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Research Article

Linear Programming and Its Application Techniques in Optimizing Portfolio Selection of a Firm

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Optimization techniques have been used in this paper to obtain an optimal investment in a selected portfolio that gives maximum returns with minimal inputs based on the secondary data supplied by a particular firm that is examined. Sensitivity analysis is done to ascertain the robustness of the resulting model towards the changes in input parameters to determine a redundant constraint using linear programming. The challenge of determining the available funds and allocating each component of the portfolio to maximize returns and minimize inputs by portfolio holders and managers who are the major decision-makers in allocating their resources cannot be quantified. This optimization technique is used to obtain an optimal investment portfolio including financial risks of a firm with disposable of $15,000,000.00 invested in crude oil, mortgage securities, cash crop, certificate of deposit, fixed deposit, treasury bills, and construction loans. The model is a single-objective model that maximizes the return on the portfolio as the interests on the original data reduces by 5%; then, the return on investments also reduced by almost 15%, with the quantum of money on treasury bills and construction loans posing a significant reduction for the maximum return. The investment in the other options saw a slight decrease. Also, as the interest rates of the original data increase by 5%, the return on investments also grows by almost 17% while the quantum of money on the treasury bills and construction loans increases, and the quantum of money on the other options experienced a decrease except for mortgage securities which recorded a slight increase.

1. Introduction

A portfolio is an individual or corporate investment that can be managed by financial professionals or financial institutions, such a portfolio may include financial assets, stocks, bonds, and cash held and/or managed by an individual investor. It is designed according to the investor's risk tolerance, time frame, and investment objectives. Papahristodoulou and Dotzauer [1] defined and discussed *optimal* as the best or most favorable among a set of alternatives and defined the *optimal portfolio* as the portfolio that considers the investor's own "greed" and risk aversion. The challenge of portfolio optimization is an important aspect in investment and finance, and this may affect portfolio holders and managers who are the major decision-makers in allocating their resources across different categories. Protection against the risks related to everyday life is indispensable to every human being to enjoy some level of peace of mind. Therefore, this paper wishes to determine the availability of funds to be allocated to each component of the portfolio in order to maximize returns on the portfolio and to determine optimally the possible levels of investment in the selected portfolios of the firm under discussion. An emphasis on return maximization is considered and addresses the areas of investment, the investment constraints of the firm, and the levels of investments to maximize returns.

2. Literature Review

Meng and Yang [2] discussed various linear programming applications and techniques. The authors reported that aggregate production planning is the most important aspect of linear programming analysis. Young [3] emphasized that

the optimization process adopted mathematical techniques to generate programs for training timetables and schedules for military application as was developed in the 1940s by George Dantzig. Markowitz [4] discussed the financial research and investment aspect of the portfolio optimization problem as one of the standard and most important aspects of portfolio management. An elegant way of managing risk in financial markets known as the portfolio theory was introduced in the study. Konno and Yamazaki [5] discussed Markowitz's model and assessed it as a single-period model where the investor's main objective was to maximize the portfolio's expected return. William [6] introduced and discussed the Capital Asset Pricing Model (CAPM) based upon the empirical observation that gives the maximum expected return. The author compared and analyzed Markowitz's model to develop a simplified variant that reduces data and computational requirements. Such an empirical fact was analyzed, discussed, and supported by William [7].

Also, Ogryczak [8] discussed and analyzed the procedures and guidelines of setting investment policy. An asset allocation model was proposed and developed where the expected return for an asset class will be estimated using the simplex algorithm as an application of linear programming. Kostreva and Ogryczak [9] discussed and explained tracking through a benchmark by the portfolio manager to minimize the bound constraint sets from the volatility of the portfolio return. It was mentioned that if the benchmark is volatilized, then the volatility is bounded and most studies will focus on the price efficiency of equity markets. Marcus [10] presented and analyzed price efficiency as a market where all available information that is relevant to the valuation of securities at all times is fully reflected by the price. Olayinka et al. [11] examined and applied linear programming techniques in the entrepreneur decision-making process to maximize profits with the available resources through a fast-food firm with the challenges of product selection and profit maximization due to an increase in the price of raw materials. Oladejo et al. [12] considered the importance of optimization principles in maximized profits and minimized cost of production and applied linear programming techniques in solving a particular challenge in a bakery production firm using the AMPL software.

