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Research Article

Chewing Khat Transmission Dynamics: A Mathematical Model and Stability Analysis

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In this study, the authors proposed a nonlinear deterministic model and stability analysis for the transmission dynamics of chewing khat. The model's solution is proved to be positive and bounded, and the basic reproduction number (R_0) is calculated using the next-generation matrix method. Following that, the authors have looked at the local and global stability of the model's khat-free and endemic equilibrium points. When $R_0 < 1$, the chewing khat-free equilibrium point is locally and globally asymptotically stable, whereas when $R_0 > 1$, the endemic equilibrium point is locally and globally asymptotically stable. The simulation results demonstrate the analytical results.

1. Introduction

Khat (Catha edulis Forsk) is a green plant of the Celesterece family (El-Setouhy et al. [1]). It is a naturally occurring psychoactive stimulant that has been chewed for centuries in Ethiopia, East Africa, and the southern Arabian Peninsula (Gebissa [2]). Different countries have different names for khat such as khat or chat in Ethiopia, miraa in Kenya, jaad in Somalia (Alem et al. [3]), and qat or gat in (Yemen Odenwald et al. [4]). Ethiopia is one of the leading countries that produce khat in East Africa. Khat grows almost in all parts of the country (Rather et al. [5]). It is a green leaf and greenish plant that is chewed to release fluids containing active chemicals that influence the user's mood (Wubneh et al. [6]).

In private or small social events, stimulation is normally produced by chewing the young flowers and vulnerable leaves. Abdeta et al. [7] reported that khat chewing has recently grown popular among high school, college, and university students. Chewing khat was more common among the uneducated, elderly, alcoholics and smokers, Muslims, and professionals, while the lowest income index and divorce were related to a lower risk (Akalu et al. [8]). Furthermore, the leaves of khat are commonly chewed culturally and socially for a variety of reasons, including improving social interaction, increasing alertness, improving mood, and decreasing the need for sleep (Douglas et al. [9]), decreasing appetite, inducing euphoria, staying active while learning, and increase self-esteem (Teni et al. [10]).

Khat was also chewed for its euphoric and stimulating effects (Adeoya-Osiguwa and Fraser [11]). Chronic khat usage, on the other hand, has been related to detrimental health effects such as cardiac arrhythmias, sleeplessness, liver damage, mouth cancer, spermatorrhea, hemorrhoids (Yahya et al. [12]), high blood pressure, gastrointestinal problems, inflammation of the esophagus and stomach, mouth ulcers, gum disease, coronary artery narrowing, and myocardial infarction (Al-Habori [13]). Various studies have identified altered stress responses, cognitive impairments, increased depressive symptoms and stress levels, and insomnia in khat users (Yitayih and van Os [14]).

Additionally, chewing khat has been related to an increased risk of stroke and mortality (Ali et al. [15]), can impair your ability to drive, and therefore increases the rate of road accidents (Sheikh et al. [16]). In addition to the physical and psychological effects, khat sourcing and chewing consumes a significant amount of time and household income (Milanovic

[17]), which severely causes gingivitis, tooth loss, stomach upset, heart complications, male impotence, insomnia, and various psychological problems. According to the WHO, persistent khat use can lead to urinary retention, impotence, oral cancer, dental decay, chronic gastritis, hemorrhoids, paralytic ileus, liver cirrhosis, high blood pressure, and blurred vision, among other health issues (Havlicek et al. [18]).

In recent decades, both mathematicians and biologists have constructed numerous mathematical models to study the transmission of various infectious diseases, according to the study of Sunthrayuth et al. [19], Fantaye and Birhanu [20], Li et al. [21], Alqarni et al. [22], Fantaye [23], Fantaye and Muchie [24], Tiwari et al. [25], Sardar et al. [26], Khajanchi et al. [27], Tiwari et al. [28], Das et al. [29], Birhanu and Fantaye [30], Das et al. [31], Misra et al. [32], and Wong et al. [33].

Wubneh et al. [6] develop and analyze a mathematical model that demonstrates the dynamics of chewing khat by dividing the population into a group of people who do not chew khat $(N(t))$, a group of people who are surrounded by khat chewers but do not chew at the moment but may chew khat in the future $(\Sigma(t))$, a group of people who chew khat $(C(t))$, a group of people includes people who only consumed khat temporarily for social, spiritual ,and recreational purposes $(T(t))$, and a group of people who chew khat constantly $(H(t))$. The findings demonstrate that increasing the conversion rate of non-khat chewers to exposed groups makes disease eradication more challenging.

George et al. [34] construct a deterministic model of miraa addiction based on three compartment classes: susceptible (S), light users (L), and addicted (A). The fundamental reproduction number (R_0) was determined, the positivity and boundedness of the solution were studied, and it was discovered that the system of equations is within the feasible range. The findings of this study will provide information on the spread of addiction to stakeholders such as the government, NACADA, rehabilitation institutions, and the general public, allowing them to take appropriate action to address the issues.

Motivated by the works of George et al. [34] and Wubneh et al. [6], in this study, we proposed a nonlinear deterministic model to study and analyze the dynamics of chewing khat. Moreover, we subdivides the total population into susceptible $S(t)$, moderate $E(t)$, addicted $I(t)$, and quitter $R(t)$ classes in which taking into account that moderate khat chewers and addicted can be joined to quitting class due to addressing the risk groups by building self-efficacy and peer resistance skills among the young khat chewers and involvement of religious institution. This study is organized as follows. In section two, we develop our new mathematical model for the stability analysis of the transmission dynamics of chewing khat. We analyze the positivity and boundedness of solutions and stability of the chewing khat-free and endemic equilibria in section three. Section four deals with numerical simulation, and finally, conclusion are given in section five.

