

SUPPLEMENTARY MATERIAL

Iteration of the Parallelogram Law in a Phasor Diagram (see Figure-3 in main paper)

A. 1st Iteration of the Parallelogram Law with Two Phasors

$$A_1 = A_2 = A$$

$$\alpha_1 = \alpha$$

$$R_1 = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\alpha_1} = \sqrt{A^2 + A^2 + 2AA\cos\alpha} = \sqrt{4A^2\cos^2\left(\frac{\alpha}{2}\right)} = 2A\cos\left(\frac{\alpha}{2}\right)$$

$$\tan\beta_1 = \frac{A_2\sin\alpha_1}{A_1+A_2\cos\alpha_1} = \frac{A\sin\alpha}{A+A\cos\alpha} = \tan\left(\frac{\alpha}{2}\right) \Rightarrow \beta_1 = \frac{\alpha}{2}$$

B. 2nd Iteration of the Parallelogram Law with Three Phasors

$$A_1 = A_2 = A_3 = A$$

$$\alpha_1 = \alpha_2 = \alpha$$

$$\begin{aligned} R_2 &= \sqrt{R_1^2 + A_3^2 + 2R_1A_3\cos(\alpha_2 + \alpha_1 - \beta_1)} \\ &= \sqrt{4A^2\cos^2\left(\frac{\alpha}{2}\right) + A^2 + 2\left(2A\cos\left(\frac{\alpha}{2}\right)\right)A\cos\left(\alpha + \alpha - \frac{\alpha}{2}\right)} \\ &= \sqrt{4A^2\cos^2\left(\frac{\alpha}{2}\right) + A^2 + 2\left(2A\cos\left(\frac{\alpha}{2}\right)\right)A\cos\left(\alpha + \alpha - \frac{\alpha}{2}\right)} \\ &= \sqrt{A^2 + 4A^2\cos\left(\frac{\alpha}{2}\right)\left(\cos\left(\frac{\alpha}{2}\right) + \cos\left(\frac{3\alpha}{2}\right)\right)} \\ &= \sqrt{A^2 + 8A^2\cos\alpha\cos^2\left(\frac{\alpha}{2}\right)} \\ &= \sqrt{A^2 + 8A^2\cos\alpha\frac{(1+\cos\alpha)}{2}} \\ &= \sqrt{A^2 + 4A^2\cos\alpha + 4A^2\cos^2\alpha} \\ &= \sqrt{A^2(1 + 2\cos\alpha)^2} \\ &= A(1 + 2\cos\alpha) \end{aligned}$$

$$\begin{aligned} \tan\beta_2 &= \frac{A_3\sin(\alpha_2 + \alpha_1 - \beta_1)}{R_1 + A_3\cos(\alpha_2 + \alpha_1 - \beta_1)} = \frac{A\sin\left(\alpha + \alpha - \frac{\alpha}{2}\right)}{2A\cos\left(\frac{\alpha}{2}\right) + A\cos\left(\alpha + \alpha - \frac{\alpha}{2}\right)} = \frac{\sin\left(\frac{3\alpha}{2}\right)}{2\cos\left(\frac{\alpha}{2}\right) + \cos\left(\frac{3\alpha}{2}\right)} = \frac{3\sin\left(\frac{\alpha}{2}\right) - 4\sin^3\left(\frac{\alpha}{2}\right)}{2\cos\left(\frac{\alpha}{2}\right) + 4\cos^3\left(\frac{\alpha}{2}\right) - 3\cos\left(\frac{\alpha}{2}\right)} \\ &= \frac{(3 - 4\sin^2\left(\frac{\alpha}{2}\right))\sin\left(\frac{\alpha}{2}\right)}{(4\cos^2\left(\frac{\alpha}{2}\right) - 1)\cos\left(\frac{\alpha}{2}\right)} = \tan\left(\frac{\alpha}{2}\right) \Rightarrow \beta_2 = \frac{\alpha}{2} \end{aligned}$$

