Hindawi
Journal of Applied Mathematics
Volume 2022, Article ID 4072418, 6 pages
https://doi.org/10.1155/2022/4072418

*Research Article*

# Triangular Equilibria in R3BP under the Consideration of Yukawa Correction to Newtonian Potential

**M. Javed Idrisi, Teklehaimanot Eshetie, Tenaw Tilahun, and Mitiku Kerebh**

*Department of Mathematics, College of Natural and Computational Science, Mizan-Tepi University, Tepi Campus, Ethiopia*

Correspondence should be addressed to M. Javed Idrisi; javed@mtu.edu.et

Received 16 January 2022; Accepted 13 June 2022; Published 25 June 2022

Academic Editor: A. R. Ashrafi



We study the triangular equilibrium points in the framework of Yukawa correction to Newtonian potential in the circular restricted three-body problem. The effects of $\alpha$ and $\lambda$ on the mean-motion of the primaries and on the existence and stability of triangular equilibrium points are analyzed, where $\alpha \in (-1, 1)$ is the coupling constant of Yukawa force to gravitational force, and $\lambda \in (0,\infty)$ is the range of Yukawa force. It is observed that as $\lambda \longrightarrow \infty$, the mean-motion of the primaries $n \longrightarrow (1+\alpha)^{1/2}$ and as $\lambda \longrightarrow 0$, $n \longrightarrow 1$. Further, it is observed that the mean-motion is unity, i.e., $n = 1$ for $\alpha = 0$, $n > 1$ if $\alpha > 0$ and $n < 1$ when $\alpha < 0$. The triangular equilibria are not affected by $\alpha$ and $\lambda$ and remain the same as in the classical case of restricted three-body problem. But, $\alpha$ and $\lambda$ affect the stability of these triangular equilibria in linear sense. It is found that the triangular equilibria are stable for a critical mass parameter $\mu_c = \mu_0 + f(\alpha, \lambda)$, where $\mu_0 = 0.0385209 \cdots$ is the value of critical mass parameter in the classical case of restricted three-body problem. It is also observed that $\mu_c = \mu_0$ either for $\alpha = 0$ or $\lambda = 0.618034$, and the critical mass parameter $\mu_c$ possesses maximum ($\mu_{c\,\max}$) and minimum ($\mu_{c\,\min}$) values in the intervals $-1 < \alpha < 0$ and $0 < \alpha < 1$, respectively, for $\lambda = 1/3$.

## 1. Introduction

The general theory of relativity is the base of Modern Physics, and its predictions are confirmed by a wide variety of experiments. Since many of the present theories of gravitation and elementary particles predict forces coupled to gravitation, the investigation for the deviations from Newtonian gravity is of interest. For instance, Muecket and Treder [1] have considered a logarithmic correction to the post-Newtonian gravitational potential acting on a satellite which is moving on the vicinity of a primary body. This correction predicts a perihelion shift that depends neither on the primary mass $Mp$ nor on the semimajor axis $a$. Similarly, another potential, which is widely used in the study of various celestial mechanics scenarios, is the Manev potential [2]. Haranas and Mioc [3] have studied the motion of a satellite in such a potential with results similar to those predicted by general relativity. This potential provided unexpected results which, statistically as well as observationally, match better the astronomical reality when compared to those of the classical Newtonian model.

The classical Newtonian model in the framework of three or more body problem in different aspects is studied by many researchers in the last decades. For instance, the effect of albedos on the infinitesimal mass in restricted three-body problem is studied by Idrisi [4, 5] and Idrisi and Ullah [6–9]. Jain and Aggarwal [10], Idrisi and Jain [11], and Idrisi and Ullah [12] have investigated the circular restricted three-body problem under the consideration Stokes drag. The effect of variable mass and the topology of basins of convergence linked to the libration points in the modified three-body problem are studied by Suraj et al. [13–15].

