Hindawi
Journal of Applied Mathematics
Volume 2017, Article ID 4612853, 5 pages
https://doi.org/10.1155/2017/4612853

Research Article

Sparse Optimization of Vibration Signal by ADMM

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Received 15 November 2016; Revised 22 January 2017; Accepted 23 January 2017; Published 15 February 2017

Academic Editor: M. Montaz Ali

In this paper, the alternating direction method of multipliers (ADMM) algorithm is applied to the compressed sensing theory to realize the sparse optimization of vibration signal. Solving the basis pursuit problem for minimizing the $L1$ norm minimization under the equality constraints, the sparse matrix obtained by the ADMM algorithm can be reconstructed by inverse sparse orthogonal matrix inversion. This paper analyzes common sparse orthogonal basis on the reconstruction results, that is, discrete Fourier orthogonal basis, discrete cosine orthogonal basis, and discrete wavelet orthogonal basis. In particular, we will show that, from the point of view of central tendency, the discrete cosine orthogonal basis is more suitable, for instance, at the vibration signal data because its error is close to zero. Moreover, using the discrete wavelet transform in signal reconstruction there still are some outliers but the error is unstable. We also use the time complex degree and validity, for the analysis of the advantages and disadvantages of the ADMM algorithm applied to sparse signal optimization. The advantage of this method is that these abnormal values are limited in the control range.

1. Introduction

The monitoring and forecast technique of mechanical fault state is mainly applied to extract or separate the fault features which can reflect the development trend of the equipment fault. Monitoring enables to predict both the tendency of the fault features, and the running healthy state, and make feasible a better maintenance scheme according to the deterioration level of the equipment. However, in order to improve the accuracy of prediction, a large amount of vibration data must be collected from the equipment which runs for long time (e.g., half a year). We always expect to store the data quickly and efficiently and operate at real time.

Compressed sensing (CS) theory [1] breakthroughs the limit of traditional Nyquist Sampling Theorem, data sampling and compression are performed at the same time, thus greatly reducing the sampling costs and storage resource. We can forecast the changing trend of the equipment state and the probable fault through the analysis of a long-time vibration signal; there results a big data processing via the long-time monitoring of equipment status.

The key technique of CS is signal reconstruction, and difference reconstruction algorithm directly affects the accuracy of the original vibration reconstruction. The most popular algorithms in signal reconstruction are: the Minimum algorithm $L0$, the orthogonal matching pursuit (OMP) algorithm [2] and its improved type, the $L1$-magi algorithm [3, 4], the weighted algorithm $L1$ [5], the Homotopy algorithm [6], the Lq-FL algorithm [7]. These reconstructions, however, have to face some cumbersome problems both on the uncertainty arising in the mathematical inverse problem, and on the computational complexity due to the deficiency of a small quantity of sparse value. There follows a group of highly undetermined equations, some approximation errors, and a very high complexity, thus making a problem with large scale data a very hard task.

In this paper we study the application of ADMM algorithm in the sparse reconstruction problem for the vibration signal of an equipment. We will show that, comparing with the other current methods, the ADMM is more accurate and has low computational costs, thus being the more suitable method in engineering applications for the long-time monitoring of an equipment status.

2. ADMM Reconstruction Compares with OPM and Dual Interior Point

ADMM blends decomposability of dual ascent and excellent convergence of Lagrange multiplier [8], it is a simple and effective new method to solve the optimization problem of separable target. ADMM is a new algorithm deduced based on augmented Lagrange [9].

Compared with augmented Lagrange, ADMM may decompose original problem into much alternating minimization subproblems; the algorithm can make full use of the advantage on the separability of the objective function. The separating minimization subproblems by ADMM can get global solutions and display solutions more easily.

In this section we give an overview of ADMM for basis pursuit:

$$\begin{aligned} \min \quad & f(x) + g(z) \\ \text{subject to} \quad & Ax + Bz = c, \end{aligned} \tag{1}$$

where variables $x \in R^n$, $z \in R^m$, $A \in R^{p\times n}$, $B \in R^{p\times m}$, $c \in R^p$; f and g are convex. We follow closely the development in Section 6.2 of Boyd et al. [10]: For (1) optimization problem, corresponding to ADMM form, $A = I$, $B = -I$, and $C = 0$.

