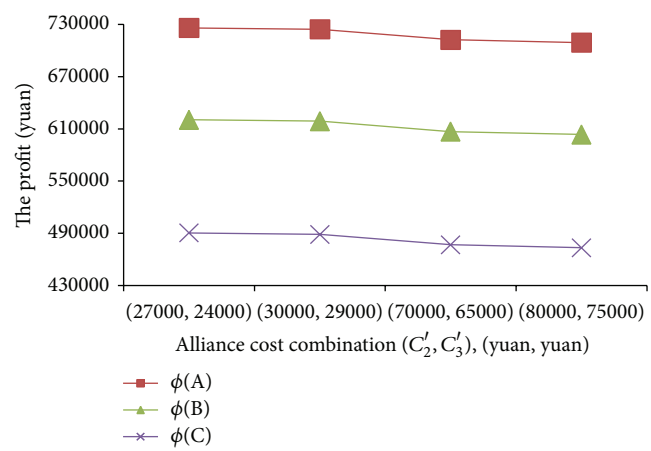

Figure 2: The influence of different alliance cost combinations on the profit distribution.

significantly smaller than alliance profit, which is also an important reason why the airlines are willing to join the alliance. The proportion of airline A's profit distribution tends to increase, such that it rises from 33.8% at (27000, 24000) to 39.5% at (80000, 75000), which is the same tendency for the proportion of C's profit. In summary, the profit of each airline increases after joining the alliance, and the larger the airline is, the greater the benefit increases. The alliance cost does not play an important role.

4. Conclusion

It is the current and future mode of airlines to join the alliance for various ways of communication and cooperation and

to seek a deeper level of development in airline industry. Flight frequencies, fares, and airline costs are always the problems that concern most airlines. In this paper, from the perspective of optimizing flight frequency, we study the flight frequency and profit distribution with both noncooperative and cooperative game before and after the alliance. We also propose a methodology to reasonably analyze alliance profit distribution using the Game Theory.

It can be concluded that the alliance can increase the overall profit for airlines; the changes in the agreement price and cost combination are more favorable to larger airlines; the results of the profit distribution accord with the actual development of airline business. With the agreement price increases, the proportion of profit distribution is increasing for larger airlines, while it has opposite effect on smaller airlines. At a fixed agreement price, the change of profit distribution is consistent with the change of the combination costs in the alliance.

In future research, we will consider the game trend and profit distribution in the network of airlines, which is closer to the airline development patterns, so as to provide more practical guidance for the progress of airline alliances.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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