2. Model Formulation

This section proposes a nonlinear deterministic model of SEIRS to describe the interaction between khat chewer classes. The total population is divided into four compartments: susceptible $S(t)$, moderate $E(t)$, addicted $I(t)$ and quitters of khat chewer $R(t)$ individuals. We assumed that:

(i) Susceptible are those who do not chew khat, and who may become moderate chewers through effective contact with moderate chewers. There is a positive recruitment rate Λ into the susceptible class. It increased with the rate γ and decreased by effective contact with the moderate chewers at a rate of α

(ii) Moderate khat chewers are individuals who chew khat for their psychostimulant and mood uplift, to combat fatigue and act against obesity due to appetite suppression, religious purpose, peer pressure, passing the time, and socialization. They do not often think about chewing khat or often feel the need to chew, but they can influence the susceptible individuals to chew khat. It is increased by susceptible individuals who turn to be moderate chewers at α rate and decreased when moderate chewers become addicted and quit at rate $\beta\delta$ and $(1-\delta)\beta$, respectively

(iii) Addicted are individuals who constantly chew khat. Due to chewing khat, the majority of addicted individuals due to chewing khat begin as susceptible chewers and then turn to moderate chewers. These people's health, work, family, and social environment are at risk. This class becomes larger as the number of addicted increases by the rate $\beta\delta$ and decreases when they join to quitter class at rate of θ

(iv) Quitters of khat chewers are individuals who are quitting of chewing khat. It is increased with the rates of θ and $(1-\delta)\beta$ and while it decreased by the rate of γ.

(v) There is a positive natural death rate μ for all classes and disease-induced death rate σ in addicted class. The parameters of the model are presented in Table 1

Based on the assumption and the flow chart shown in Figure 1 for the khat chewing dynamics, we have the following system of nonlinear ordinary differential equations:

$$\frac{dS}{dt} = \Lambda - \alpha\frac{\mathrm{SI}}{N} + \gamma R - \mu S, \tag{1}$$

$$\frac{dE}{dt} = \alpha\frac{\mathrm{SI}}{N} - (\beta+\mu)E, \tag{2}$$

$$\frac{dI}{dt} = \beta\delta\mathrm{E} - (\sigma+\theta+\mu)I, \tag{3}$$

$$\frac{dR}{dt} = \theta I + (1-\delta)\beta E - (\gamma+\mu)R, \tag{4}$$

with

$$S(0) > 0, E(0) \geq 0, I(0) \geq 0, R(0) \geq 0. \tag{5}$$

Table 1: The parameter values of model.

Parameters	Description	Value
Λ	Recruitment rate	800
α	Effective contact rate	0.06
β	Rate at which $E(t)$ joins to $I(t)$ and $R(t)$	0.01
δ	Proportion of individuals that joins to $I(t)$ and $R(t)$ from $E(t)$	0.0007
θ	The recovery rate	0.02
γ	The rate of $R(t)$ becomes $S(t)$	0.0012
σ	Disease-induced death rate	0.005
μ	Natural death rate	0.08

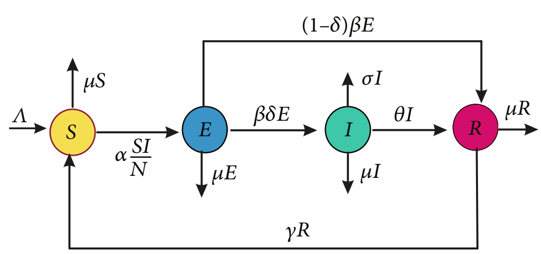

Figure 1: Flow chart of the model.

3. Model Analysis

This section studies the solution of Equation (1) in the epidemiologically feasible region.

$$\Omega = \left\{ (S, E, I, R) \in R_+^4 : 0 \le S(t) + E(t) + I(t) + R(t) \le \frac{\Lambda}{\mu} \right\}. \quad (6)$$

3.1. Positivity of Solution of the Model. Since model Equation (1) deals with the human population, it is necessary to prove that all state variables and associated parameters are positive. The following theorem will recognize this:

Theorem 1. *Let $S(0) > 0, E(0) > 0, I(0) > 0$ and $R(0) > 0$ be the initial solution of model Equation (1); then, $S(t), E(t), I(t)$ and $R(t)$ are positive in R^4 for all $t > 0$.*

Proof. Consider

$$\tau = \sup \{ t > 0 : S(t) > 0, E(t) > 0, I(t) > 0, R(t) > 0 \}. \quad (7)$$

Since $S(t), E(t), I(t)$, and $R(t)$ are continuous, we deduce that $\tau > 0$. If $\tau = +\infty$, then positivity holds, but, if $0 < \tau < +\infty$, $S(\tau) = 0$ or $E(\tau) = 0$ or $I(\tau) = 0$ or $R(\tau) = 0$. Now, from the first equation of system (1), we have

$$\frac{dS}{dt} + (\mu + c(t))S(t) = \Lambda + \gamma R, \quad (8)$$

where $c(t) = \alpha I/N$. So that, by integrating and simplifying, Equation (8) becomes

$$S(\tau) = D_1 S(0) + D_1 \int_0^{\tau} e^{\int (\mu + c(t))dt} . (\Lambda + \gamma R) dt > 0, \quad (9)$$

where $D_1 = e^{-(\mu\tau + \int_0^{\tau} c(w)dw)} > 0$, $S(0) > 0$, and from the definition of τ, we have $R(t) > 0$, and then, the solution $S(\tau) > 0$, and hence, $S(\tau) \ne 0$. Again, from the second equation of system (1), we can obtain that

$$\frac{dE}{dt} + (\beta + \mu)E = \alpha c(t). \quad (10)$$

Then, by integrating and simplifying Equation (10), we get

$$E(\tau) = G_1 E(0) + G_1 \int_0^{\tau} e^{\int (\beta + \mu)dt} . (c(t)S(t)) dt > 0, \quad (11)$$

where $G_1 = e^{-(\beta\tau + \mu\tau)} > 0$, $E(0) > 0$, and from the above $S(t) > 0$; then, the solution $E(\tau) > 0$, and hence, $E(\tau) \ne 0$. Similarly, $I(\tau) > 0$; hence, $I(\tau) \ne 0$ and $R(\tau) > 0$; hence, $R(\tau) \ne 0$. Thus, based on definition, τ is not finite which means $\tau = +\infty$, and as a result, all the solutions of model Equation (1) are nonnegative. □

3.2. Invariant Region. It is essential to prove that all solution of Equation (1) system with positive initial data will remain positive for all time $t \ge 0$. So, we will establish the following lemma:

