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Research Article

The Implied Risk Neutral Density Dynamics: Evidence from the S&P TSX 60 Index

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The risk neutral density is an important tool for analyzing the dynamics of financial markets and traders' attitudes and reactions to already experienced shocks by financial markets as well as the potential ones. In this paper, we present a new method for the extraction information content from option prices. By eliminating bias caused by daily variation of contract maturity through a completely nonparametric technique based on kernel regression, we allow comparing evolution of risk neutral density and extracting from time continuous indicators that detect evolution of traders' attitudes, risk perception, and belief homogeneity. This method is useful to develop trading strategies and monetary policies.

1. Introduction

The risk neutral density is an important tool for analyzing the dynamics of financial markets and traders' attitudes and reactions to already experienced shocks by financial markets as well as the potential ones.

Interest in RND continues to grow mainly due to the proliferation of financial crises. Indeed, one of the virtues of these densities is the ability to predict crises through extreme shock probabilities revealed by densities flattening coefficients. One of the possible applications of RND is to extract traders' risk aversion with regard to the underlying historical density. The estimation of the bias between the two densities gives an idea about the market risk premium; indeed the market risk premium should be positive under the assumption that investors are risk averse. As to the specific magnitude and its volatility over time, several key results emerge from the academic literature (Dimson et al. (2003), Cochrane (2005), van Binsbergen et al. (2012), and Berg (2012)), which will help to develop more effective trading strategies. These densities are also an important tool for developing monetary policies. Indeed, financial authorities are using these densities to estimate the effectiveness of monetary policies through direct observation of changes in agents' attitudes and beliefs regarding future maturities.

The RND allows comparing expectations with the average market opinion, which will help investors to explicitly adjust their positions according to market trends revealed by these densities. In this sense, the risk neutral density is used to develop confidence intervals for determined predictions based on standard deviations of the skewed distribution in a similar approach to the concept of value-at-risk.

From the end of the 70s, the research on RND has proliferated thanks to the martingale presentation of Harrison [1] writing option price as a function of underlying expected return. The work of Breeden and Letzenberger constitutes the core of future research addressing the RND showing that this density is simply the second derivative of the option price with respect to the option strike. The existence of small number of option contracts at different strike prices for the same maturity is the major problem of RND curve drawing.

Further works have proposed several methods to solve this problem. Some authors have assumed a definite form of the density (log-normal mixture). Others have made assumptions on the underlying diffusion model (jump model). There are also completely nonparametric models without any assumptions neither on the underlying diffusion process nor on the shape of density, and methods called semiparametric mainly use nonparametric technology and are assisted by some assumptions due to convergence problems. Several authors have presented deep analysis of the dynamics of financial markets and traders' behavior based on RND. For instance, Gemmill and Saflekos [2] exploited densities extracted from options on FTSE to study the market expectations during the British elections. When analyzing the information content of option prices during the crash of the Hong Kong Stock Exchange in 1997, Souissi and Aloulou [3] deduced that CAC40 implied volatility is a good estimator of future volatility. Lynch and Panigirtzoglou [4] analyzed the densities extracted from S&P 500 index options during the period 1985–2001. They found that densities responded to events of the steps, but they were less convenient to predict.

Taylor et al. [5] showed a relationship between the risk neutral skew and the size of the firm, systemic risks, market and firm volatility, liquidity, and the leverage effect ratio. Hamidieh (2010) examined the density tails during the second half of 2008; he found that the left tail was thinner at the peak of the crisis. Birru and Figlewski [6] examined the RND in the S&P 500 during the subprime crisis. They showed that the arbitrage kept the densities average closely related to the market index. They also found a strong density reaction to the index movement. The central portions of densities amplified this change of more than 50% in some cases, especially in hawsers periods.

Melick and Thomas (1998) made two warnings about the estimated densities. The first is related to the functional form of the estimated density, leading to a particular distribution. The simplest example is the density extracted via the Black-Scholes model whose distribution is always symmetrical whatever the level of investor's loss aversion is. The second is related to the "unknown extent" in which the attitude to risk is incorporated in the prices of options contracts which complicates the interpretation of the estimated densities.

The two authors added that the analyses at the Federal Reserve based on DNR were generally built on "instant" comparisons involving short periods during which it could be assumed that the risk attitude did not change according to the marginalization of foreshortening maturity changes. These comparisons were useful only for very short periods which did not allow detecting changes in agent's attitudes and beliefs. According to Melick and Thomas, eliminating bias caused by daily variation time to maturities would make it possible to compare the evolution of densities and thus to extract continuous indicators reflecting changes in attitudes and expectations.

In this context, we will develop in the second section a new approach to better exploit the option prices information content through the elimination of bias caused by the daily variation of maturities. This will allow us to compare the risk neutral densities and extract continuous indicators reflecting attitudes (especially risk perception) and operative characteristics such as the heterogeneity of their beliefs.

2. Risk Neutral Density

One of the basic assumptions of the popular models in finance is the absence of arbitrage opportunity. This means that it would be no way to gain some wealth in a future date without initial cost. In a perfect market, two portfolios generating same cash flows should have the same value at any time. Accidentally, arbitrage opportunities can be detected due to market imperfections which are rapidly depleted by market players whose concern is to enjoy these abnormalities.