C. 3rd Iteration of the Parallelogram Law with Four Phasors

$$A_1 = A_2 = A_3 = A_4 = A$$

$$\alpha_1 = \alpha_2 = \alpha_3 = \alpha$$

$$\begin{aligned} R_3 &= \sqrt{R_2^2 + A_4^2 + 2R_2A_4 \cos(\alpha_3 + \alpha_2 + \alpha_1 - (\beta_2 + \beta_1))} \\ &= \sqrt{A^2(1 + 2\cos\alpha)^2 + A^2 + 2A(1 + 2\cos\alpha)A \cos\left(\alpha + \alpha + \alpha - \left(\frac{\alpha}{2} + \frac{\alpha}{2}\right)\right)} \\ &= \sqrt{A^2((1 + 2\cos\alpha)^2 + 1 + 2(1 + 2\cos\alpha) \cos 2\alpha)} \\ &= \sqrt{A^2(1 + 4\cos\alpha + 4\cos^2\alpha + 1 + 2 \cos 2\alpha + 4\cos\alpha \cos 2\alpha)} \\ &= \sqrt{A^2(2 + 4\cos\alpha + 4\cos^2\alpha + 2(2\cos^2\alpha - 1) + 4\cos\alpha(2\cos^2\alpha - 1))} \\ &= \sqrt{A^2(8\cos^2\alpha + 8\cos^3\alpha)} \\ &= \sqrt{8A^2\cos^2\alpha(1 + \cos\alpha)} \\ &= \sqrt{16A^2\cos^2\alpha \cos^2\left(\frac{\alpha}{2}\right)} \\ &= 4A \cos\alpha \cos\left(\frac{\alpha}{2}\right) \end{aligned}$$

$$\begin{aligned} \tan\beta_3 &= \frac{A_4 \sin(\alpha_3 + \alpha_2 + \alpha_1 - (\beta_2 + \beta_1))}{R_2 + A_4 \cos(\alpha_3 + \alpha_2 + \alpha_1 - (\beta_2 + \beta_1))} = \frac{A \sin\left(\alpha + \alpha + \alpha - \frac{\alpha}{2} - \frac{\alpha}{2}\right)}{A(1 + 2\cos\alpha) + A \cos\left(\alpha + \alpha + \alpha - \frac{\alpha}{2} - \frac{\alpha}{2}\right)} = \frac{\sin 2\alpha}{1 + 2\cos\alpha + \cos 2\alpha} \\ &= \frac{2\sin\alpha \cos\alpha}{1 + 2\cos\alpha + 2\cos^2\alpha - 1} = \frac{\sin\alpha}{1 + \cos\alpha} = \frac{2 \sin\left(\frac{\alpha}{2}\right) \cos\left(\frac{\alpha}{2}\right)}{2\cos^2\left(\frac{\alpha}{2}\right)} = \tan\left(\frac{\alpha}{2}\right) \Rightarrow \beta_3 = \frac{\alpha}{2} \end{aligned}$$

D. 4th Iteration of the Parallelogram Law with Five Phasors

$$A_1 = A_2 = A_3 = A_4 = A_5 = A$$

$$\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = \alpha$$

$$\begin{aligned} R_4 &= \sqrt{R_3^2 + A_5^2 + 2R_3A_5 \cos(\alpha_4 + \alpha_3 + \alpha_2 + \alpha_1 - (\beta_3 + \beta_2 + \beta_1))} \\ &= \sqrt{16A^2\cos^2\alpha \cos^2\left(\frac{\alpha}{2}\right) + A^2 + 2(4A \cos\alpha \cos\left(\frac{\alpha}{2}\right))A \cos\left(\alpha + \alpha + \alpha + \alpha - \left(\frac{\alpha}{2} + \frac{\alpha}{2} + \frac{\alpha}{2}\right)\right)} \\ &= \sqrt{A^2(16\cos^2\alpha \cos^2\left(\frac{\alpha}{2}\right) + 1 + 8\cos\alpha \cos\left(\frac{\alpha}{2}\right) \cos\left(\frac{5\alpha}{2}\right))} \\ &= \sqrt{A^2(16\cos^2\alpha \frac{(1 + \cos\alpha)}{2} + 1 + 8\cos\alpha \frac{(\cos 3\alpha + \cos 2\alpha)}{2})} \\ &= \sqrt{A^2(8\cos^2\alpha + 8\cos^3\alpha + 1 + 4\cos\alpha \cos 3\alpha + 4\cos\alpha \cos 2\alpha)} \end{aligned}$$