Lemma 2. *All feasible solutions $S(t), E(t), I(t)$, and $R(t)$ of the system of Equation (1) are bounded for the system.*

$$\Omega = \left\{ (S, E, I, R) \in R_+^4 ; 0 \le S(t) + E(t) + I(t) + R(t) : N(t) \le \frac{\Lambda}{\mu} \right\}. \quad (12)$$

Proof. Differentiating the total human population, $N(t) = S(t) + E(t) + I(t) + R(t)$ with respect to time of our model Equation (1), we have

$$\frac{dN}{dt} = \frac{dS}{dt} + \frac{dE}{dt} + \frac{dI}{dt} + \frac{dR}{dt}. \quad (13)$$

Substituting all state equations from the system of Equation (1) in Equation (13), we get

$$\frac{dN}{dt} = \Lambda - \mu N(t) - \sigma I(t), \tag{14}$$

implies that

$$\frac{dN}{dt} \leq \Lambda - \mu N(t). \tag{15}$$

By rearranging and then multiplying both sides of Equation (15) by integrating factor $e^{\mu t}$ and after some simplification, we have

$$N(t) \leq \frac{\Lambda}{\mu} + N(0)e^{-\mu t}, \tag{16}$$

where $N(0)$ is the initial values of the total human population,

From Equation (16), we have $\lim \sup_{t\longrightarrow\infty} N(t) \leq \Lambda/\mu$. Thus, Ω is positively invariant (all solutions in Ω remain in Ω for all time). □

Remark 1. In the region Ω, our Equation (1) model is mathematically and epidemiologically well posed.

3.3. Khat Chewing-Free Equilibrium Point of the Model. The khat chewing-free equilibrium points, E_0 of model Equation (1), are steady-state solutions, where there is no khat chewer in the community. In the absence of chewing khat, we have $E = 0$ and $I = 0$. Then, we obtain

$$E_0 = \left(S^0, E^0, I^0, R^0\right) = \left(\frac{\Lambda}{\mu}, 0, 0, 0\right). \tag{17}$$

3.3.1. The Basic Reproduction Number (R_0). The basic reproduction number R_0 measures the expected number of secondary khat chewers that result from one newly khat chewer individuals introduced into a susceptible population. We can obtain R_0 of the system of Equation (1) using the next-generation matrix method as described by Van den Driessche and Watmough [35]. Then, we calculate the square matrix $\mathcal{F}$, which represents the rate of new khat-chewing devices (new infections) and $\mathcal{V}$ which represents the rate of individual transmission. Both matrices have a order of $n \times n$, where n is the total number of infected compartments. We obtain a new subsystem and a new infected compartment and transfer compartment, designated as $E(t)$ and $I(t)$, respectively.

$$\begin{aligned} \frac{dE}{dt} &= \alpha\frac{\mathrm{SI}}{N} - (\beta + \mu)E, \\ \frac{dI}{dt} &= \beta\delta E - (\sigma + \theta + \mu)I. \end{aligned} \tag{18}$$

That is

$$\begin{pmatrix} \frac{dE}{dt} \\ \frac{dI}{dt} \end{pmatrix} = \begin{pmatrix} \alpha\frac{\mathrm{SI}}{N} \\ 0 \end{pmatrix} - \begin{pmatrix} (\beta+\mu)E \\ -\beta\delta E + (\sigma+\theta+\mu)I \end{pmatrix}. \tag{19}$$

Now, let $x = (E, I)$, and then, the above system of equation can be rewritten as

$$\frac{dx}{dt} = \mathcal{F}(x) - \mathcal{V}(x), \tag{20}$$

where

$$\begin{aligned} \mathcal{F}(x) &= \begin{pmatrix} \alpha\frac{\mathrm{SI}}{N} \\ 0 \end{pmatrix}, \\ \mathcal{V}(x) &= \begin{pmatrix} (\beta+\mu)E \\ -\beta\delta E + (\sigma+\theta+\mu)I \end{pmatrix}. \end{aligned} \tag{21}$$

The Jacobian matrices of $\mathcal{F}(x)$ and $\mathcal{V}(x)$ at disease free equilibrium point, E_0, is given by

$$\begin{aligned} F &= \begin{pmatrix} 0 & \alpha \\ 0 & 0 \end{pmatrix}, \\ V &= \begin{pmatrix} (\beta+\mu) & 0 \\ -\beta\delta & (\sigma+\theta+\mu) \end{pmatrix}, \\ V^{-1} &= \begin{pmatrix} \frac{1}{\beta+\mu} & 0 \\ -\frac{\beta\delta}{(\beta+\mu)(\sigma+\theta+\mu)} & \frac{1}{\sigma+\theta+\mu} \end{pmatrix}. \end{aligned} \tag{22}$$

As a result, FV^{-1} becomes

$$FV^{-1} = \begin{pmatrix} \frac{\alpha\beta\delta}{(\beta+\mu)(\sigma+\theta+\mu)} & \frac{\alpha}{\sigma+\theta+\mu} \\ 0 & 0 \end{pmatrix}. \tag{23}$$

So that the eigenvalues of matrix FV^{-1} are $\lambda = 0$ and $\lambda = \alpha\beta\delta/(\beta+\mu)(\sigma+\theta+\mu)$. Using the method of next-generation matrix, the basic reproduction number, R_0 is the spectral radius of FV^{-1} or the dominant eigenvalue of FV^{-1} and thus, the basic reproduction number R_0 is given by

$$R_0 = \frac{\alpha\beta\delta}{(\beta+\mu)(\sigma+\theta+\mu)}. \tag{24}$$

3.4. Chewing Khat Endemic Equilibrium Point of the Model. Khat chewing will persist in the population, and the chewing khat endemic equilibrium point of the model is given by $E_1 = (S^*, E^*, I^*, R^*)$. Now, a system of Equation