Let $f(x) = \|\hat{s}\|_1$ and $g(z) = \|z\|_1$, then ADMM for basis pursuit can be written as follows:

$$\begin{aligned} \min \quad & f(\hat{s}) + g(z) = \min \|\hat{s}\|_1 + \|z\|_1 \\ \text{subject to} \quad & \hat{s} - z = 0, \end{aligned} \tag{2}$$

where $f \in \{\hat{s} \in R^n \mid A\hat{s} = b\}$ and satisfies $f(\hat{s}) = \{0, A\hat{s} = b; +\infty, A\hat{s} \neq b\}$. Obviously, $\|\hat{s}\|_1$ and $\|z\|_1$ are convex, and ADMM basis pursuit solution of $\hat{s}^{k+1}$- recursion is

$$\begin{aligned} \hat{s}^{k+1} &:= \prod \left(z^k - u^k\right), \\ z^{k+1} &:= S_{1/\rho}\left(\hat{s}^{k+1} + u^k\right), \\ u^{k+1} &:= u^k + \hat{s}^{k+1} - z^{k+1}. \end{aligned} \tag{3}$$

In (3), u^{k+1} is dual variable for $\hat{s}$ and is also called Lagrange operator. $S_{1/\rho}$ is a soft threshold and a proximity operator of ℓ_1 norm. $\prod$ is the projection onto $\{\hat{s} \in R^n \mid A\hat{s} = b\}$, and $\hat{s}^{k+1}$ recursion form is a minimum Euclidean norm problem under linear equality-constrained, which is as follows:

$$\begin{aligned} \hat{s}^{k+1} := & \left(I - A^T\left(AA^T\right)^{-1} A\right)\left(z^k - u^k\right) \\ & + A^T\left(AA^T\right)^{-1} b. \end{aligned} \tag{4}$$

Proof. In terms of (4),

$$\hat{s}^{k+1} = \arg\min\left(f(x) + \frac{1}{2}\left\|A\hat{s} + Bz^{k+1} - c + u^k\right\|_2^2\right) \tag{5}$$

corresponding to ADMM for basis pursuit, $A = I$, $B = -I$, and $C = 0$; then,

$$\hat{s}^{k+1} = \arg\min\left(f(x) + \frac{1}{2}\left\|\hat{s} - z^k + u^k\right\|_2^2\right), \tag{6}$$

where $f \in \{\hat{s} \in R^n \mid A\hat{s} = b\}$ is called indicator function and satisfies

$$f(\hat{s}) = \begin{cases} 0 & A\hat{s} = b \\ +\infty & A\hat{s} \neq b. \end{cases} \tag{7}$$

Equation (6) is equivalent to $\min (1/2)\|x - (z^k - u^k)\|_2^2$ s.t. $Ax = b$ (construction Lagrangian function). For (6), we get $L = (1/2)\|\hat{s} - z^k + u^k\|_2^2 + y^T(Ax - b)$ and solve maximum value

$$\begin{aligned} & \frac{\partial L}{\partial x} = x - \left(z^k - u^k\right) + A^T y = 0 \\ & \frac{\partial L}{\partial y} = Ax - b = 0 \\ & \qquad \Downarrow \\ & x = z^k - u^k + A^T y \\ & Ax = b. \end{aligned} \tag{8}$$

Get $A[(z^k - u^k) - A^T y] = b$, and $y = (AA^T)^{-1}A(z^k - u^k) - (AA^T)^{-1}b$ inserts into $x = z^k - u^k + A^T y$, and we get

$$\begin{aligned} x = & \left(z^k - u^k\right) \\ & + A^T\left[\left(AA^T\right)^{-1} A\left(z^k - u^k\right) - \left(AA^T\right)^{-1} b\right] \\ = & \left(I - A^T\left(AA^T\right)^{-1} A\right)\left(z^k - u^k\right) - A^T\left(AA^T\right)^{-1} b. \end{aligned} \tag{9}$$

By caching factorization of AA^T, subsequent projections are much cheaper than the first one. ADMM for basis pursuit can been interpreted as reducing the solution of a least ℓ_1. In the paper, the test bearing was used to study only included one kind of surface fault: the bearing was damaged on the outer, sampling frequency is 20 KHz, signal series is 1024 sampling data, original signal waveform is shown as Figure 1(a). Sparse by FFT, DCT, WDT, the reconstruction signal of ADMM for basis pursuit are shown as Figures 1(b), 1(c), and 1(d).

The updates step of ADMM for basis pursuit is shown in Figure 2. x-axis is the number of iteration steps, and y-axis is the value of object function $f(x^k) + g(z^k)$. When the object function is enough small, exit update process, the result is optimal solution.