$$\begin{aligned}
 &= \sqrt{A^2(8\cos^2\alpha + 8\cos^3\alpha + 1 + 4\cos\alpha(4\cos^3\alpha - 3\cos\alpha) + 4\cos\alpha(2\cos^2\alpha - 1))} \\
 &= \sqrt{A^2(8\cos^2\alpha + 8\cos^3\alpha + 1 + 16\cos^4\alpha - 12\cos^2\alpha + 8\cos^3\alpha - 4\cos\alpha)} \\
 &= \sqrt{A^2(16\cos^4\alpha + 16\cos^3\alpha - 4\cos^2\alpha - 4\cos\alpha + 1)} \\
 &= \sqrt{A^2(16\cos^4\alpha + 16\cos^3\alpha - 8\cos^2\alpha + 4\cos^2\alpha - 4\cos\alpha + 1)} \\
 &= \sqrt{A^2(16\cos^4\alpha + 4\cos^2\alpha + 1 + 16\cos^3\alpha - 4\cos\alpha - 8\cos^2\alpha)} \\
 &= \sqrt{A^2(4\cos^2\alpha + 2\cos\alpha - 1)^2} \\
 &= A(4\cos^2\alpha + 2\cos\alpha - 1)
 \end{aligned}$$

$$\begin{aligned}
 \tan\beta_4 &= \frac{A_5 \sin(\alpha_4 + \alpha_3 + \alpha_2 + \alpha_1 - (\beta_3 + \beta_2 + \beta_1))}{R_3 + A_5 \cos(\alpha_4 + \alpha_3 + \alpha_2 + \alpha_1 - (\beta_3 + \beta_2 + \beta_1))} = \frac{A \sin(\alpha + \alpha + \alpha + \alpha - \frac{\alpha}{2} - \frac{\alpha}{2} - \frac{\alpha}{2})}{4A \cos\alpha \cos(\frac{\alpha}{2}) + A \cos(\alpha + \alpha + \alpha - \frac{\alpha}{2} - \frac{\alpha}{2} - \frac{\alpha}{2})} \\
 &= \frac{\sin(\frac{5\alpha}{2})}{4\cos\alpha \cos(\frac{\alpha}{2}) + \cos(\frac{5\alpha}{2})} = \frac{\sin(2\alpha + \frac{\alpha}{2})}{4\cos\alpha \cos(\frac{\alpha}{2}) + \cos(2\alpha + \frac{\alpha}{2})} = \frac{\sin(2\alpha)\cos(\frac{\alpha}{2}) + \cos(2\alpha)\sin(\frac{\alpha}{2})}{4\cos\alpha \cos(\frac{\alpha}{2}) + \cos(2\alpha)\cos(\frac{\alpha}{2}) - \sin(2\alpha)\sin(\frac{\alpha}{2})} \\
 &= \frac{4\sin(\frac{\alpha}{2})\cos(\frac{\alpha}{2})\cos\alpha \cos(\frac{\alpha}{2}) + \cos(2\alpha)\sin(\frac{\alpha}{2})}{4\cos\alpha \cos(\frac{\alpha}{2}) + \cos(2\alpha)\cos(\frac{\alpha}{2}) - 4\sin(\frac{\alpha}{2})\cos(\frac{\alpha}{2})\cos\alpha \sin(\frac{\alpha}{2})} = \left(\frac{\sin(\frac{\alpha}{2})}{\cos(\frac{\alpha}{2})}\right) \left(\frac{4\cos\alpha \cos^2(\frac{\alpha}{2}) + \cos(2\alpha)}{4\cos\alpha + \cos(2\alpha) - 4\cos\alpha \sin^2(\frac{\alpha}{2})}\right) \\
 &= \left(\tan\left(\frac{\alpha}{2}\right)\right) \left(\frac{4\cos\alpha \cos^2(\frac{\alpha}{2}) + \cos(2\alpha)}{4\cos\alpha + \cos(2\alpha) - 4\cos\alpha(1 - \cos^2(\frac{\alpha}{2}))}\right) = \left(\tan\left(\frac{\alpha}{2}\right)\right) \left(\frac{4\cos\alpha \cos^2(\frac{\alpha}{2}) + \cos(2\alpha)}{4\cos\alpha + \cos(2\alpha) - 4\cos\alpha + 4\cos\alpha \cos^2(\frac{\alpha}{2})}\right) \\
 &= \left(\tan\left(\frac{\alpha}{2}\right)\right) \left(\frac{4\cos\alpha \cos^2(\frac{\alpha}{2}) + \cos(2\alpha)}{\cos(2\alpha) + 4\cos\alpha \cos^2(\frac{\alpha}{2})}\right) = \tan\left(\frac{\alpha}{2}\right) \Rightarrow \beta_4 = \frac{\alpha}{2}
 \end{aligned}$$