(1) can be rewritten as:

$$\Lambda - \alpha \frac{S^* I^*}{N} + \gamma R^* - \mu S^* = 0, \tag{25}$$

$$\alpha \frac{S^* I^*}{N} - (\beta + \mu)E^* = 0, \tag{26}$$

$$\beta\delta E^* - (\sigma + \theta + \mu)I^* = 0, \tag{27}$$

$$\theta I^* + (1 - \delta)\beta E^* - (\gamma + \mu)R^* = 0. \tag{28}$$

From the second and third equation of system Equation (25), we have

$$\alpha \frac{S^* I^*}{N} - (\beta + \mu)E^* = 0, \tag{29}$$

$$\beta\delta E^* - (\sigma + \theta + \mu)I^* = 0. \tag{30}$$

Then, by dividing Equation (29) by Equation (30), we get

$$S^* = \frac{\Lambda}{\mu R_0}. \tag{31}$$

Again by adding the first and second equation of system Equation (25), we have

$$R^* = \frac{(\beta + \mu)E^*}{\gamma} + \frac{\Lambda}{\gamma}\left(\frac{1}{R_0} - 1\right). \tag{32}$$

From Equation (30), we obtain

$$I^* = \frac{\beta\delta E^*}{\sigma + \theta + \mu}. \tag{33}$$

Substituting Equation (33) in the last equation of Equation (33), we get

$$R^* = \frac{\beta\delta\theta E^*}{(\gamma + \mu)(\sigma + \theta + \mu)} + \frac{(1 - \delta)\beta E^*}{\gamma + \mu}. \tag{34}$$

Equating Equations (32) and (34), we get

$$E^* = \frac{(\gamma + \mu)(\sigma + \theta + \mu)\Lambda(1 - 1/R_0)}{\beta\gamma\delta(\sigma + \mu) + \mu(\sigma + \theta + \mu)(\beta + \gamma + \mu)}. \tag{35}$$

Substituting Equation (35) in Equation (33), we obtain

$$I^* = \frac{\beta\delta(\gamma + \mu)\Lambda(1 - 1/R_0)}{\beta\gamma\delta(\sigma + \mu) + \mu(\sigma + \theta + \mu)(\beta + \gamma + \mu)}. \tag{36}$$

Again substituting Equation (35) in the last equation of system Equation (34), we obtain

$$R^* = \frac{[\beta\delta\theta + (1 - \delta)\beta(\sigma + \theta + \mu)](\gamma + \mu)\Lambda(1 - 1/R_0)}{(\gamma + \mu)[\beta\gamma\delta(\sigma + \mu) + \mu(\sigma + \theta + \mu)(\beta + \gamma + \mu)]}. \tag{37}$$

3.5. Local Stability of the Chewing Khat-Free Equilibrium Point. The local stability of the chewing khat-free equilibrium point will be obtained by using the linearized form of the system Equation (1) at the steady state.

Theorem 4. *The equilibrium solution E_0 of the nonlinear system of Equation (1) is locally asymptotically stable, if $R_0 < 1$.*

Proof. The Jacobian matrix of system Equation (1) at chewing khat-free equilibrium point $E_0 = ((\Lambda/\mu), 0, 0, 0)$ is

$$J(E_0) = \begin{pmatrix} -\mu & 0 & -\alpha & \gamma \\ 0 & -(\beta + \mu) & \alpha & 0 \\ 0 & \beta\delta & -(\sigma + \theta + \mu) & 0 \\ 0 & (1 - \delta)\beta & \theta & -(\gamma + \mu) \end{pmatrix}. \tag{38}$$

The characteristic equation of Equation (29) at chewing khat-free equilibrium point, E_0

$$|J(E_0) - \lambda I_4| = \begin{vmatrix} -\mu - \lambda & 0 & -\alpha & \gamma \\ 0 & -(\beta + \mu) - \lambda & \alpha & 0 \\ 0 & \beta\delta & -(\sigma + \theta + \mu) - \lambda & 0 \\ 0 & (1 - \delta)\beta & \theta & -(\gamma + \mu) - \lambda \end{vmatrix} = 0. \tag{39}$$

Equation (39) can be rearranged and simplified as

$$(-\mu - \lambda)(-(\gamma + \mu) - \lambda)\left[\lambda^2 + k_1\lambda + k_2\right] = 0. \tag{40}$$

Here, from Equation (40), we obtain

$$\begin{aligned} \lambda_1 &= -\mu < 0, \\ \lambda_2 &= -(\gamma + \mu) < 0. \end{aligned} \tag{41}$$

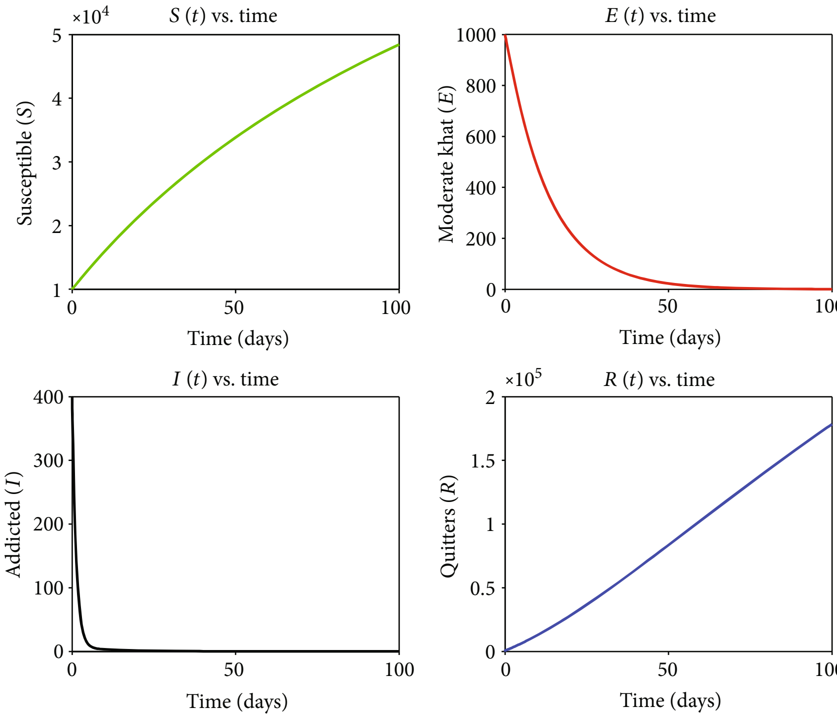

FIGURE 2: Time series plot of state variables for $R_0 = 0.0021 < 1$.