These agents, through their actions, bring prices back to the no-arbitrage price. So the opportunities are disappearing rapidly, which further legitimizes the general assumption of no arbitrage. The direct consequence of this assumption is the existence of a single measure called martingale probability. It follows that asset prices are equal to their expected return.

Breeden and Litzenberger [7] were the first to identify a relationship between the option price and the distribution of the underlying price. They extracted RND by deriving twice the price of a derivative with respect to the strike.

According to these authors, the price of the call according to the no-arbitrage assumption had to be equal to the expected future returns:

$$\begin{aligned} &C(t,T,S,X) \\ &\quad = e^{-rt(t)(T-t)} \int_0^\infty \max(S_t - Xc, 0)\, q(S_T)\, dS_T \\ &\quad \cdot P[a \le x \le b], \\ &C(t,T,S,X) = e^{-rt(t)(T-t)} \int_{Xc}^\infty (S_t - Xc)\, q(S_t)\, dS_t \end{aligned} \tag{1}$$

where T is time to maturity, C is option price, r is free risk interest rate, S is underlying price, Xc is strike, and $q(\cdot)$ is probability density.

The first derivative of the European call option with respect to the exercise price is

$$\begin{aligned} \frac{\partial C}{\partial Xc} &= -e^{-rt} \int_{Xc}^\infty q(S_t)\, dS_t \\ &= -e^{-rt}\left[1 - \int_\infty^{Xc} q(S_t)\, dS_t\right] \\ &= -e^{-rt}\left[1 - Q(S_t)\right]\Big|_{S_t = Xc} \\ \frac{\partial C}{\partial Xc} &= -e^{-rt} P(S_t > Xc). \end{aligned} \tag{2}$$

The first derivative of the European call option with respect to the exercise price is equal to the probability that the option is on the money multiplied by −1.

From this result, we can notice that when the option is deep out of the money, the option price is not significantly affected by strike variation

$$\frac{\partial C}{\partial Xc} \cong 0. \tag{3}$$

In the opposite case, if the strike is much lower than the underlying price, the option has very high chance of being exercised. Strike variation of one currency unit will cause a change in the opposite direction of the option price of one updated currency unit

$$\frac{\partial C}{\partial Xc} \cong -e^{-rt}. \tag{4}$$

When we drift again the price of the call to Strike, we obtain the Breeden and Litzenberger formula

$$\left.\frac{\partial 2C}{\partial {X_c}^2}\right|_{S_t=Xc} = -e^{-rt} q\left(S_t\right). \quad (5)$$

This approach requires digital approximation using the method of finite difference. This presupposes the existence of sufficient number of listed options contracts for the same maturity with different strikes. However, the major limitation of this method revolves around the robustness of the empirical application due to insufficient exercise price quotes.

To solve this problem, some authors introduced mainly two ideas. The first was to make nonparametric interpolation to obtain sufficient observations to the extraction of second derivatives. The second way was to make assumptions about the underlying diffusion process or the shape of the density to grant (structural models). Jackwerth [8] and Figlewski [9] provided a detailed review of the work on the extraction of RND.

Among these works, we could cite those of Jarrow and Rudd [10] who developed option pricing method assuming that the underlying asset did not follow a log-normal. These authors obtained the risk neutral density by Edgeworth expansion around the log-normal distribution. Corrado and Su [11] used the Jarrow and Rudd method to determine the third and the fourth moments of the SP500 options prices.

Shimko (1993) estimated the risk neutral density by smoothing smile. Heston [12] offered a quasi-analytical solution for option prices in a context of stochastic volatility. Rubinstein [13] developed an implicit tree to estimate asset price from options prices, minimizing the gap between the implied probability of the tree and the probabilities obtained from the Cox and Ross tree (1976). Jackwerth and Rubinstein [14] proposed a method based on binomial trees. Madan and Milne [15] and Abken et al. [16] obtained the risk neutral density from Hermite polynomial approximation. Chiarell [17] used the work of Fourier-Hermite to assess the EU and US share options. Bahra [18] and Sherrick et al. [19] introduced a nonparametric method to evaluate European options. These authors presented the risk neutral density as a linear combination of log-normal densities. Melik and Thomas [20] adjust this method for American options. Adir and Rockinger [21] determined the option price formulas by using the Kummer hypergeometric functions. Backus [22] used the Gram-Charlier expansion series to approximate the conditional distribution of the underlying price. Ait-Sahalia [23] and Ait-Sahalia and Lo [24, 25] obtained the risk neutral density from the S&P 500 options by the kernel estimation technique. Bondarenko [26] proposed a nonparametric method based on a positive convolution approximation. Rompolis and Tzavalis [27] used a new density extraction method based on exponential Gram-Charlier series to ensure always positive values. Monteiro and Vecentec [28] presented a new approach for estimating RND achieved in the space of cubic splines ensuring positivity for the polynomial functions. Rompolis [29] suggested a new method of using the principle of entropy maximization. Figlewski [9] combined smoothing techniques to reflect the bid-ask spread and a new method of completing the density with tails from a generalized extreme value distribution. Fabozzi et al. [30] applied the Levy process to estimate the risk neutral density.