The Yukawa potential was proposed by Yukawa in 1935 [16] to modify the Newtonian one is an effective nonrelativistic potential describing the strong interactions between particles. Let us consider a two-body problem describing the motion of a secondary body of mass $m$ under the influ-

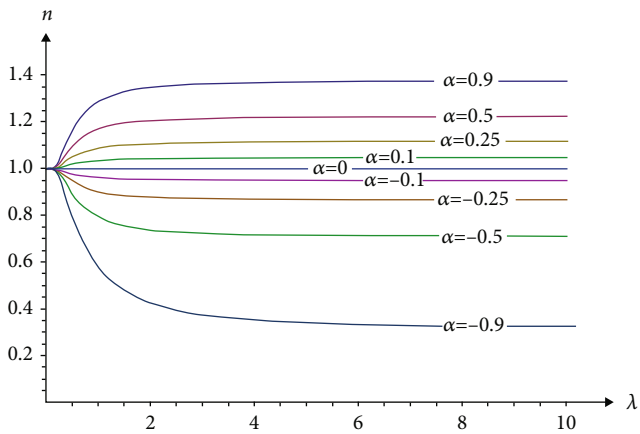


Figure 1: Variation in mean-motion curves.

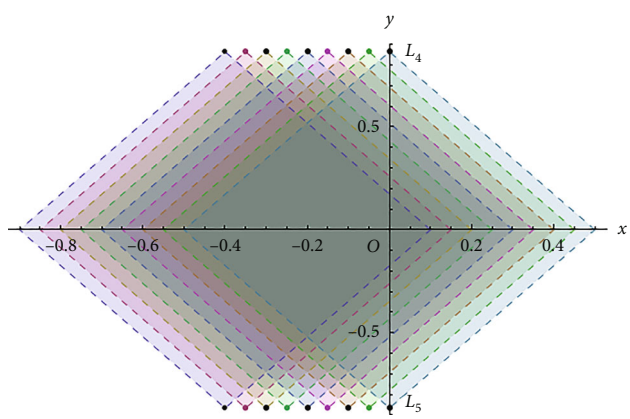


Figure 2: Triangular equilibria $L_{4,5}$ for various values of $\mu$.

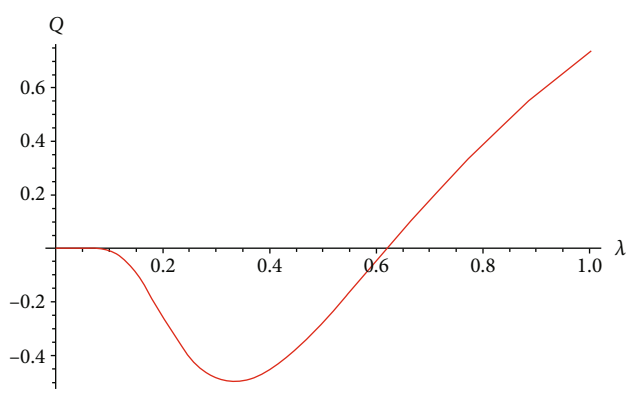


Figure 3: $Q$ versus $\lambda$.

ence of massive primary of mass $M$. The effects of gravity on the primary $m$ in the presence of the Yukawa correction can be described in terms of the modified potential energy [17]

$$V(r) = -\frac{GMm}{r}\left(1 + \alpha e^{-r/\lambda}\right) = -\frac{GMm}{r} - \frac{GMm}{r}\alpha e^{-r/\lambda} = V_N(r) + V_Y(r), \tag{1}$$

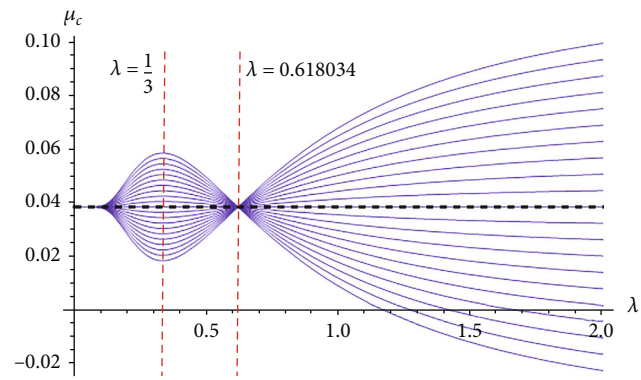


Figure 4: $\mu_c$ versus $\lambda$ for different values of $\alpha$ (the dashed line represents $\mu_c$ for $\alpha = 0$).