Numerical instantiations of closed form expressions:

$$R_n = A \frac{\sin\left(\frac{(n+1)\alpha}{2}\right)}{\sin\left(\frac{\alpha}{2}\right)} ; \beta_n = \frac{\alpha}{2}$$

A. For $n = 1$,

$$R_1 = A \frac{\sin \alpha}{\sin\left(\frac{\alpha}{2}\right)} = A \frac{2 \sin\left(\frac{\alpha}{2}\right) \cos\left(\frac{\alpha}{2}\right)}{\sin\left(\frac{\alpha}{2}\right)} = 2A \cos\left(\frac{\alpha}{2}\right)$$

$$\beta_1 = \frac{\alpha}{2}$$

B. For $n = 2$,

$$R_2 = A \frac{\sin\left(\frac{3\alpha}{2}\right)}{\sin\left(\frac{\alpha}{2}\right)} = A \left(\frac{3\sin\left(\frac{\alpha}{2}\right) - 4\sin^3\left(\frac{\alpha}{2}\right)}{\sin\left(\frac{\alpha}{2}\right)} \right) = A \left(3 - 4\sin^2\left(\frac{\alpha}{2}\right) \right) = A \left(3 - 4 \frac{(1-\cos\alpha)}{2} \right) = A(1 + 2\cos\alpha)$$

$$\beta_2 = \frac{\alpha}{2}$$

C. For $n = 3$,

$$R_3 = A \frac{\sin(2\alpha)}{\sin\left(\frac{\alpha}{2}\right)} = A \frac{2 \sin \alpha \cos \alpha}{\sin\left(\frac{\alpha}{2}\right)} = A \frac{4 \sin\left(\frac{\alpha}{2}\right) \cos\left(\frac{\alpha}{2}\right) \cos \alpha}{\sin\left(\frac{\alpha}{2}\right)} = 4A \cos \alpha \cos\left(\frac{\alpha}{2}\right)$$

$$\beta_3 = \frac{\alpha}{2}$$

D. For $n = 4$,

$$R_4 = A \frac{\sin\left(\frac{5\alpha}{2}\right)}{\sin\left(\frac{\alpha}{2}\right)} = A \frac{\sin\left(2\alpha + \frac{\alpha}{2}\right)}{\sin\left(\frac{\alpha}{2}\right)} = A \left(\frac{\sin(2\alpha) \cos\left(\frac{\alpha}{2}\right) + \cos(2\alpha) \sin\left(\frac{\alpha}{2}\right)}{\sin\left(\frac{\alpha}{2}\right)} \right) = A \left(\frac{4 \sin\left(\frac{\alpha}{2}\right) \cos\left(\frac{\alpha}{2}\right) \cos \alpha \cos\left(\frac{\alpha}{2}\right) + (2\cos^2\alpha - 1) \sin\left(\frac{\alpha}{2}\right)}{\sin\left(\frac{\alpha}{2}\right)} \right)$$

$$= A \left(4 \cos \alpha \cos^2\left(\frac{\alpha}{2}\right) + 2\cos^2\alpha - 1 \right) = A \left(4 \cos \alpha \frac{(1+\cos\alpha)}{2} + 2\cos^2\alpha - 1 \right) = A(4\cos^2\alpha + 2\cos\alpha - 1)$$

$$\beta_4 = \frac{\alpha}{2}$$

Phasor Diagrams and Regular Polygons

Remark-1: A phasor diagram composed of $N = \{3, 4, 5, 6, \dots\}$ phasors of equal magnitude A and equal angular spacing $\alpha = \frac{2\pi}{N} = \left\{ \frac{2\pi}{3}, \frac{2\pi}{4}, \frac{2\pi}{5}, \frac{2\pi}{6}, \dots \right\}$, is equivalent to a regular (convex) polygon of side length A and external (or central) angle α .ⁱ