Several authors were interested in comparing RND extraction methods. Jondeau [31] compared the log-normal mixing method, Heston model, jump model, and Edgewerth approximating. According to their study on options on exchange rates, they found that the mixing log-normal method gave the best results. However, in times of crises, density extracted from jumping model offered the best results.

Coutant [32] carried out the comparison between the entropy maximizing method, the mixture of log-normal distribution, and the Gram-Charlier expansion based on three criteria: these are robustness of estimation, convergence speed, and ease of application. They argued that the method which is based on the Gram-Charlier expansion provides the most stable results for options on interest rates. Bliss and Panigrizoglou [33] analyzed the robustness of the results provided by the Shimko method and that of the log-normal mixture. They showed that the smile smoothing method provided a slightly better performance. However, this method could not cover the leptokurtic distribution after a certain level of exercise prices.

Hamdi and Lemennicier (2011) compared six methods of RND extraction. They find that mixing log-normal method as well as Hermite polynomials and jumping model fits best historical densities.

3. Maturity Effect on Risk Neutral Density

The effect of maturity on traders' expectations is capital. The more the deadline is approaching, the more traders are confident in their expectations and show less risk aversion. Humphreys [34] suggested that uncertainty embodied in the densities tended to decline as we approached the expiration date and a very few exchanges usually took place on days immediately preceding this date.

Figure 1 shows the effect of the maturity on the traders' expectations. Indeed, the distributions of probability densities show that traders are more confident in their expectations for the first date.

According to Table 1, the market is twice more confident on its expectation about underlying price expectation on the first maturity than the second farthest of 34 days. Also, note that the probability granted by the market to an extreme disturbance increases by more than 200 times. To analyze the dynamics of changes in attitudes, in particular risk perception, it is imperative to eliminate bias caused by daily maturity variation, which does not allow comparing the evolution of densities.

Melik and Thomas [20] discussed the problem of daily change in the maturity options as well as that of replacing contracts which reached maturity. They suggested two "possible" methods to correct these problems. One idea was to incorporate the dependence of maturity explicitly in the functional of the probability density. Butler and Davies [35] applied a correction of this kind to the implied probability

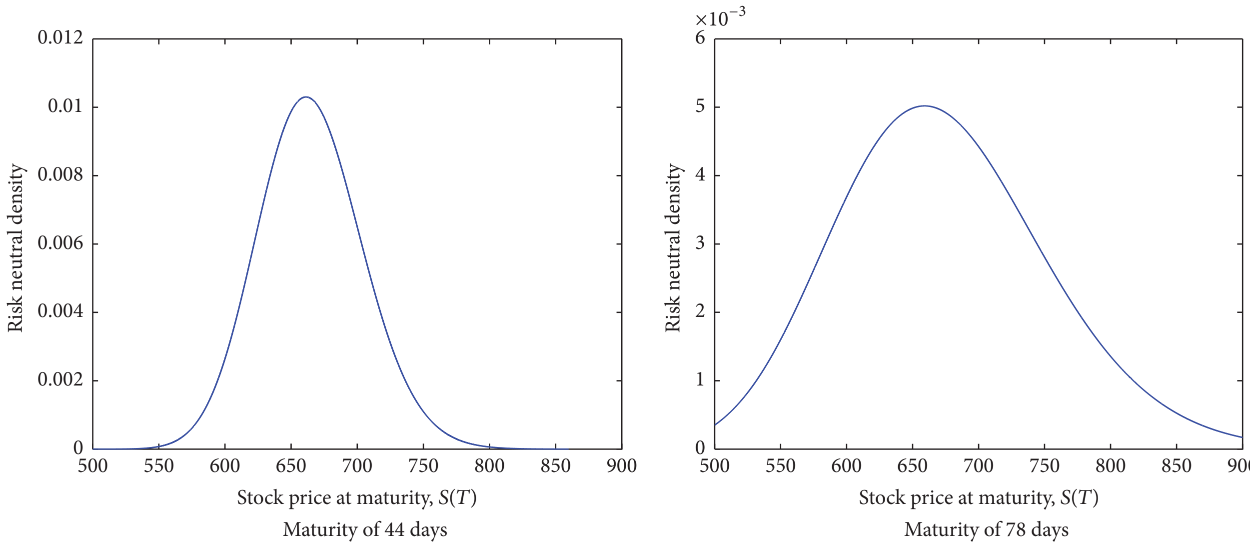

Figure 1: RND 3 July 2012 extracted from S&P TSX 60 index.

Table 1: Maturity effect on traders' expectations.

	Maturity of 44 days	Maturity of 78 days
Most anticipated value (m)	661.870	659.160
$P(x) = m$	0.0102	0.0050
$P(x \leq m) - P(x \geq m)$	0.0466	0.0977
$P(x \leq 540) - P(x \geq 860)$	0.00019	0.04133

densities in the interest rate contracts on three-month Eurosterling. The major drawback of this idea was that, using the Black-Scholes model, the density would be a log-normal, so it did not reflect the distribution asymmetry and the fear of extreme shock. The second method is to freely estimate probability densities and to correct the results under the dependence of maturity. The disadvantage of this technique was that the correction will be very arbitrary and that it will be hard to check result's robustness.