Here, $\lambda_1 < 0$ and $\lambda_2 < 0$. Furthermore, using Routh-Hurwitz criteria, the last equation of Equation (40) has strictly negative real part since $k_1 = 2\mu + \beta + \sigma + \theta > 0$ and $k_2 = (\beta + \mu)(\sigma + \theta + \mu) - \alpha\beta\delta = \alpha\beta\delta[(\beta + \mu)(\sigma + \theta + \mu)/\alpha\beta\delta - 1] = \alpha\beta\delta[1/R_0 - 1] > 0$ if $R_0 < 1$. Hence, with $R_0 < 1$, our system of Equation (1) at the chewing khat-free equilibrium point E_0 offers all eigenvalues with real negative part, and thus, it is locally asymptotically stable. □

3.6. Global Stability of the Khat Chewer Free Equilibrium Point. We used the method proposed by Iggidr et al. [36] to investigate the global stability. Based on Iggidr et al. [36], the system of Equation (1) can be rewritten as the following form:

$$\frac{dX}{dt} = H\left(X - X_{E_0,n}\right) + H_1 Y, \tag{42}$$

$$\frac{dY}{dt} = H_2 Y. \tag{43}$$

where $X = (S, R)$ which represents the number of uninfected individuals, while $Y = (E, I)$ which represents the number of infected individuals, and E_0, n represent a vector at disease free equilibrium point pf the same length as X. Based on [20], the chewing khat-free equilibrium point, E_0, is globally asymptotically stable if the following two conditions are fulfilled:

(i) (C_1): H should be a matrix with real negative eigenvalues.

(ii) (C_2): H_2 should be Metzler matrix.

Theorem 5. *The chewing khat-free equilibrium point, E_0, is globally asymptotically stable if $R_0 \le 1$.*

Proof. From system of Equation (1), we have

$$X = (S, R),\ Y = (E, I),\ E_0 = \left(\frac{\Lambda}{\mu}, 0, 0, 0\right)^T, \quad X_{E_0,n} = \left(\frac{\Lambda}{\mu}, 0\right)^T. \tag{44}$$

The system of Equation (1) together with Equation (42) can be written as

$$\begin{pmatrix} \Lambda - \alpha \dfrac{SI}{N} + \gamma R - \mu S \\ \theta I + (1-\delta)\beta E - (\gamma + \mu) R \end{pmatrix} = H \begin{pmatrix} S - \dfrac{\Lambda}{\mu} \\ R \end{pmatrix} + H_1 \begin{pmatrix} E \\ I \end{pmatrix}, \quad \begin{pmatrix} \alpha \dfrac{SI}{N} - (\beta + \mu) E \\ \beta\delta E - (\sigma + \theta + \mu) I \end{pmatrix} = H_2 \begin{pmatrix} E \\ I \end{pmatrix}. \tag{45}$$

Using uninfected entry of Jacobian matrix of model Equation (1) and the representation in Equation (42), the matrices H, H_1, H_2 are

$$H = \begin{pmatrix} -\mu & \gamma \\ 0 & -(\gamma+\mu) \end{pmatrix},$$
$$H_1 = \begin{pmatrix} 0 & -\frac{\alpha S}{N} \\ (1-\delta)\beta & \theta \end{pmatrix}, \quad (46)$$
$$H_2 = \begin{pmatrix} -(\beta+\mu) & \frac{\alpha S}{N} \\ \beta\delta & -(\sigma+\theta+\mu) \end{pmatrix}.$$

This implies that the sufficient conditions are satisfied, Therefore, chewing khat-free equilibrium point, E_0, is globally asymptotically stable if $R_0 \le 1$. □

3.7. Local Stability of Chewing Khat Endemic Equilibrium Point. In this section, we apply the Jacobian stability approach to prove the local stability of chewing khat endemic equilibrium state.

Theorem 6. *The chewing khat endemic equilibrium, E_1, of the model is locally asymptotically stable when $R_0 > 1$.*

Proof. The Jacobian matrix of system Equation (1) at chewing khat endemic equilibrium point $E_1 = (S^*, E^*, I^*, R^*)$ is

$$J(E_1) = \begin{pmatrix} -\left(\alpha\frac{I^*}{N}+\mu\right) & 0 & -\alpha\frac{S^*}{N} & \gamma \\ \alpha\frac{I^*}{N} & -(\beta+\mu) & \alpha\frac{S^*}{N} & 0 \\ 0 & \beta\delta & -(\sigma+\theta+\mu) & 0 \\ 0 & (1-\delta)\beta & \theta & -(\gamma+\mu) \end{pmatrix}. \quad (47)$$

The characterstic equation of Equation (47) at (E_1) is $|J(E_1) - \lambda I_4| = .$

$$\begin{vmatrix} -\left(\alpha\frac{I^*}{N}+\mu\right)-\lambda & 0 & -\alpha\frac{S^*}{N} & \gamma \\ \alpha\frac{I^*}{N} & -(\beta+\mu)-\lambda & \alpha\frac{S^*}{N} & 0 \\ 0 & \beta\delta & -(\sigma+\theta+\mu)-\lambda & 0 \\ 0 & (1-\delta)\beta & \theta & -(\gamma+\mu)-\lambda \end{vmatrix}. \quad (48)$$

Equation (48) can be simplified as

$$P(\lambda) = a_4\lambda^4 + a_3\lambda^3 + a_2\lambda^2 + a_1\lambda + a_0, \quad (49)$$