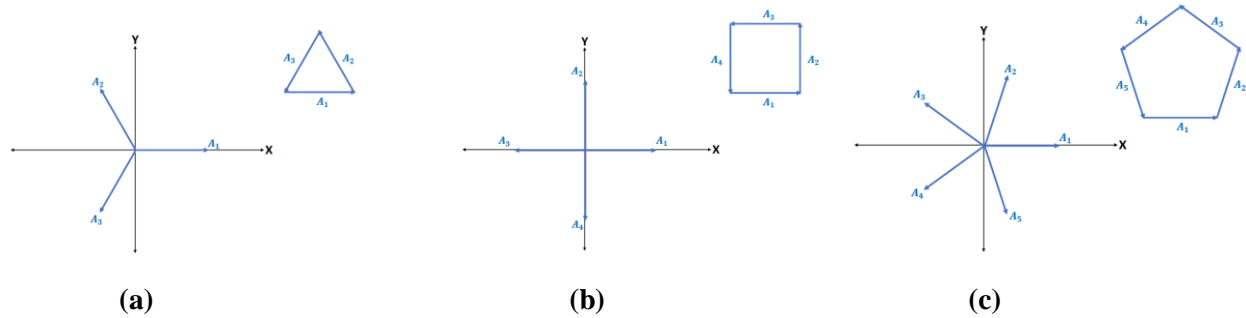


Figure S4: (a) Three equal phasors spaced $2\pi/3$ apart is equivalent to an equilateral triangle, (b) Four equal phasors spaced $2\pi/4$ apart is equivalent to a square, (c) Five equal phasors spaced $2\pi/5$ apart is equivalent to a pentagon.

Remark-2: A regular (convex) polygon with $N \geq 4$ sides, has a total of $N - 3$ diagonals sharing a common vertex (excluding the two adjacent sides). These diagonals divide the polygon into $N - 2$ triangles.ⁱⁱ

Remark-3: Theorem-1 of the main paper pertains in full generality to phasors for any value of the angular spacing α . But the same statement may be rephrased to also fit the special case of regular polygons, wherein α (denoting the external or central angle) takes up only specific values $\left\{ \frac{2\pi}{3}, \frac{2\pi}{4}, \frac{2\pi}{5}, \frac{2\pi}{6}, \dots \right\}$:

The $N - 3$ diagonals of a regular (convex) polygon having $N \geq 4$ sides, side length A , and sharing a common vertex V , divides the associated internal angle into $N - 2$ congruent angles, each a half-measure of the external angle α . The length of these diagonals may be determined from the expression $R_n = A \frac{\sin\left(\frac{(n+1)\pi}{N}\right)}{\sin\left(\frac{\pi}{N}\right)}$, where $n = \{1, 2, 3, \dots, N - 3\}$ is the diagonal number relative to V .^{iii, iv}

Total Fringe Count Formula in The Double-Slit Experiment:

$$\Omega(2) = 2 \left\lfloor \frac{d}{\lambda} \right\rfloor + 1$$

A detailed description of the elements that go into the proof of the above expression like definitions, theorems etc. may be found in Ref. [24] listed at the end of the main paper. Only a synopsis is presented below. The first important consideration to be taken into account is that the two slits in the double-slit experiment behave as synchronous point sources, emanating circular wavefronts of light of the same frequency, in an in-phase manner at their respective spatial locations. Next, the *first point of contact* (call them meeting points, for short) between any pair of wavefronts, one from either source, can occur only along the line segment joining the two sources (permissible zone) and never on the outward extensions (forbidden zones). This is a direct consequence of theorem-2 in the supplementary material of Ref. [24]. Furthermore, these meeting points are uniformly spaced, at intervals equal to half the wavelength of light as measured from either side of the midline (see Fig. S1).