The authors were faced with the daily variation problem maturities in option contracts when exploring two areas, the first topic being the study of the predictive power of implied volatility of future changes in assets and the second being the dynamic analysis of daily changes in attitudes of agents through risk neutral densities. Panigirtzoglou and Proudman [36] developed a method for obtaining a constant maturity series based on an interpolation of the implicit volatilities. This technique consisted of obtaining a smooth function of the smile via the technique of cubic spline. This method has been used by several subsequent works in analyzing the behavior of agents via the RND evolution. Indeed, this technique allowed Bliss and Panigirtzoglou [37] to estimate the implied risk aversion at different horizons. They showed that for the FTSE 100 and S&P 500, the degree of risk aversion decreased greatly with the forecast horizon and was lower during periods of high market volatility. Panigirtzoglou and Skiadopoulos [38] presented a new approach to modeling the dynamics of implied distributions obtained through constant maturity option prices on the S&P 500. They applied a principal component analysis and "Monte Carlo" simulation to model the evolution of the entire distribution over time. Lynch and Panigirtzoglou [4] analyzed the evolution of the risk neutral densities extracted from constant maturity option prices. Kostakis, Panigirtzoglou, and Skiadopoulos (2011) developed an approach that used the constant maturity options price for a better allocation and portfolio management on the S&P 500.

The major drawback of the method of Panigirtzoglou and Proudman [36] on which all this work was based was that, to get the price of options for a fixed maturity horizon, the authors were forced to go through a pricing model (Black-Scholes model) to extract the price from implied volatilities. Indeed, this model and even every other pricing model were subject to much criticism. We developed a new option price of obtaining constant maturity approach fully nonparametric which has the advantage of not resorting to any pricing model.

4. Constant Maturity Series

To compare the daily evolution of the risk neutral densities and collect time continuous indicators measuring the above variables, we developed an approach similar to that of Panigirtzoglou and Proudman [36]. These authors developed a method to obtain a series of constant maturity option prices based on an interpolation of the implicit volatilities with cubic spline.

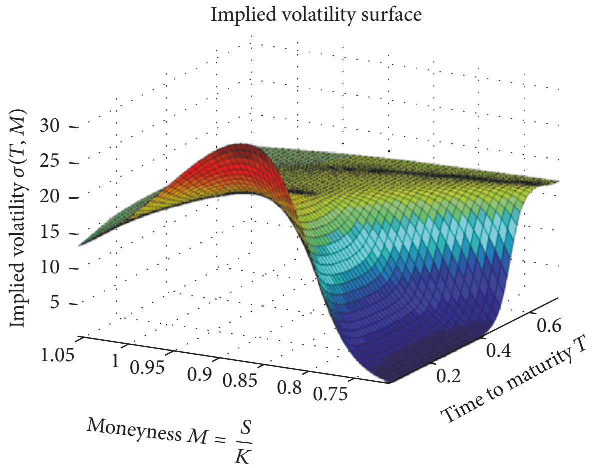

Figure 2: Options on S&P TSX 60 January 10, 2011, implied volatility surface.

The major drawback of the method of Panigirtzoglou Proudman [36] related to the fact that these authors were forced to go through a pricing model (Black-Scholes model [22]) to extract prices from implied volatilities. Indeed, this model and even every other pricing model were subject to much criticism.

We have developed a new fully nonparametric approach to obtain constant maturity option price series which has the advantage of not resorting to any pricing model inspired from Ait-Sahalia and Lo [25] using kernel smoothing. This allowed us to compare the probabilities extracted from risk neutral densities and create a time continuous indicator reflecting the evolution of the traders' risk perception and their homogeneity belief degree. The method of Ait-Sahalia and Lo [25] supposed that this surface was smooth enough that the value at a given time could be calculated by taking the weighted average of all neighboring points. The weight given to each neighboring point decreased as the point was located farther from the target point. To calculate the corresponding implied volatility, the authors used a kernel density function which included smoothing parameter that indicated a weighted average of the neighboring points to include. On this volatility surface, the corresponding value of implied volatility could be identified for each couple (S/K ratio, remaining time to maturity) (see Figure 2).

Kermiche [39] extracted from the volatility surface corresponding smile curves for one-, three-, and six-month maturity. Then, she uses the Black-Scholes model for the corresponding options prices.

We used the method of Ait-Sahalia and Lo [25] to directly build a price surface to avoid any bias in pricing, and we adapted the smoothing parameters used by these authors. Indeed, their goals were to build a sufficiently smooth surface to be interpretable. The choice of parameters of the kernel function is forced to produce a sufficiently smooth surface while getting less biased values possible. Our goal is just to get the least biased option prices. We try to optimize the choice of parameters in this direction regardless of the constraint of having smooth surfaces (see Figure 2).