where $V_N(r)$ is the Newtonian potential between the two bodies $M$ and $m$, $V_Y(r)$ is the Yukawa correction to the Newtonian potential, $r$ is the distance between $m$ and $M$, $G$ is the Newtonian gravitational constant, $\alpha \in (-1, 1)$ is the coupling constant of the Yukawa force to the Gravitational force, and $\lambda \in (0,\infty)$ is the range of the Yukawa force [18]. Therefore, the corresponding force between $M$ and $m$ can be expressed as

$$\vec{F}(r) = \frac{GMm}{r^2}\left\{1 + \alpha\left(1 + \frac{r}{\lambda}\right)e^{-r/\lambda}\right\}\vec{r}. \tag{2}$$

As $\alpha \longrightarrow 0$, the Newtonian gravitational force can be obtained. Kokubun [18] has studied restricted three-body problem including a Yukawa term to the Newtonian gravitational potential. He observed that the modified gravitational potential changes some important aspects of restricted three-body problem. Because of coupling constant $\alpha$, motions obtained in pure Newtonian case are qualitatively different when Yukawa term is included. Kolosnitsyn and Melnikov [19] have found that, for the artificial Earth satellites LAGEOS and LAGEOS II, a minimum value of the Yukawa coupling constant is $\alpha_{\min} = 1.38 \times 10^{-11}$ for $\lambda = 6.081 \times 10^6$ m. Massa [20] has studied the Newton + Yukawa type potential in the framework of the Sciama linear approach to Mach's principle. The Yukawa-type effects in satellite dynamics are studied by Haranas and Ragos [21]. Pricopi [22] has studied the stability of celestial orbits under the influence Yukawa potential in the two-body problem. Haranas et al. [23] have studied the circular and elliptical orbits of Earth under the consideration of Yukawa potential combined with Poynting-Robertson effect. Cavan et al. [24] have studied the dynamics and stability of the two-body problem with Yukawa correction.

In the present work, we consider a circular restricted three-body problem with Yukawa correction to Newtonian potential and studied the effects of $\alpha$ and $\lambda$ on the existence and stability of triangular equilibrium points, where $\alpha \in (-1, 1)$ is the coupling constant of Yukawa force to Gravitational force, and $\lambda \in (0,\infty)$ is the range of Yukawa force.

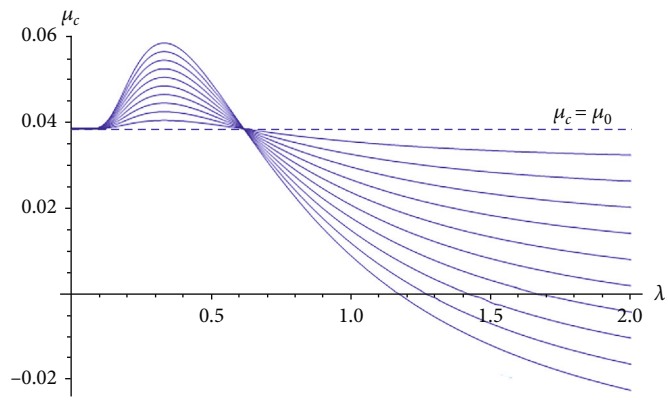


Figure 5: $\mu_c$ versus $\lambda$ for $\alpha < 0$.