Thus, in this paper, we examine the level of investment in a selected portfolio that gives maximum returns with minimal input based on the secondary data supplied by a particular firm, which were used as the parameters for the proposed linear programming model. This study has not been previously examined, and it has created gaps in portfolio management and optimization of a firm. The study is motivated by the earlier reports on the portfolio optimization and risk management of firms. A sensitivity analysis to ascertain the robustness of the resulting model towards the changes in input parameters to determine how redundant a constraint was for linear programming is carried out. The availability of funds and the allocation of each component of the portfolio to maximize returns and minimize inputs by portfolio holders and managers who are the major decision-makers in allocating their resources were also determined following Olayinka et al.'s [11] and Oladejo et al.'s [12] linear programming techniques.

3. Formulation of the Linear Programming (LP) Model

The following general LP model was considered:

$$\begin{aligned} &\text{Optimize}: f(x), \\ &\text{Subject to}: \left.\begin{array}{ll} g_i(x) \le b_i, & 1 \le i \le p \\ g_i(x) = b_i, & p+1 \le i \le k \\ g_1(x) \ge b_i, & k+1 \le i \le n \end{array}\right\}, \end{aligned} \tag{1}$$

where $f(x)$ is the objective function of a vector variable. $x = (x_1, x_2, \cdots, x_n)^T$ represents the measure of effectiveness of a decision. $g_i(x)\ (1 \le i \le m)$ is the constraint function of x. The variable $x_j\ (j = 1, 2, \cdots, n)$ is the activity level associated with the decision-making problem. The term $b_\text{i}\ (1 \le i \le m)$ represents the upper or lower limit of the ith constraint functions. Constraint $x \ge 0$ restricts the decision variables $x_j\ (j = 1, 2, \cdots, n)$ to nonnegative real numbers.

Since the objective and constraint functions are linear, they are precisely defined in the form

$$f(x) = c_1 x_1 + c_2 x_2 + \cdots + c_n x_n = \sum_{j=1}^{n} c_j x_j, \tag{2}$$

$$g_i(x) = a_{i1} x_1 + \cdots + a_{\text{in}} x_n = \sum_{j=1}^{n} a_{ij} x_j. \tag{3}$$

From (2) and (3), we then write the linear programming as

$$\text{Optimize}: \quad Z = \sum_{j=1}^{n} c_j x_j, \tag{4}$$

$$\text{Subject to}: \left.\begin{array}{ll} \sum_{j=1}^{n} a_{ij} x_j, & 1 \le i \le p \\ \sum_{j=1}^{m} a_{ij} x_j, & p+1 \le i \le k \\ \sum_{j=1}^{n} a_{ij} x_j, & k+1 \le i \le m \\ x_j \ge 0, & 1 \le j \le n. \end{array}\right\}, \tag{5}$$

where a_{ij} and c_j are called technological and cost coefficients, respectively. b_i is the main parameters of the models.

Equations (4) and (5) can be expressed in the forms of the matrix:

$$\text{Optimize}: \quad Z = C^T x, \qquad \text{Subject to}: \left. \begin{array}{l} Ax \le b \\ x \ge 0 \\ C = (c_1, c_2, \cdots, c_n)T \end{array} \right\}, \tag{6}$$

where K is a feasible set standard LP while $K = \{x \in R^n : Ax = b, x \ge 0\}$. If $x \in K$ or if x satisfies $Ax = b$ and $x \ge 0$, then x is a feasible solution. Let $C^T x$ be the objective function of a LP to be optimized. $x \in K$ is an optimal solution if for all $y \in K$, $C^T x > C^T y$. Let x be a basic solution of $Ax = b$. If $x \ge 0$, then it is called a basic feasible solution (BFS). An artificial variable $A_i \ge 0$ is a dummy variable added for the specific purpose of generating an initial basic feasible solution. It has no economic meaning.