The principle of the kernel regression is based on smoothing techniques. It seeks to estimate the link function $f(x_i)$ at any point x. This method is developed by Nadaraya and Watson [40].

The kernel estimator (kernel estimate) of the link function evaluated at the point x_0 noted $\widehat{f}(x_0)$ is defined by

$$\widehat{f}(x_0) = \sum_{i=1}^{N} w_i(x_0)\, y_i \tag{6}$$

with

$$w_i(x_0) = \frac{k((x_i - x_0)/\gamma)}{\sum_{i=1}^{N} k((x_i - x_0)/\gamma)}, \tag{7}$$

where k is a kernel function, $\gamma > 0$ is a smoothing parameter (bandwidth parameter), and N is the size of the sample used for estimation.

The link function evaluated at point x_0 is the weighted sum of the observations with y_i, where the weights $w_i(x_0)$ are dependent on x_0.

The $w_i(x_0)$ function where $w(x_0; x_i)$ defines the weight to be assigned to the couple of observations $(x_i; y_i)$ in the value of the link function evaluated at x-axis of x_0. Generally, the more the points x_i are close to x_0, the more the weight will be important: $w(x_0; x_i)$ decreases with distance $(x_i - x_0)$. These weights depend on kernel function that represents the probability density functions.

A kernel function $k((x_i - x_0)/\gamma) = k(u)$ satisfies the following properties:

(i) $K(u) \geq 0$.

(ii) $\int K(u) d_u = 1$.

(iii) $K(u)$ reaches pts maximum in 0 when $x_i = x_0$ and decreases with the distance $(x_i - x_0)$.

(iv) $K(u)$ is symmetrical: kernel does not depend on the distance $(x_i; x_0)$ and on the sign of $x_i - x_0$.

Different kernel functions can be used (uniform, triangular, quadratic, BiWeight, Epanechnikov, and Triweight). Generally, the choice of the kernel function slightly influences the estimation results. The only notable exception is related to the use of a uniform kernel function that can yield different results from other functions.

The parameter γ represents the distance beyond which the observations x_i have a light weight in the value of $w_i(x_0)$. This parameter represents the radius of the values of x_i window around x_0, the weight of which is significantly influential in computing $m(x_0) = f(x_0)$. This window magnitude is 2.

The choice of γ corresponds to an arbitration smoothing/bias. In our sample, there are always options with an initial maturity of 1 and 2 months. Therefore, there is always a close observation of less than 15 days of the estimated value. In our study, since there is no advantage of having smooth surfaces but a less biased interpolation, we set the smoothing parameter as 1/2 of the maximum distance that can separate the little reckoning of observation:

$$\gamma = \frac{(15/365)}{2} \Longrightarrow \gamma = 0.020. \tag{8}$$

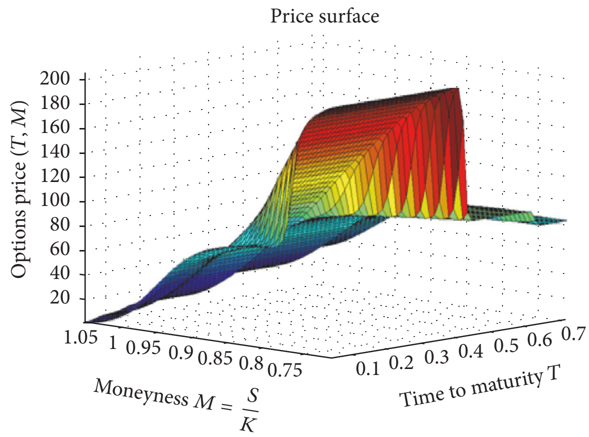

FIGURE 3: Options on S&P TSX 60 January 10, 2011, surface price.

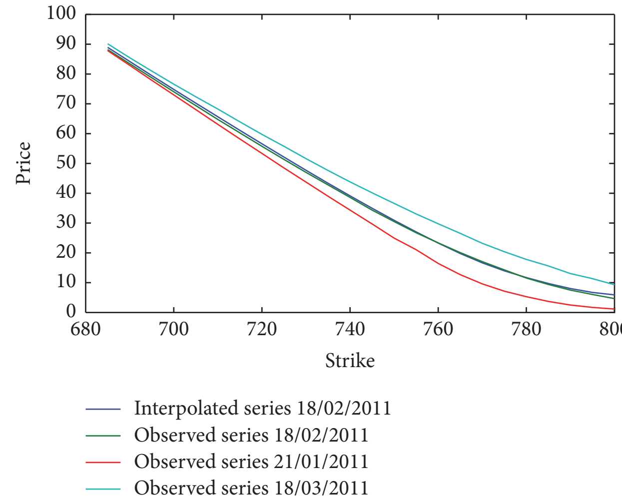

FIGURE 4: The figure calls interpolated and observed Series on 04/01/2011 (maturity 18/02/2011).