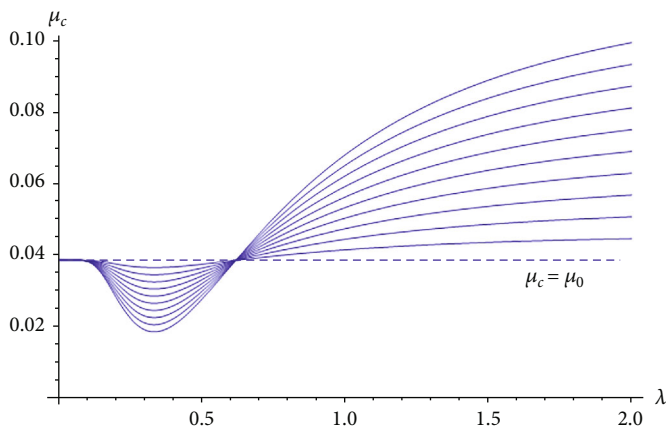


Figure 6: $\mu_c$ versus $\lambda$ for $\alpha > 0$.

## 2. Equations of Motion

The equations of motion of the infinitesimal mass in a barycentric synodic coordinate system $(x, y)$ and dimensionless variables are [18]:

$$\ddot{x} - 2n\dot{y} = U_x, \ddot{y} + 2n\dot{x} = U_y, \tag{3}$$

and the potential function $U$ can be expressed as

$$U = \frac{n^2}{2}\left(x^2 + y^2\right) + \sum_{i=1}^{2} \frac{m_i}{r_i}\left(1 + \alpha e^{-r_i/\lambda}\right), \tag{4}$$

$|\alpha| < 1$ is the coupling constant of Yukawa force to Gravitational force, and $\lambda \in (0,\infty)$ is the range of Yukawa force,

$$m_1 = \mu, m_2 = 1 - \mu, \tag{5}$$

$$r_1 = \sqrt{(x-\mu)^2 + y^2}, \tag{6}$$

$$r_2 = \sqrt{(x+1-\mu)^2 + y^2}, \tag{7}$$

$0 < \mu \leq 1/2$ is the ratio of mass of the smaller primary $(m_2)$ to the total mass of the primaries $(m_1 + m_2)$, $n$ is the mean-motion of the primaries, and $r_1$ and $r_2$ are the distances of infinitesimal mass from the more massive and less massive primaries, respectively.

The integral analogous to Jacobi integral is

$$v^2 = 2U - C, \tag{8}$$

$v$ is the velocity of infinitesimal mass, and Eqn. (8) is Jacobi integral associated with the problem. The square of the velocity cannot be negative, therefore, $v^2 = 2U - C \geq 0$. Thus, the motion of the infinitesimal mass is possible in the region where $U \geq C/2$, and $C$ is known as Jacobi constant.

## 3. Mean-Motion of the Primaries

To maintain the configuration, the sum of the mutual gravitational forces must be equal to the centrifugal force, i.e.,

$$m_1\rho_1 n^2 = \frac{Gm_1m_2}{R^2}\left[1 + \alpha\left(1 + \frac{R}{\lambda}\right)e^{-R/\lambda}\right], \tag{9}$$

$$m_2\rho_2 n^2 = \frac{Gm_2m_1}{R^2}\left[1 + \alpha\left(1 + \frac{R}{\lambda}\right)e^{-R/\lambda}\right], \tag{10}$$

where $R = \rho_1 + \rho_2$, $\rho_1$ and $\rho_2$ are the distances of $m_1$ and $m_2$ from the center of mass of the system, respectively. On

simplifying and then adding Eqns. (9) and (10), we have

$$(\rho_1+\rho_2)n^2=\frac{G(m_1+m_2)}{R^2}\left[1+\alpha\left(1+\frac{R}{\lambda}\right)e^{-R/\lambda}\right]. \quad (11)$$

Using the terminology of restricted three-body problem, i.e., $R=\rho_1+\rho_2=1$, $m_1+m_2=1$, and $G=1$, the mean-motion of the primaries is given by

$$n^2=1+\alpha\Lambda,\ \Lambda=\left(1+\frac{1}{\lambda}\right)e^{-1/\lambda}. \quad (12)$$