where

$$\begin{aligned}
a_4 &= 1,\\
a_3 &= 4\mu+\sigma+\theta+\beta+\gamma+\frac{\mu\alpha\beta\delta(\gamma+\mu)(R_0-1)}{[\beta\delta\gamma(\sigma+\mu)+\mu(\sigma+\theta+\mu)(\beta+\gamma+\mu)]R_0},\\
a_2 &= (3\mu+\sigma+\theta+\beta+\gamma)\mu+(2\mu+\sigma+\theta+\beta)(\gamma+\mu)+\frac{\mu\alpha\beta\delta(3\mu+\sigma+\theta+\beta+\gamma)(\gamma+\mu)(R_0-1)}{[\beta\delta\gamma(\sigma+\mu)+\mu(\sigma+\theta+\mu)(\beta+\gamma+\mu)]R_0},\\
a_1 &= (2\mu+\sigma+\theta+\beta)(\gamma+\mu)\mu+\frac{\mu\alpha\beta\delta(2\mu+\sigma+\theta+\beta)(\gamma+\mu)^2(R_0-1)}{[\beta\delta\gamma(\sigma+\mu)+\mu(\sigma+\theta+\mu)(\beta+\gamma+\mu)]R_0}+\frac{\mu\alpha\beta^2\delta(\gamma+\mu)(\delta\alpha+\gamma(1-\delta))(R_0-1)}{[\beta\delta\gamma(\sigma+\mu)+\mu(\sigma+\theta+\mu)(\beta+\gamma+\mu)]R_0},\\
a_0 &= \frac{\mu\alpha\beta^2\delta[\alpha\delta(\gamma+\mu)+\delta\gamma(\sigma+\mu)+\gamma(\sigma+\theta+\mu)](\gamma+\mu)(R_0-1)}{[\beta\delta\gamma(\sigma+\mu)+\mu(\sigma+\theta+\mu)(\beta+\gamma+\mu)]R_0},\\
a_3a_2-a_1 &= (3\mu+\sigma+\theta+\beta+\gamma)((4\mu+\sigma+\theta+\beta+\gamma)\mu+(\gamma+\mu)(2\mu+\sigma+\beta+\theta))\\
&\quad+\frac{\mu\alpha\beta\delta(\gamma+\mu)[(3\mu+\sigma+\theta+\beta)(4\mu+\sigma+\theta+\beta+\gamma)+(\delta\alpha+\gamma(1-\delta)\beta)](R_0-1)}{[\beta\delta\gamma(\sigma+\mu)+\mu(\sigma+\theta+\mu)(\beta+\gamma+\mu)]R_0}\\
&\quad+\frac{(3\mu+\sigma+\theta+\beta+\gamma)(\mu\alpha\beta\delta(\gamma+\mu)(R_0-1))^2}{([\beta\delta\gamma(\sigma+\mu)+\mu(\sigma+\theta+\mu)(\beta+\gamma+\mu)]R_0)^2}>0,\\
a_1a_2a_3-\left(a_0a_3^2+a_1^2\right) &= (4\mu+\sigma+\theta+\beta+\gamma)((2\mu+\sigma+\theta+\beta)(\gamma+\mu)\mu)(3\mu+\sigma+\beta+\theta+\gamma)\mu+(2\mu+\sigma+\theta+\beta)(\gamma+\mu)\\
&\quad+\frac{[(3\mu+\sigma+\beta+\theta+\gamma)\mu+(2\mu+\sigma+\theta+\beta)(\gamma+\mu)]\mu\alpha\beta\delta(\gamma+\mu)(R_0-1)}{[\beta\delta\gamma(\sigma+\mu)+\mu(\sigma+\theta+\mu)(\beta+\gamma+\mu)]R_0}\\
&\quad+\frac{(\mu\alpha\beta\delta)^2(2\mu+\sigma+\beta+\theta)(3\mu+\sigma+\beta+\theta+\gamma)(\gamma+\mu)^3(R_0-1)^2}{([\beta\delta\gamma(\sigma+\mu)+\mu(\sigma+\theta+\mu)(\beta+\gamma+\mu)]R_0)^2}\\
&\quad+\frac{((\mu\alpha\beta\delta)(R_0-1))^2[\alpha\sigma(\sigma+\mu)+\gamma(\sigma+\theta+\mu)+\sigma\gamma(\gamma+\mu)]}{([\beta\delta\gamma(\sigma+\mu)+\mu(\sigma+\theta+\mu)(\beta+\gamma+\mu)]R_0)^2}>0.
\end{aligned} \quad (50)$$

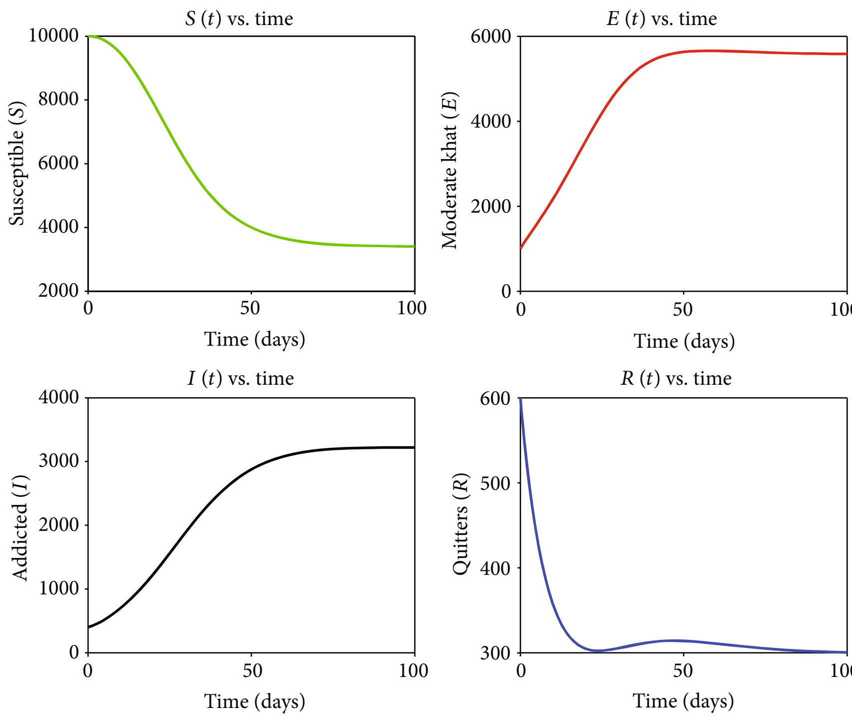

Figure 3: Time series plot of state variables for $R_0 = 6.0217 > 1$.