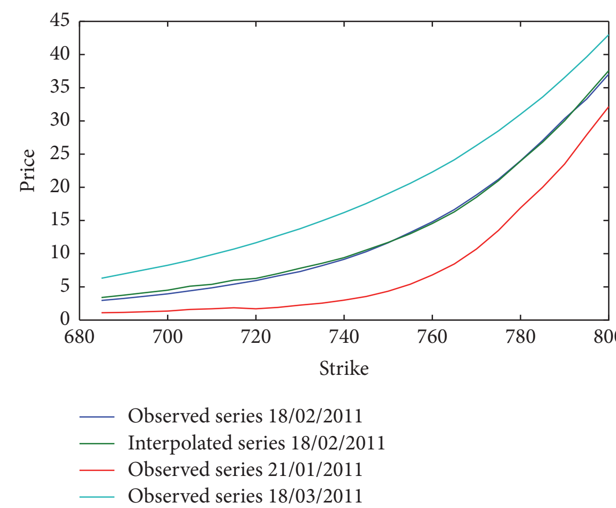

FIGURE 5: The figure puts interpolated and observed Series on 04/01/2011 (maturity 18/02/2011).

In the obtained surface, we can get every day the price of option contracts defined by the pair maturity, mean (Figure 3).

This technique has two advantages. Firstly it is performed directly on price and not on implied volatilities, which helps avoid any bias caused by the use of a model of pricing such as that of Black and Scholes [22]. Secondly, it optimizes the choice s of the kernel function parameter in order to obtain the least biased possible prices.

5. Robustness Check

We use kernel technique to obtain a series of constant maturity (one month or 22 working days) from the index options series S&P TSX 60 traded on the Canadian stock exchange. To test the robustness of the method, we perform an interpolation of the price of options maturing on the last Friday of the second coming months, through the price of other options traded on the market. Then, we compare the series obtained via the kernel technique and the observed series. Figures 4 and 5 show the observed series and those interpolated calls and puts on 04/01/2011 by the deadline of 18/02/2011.

The interpolation results seem to be acceptable. Indeed, the interpolated series and the observed are nearly coincident. We calculate for 30 random chosen days the mean error of the interpolated price compared to that observed for the puts and calls traded.

Table 2 shows the absolute values of the average differences between the observed prices and the interpolated calls and puts; indeed 93.33% of interpolated observations are acceptable at the 95% threshold. The interpolation error average is 3.6% which justifies the use of kernel method.

6. Possible Application

The presented method is helpful to deeply analyze financial market dynamics and traders beliefs evolutions. We apply this method on Canadian market during period from 1 January 2012 to 31 December 2013 to extract 3 different indicators related to the evolution of asymmetry perception, extreme risk fear, and belief heterogeneity.

6.1. Asymmetry Perception. Asymmetry perception can be obtained comparing the lower anticipated value at the maturity to the highest index values and this is summarized in Figure 6.

We collected from obtained densities the difference between these two corresponding probabilities:

$$P(x \leq m) - P(x \geq m) = \int_0^m f(x)\,dx - \int_m^\infty f(x)\,dx \quad (9)$$

with m being most anticipated value.

The evolution of the series during the study period is summarized in Figure 7. Statistics are presented in Table 3.

On average probability given by the market that the underlying has a value less than the most anticipated value is

Table 2: The table puts and calls estimation error average.

Day	Puts error average	Calls error average	Error average
1	0.0345987	0.0215185	0.0280586
2	0.0259826	0.0416025	0.0337926
3	0.0417035	0.0279804	0.0348420
4	0.0219605	0.0319890	0.0269747
5	0.0415346	0.0451279	0.0433313
6	0.0322403	0.0350111	0.0336257
7	0.0510708	0.0287885	0.0399296
8	0.0330091	0.0407680	0.0368885
9	0.0497968	0.0504886	0.0501427
10	0.0451203	0.0238962	0.0345082
11	0.0462980	0.0305246	0.0384113
12	0.0209187	0.0289141	0.0249164
13	0.0433114	0.0213045	0.0323079
14	0.0258682	0.0387042	0.0322862
15	0.0315140	0.0407331	0.0361236
16	0.0422745	0.0200599	0.0311672
17	0.0210142	0.0392219	0.0301181
18	0.0413103	0.0532919	0.0473011
19	0.0396415	0.0252387	0.0324401
20	0.0353776	0.0497175	0.0425475
21	0.0358715	0.0338359	0.0348537
22	0.0216254	0.0433042	0.0324648
23	0.0444826	0.0285212	0.0365019
24	0.0239272	0.0403968	0.0321620
25	0.0482972	0.0206399	0.0344686
26	0.0376054	0.0398404	0.0387229
27	0.0321795	0.0400345	0.0361070
28	0.0477441	0.0408422	0.0442932
29	0.0446228	0.0301299	0.0373764
30	0.0511225	0.0409829	0.0460527
Mean	0.0370675	0.0351136	0.0360906

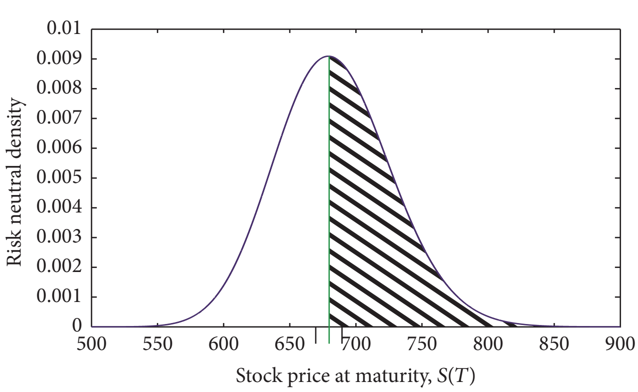

Figure 6: Asymmetry perception.