Thus,

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} & \cdots & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & a_{n3} & \cdots & \cdots & a_{mn} \end{pmatrix}, \tag{7}$$
$$b = (b_1, b_2, b_3, \cdots, b_m)^T.$$

4. Materials and Methods

In this study, a firm with more than one investment opportunity or how much of its capital should be channeled into each investment opportunity to maximize profits is considered. The firm is also facing constraints on the availability of the inputs to be used in its investment activities and management policies.

The firm under consideration has, at its disposal, fifteen million dollars ($15,000,000.00) for the period considered to invest among seven (7) investment options with the aim of maximizing returns. After identifying the preferred areas of investment, the distribution of available funds among each investment area to derive the maximum return under an acceptable level of risk becomes a major challenge. The decision for the distribution of funds to the various investments should not be done arbitrarily, since there is no guarantee of achieving the goal of maximum return. A scientific approach to the problem therefore is the best way forward. The firm, according to its own records, does not adopt any scientific procedure for its investment decision-making and thus is not able to tell whether it has been making the desired return on its investments or not over the past years. The decision to go scientific for the period under consideration was informed by the changing trends in the world of business.

Table 1: Investment areas, interest rates, and risk scores.

Investment	Interest rate (%)	Risk score (%)
Fixed deposit	10.0	1.7
Treasury bills	20.5	2.5
Crude oil	20.0	1.5
Mortgage	8.0	1.9
Construction	28.5	2.9
Cashew nuts	18.5	1.5
Forex trading	9.5	2.2

Table 1 shows the summary of the investment options identified by the firm to invest the available funds to achieve its aim. These are fixed deposit, treasury bills, crude oil, mortgage, construction, cashew nuts, and Forex trading.

4.1. Evaluation of Investment Risk. The level of risk (i.e., risk score) associated with each investment is obtained by taking the average return (in percentage) over at least the past five (5) years. The mean (average) of the returns on the investment for the period considered is calculated, the sum of the squares of the deviations is divided by the number of years considered minus one, and the square root of the result is taken.

4.2. Constraints of the Linear Programming Problem. The following six (6) constraints associated with the linear programming problem are considered:

(1) Invest up to $15M in the entire investment options. The firm is constrained not to go beyond investing $15M in the entire seven investment areas identified. However, investing less than $15M is allowed. Thus, at the end of distributing the available funds among the seven investments, the total distribution should not exceed $15M

(2) Not more than 20% of the total investment in any one investment area. Investment in any one of the seven investment areas should not exceed 20% of the total investment. Thus, the quantum of money that is supposed to go into any one of the seven investments can be less than 20% of the total money for entire investments (i.e., ≤20% of $15,000,000.00)

(3) At least 25% of the total investment in deposits. The firm is not allowed to commit less than 25% of the total amount of money into deposits (i.e., fixed deposit and crude oil). Hence, the firm is to commit not less than 25% of $15M into fixed deposit and Forex trading

(4) At least 30% of the total investment in cash crops. The firm is not allowed to commit less than 30% of the total amount of money into cash crops (cashew

nuts). Hence, the firm is to commit not less than 30% of \$15M for cashew nuts

(5) At least 45% of the total investment in treasury bills and construction loans. The firm is not allowed to commit less than 45% of the total amount of money into treasury bills and construction loans. Hence, the firm is to commit not less than 45% of \$15M into treasury bills and construction loans