Now, as $\lambda\longrightarrow\infty$, $\Lambda\longrightarrow 1$ and hence $n\longrightarrow(1+\alpha)^{1/2}$. Also, as $\lambda\longrightarrow 0$, $\Lambda\longrightarrow 0$, and then $n\longrightarrow 1$. The mean-motion curves with respect to $\lambda$ for different values of coupling constant $\alpha$ are plotted in Figure 1. It is observed that the mean-motion is unity, i.e., $n=1$ for $\alpha=0$, $n>1$ if $\alpha>0$ and $n<1$ when $\alpha<0$.

## 4. Triangular Equilibria

The triangular equilibrium points are the solution of the Eqns. $U_x=0$ and $U_y=0$ for $y\neq 0$, i.e.,

$$\begin{aligned} n^2x-\frac{(1-\mu)(x-\mu)}{r_1^3}\left[1+\alpha\left(1+\frac{r_1}{\lambda}\right)e^{-r_1/\lambda}\right] \\ -\frac{\mu(x+1-\mu)}{r_2^3}\left[1+\alpha\left(1+\frac{r_2}{\lambda}\right)e^{-r_2/\lambda}\right]=0, \end{aligned} \quad (13)$$

$$n^2-\frac{(1-\mu)}{r_1^3}\left[1+\alpha\left(1+\frac{r_1}{\lambda}\right)e^{-r_1/\lambda}\right]-\frac{\mu}{r_2^3}\left[1+\alpha\left(1+\frac{r_2}{\lambda}\right)e^{-r_2/\lambda}\right]=0. \quad (14)$$

The solution of Eqns. (13) and (14) is given by

$$r_i^3=\frac{1}{n^2}\left[1+\alpha\left(1+\frac{r_i}{\lambda}\right)e^{-r_i/\lambda}\right],\ i=1,2. \quad (15)$$

Since $r_i=1$ satisfies Eqns. (15) and thus the coordinates of triangular equilibrium points are $x_0=\mu-1/2$ and $y_0=\pm\sqrt{3}/2$ which are independent of $\alpha$ and $\lambda$. Hence, there is no effect of $\alpha$ and $\lambda$ on the locations of triangular equilibria. The triangular equilibria affected by only mass parameter $\mu$ and for each value of $\mu$ in the interval $(0,1/2]$ there exist a pair of triangular equilibrium points $L_{4,5}(x_0,y_0)$ symmetric with respect to $x$-axis and form equilateral triangles with the primaries and as $\mu\longrightarrow 1/2$, $L_{4,5}$ approaches to $y$-axis (Figure 2).

## 5. Stability of Triangular Equilibria

The variational equations of motion can be obtained by plugging $x=x_o+\delta_1$ and $y=y_o+\delta_2$ in Eqs. (3), where $(x_0, y_0)$ are the coordinates of $L_4$ and $\delta_i<<1$, $i=1,2$, i.e.,

$$\left.\begin{aligned} \ddot{\delta}_1-2n\dot{\delta}_2&=\delta_1\overset{o}{U}_{xx}+\delta_2\overset{o}{U}_{xy} \\ \ddot{\delta}_2+2n\dot{\delta}_1&=\delta_1\overset{o}{U}_{xy}+\delta_2\overset{o}{U}_{yy} \end{aligned}\right\}. \quad (16)$$