higher than the probability of exceeding this value by 4%. This reference to the asymmetry of the price of options is relative to market expectations and reflected the margin of safety

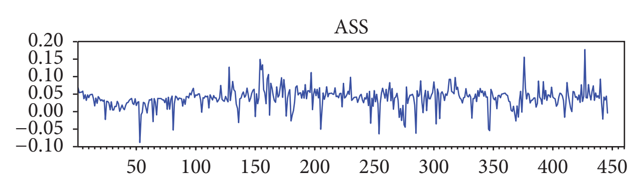

Figure 7: Asymmetry perception evolution in 2012-2013.

required by investors to hedge against the risk of adverse changes in asset prices financial.

The causality test (Table 4) shows that the fear of asymmetry is one of the influential variables in the overall risk perception in the markets. Conversely, this fear of asymmetry increases with market volatility. Indeed, if the market is quite volatile and risky, agents require a higher margin to compensate for the risk resulting in an asymmetry in the prices of options contracts.

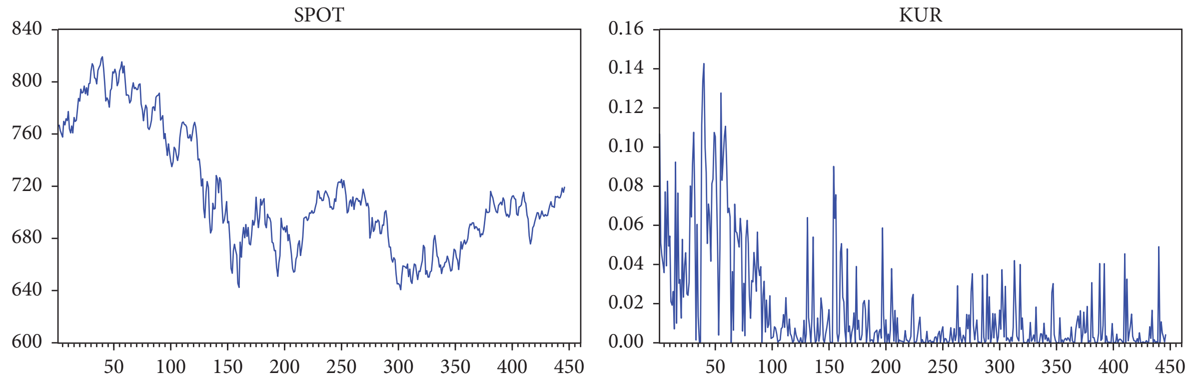

FIGURE 8: Extreme risk fear and index evolution in 2012-2013.

TABLE 3: Asymmetry perception evolution statistics in 2012-2013.

Statistics	Values
Mean	0.038821
Median	0.039917
Maximum	0.177359
Minimum	−0.088242
Std. Dev.	0.029453
Skewness	−0.196402
Kurtosis	6.717279
Jarque-Bera	259.6549
Probability	0.000000
Sum	17.31427
Sum Sq. Dev.	0.386036

TABLE 4: Asymmetry perception and VIX causality test in 2012-2013.

Hypothesis	F-statistic
VIX does not cause SSA Granger	10.8958
SSA does not cause VIX Granger	3.34954

6.2. Extreme Risk Fear. We have assimilated the leptokurtic effect of the probability distribution that has values below 540 or above 860. Mathematically,

$$P(x \leq 540) + P(x \geq 860) = \int_{0}^{540} f(x)\, dx + \int_{860}^{\infty} f(x)\, dx. \tag{10}$$

The graphic analysis (Figure 8) shows that during the first half of 2012 the agents develop a growing fear of underlying extreme variation. This extreme variation has occurred and there has been a sharp drop in the SP TSX60.

This indicator could prevent a quarter rather the loss of almost 25% of the of the index value. Indeed, the S&P TSX 60 index fell from 819.25 on 04/03/2012 to 642.34 on 04/10/2012.

When we explore Table 5, we can conclude that the market gives 1.7% chance achieving extreme shock; this probability has reached almost 15% in the first half of 2012 expressing a growing fear of sudden drop in the index over.

TABLE 5: Extreme risk fear statistics in 2012-2013.

Statistics	Values
Mean	0.017088
Median	0.004684
Maximum	0.142669
Minimum	$2.77E-17$
Std. Dev.	0.026435
Skewness	2.095155
Kurtosis	7.198268
Jarque-Bera	653.8389
Probability	0.000000
Sum	7.621446
Sum Sq. Dev.	0.310977

TABLE 6: Extreme risk fear and VIX causality test in 2012-2013.