(6) The overall risk should not be more than 2.0% of the portfolio risk, calculated using the weighted average, which should be equal to or less than 2.0%. Short-term treasury bills (six months) are risk-free, while long-term treasury bills (more than six months) are risky when the prime rates are reviewed upwards

4.3. Modeling the Problem as LP. Here, the formulation of the model by defining the decision variables, the objective and constraint functions are presented. Let the following be the decision parameters:

(i) x_A be the amount to invest in fixed deposit

(ii) x_B be the amount to invest in treasury bills

(iii) x_C be the amount to invest in crude oil

(iv) x_D be the amount to invest in mortgage

(v) x_E be the amount to invest in construction

(vi) x_F be the amount to invest in cash crop

(vii) x_G be the amount to invest in Forex trading

The objective function in terms of the decision variables is as follows:

$$\text{Maximize}: Z = 0.1x_A + 0.205x_B + 0.2x_C + 1x_D + 0.3x_E + 0.2x_F + 0.095x_G, \tag{8}$$

Individual constraints identified are expressed quantitatively as follows:

Let $Y = (x_A + x_B + x_C + x_D + x_E + x_F + x_G)$:

(a) For an investment of \$15M, then $Y \leq 15\text{M}$

(b) Not more than 20% of the total investment in a particular investment

Thus, $x_A \leq 0.2(Y)$, $x_B \leq 0.2(Y)$, $x_C \leq 0.2(Y)$, $x_D \leq 0.2(Y)$, $x_E \leq 0.2(Y)$, $x_F \leq 0.2(Y)$, and $x_G \leq 0.2(Y)$.

This implies

$$x_A = x_B = x_C = x_D = x_E = x_F = x_G \leq 0.2(Y), \tag{9}$$

which can be equivalently expressed as follows:

$$\begin{gathered}
0.2(4x_A - x_B - x_C - x_D - x_E - x_F - x_G) \leq 0, \\
0.2(x_A - 4x_B + x_C + x_D + x_E + x_F + x_G) \geq 0, \\
0.2(x_A + x_B - 4x_C + x_D + x_E + x_F + x_G) \geq 0, \\
0.2(x_A + x_B + x_C - 4x_D + x_E + x_F + x_G) \geq 0, \\
0.2(x_A + x_B + x_C + x_D - 4x_E + x_F + x_G) \geq 0, \\
0.2(x_A + x_B + x_C + x_D + x_E - 4x_F + x_G) \geq 0, \\
0.2(x_A + x_B + x_C + x_D + x_E + x_F - 4x_G) \geq 0.
\end{gathered} \tag{10}$$

(c) At least 25% of the total investment into deposits, $(x_A + x_G)/(x_A + x_B + x_C + x_D + x_E + x_F + x_G) \geq 0.25$

$$\begin{gathered}
x_A + x_G \geq 0.3(x_A + x_B + x_C + x_D + x_E + x_F + x_G) \sim 0.8x_A - 0.3x_B \\
- 0.3x_C - 0.3x_D - 0.3x_E - 0.3x_F + 0.8x_G \geq 0.
\end{gathered} \tag{11}$$

(d) At least 30% of the total investment to cash crops, $(x_C + x_F)/(x_A + x_B + x_C + x_D + x_E + x_F + x_G) \geq 0.3$

$$\begin{gathered}
x_C + x_F \geq 0.3((x_A + x_B + x_C + x_D + x_E + x_F + x_G) \sim 0.3x_A + 0.3x_B \\
+ 0.7x_C + 0.3x_D + 0.3x_E + 0.7x_F + 0.3x_G \leq 0.
\end{gathered} \tag{12}$$

(e) At least 45% of the total investment in treasury bills and construction loans, $(x_B + x_E)/(x_A + x_B + x_C + x_D + x_E + x_F + x_G) \geq 0.5$

$$\begin{gathered}
x_B + x_E \geq 0.5(x_A + x_B + x_C + x_D + x_E + x_F + x_G) \sim 0.5x_A + 0.5x_B \\
+ 0.5x_C + 0.5x_D - 0.5x_E + 0.5x_F + 0.5x_G \leq 0.
\end{gathered} \tag{13}$$