Figures 3 and 4 show a signal reconstruction by dual interior point method and OPM, respectively. □

3. Sparse Basis Selection during Reconstruction of ADMM

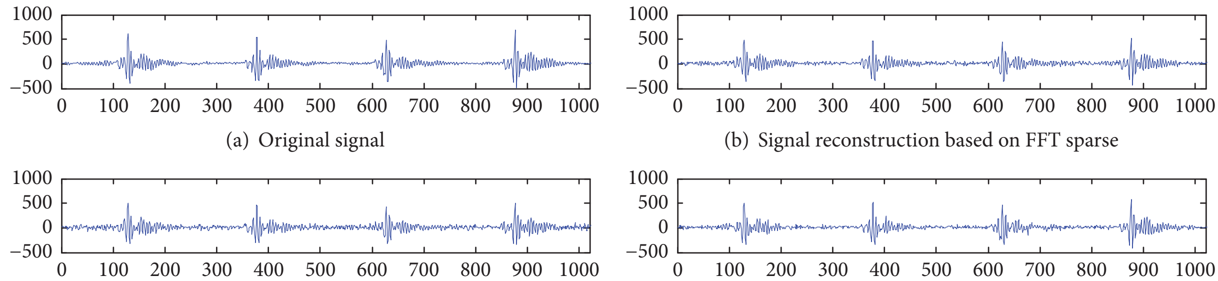
When we choose the different sparse bases during the process of reconstruction of ADMM, the error would be different and we compare them in two ways: central tendency and dispersion tendency.

(A) One of the measurement indexes of central tendency is arithmetic average and the equation is

$$\bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{n} = \frac{\sum x}{n}, \tag{10}$$

where $x_1, x_2, \ldots, x_n$ are the values of variables.

(B) Another measurement index of central tendency is median. The median of a finite list of numbers can be



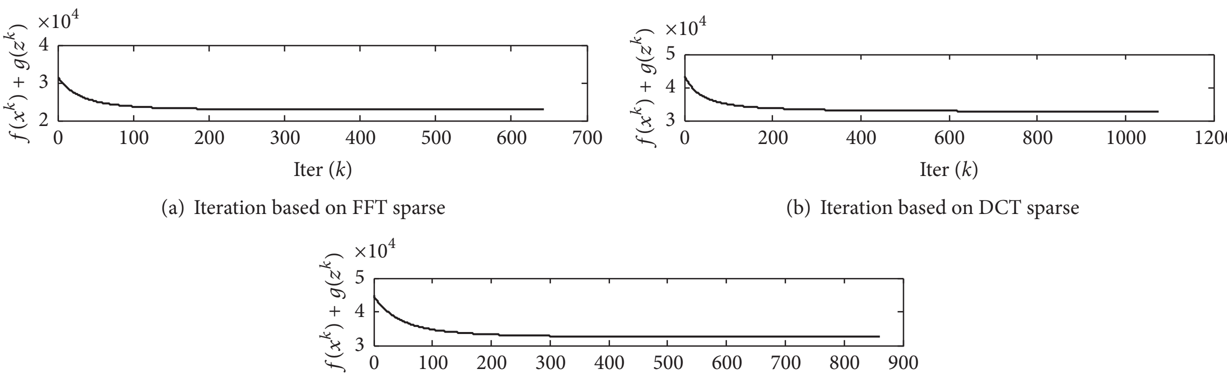
(a) Original signal

(b) Signal reconstruction based on FFT sparse

(c) Signal reconstruction based on DCT sparse

(d) Signal reconstruction based on DWT sparse

Figure 1: Reconstruction on ADMM for basis pursuit.

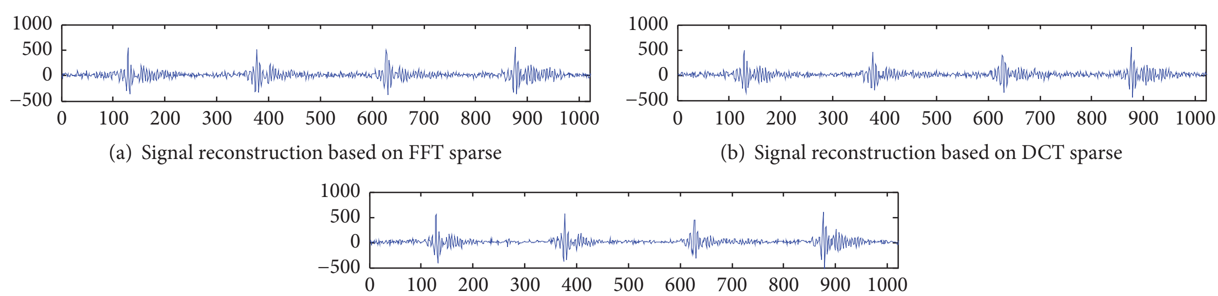


(a) Iteration based on FFT sparse

(b) Iteration based on DCT sparse

(c) Iteration based on DWT sparse

Figure 2: Iteration of ADMM for basis pursuit.