As $\delta_i<<1$ and $|\alpha|<1$, therefore, consider only linear terms in $\delta_1$, $\delta_2$, and $\alpha$, the characteristic equation corresponding to Eqn. (16) is given by

$$\kappa^4+\left(4n^2-\overset{o}{U}_{xx}-\overset{o}{U}_{yy}\right)\kappa^2+\overset{o}{U}_{xx}\overset{o}{U}_{yy}-\left(\overset{o}{U}_{xy}\right)^2=0, \quad (17)$$

where

$$\begin{aligned} \overset{o}{U}_{xx}&=\frac{3}{4}+\frac{(3\lambda^2+3\lambda+1)}{4\lambda^2}\alpha e^{-1/\lambda}, \\ \overset{o}{U}_{xy}&=\frac{3\sqrt{3}}{2}\left(\mu-\frac{1}{2}\right)+\frac{\sqrt{3}(2\mu-1)(3\lambda^2+3\lambda+1)}{4\lambda^2}\alpha e^{-1/\lambda}, \\ \overset{o}{U}_{yy}&=\frac{9}{4}+\frac{3(3\lambda^2+3\lambda+1)}{4\lambda^2}\alpha e^{-1/\lambda}. \end{aligned} \quad (18)$$

The quadratic equation corresponding to Eqn. (17) is given by

$$\Lambda^{*2}+a\Lambda^*+b=0, \quad (19)$$

where

$$\Lambda^*=\kappa^2,\ a=4n^2-\overset{o}{U}_{xx}-\overset{o}{U}_{yy},\ b=\overset{o}{U}_{xx}\overset{o}{U}_{yy}-\left(\overset{o}{U}_{xy}\right)^2. \quad (20)$$

The roots of Eqn. (19) are

$$\Lambda^*_{1,2}=\frac{1}{2}\left(-a\pm\sqrt{a^2-4b}\right). \quad (21)$$

The motion near the equilibrium point $L_4(x_0,y_0)$ is said to be bounded if $D=a^2-4b\geq 0$, i.e., $27\mu^2-27\mu+1+Q\alpha\geq 0$, which gives

$$\mu\leq\mu_c=\mu_0+\frac{Q\alpha}{3\sqrt{69}}\ ;Q=\frac{2(\lambda^2+\lambda-1)e^{-1/\lambda}}{\lambda^2}, \quad (22)$$

where $\mu_0=0.0385209\cdots$ is the critical mass parameter in the classical restricted three-body problem [25]. Thus, the triangular equilibria $L_{4,5}$ are linearly stable for the critical mass parameter $\mu_c$ defined in Eqn. (22). As shown in Figure 3, $Q<0$ in the interval $0<\lambda<0.618034$, $Q=0$ if $\lambda=0.618034$ and $Q>0$ in the interval $0.618034<\lambda<\infty$. Thus, $\mu_c=\mu_0$ either for $\alpha=0$ or $\lambda=0.618034$. Figure 4, $\lambda=1/3$ is a critical point at which the critical mass parameter $\mu_c$ has maximum and minimum values in the interval $-1<\alpha<1$.

Figure 5 shows that $\mu_c > \mu_0$ in the interval $0 < \lambda < 0.618034$ and $\mu_c < \mu_0$ in the interval $0.618034 < \lambda < \infty$ for all $\alpha \in (-1, 0)$. Also, $\mu_c$ has maximum value at $\lambda = 1/3$ for all $\alpha \in (-1, 0)$, thus $\lambda = 1/3$ is a point of maxima if $-1 < \alpha < 0$.

Similarly, $\mu_c < \mu_0$ in the interval $0 < \lambda < 0.618034$ and $\mu_c > \mu_0$ in the interval $0.618034 < \lambda < \infty$ for all $\alpha \in (0, 1)$. The critical mass parameter $\mu_c$ has minimum value at $\lambda = 1/3$ for all $\alpha \in (0, 1)$ (Figure 6), and hence, $\lambda = 1/3$ is a point of minima if $-1 < \alpha < 0$.