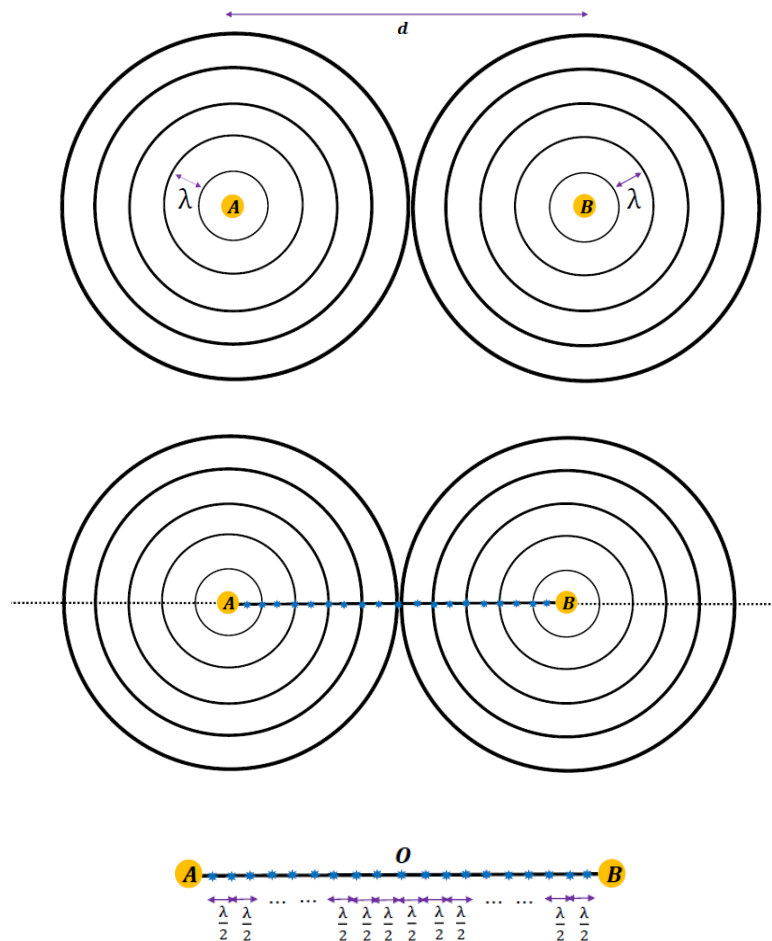


Figure S1. Above: Two synchronous point sources A and B (yellow) separated by a distance d , emanate circular ripples of light of wavelength λ (black). Middle: The meeting points (blue stars) between wavefronts occur over the line segment AB (permissible zone). The forbidden zones (dotted lines) extend outwards in opposite directions from A and B. Below: Meeting points occur at uniform intervals of half-wavelength relative to the center O.

Algebraic Notation

If d and λ be the distance between two sources A and B and the wavelength of light respectively, then it follows that a total of $\frac{\binom{d}{2}}{\binom{\lambda}{2}} = \frac{d}{\lambda}$ meeting points occupy each half OA and OB of the line segment AB with center O . Since the number of meeting points has to necessarily be a positive integer, it is convenient to express the quantity $\frac{d}{\lambda}$ using the notation popularized by Graham, Knuth, & Patashnik in their book *Concrete Mathematics*^v: $\frac{d}{\lambda} = \lfloor \frac{d}{\lambda} \rfloor + \{ \frac{d}{\lambda} \}$, where $\lfloor \frac{d}{\lambda} \rfloor$ and $\{ \frac{d}{\lambda} \}$ represents the integer part and the fractional part of $\frac{d}{\lambda}$ respectively (or equivalently, the quotient and remainder portion of the division process). Note that $0 \leq \{ \frac{d}{\lambda} \} < 1$.