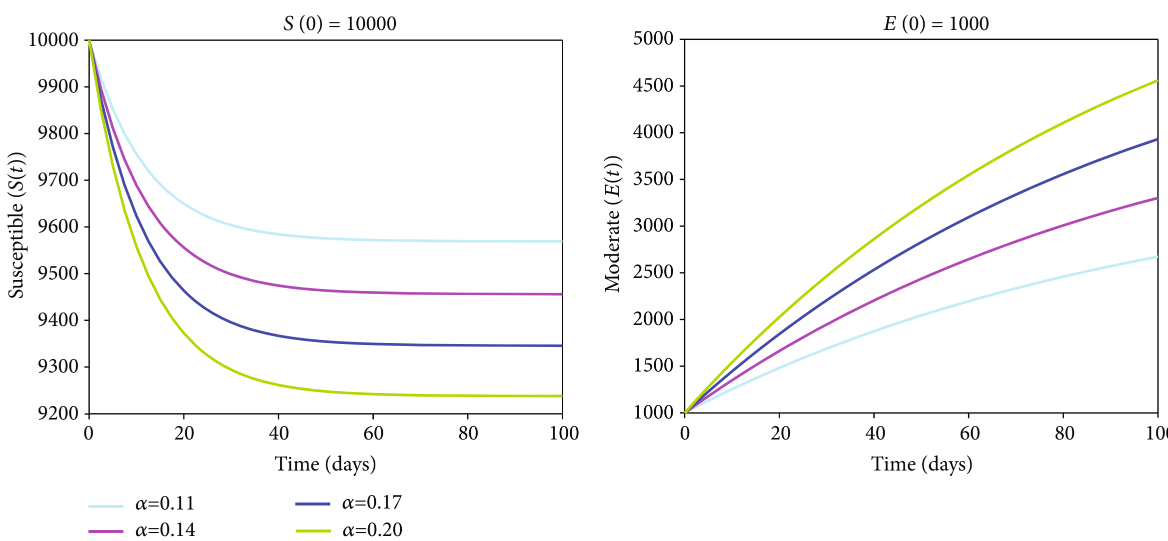

Figure 4: Susceptible and moderate khat chewer population with varying effective contact rate (α).

Using the Routh-Hurwitz criterion, all roots of characteristic polynomial have negative real parts if and only if $a_4 > 0, a_3 > 0, a_2 > 0, a_1 > 0, a_0 > 0, a_3 a_2 - a_1 > 0, a_1 a_2 a_3 - (a_0 a_3^2 + a_1^2) > 0$ for $R_0 > 1$. Thus, the chewing khat endemic equilibrium point, E_1, is locally asymptotically stable. □

4. Numerical Simulation

In this section, we verify our work numerically using MATLAB ODE solvers. Our simulations examine the effect of different combinations of parameters of the model on the transmission dynamics of chewing khat. The description and values for parameters, which are chosen arbitrarily, are shown on Table 1. The simulations and analysis made are based on these initial conditions: $S(0) = 10000, E(0) = 1000, I(0) = 400, R(0) = 600$.

The time series plot of state variables for $R_0 < 1$ and $R_0 > 1$ is shown in Figures 2 and 3. From Figure 2, we observe that susceptible and quitter individuals are increasing asymptotically to the disease free equilibrium point, while

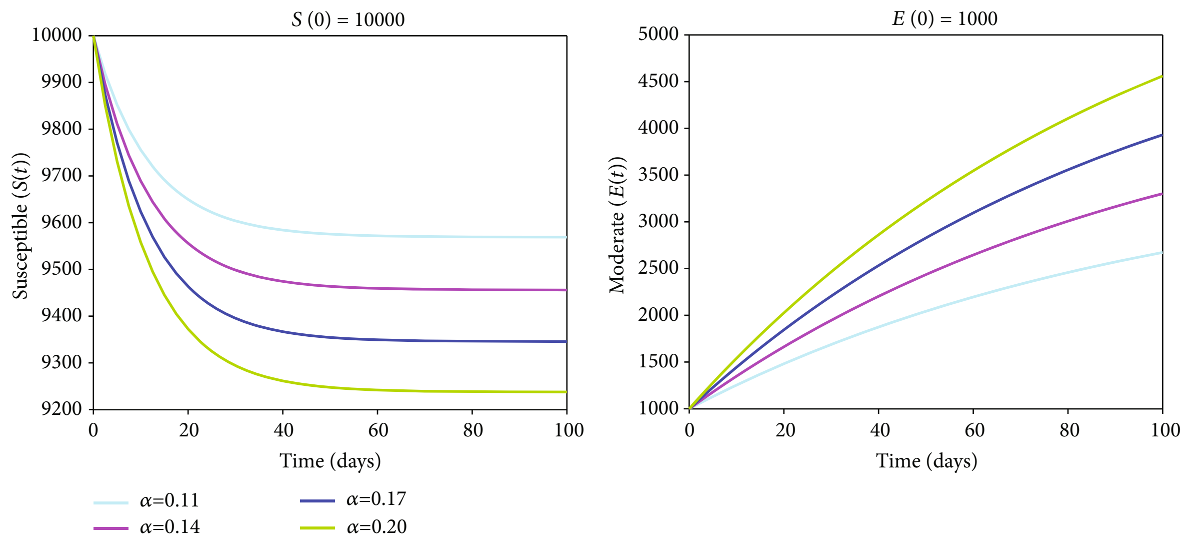

Figure 5: Moderate khat chewer and addicted population with varying rate at which $E(t)$ joins to $I(t)$ and $R(t)$, β.

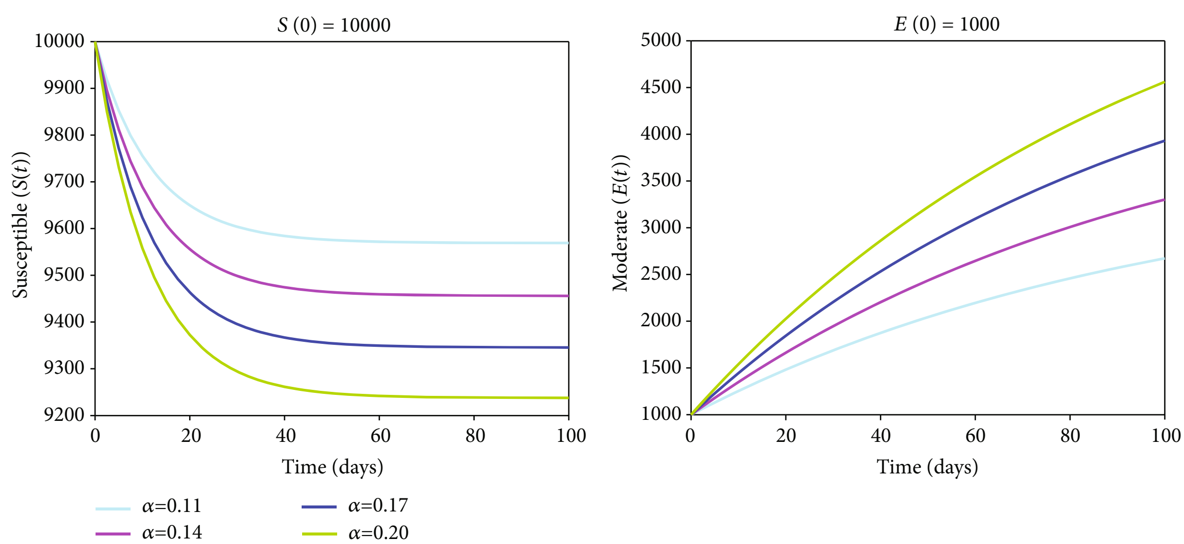

Figure 6: Addicted and quitter population with varying recovery rate (θ)