Thus, it can be easily verified that the critical mass parameter $\mu_c$ at $\lambda = 1/3$ possesses maximum ($\mu_{c\ \max}$) and minimum ($\mu_{c\ \min}$) values in the intervals $-1 < \alpha < 0$ and $0 < \alpha < 1$, respectively, as given below:

$$\mu_c = \mu_0 + \frac{10\alpha e^{-3}}{3\sqrt{69}} = \begin{cases} \mu_{c\ \max}, & -1 < \alpha < 0, \\ \mu_{c\ \min}, & 0 < \alpha < 1. \end{cases} \tag{23}$$

## 6. Conclusion

In this paper, we studied the triangular equilibrium points in the framework of Yukawa correction to Newtonian potential in circular restricted three-body problem. First, the effects of $\alpha$ and $\lambda$ on the mean-motion of the primaries has been shown, where $\alpha \in (-1, 1)$ is the coupling constant of Yukawa force to gravitational force, and $\lambda \in (0,\infty)$ is the range of Yukawa force. We observed that as $\lambda \longrightarrow \infty$, the mean-motion of the primaries $n \longrightarrow (1+\alpha)^{1/2}$ and as $\lambda \longrightarrow 0$, $n \longrightarrow 1$. Further, we observed that the mean-motion is unity, i.e., $n = 1$ for $\alpha = 0$, $n > 1$ if $\alpha > 0$ and $n < 1$ when $\alpha < 0$ (Figure 1).

After a detailed analysis of mean-motion of the primaries, we have investigated the triangular equilibria and found that the locations of triangular equilibria are not affected by $\alpha$ and $\lambda$ and remain the same as in the classical case of circular restricted three-body problem (Figure 2). However, the positions of the triangular equilibria are not affected by $\alpha$ and $\lambda$ but their stability is influenced by $\alpha$ and $\lambda$ and which is quite different from the classical case. It is found that the triangular equilibria are stable for a critical mass parameter $\mu_c$ defined in Eqn. (22), and it is observed that $\mu_c = \mu_0$ either for $\alpha = 0$ or $\lambda = 0.618034$, where $\mu_0 = 0.0385209\cdots$ is the value of critical mass parameter in the classical case of restricted three-body problem. Figure 4, 5, and 6 show that $\lambda = 1/3$ is a critical point at which the critical mass parameter $\mu_c$ has maximum and minimum values in the interval $-1 < \alpha < 0$ and $0 < \alpha < 1$, respectively. The maximum ($\mu_{c\ \max}$) and minimum ($\mu_{c\ \min}$) values of critical mass parameter in the intervals $-1 < \alpha < 0$ and $0 < \alpha < 1$ for $\lambda = 1/3$ are given in Eqn. (23).

## Data Availability

The data used to support the findings of this study are included in this research article. For simulation, we have used data from other research papers which are properly cited.

## Conflicts of Interest

The authors declare that there is no conflict of interests regarding publication of this manuscript.

## References


[1] J. P. Muecket and H. J. Treder, "The perihelion advance according to a post-Newtonian gravitational law with logarithmic correction term," *Astronomische Nachrichten*, vol. 298, no. 2, pp. 65–67, 1977.

[2] G. Maneff, "La gravitation et le principie de l'égalité de l'action et de la réaction," *Comptes Rendus de l'Académie des Sciences Serie IIa, Sciences de la Terre Planetes*, vol. 178, pp. 2159–2161, 1924.

[3] I. Haranas and V. Mioc, "Manev potential and satellite orbits," *Romanian Astronomical Journal*, vol. 19, pp. 153–166, 2009.

[4] M. J. Idrisi, "A study of libration points in CR3BP under albedo effect," *International Journal of Advanced Astronomy*, vol. 5, no. 1, pp. 1–6, 2016.

[5] M. J. Idrisi, "A study of libration points in modified CR3BP under albedo effect when smaller primary is an ellipsoid," *Journal of Astronautical Sciences*, vol. 64, no. 4, pp. 379–398, 2017.

[6] M. J. Idrisi and M. Ullah, "Restricted three-body problem with albedo effect when smaller primary is an oblate spheroid," *The Global Journal of Science Frontier Research*, vol. 17, pp. 9–24, 2017.

[7] M. J. Idrisi and M. S. Ullah, "Non-collinear libration points in ER3BP with albedo effect and oblateness," *Journal of Astrophysics and Astronomy*, vol. 39, no. 3, p. 28, 2018.