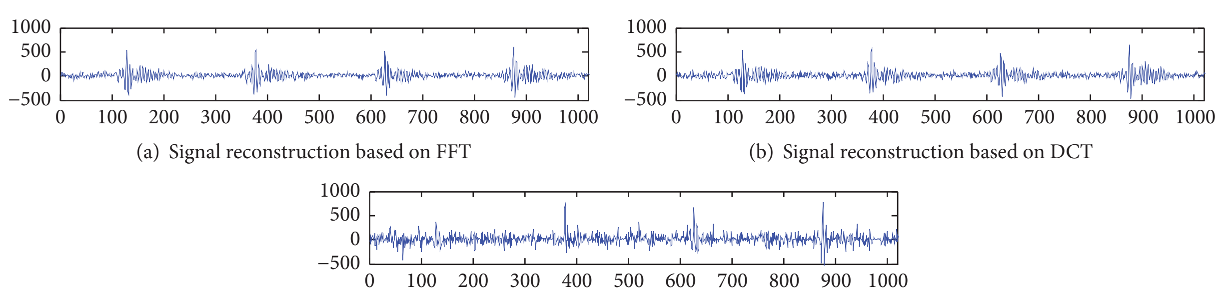


(a) Signal reconstruction based on FFT sparse

(b) Signal reconstruction based on DCT sparse

(c) Signal reconstruction based on DWT sparse

Figure 3: Signal reconstruction by dual interior point.

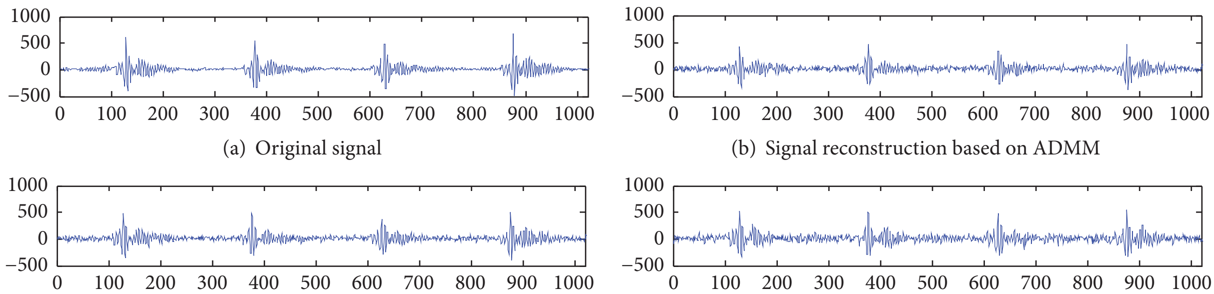


(a) Signal reconstruction based on FFT

(b) Signal reconstruction based on DCT

(c) Signal reconstruction based on DWT

Figure 4: Original signal reconstruction by OPM.