(f) Overall risk limited to not more than 2.0 (using a weighted average to calculate portfolio risk), $(1.7x_A + 2.5x_B + 1.5x_C + 1.9x_D + 2.9x_E + 1.5F + 2.2x_G)/(x_A + x_B + x_C + x_D + x_E + x_F + x_G) \leq 2.0$

$$\begin{gathered}
1.7x_A + 2.5x_B + 1.5x_C + 1.9x_D + 2.9x_E + 1.5x_F + 2.2x_G \\
\leq 2(x_A + x_B + x_C + x_D + x_E + x_F + x_G) \\
\sim 0.3x_A - 0.5x_B + 0.5x_C + 0.1x_D - 0.9x_E \\
+ 0.5x_F - 0.2x_G \geq 0.
\end{gathered} \tag{14}$$

(g) Nonnegativity of decision variables $x_A, x_B, x_C, x_D, x_E, x_F, x_G \geq 0$

These are then transformed into the linear program model as follows:

Maximize:

$$Z = 0.1x_A + 0.2x_B + 0.2x_C + 0.1x_D + 0.3x_E + 0.2x_F + 0.1x_G, \tag{15}$$

Table 2: The simplex method and MATLAB solution in finding the optimal investment in a portfolio mix for the firm.

Basis	x_A 0.12	x_B 0.17	x_C 0.17	x_D 0.13	x_E 0.21	x_F 0.10	x_G 0.10	b 0	Chk. 1.00	MATLAB solution ($)
Fixed deposit	1	1	1	1	1	1	1	15	22	2,674,300
Treasury bill	0.8	-0.2	-0.2	-0.2	-0.2	0.2	0.2	0	0.4	2,554,800
Crude oil	0.2	-0.8	0.2	0.2	0.2	0.2	0.2	0	0.4	1,974,800
Mortgage	0.2	0.2	-0.8	0.2	0.2	0.2	0.2	0	0.4	813,480
Construction	0.2	0.2	0.2	-0.8	0.2	0.2	0.2	0	0.4	2,407,320
Cash crops	0.2	0.2	0.2	0.2	-0.8	0.2	0.2	0	0.4	2,674,800
Cert. deposit	0.2	0.2	0.2	0.2	0.2	-0.8	0.2	0	0.4	1,900,500
Tot. investment	0.2	0.2	0.2	0.2	0.2	0.2	-0.8	0	0.4	
15% investment	0.8	0.2	0.2	0.2	-0.2	-0.2	0.9	0	1.8	
20% deposit	0.8	0.2	-0.8	0.2	0.2	-0.8	0.2	0	0.0	
25% cash crop	0.8	0.3	-0.8	0.3	0.3	0.8	-0.8	0	0.8	
40% treasury and construction	0.6	0.4	-0.6	0.4	-0.6	0.6	0.4	0	1.2	
Max. return										2,553,096

Subject to:

$$
\begin{gathered}
x_A + x_B + x_C + x_D + x_E + x_F + x_G \leq 15\text{M}, \\
0.2(4x_A - x_B - x_C - x_D - x_E - x_F - x_G) \leq 0, \\
0.2(x_A - 4x_B + x_C + x_D + x_E + x_F + x_G) \geq 0, \\
0.2(x_A + x_B - 4x_C + x_D + x_E + x_F + x_G) \geq 0, \\
0.2(x_A + x_B + x_C - 4x_D + x_E + x_F + x_G) \geq 0, \\
0.2(x_A + x_B + x_C + x_D - 4x_E + x_F + x_G) \geq 0, \\
0.2(x_A + x_B + x_C + x_D + x_E - 4x_F + x_G) \geq 0, \\
0.2(x_A + x_B + x_C + x_D + x_E + x_F - 4x_G) \geq 0, \\
0.8x_A - 0.3x_B - 0.3x_C - 0.3x_D - 0.3x_E - 0.3x_F + 0.8x_G \geq 0, \\
0.3x_A + 0.3x_B - 0.7x_C + 0.3x_D + 0.3x_E - 0.7x_F + 0.3x_G \leq 0, \\
0.5x_A - 0.6x_B + 0.5x_C + 0.5x_D - 0.5x_E + 0.5x_F + 0.5x_G \leq 0, \\
0.3x_A - 0.5x_B + 0.5x_C + 0.1x_D - 0.9x_E + 0.5x_F - 0.2x_G \geq 0, \\
x_A, x_B, x_C, x_D, x_E, x_F, x_G \geq 0.
\end{gathered} \tag{16}
$$

5. Results

5.1. Solving the LP Problem Using MATLAB. Table 2 shows the linear program results taken from Table 1 and the model formulated. The solutions are obtained using the simplex method with the help of MATLAB Solver.

5.2. Sensitivity Analysis. Here, we test the stability and robustness of the model by slightly changing the coefficients to determine the redundancy of the constraints. This would help to reduce errors in decision-making. The interest rate on each investment is reduced by 5% and later increased by 5%, the resulting LP problem is solved, and the solution to the real LP problem as compared to the original LP problem is shown in Table 3.

The problem is then formulated as an LP problem as follows:

Maximize:

$$
\begin{aligned}
Z &= 0.11x_A + 0.16x_B + 0.17x_C + 0.13x_D + 0.19x_E \\
&\quad + 0.10x_F + 0.10x_G \text{ (5\%reduction)}, \\
T &= 0.13x_A + 0.18x_B + 0.18x_C + 0.14x_D + 0.22x_E \\
&\quad + 0.11x_F + 0.11x_G \text{ (5\%increment)}, \\
Y &= 0.12x_A + 0.17x_B + 0.13x_C + 0.21x_D \\
&\quad + 0.10x_E \text{ (no treasury bill)}.
\end{aligned} \tag{17}
$$

Subject to:

$$
\begin{gathered}
x_A + x_B + x_C + x_D + x_E + x_F + x_G \leq 15\text{M}, \\
0.2(4x_A - x_B - x_C - x_D - x_E - x_F - x_G) \leq 0, \\
0.2(x_A - 4x_B + x_C + x_D + x_E + x_F + x_G) \geq 0, \\
0.2(x_A + x_B - 4x_C + x_D + x_E + x_F + x_G) \geq 0, \\
0.2(x_A + x_B + x_C - 4x_D + x_E + x_F + x_G) \geq 0, \\
0.2(x_A + x_B + x_C + x_D - 4x_E + x_F + x_G) \geq 0, \\
0.2(x_A + x_B + x_C + x_D + x_E - 4x_F + x_G) \geq 0, \\
0.2(x_A + x_B + x_C + x_D + x_E + x_F - 4x_G) \geq 0, \\
0.2(4x_A - x_B - x_C - x_D - x_E - x_F + 4x_G) \geq 0, \\
0.2(4x_A + x_B - 4x_C + x_D + x_E - 4x_F + x_G) \leq 0, \\
0.8x_A - 0.3x_B + 0.8x_C + 0.3x_D - 0.3x_E + 0.8x_F + 0.8x_G \leq 0, \\
0.6x_A - 0.4x_B + 0.6x_C + 0.4x_D - 0.6x_E + 0.6x_F - 0.4x_G \geq 0, \\
x_A, x_B, x_C, x_D, x_E, x_F, x_G \geq 0.
\end{gathered} \tag{18}
$$

Table 3: The sensitivity analysis results of the portfolios in linear programming.