[8] M. J. Idrisi and M. S. Ullah, "A study of albedo effects on libration points in the elliptic restricted three-body problem," *Journal of Astronautical Sciences*, vol. 67, no. 3, pp. 863–879, 2020.

[9] M. J. Idrisi and M. S. Ullah, "Out-of-plane equilibrium points in the elliptic restricted three-body problem under albedo effect," *New Astronomy*, vol. 89, article 101629, 2021.

[10] M. Jain and R. Aggarwal, "A study of non-collinear libration points in restricted three body problem with stokes drag effect when smaller primary is an oblate spheroid," *Astrophysics and Space Science*, vol. 358, no. 2, p. 51, 2015.

[11] M. J. Idrisi and M. Jain, "Restricted three-body problem with stokes drag effect when less massive primary is an ellipsoid," *International Journal of Advanced Astronomy*, vol. 4, no. 1, pp. 61–67, 2016.

[12] M. J. Idrisi and M. S. Ullah, "A study of non-collinear libration points under the combined effects of radiation pressure and Stokes drag," *New Astronomy*, vol. 82, article 101441, 2021.

[13] M. S. Suraj, R. Aggarwal, M. C. Asique, and A. Mittal, "On the modified circular restricted three-body problem with variable mass," *New Astronomy*, vol. 84, article 101510, 2021.

[14] M. S. Suraj, R. Aggarwal, M. C. Asique, and A. Mittal, "On the basins of convergence in the magnetic-binary problem with angular velocity," *Computational and Mathematical Methods*, vol. 3, no. 4, p. 1161, 2021.

[15] M. S. Suraj, S. Alhowaity, R. Aggarwal, M. C. Asique, A. Mittal, and M. Jain, "On the topology of basins of convergence linked to libration points in the modified R3BP with oblateness," *New Astronomy*, vol. 94, article 101776, 2022.

[16] H. Yukawa, "On the interaction of elementary particles," *Proceedings of the Physico-Mathematical Society of Japan 3rd Series*, vol. 17, pp. 48–57, 1935.

[17] E. Fischbach, C. Talmadge, and D. E. Krause, "Exponential models of non-Newtonian gravity," *Physical Review D*, vol. 43, no. 2, pp. 460–467, 1991.

[18] F. Kokubun, "Restricted problem of three bodies with Newtonian+Yukawa potential," *International Journal of Modern Physics D*, vol. 13, no. 5, pp. 783–806, 2004.

[19] N. I. Kolosnitsyn and V. N. Melnikov, "Test of the inverse square law through precession of orbits," *General Relativity and Gravitation*, vol. 36, no. 7, pp. 1619–1624, 2004.

[20] C. Massa, "Mach's principle and Yukawa-like corrections to the Newtonian potential as a bound for the mean energy density of the universe," *Astrophysics and Space Science*, vol. 315, no. 1-4, pp. 285-286, 2008.

[21] I. Haranas and O. Ragos, "Yukawa-type effects in satellite dynamics," *Astrophysics and Space Science*, vol. 331, no. 1, pp. 115–119, 2011.

[22] D. Pricopi, "Stability of the celestial body orbits under the influence of Yukawa potential," *Astrophysics and Space Science*, vol. 361, no. 8, p. 277, 2016.

[23] I. Haranas, O. Ragos, I. Gkigkitzis, I. Kotsireas, C. Martz, and S. V. Middekoop, "The Poynting-Robertson effect in the Newtonian potential with a Yukawa correction," *Astrophysics and Space Science*, vol. 363, no. 1, p. 3, 2018.

[24] E. Cavan, I. Haranas, I. Gkigkitzis, and K. Cobbett, "Dynamics and stability of the two body problem with Yukawa correction," *Astrophysics and Space Science*, vol. 365, no. 2, p. 36, 2020.

[25] V. Szebehely, *Theory of Orbits, The Restricted Problem of Three Bodies*, Academic Press, New York and London, 1967.