the moderate and addicted chewers are decreasing asymptotically to the disease free equilibrium point. The existence of such condition is due to the fact that $R_0 = 0.0021$ which is less than one. This supports theorem that the stability of khat chewer free equilibrium point exists when $R_0 < 1$, that is, if $R_0 < 1$, and then on average, one addicted individuals produce less than one newly khat chewer over the course of its chewers period. From Figure 3, we observe that susceptible and quitter individuals are decreased due to influence of moderate and addicted individuals, and then, they become chewers; as a result, the moderate and addicted individuals are increased. Therefore, moderate and addicted khat chewers are increased, and the khat chewer endemic equilibrium point exists and stable. The existence of this condition is due to the fact that $R_0 = 6.0217$ which is greater than one. This supports theorem that the stability of khat chewers endemic equilibrium point exists when $R_0 > 1$, that is, if $R_0 > 1$, each moderate and addicted chewer produces on average more than one new khat chewer; then, khat chewers will be able to spread in the given society.

From Figure 4, we observe that as the effective contact rate, α, increases, the moderate chewer individuals are increased, while, the susceptible individuals are decreased due to the influence of moderate chewer individuals. It suggests that as the effective contact rate decreases, moderate individuals who chew khat to combat fatigue and act against obesity due to appetite suppression, peer pressure, passing the time, and socialization decrease, and they cannot influence the susceptible individuals to chew khat, and as a result, the khat chewer individuals are eliminated or removed from the community.

Moreover, from Figure 5, we observe that as the rate at which $E(t)$ joins to $I(t)$ and $R(t)$, β, increases, moderate chewer individuals decreases, while addicted individuals are increased. From Figure 6, we observe that as the rate at which recovery rate, θ, increases, the addicted individuals

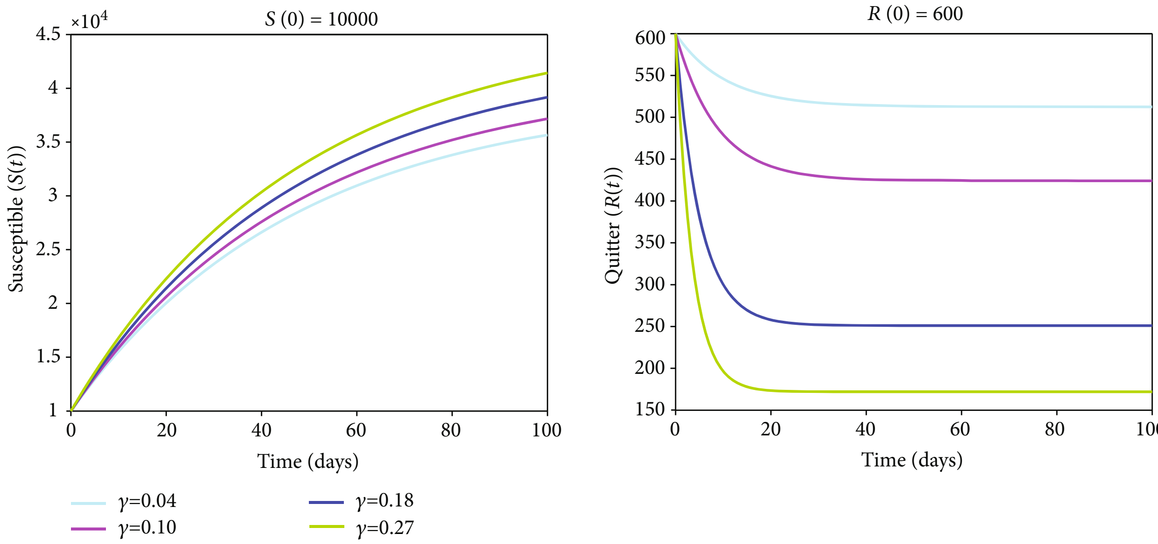

Figure 7: Susceptible and quitter population with varying the rate at which $R(t)$ becomes $S(t)$, γ

are decreased, while the quitter individuals are increased. This is due to economic, social (isolation) and health concerns, media awareness, and stage-based treatments. Finally, from Figure 7, we observe that as the rate at which $R(t)$ becomes to $S(t)$, γ, increases, the susceptible individuals are increased, while the quitter individuals are decreased.

5. Conclusion

In this paper, a deterministic mathematical model on the transmission dynamics of khat chewing was developed. We first showed that there exists a domain where the model is epidemiological and mathematically well posed. The basic reproduction number (R_0), chewing khat-free (E_0), and endemic equilibrium point (E_1) are computed. Furthermore, we proved that for $R_0 < 1$, then the chewing khat-free equilibrium point (E_0) is locally and globally asymptotically stable. We also showed that, chewing khat of an endemic equilibrium point (E_1) exists and locally and globally asymptotically stable for $R_0 > 1$. Finally, the simulation result shows that, as the contact rate, α, as well as the rate at which $E(t)$ joins to $I(t)$, β, decreases, and as the recovery rate, θ, increases, the addicted individuals decreased from the society. Designing optimal control would be considered in a future study.

Data Availability

The data used to support the findings of this study are included in the article. Actually, we used data from other papers for the simulation. The papers are properly cited.

Disclosure

The research did not receive specific funding but was performed as part of the employment of the authors at Hawassa University and Debre Tabor University.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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