Hypothesis	F-statistic
VIX does not cause LEP (Granger)	12.2352
LEP does not cause VIX (Granger)	1.96869

The Granger causality summarized in Table 6 shows that the fear of extreme shock is affected by changes in the market volatility index. However, this fear does not significantly influence the VIX.

6.3. Belief Heterogeneity. Options prices, like any financial asset, are the result of balancing by operative on the market. The more traders are heterogeneous, the more their expectations about the underlying future distributions are divergent. This translates into flat or multimodal densities. In this case, the density is characterized by a high dispersion and thus a lower concentration of the percentage given by the market around the maximum of the distribution.

In the opposite case, the market is characterized by high homogeneity, which leads to more convergent expectations, resulting in curves and acute united modal densities. The probability of density in this case is much less scattered and shows strong percentage concentration around the most expected value. At the end of detecting the daily change of the beliefs heterogeneity in the Canadian market, we set a

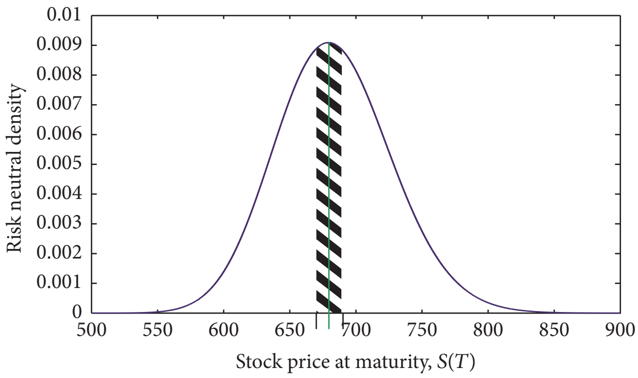

FIGURE 9: $[m-10, m+10]$ confidence interval.

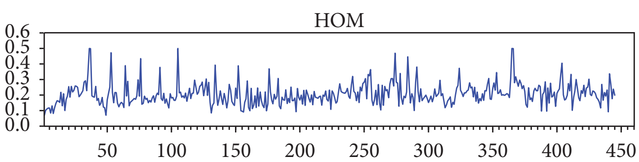

FIGURE 10: Belief homogeneity evolutions 2012-2013.

confidence interval ranging from $m - 10$ to $m + 10$, where m is the most expected value.

$$\int_{m-10}^{m+10} f(x)\,dx = P[m-10 \leq x \leq m+10]\,d. \tag{11}$$

The indicator obtained in Figure 9 informs about probability granted, since the underlying at maturity ends in the interval predefined; that is to say the underlying does not deviate more than 10 monetary units from the most anticipated value (which is from 1 to 1.5% of the value of the index).

According to Table 7 we conclude that, during the study period, we found that on average 20% of traders developed convergent expectations and formed the core of market average opinion. It should be noted that during this period we noticed homogeneity beliefs peak of 50%.

The obtained indicator was volatile reflecting very dynamic reactions of traders in Canadian market which would be interesting to study more closely (Figure 10).

This indicator could be used as a proxy of switching evolution during this period.

7. Conclusion

The risk neutral densities offer investors as well as the monetary authorities a powerful tool for analyzing the dynamics of financial markets and the response of the people operating on potential shocks or already experienced by financial markets. Indeed they can test, monitor, and adjust the adopted monetary policies and ensure the credibility of the institutions through direct observations of their influences on the beliefs of agents as future maturities.

The risk neutral densities also allow investors to compare expectations about future developments of financial assets with the average opinion of the market, which will allow investors to adjust their positions according to market trends explicitly revealed by these densities. However, the risk neutral densities allow only instantaneous or bearing analysis on short periods during which it can marginalize foreshortening of the maturity of the contracts. The creation of a virtual option spectrum obtained tradable options on the Canadian stock exchange via technical "kernel" which has allowed us to eliminate the bias caused by the daily variation of maturity. This legitimates comparison of daily densities obtained to analyze the changing beliefs and attitudes operative on financial markets.

TABLE 7: Homogeneity belief statistics in 2012-2013.

Statistics	Values
Mean	0.207012
Median	0.196507
Maximum	0.499756
Minimum	0.070050
Std. Dev.	0.071223
Skewness	1.350903
Kurtosis	6.248885
Jarque-Bera	331.0617
Probability	0.000000
Sum	92.12052
Sum Sq. Dev.	2.252315

This offers an advanced tool to extract advantage of information content of option prices. The elimination of the bias caused by the variation of maturities has also allowed us to examine the evolution of operative attitudes to risk more closely. Indeed, we have created two continuous indicators over time. The first indicator reflects the operative risk perception in the market and their fear of adverse price change relative to their expectations inferred from the asymmetric densities obtained.

The second indicator reflects the evolution of the fear of extreme shock of realization of investors exhibited by the leptokurtic distribution densities. Both indicators reveal two important components in the formation of attitudes and behaviors of agents to risk. The elimination of daily stress variation maturities has allowed us also to develop a continuous indicator over time of the evolution of the heterogeneity beliefs operative in the Canadian market. This will allow studying the effect of this heterogeneity on the dynamics of asset prices as well as market stability.

Conflicts of Interest

The author declares that there are no conflicts of interest regarding the publication of this paper.

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