Half-Wavelength Intervals

The distances of the half-wavelength intervals along the OA and OB directions as measured from the center O may be labelled $\{0, \frac{\lambda}{2}, 1 \cdot \frac{\lambda}{2}, 2 \cdot \frac{\lambda}{2}, 3 \cdot \frac{\lambda}{2}, \dots, \lfloor \frac{d}{\lambda} \rfloor \cdot \frac{\lambda}{2}, \lfloor \frac{d}{\lambda} \rfloor \cdot \frac{\lambda}{2}, (\lfloor \frac{d}{\lambda} \rfloor + 1) \cdot \frac{\lambda}{2}, (\lfloor \frac{d}{\lambda} \rfloor + 2) \cdot \frac{\lambda}{2}, (\lfloor \frac{d}{\lambda} \rfloor + 3) \cdot \frac{\lambda}{2}, \dots\}$. Clearly, if $\frac{d}{\lambda} \notin \mathbb{Z}^+$ then the quantity $\lfloor \frac{d}{\lambda} \rfloor \cdot \frac{\lambda}{2}$ represents the distance of the final half-wavelength interval from the center O lying wholly on AB , while the quantity $\lfloor \frac{d}{\lambda} \rfloor \cdot \frac{\lambda}{2}$ representing the half-wavelength interval that immediately succeeds it, lies partly on AB and partly on its outward extensions. However, if $\frac{d}{\lambda} \in \mathbb{Z}^+$ then both these quantities $\lfloor \frac{d}{\lambda} \rfloor \cdot \frac{\lambda}{2}$ and $\lfloor \frac{d}{\lambda} \rfloor \cdot \frac{\lambda}{2}$ are exactly equal to half the inter-source separation distance $\frac{d}{2}$, since $\lfloor x \rfloor = \lfloor x \rfloor \forall x \in \mathbb{Z}$. The quantities $\{(\lfloor \frac{d}{\lambda} \rfloor + 1) \cdot \frac{\lambda}{2}, (\lfloor \frac{d}{\lambda} \rfloor + 2) \cdot \frac{\lambda}{2}, (\lfloor \frac{d}{\lambda} \rfloor + 3) \cdot \frac{\lambda}{2}, \dots\}$ represent the distances of the half-wavelength intervals that lie wholly on the outward extensions of AB .

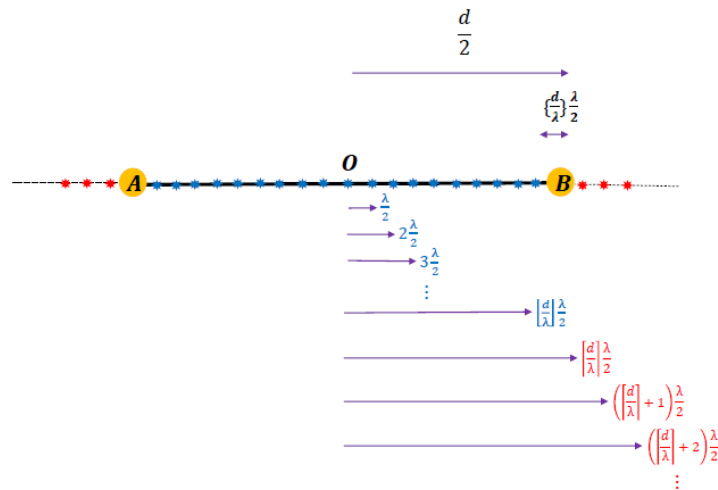


Figure S2. Distances of half-wavelength intervals are measured from the center O . Half-wavelength intervals are depicted along the OB direction in the forbidden zone (red) and the permissible zone (blue). For the sake of visual clarity, the corresponding labelling scheme along the OA direction is not shown.