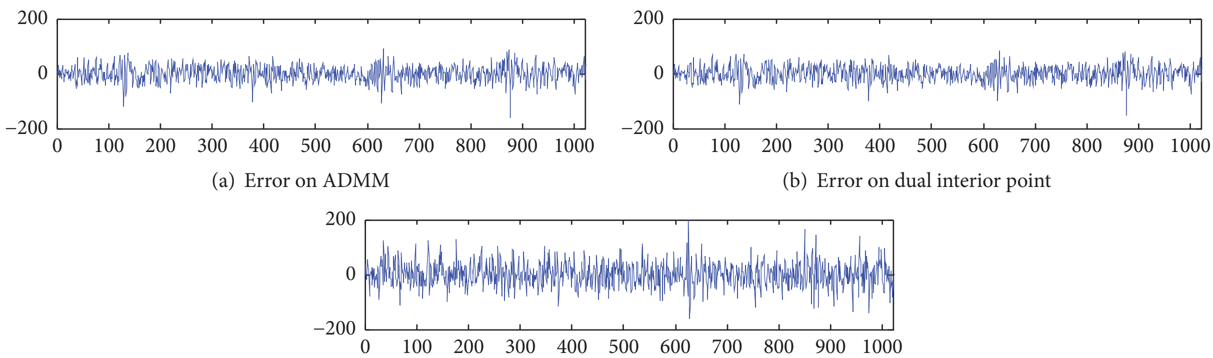
(a) Original signal

(b) Signal reconstruction based on ADMM

(c) Signal reconstruction based on dual interior point

(d) Signal reconstruction based on OMP

Figure 5: Contrast waveform of three reconstruction algorithms.



(a) Error on ADMM

(b) Error on dual interior point

(c) Error on OMP

Figure 6: Error contrast waveform of three reconstruction algorithms.

found by arranging all the observations from lowest value to highest value and picking the middle one. The numbers of lower values are equal to the higher values in general and the most common way to median is the direct method which arranges the data from large to small. The equations are as follows:

$$n \text{ is odd: } M = X_{((n+1)/2)},$$
$$n \text{ is even: } \frac{\left(X_{n/2} + X_{(n/2+1)}\right)}{2}. \tag{11}$$

(C) Dispersion tendency (variation index) reflects the difference between each individual value. The greater the degree of data separation, the greater the variation index. The dispersion tendency indexes include range, variance and standard deviation which could measure the error of reconstruction. Among them, range is received by the difference value of the maximum and the minimum, so it reflects the difference of the overall scope.

From Table 1, we can see that the reconstruction error of the DCT of vibration signal is closest to zero. From the dispersion tendency, the reconstruction error of FFT shows little batch difference, while it has the abnormal value compared with DWT which is instability. The results of DCT are in between.

Table 1: The error trend indicator during reconstruction of ADMM.

Category	FFT	DCT	DWT
Mean	0.3035	$8.48E-14$	$8.48E-14$
Median	−0.4562	−0.4562	−0.4562
Range	273.9749	230.0274	216.3107
Variance	477.3905	699.6706	807.7146
Standard deviation	21.8493	26.4513	28.4203

4. The Comparison between the ADMM and Other Algorithms

We compare the ADMM with primal-dual interior point algorithm and orthogonal matching pursuit (OMP) in the time complexity and error which reflect the effect of reconstruction. The results are shown in Table 2.

Figure 5 shows the effect contrast of different restructuring algorithms.

From the error indicator of Table 2, in the centralized tendency, the mean and median of ADMM, respectively, are $1.85967e-13$ and −0.2223, while the OMP are $1.2141e-7$ and 0.1803 and the primal-dual interior point algorithms are −0.0330 and −0.5372. All of them are close to zero and the values of ADMM are the least. It means that the effect of refactoring of ADMM is the best and primal-dual interior point algorithm is worst in the centralized tendency. Figure 6 shows the error contrast waveform of three reconstructions.

Table 2: The comparison between the ADMM and other algorithms.

Evaluation criteria	ADMM	OMP	Primal-dual interior point algorithm
Execution time (s)	1.784414	23.718855	2.310212
Mean	$1.86E-13$	$1.21E-07$	−0.033
Median	−0.2223	0.1803	−0.5372
Range	248.0199	244.985	306.9224
Variance	737.01444	748.0127	$1.48E-13$
Standard deviation	27.148	27.3498	38.4285

Although the range of ADMM is not the minimum of three, the variance and the standard deviation are quite small which indicate the superiority of ADMM. So, ADMM possesses the good performance in sparse reconstruction compared with primal-dual interior point algorithm and OMP.

5. Conclusion

In the aspect of error, the mean of ADMM is the minimum and the median is after OMP in view of the central tendency. It indicates the reconstruction error of ADMM is the best and the OMP takes second place. And in view of the dispersion tendency, despite the range of ADMM is not the minimum of three, the variance and the standard deviation are quite small. So, ADMM possesses the good performance in sparse reconstruction compared with primal-dual interior point algorithm and OMP. The insufficient of the algorithm is the present of abnormal value which is not obvious through range. As a result, ADMM sparse optimization algorithm to deal with the problem of sparse reconstruction in compressed sensing has good performance.

Competing Interests

The author declares that they have no competing interests.

Acknowledgments

This research is supported by Natural Science Foundation of Shanghai under the Project Grant nos. 14ZR1418500 and 14ZR1418400.

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