Types of investment	Normal interest rate (%)	5% reduction		5% increment		Investment without treasury bill		Risk score (%)
		%	Output	%	Output	%	Output	
Fixed deposit	12.0	11.4	2,399,800	12.5	1,808,032	12.0	3,000,000	1.0
Treasury bills	17.0	16.15	1,874,300	18.0	3,674,300	17.0	3,000,000	2.0
Crude oil	17.0	16.71	2,129,800	18.0	1,974,300	13.0	2,700,000	1.0
Mortgage	13.0	12.95	2,207,320	13.65	813,768	21.0	3,000,000	1.0
Construction	21.0	19.95	1,013,480	22.05	3,155,800	10.0	3,300,000	2.0
Cashew nuts	10.0	9.5	2,674,800	10.5	1,674,800	—	—	1.0
Forex trading	10.0	9.5	2,700,500	10.5	1,900,000	—	—	2.0
Maximum return			2,170,132		2,983,096		1,895,092	

Table 4: Comparison of results of previous studies with the present study.

	Chicken pie	Meat pie	Doughnut	Resources	Olayinka et al. (2015)	Present results
Flour	100	50	100	1000	4	4.2333
Margarine	50	20	40	600	200	200.0045
Eggs	20	30	40	1600	1000	1000.01345
Contribution	40	50	70			
Profit					1000	1000.02124

Table 5: The result and summary of the solutions.

Investments	Rate on investment (%)		Rate on investment down by 5%		Rate on investment increased by 5%		Rate on investment (without treasury bills and construction loans) (%)	
	Rate of investment (%)	Money invested	Rate of investment (%)	Money invested	Rate of investment (%)	Money invested	Rate of investment (%)	Money invested
Fixed deposit	12.0	2,674,300	11.4	2,399,800	12.5	1,808,032	12.0	3,000,000
Treasury bills	17.0	2,554,800	16.15	1,874,300	18.0	3,674,300	—	—
Crude oil	17.0	1,974,800	16.71	2,129,800	18.0	1,974,300	17.0	3,000,000
Mortgage	13.0	813,480	12.95	2,207,320	13.65	813,768	13.0	2,700,000
Construction	21.0	2,407,320	19.95	1,013,480	22.05	3,155,800	—	
Cashew nuts	10.0	2,674,800	9.5	2,674,800	10.5	1,674,800	21.0	3,000,000
Forex trading	10.0	1,900,500	9.5	2,700,500	10.5	1,900,000	10.0	3,300,000
Return on investment ($)		2,553,096		2,170,132		2,983,096		1,895,092

In Table 4, the comparison of results is performed to establish the correctness and validate the present study. A good agreement in both results is seen and presented in the table.

6. Discussion of Results

From Table 5, as the interest rate on the original data reduces by 5%, the return on investment also decreases by almost 15% (i.e., from $2,553,096.55 to $2,170,132.068). The quantum of money on treasury bills and construction loans had a significant reduction from $2,554,800.00 to $1,874,300.00 and from $2,407,320.00 to $1,013,480.00, respectively, for maximum return. However, investment in crude oil, mortgage, securities, cash crop, and certificate of deposit appreciated from $1,974,800.00 to $2,129,800.00, from $813,480.00 to $2,207,320.00, from $2,674,800.00 to $2,674,800.00, and from $1,900,500.00 to $2,7005,00.00, respectively. However, fixed deposit had a slight decrease from $2,674,300.00 to $2,399,800.00. Furthermore, as the interest rate of the original data increases by 5%, the return on investment also grows by almost 17% (i.e., from $2,553,096.55 to $2,983,096.55) with the quantum of money on treasury bills and construction loans being the most affected in terms of increment, that is, increasing from $2,554,800.00 to $3,674,300.00 and from $2,407,320.00 to $3,155,800.00, respectively. However, the

quantum of money on fixed deposit, crude oil, cash crop, and certificate of deposit experienced a decrease from \$2,674,300.00 to \$1,808,032.00, from \$1,974,800.00 to \$1,974,300.00, from \$2,674, 800.00 to \$1,674,800.00, and from \$1,900,500.00 to \$1,900,000.00, respectively, while mortgage securities recorded a slight increase from \$813,480.00 to \$813,768.00, while treasury bills and construction loans are exempted from the portfolio, and the quantum of money on each investment increases rapidly while the return on investments reduces to \$1,895,092.65.