Meeting-Points

It was previously asserted that the meeting points between circular wavefronts occur at half-wavelength intervals from the center O , exclusively along AB (the permissible zone) and not the outward extensions of OA or OB (the forbidden zones). It therefore logically follows that the meeting points on either side of the center O can be serially ordered $\{1, 2, 3, \dots, \lfloor \frac{d}{\lambda} \rfloor\}$, while O itself corresponds to the zeroth order meeting point. Recall from Ref. [24], that each meeting point geometrically represents the vertex of a hyperbola. And each hyperbola vertex corresponds to a bright fringe caught on a distant screen. A direct one-to-one correspondence thus exists between the number of meeting points, hyperbola vertices and bright fringes. This implies that the total fringe count on the screen can be known by simply adding up all of the meeting points along AB . Hence,

$$\Omega(2) = \left\lfloor \frac{d}{\lambda} \right\rfloor + 1 + \left\lfloor \frac{d}{\lambda} \right\rfloor = 2 \left\lfloor \frac{d}{\lambda} \right\rfloor + 1$$

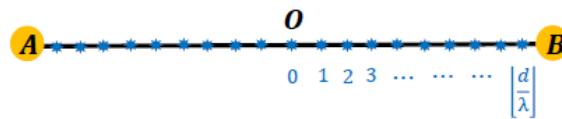


Figure S3. Ordering of meeting points along the OB direction, starting from the center O . For the sake of visual clarity, the corresponding labelling scheme along the OA direction is not shown.

Conventional Calculation

The textbook approach for arriving at the same result employs the *approximate* path difference formula $\delta = d \sin \theta$. A bright fringe occurs where the path difference is equal to an integral multiple of the wavelength: $\delta = n\lambda$, where $n \in \mathbb{Z}$, which is the condition for constructive interference. The angle θ is bounded within the closed interval $[0^\circ, 90^\circ]$. Clearly, the order n is maximized when $\theta = 90^\circ$ and minimized when $\theta = 0^\circ$. That is, $n_{max} = d/\lambda$ and $n_{min} = 0$. But the order of a fringe must always be an integer number. It therefore, becomes necessary to introduce either a floor or a ceiling function for n_{max} . The correct solution is the floor function. However, there is no formal justification for employing this particular operation, only a practical one. To elaborate on this point further, say that using the path difference formula we arrive at a maximum order of 7.3 for a bright fringe. The ‘integer mandate’ implies that the order can only be either $\lceil 7.3 \rceil = 7$ or $\lfloor 7.3 \rfloor = 8$. We accept the former and reject the latter on physical grounds. That there can be only 7 fully formed bright fringes occupying one half of the screen, while the 8th order fringe is only partially formed ($\approx 30\%$). Whereas, our treatment introduces the floor function as a natural consequence of the underlying geometry of circles (wavefronts of light), hyperbolas (regions of constructive interference) and straight lines (inter-source separation and screen).

ⁱ French AP. Vibrations and waves: The MIT Introductory Physics Series. (WW Norton & Company Inc., New York, 1971) pp. 284-288

ⁱⁱ <https://www.mathopenref.com/polygondiagonal.html>

ⁱⁱⁱ Fontaine A, Hurley S. Proof by picture: Products and reciprocals of diagonal length ratios in the regular polygon. *Forum Geometricorum*, 2006, Vol. 6, pp97-101.

^{iv} <https://ericrowland.github.io/investigations/polygons.html> : In both references iii and iv, the formula for the length of the diagonal d_k of a regular (convex) N -gon was arrived at by applying the law of sines to the k^{th} triangular division of the polygon, $d_k = s \frac{\sin(\frac{k\pi}{N})}{\sin(\frac{\pi}{N})}$, where $k = \{2,3,4, \dots, N - 2\}$ and s is the side length. In contrast, the

expression for diagonal length was proven here using the PMI in conjunction with the parallelogram law of vector addition. The advantage of the latter formula over the former is the more meaningful range of values that the index takes (beginning with least diagonal number 1 and ending with the greatest diagonal number $N - 3$).

^v Graham RL, Knuth DE, Patashnik O. (1989). Concrete mathematics: a foundation for computer science. *Addison-Wesley Publishing Company Inc.*, 2nd Ed., p70.

Dedication

To my beloved parents – Mr. Thomas Varghese & Dr. Annie Susan Thomas for their steady and unfailing love so freely given in every phase of my life. If one fine morning, I wake up to hear the good news that all my labors have finally borne fruit, then it is my heartfelt prayer that that day might bring them the greatest honor and joy. I love you both so dearly, my Acha and Ama. Thankyou for teaching me the ways of the Lord in my childhood years; Oh, how precious they are! They have guided my walk, watched over me while asleep, and spoken to me when awake.