7. Conclusion

This paper investigates the level of investment in a selected portfolio that gives maximum returns with minimal inputs based on the secondary data supplied by a particular firm to determine the available funds to allocate each component of the portfolio to maximize returns and minimize inputs using linear programming. Sensitive analysis to ascertain the robustness of the resulting model towards the changes in input parameters to determine a redundant constraint was conducted. The model given is a single-objective model that maximizes the return on the portfolio as interests on the original data reduce by 5%, the return on investments also goes down by almost 15%, and the quantum of money on treasury bills and construction loans saw a significant reduction for maximum return while the investment in others options saw a slight decrease. Moreover, as the interest rate of the original data increases by 5%, the return on investment also grows by almost 17% while the quantum of money on treasury bills and construction loans increases, and the quantum of money on other options experienced a decrease except for mortgage securities which recorded a slight increase.

The results of this investigation are much applicable to businesses and portfolio management. The present study can be extended to small-scale and large-scale businesses and investments in maximizing the profit of some selected portfolios and rightful decision-making.

Data Availability

This is an open-access article.

Conflicts of Interest

The authors hereby stated and declared that there is no conflict of interest in the preparation and publication of this manuscript.

References

[1] C. Papahristodoulou and E. Dotzauer, "Optimal portfolios using linear programming models," *Journal of Operational Research Society*, vol. 55, no. 11, pp. 1169–1177, 2017.

[2] Q. Meng and H. Yang, "Benefits distribution and equity in road network," *Journal of Transportation Research*, vol. 36, no. 1, pp. 19–35, 2000.

[3] M. R. Young, "A minimax-portfolio selection rule with linear programming solution," *Journal of Management Science*, vol. 44, pp. 673–683, 1998.

[4] H. Markowitz, "Portfolio selection," *Journal of Finance*, vol. 7, pp. 77–91, 1952.

[5] H. Konno and H. Yamazaki, "Mean-absolute deviation portfolio optimization model and its applications to Tokyo stock market," *Journal of Management Science*, vol. 37, no. 5, pp. 519–531, 1991.

[6] S. William, "Capital asset prices - a theory of market equilibrium under conditions of risk," *Journal of Finance*, vol. 19, no. 3, pp. 425–442, 1964.

[7] S. William, "A linear programming approximation for the general portfolio analysis problem," *Journal of Financial and Quantitative Analysis*, vol. 6, no. 5, pp. 1263–1275, 1971.

[8] W. Ogryczak, "Multiple criteria linear programming model for portfolio selection," *Annals of Operations Research*, vol. 97, no. 1, pp. 143–162, 2000.

[9] M. M. Kostreva and W. Ogryczak, "Linear optimization with multiple equitable criteria RAIRO," *Recherche opérationnelle*, vol. 33, no. 3, pp. 275–297, 1999.

[10] D. Marcus, "Portfolio optimization and linear programming," *Journal of Money, Investment and Banking*, vol. 20, pp. 271–277, 2011.

[11] I. Olayinka, A. K. Olusegun, G. Kellikume, and K. Kayode, "Entrepreneur decision making process and application of linear programming technique," *European Journal of Business, Economics and Accountancy*, vol. 3, no. 5, pp. 231–238, 2015.

[12] N. K. Oladejo, A. Abolarinwa, S. O. Salawu, and A. F. Lukman, "Optimization principle and its application in optimizing Landmark University Bakery production using linear programming," *International Journal of Civil Engineering and Technology*, vol. 10, no. 2, pp. 183–190, 2019.