

Research Article

Modeling and Mathematical Analysis of Liquidity Risk Contagion in the Banking System

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Received 19 October 2021; Revised 18 May 2022; Accepted 31 May 2022; Published 20 June 2022

Academic Editor: Bruno Carpentieri

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In recent times, all world banks have been threatened by the liquidity risk problem. This phenomenon represents a devastating financial threat to banks and may lead to irrecoverable consequences in case of negligence or underestimation. In this article, we study a mathematical model that describes the contagion of liquidity risk in the banking system based on the SIR epidemic model simulation. The model consists of three ordinary differential equations illustrating the interaction between banks susceptible or affected by liquidity risk and tending towards bankruptcy. We have demonstrated the bornness and positivity of the solutions, and we have mathematically analyzed this system to demonstrate how to control the banking system's stability. Numerical simulations have been illustrated by using real data to support the analytical results and prove the effects of different system parameters studied on the contagion of liquidity risk.

1. Introduction

The financial crisis of 2007-2008 was marked by a liquidity crisis and sometimes by solvency crises at the level of both banks and states and a shortage of credit for businesses. It began in July 2007 and was triggered by the deflation of price bubbles (including the American real estate bubble of the 2000s) and the significant losses of financial institutions caused by the subprime crisis. It is part of the "Great Recession" that began in 2008 and whose effects will be felt beyond 2010. The global banking crisis, which began in the summer of 2007 [1, 2], reminded us of and called into question the management of banking risks by general and liquidity risk until now neglected in favor of other risks such as credit risk or market risk. It is noteworthy that the international harmonization of banking regulations, through Basel I (1988) or Basel II (2004), excluded the risk of liquidity from its scope. Liquidity is the ability to meet obligations due at a future date.

Liquidity risk contagion in banking systems occurs when banks experience liquidity shortages and other banks withdraw funds because they believe their counterparty may not repay on time, leading to an interbank domino effect. The interbank market is presented as a network where banks are linked together via interbank deposits and lines of credit [3]. Hence, we can distinguish two types of contagion: contagion via interbank deposits and contagion via the line of credit. The first type of contagion occurs when a bank's risk causes losses on creditors' claims. Contagion through lines of credit corresponds to the second mechanism. It intervenes when there are banks that depend financially on lines of credit, which may subsequently run out. This phenomenon can have a worldwide effect. Based on reference [4], contagion is the probability of a crisis triggering in one country following a similar event in another country. In addition, the transmission of external shocks to emerging countries is accompanied by the amplification of disturbances. Therefore, a minor change in industrial countries can influence

other countries because capital flows to emerging countries are sensitive to fluctuations in interest rates on international financial markets. This mechanism plays a vital role in triggering contagious crises.

Liquidity risk contagion in the banking system is a global concern that many researchers and practitioners have well documented. However, the current literature on liquidity risk contagion has not fully addressed how this phenomenon can be modeled mathematically using differential equations. This paper introduces modeling and analyzing liquidity risk contagion with numerical modeling using differential equations.

Numerical modeling minimizes a problem on a computer to have a model that can be easily manipulated to operate various modifications to improve the problem in question. This process also serves to subject all kinds of constraints to the virtual projection of the studied problem and analyze the impacts that will allow changes to be made to optimize the result. Numerical modeling can be used in many areas such as medicine [5, 6], economic [7], environment [8], transport [9], and finance [10, 11]. We note that, for each area, there have been a number of prominent illustrative modeling applications that were proposed and developed in recent years, especially modeling infectious epidemics [12], urban crim [13], pricing of different types of options [14], chaotic systems [15], nonlinear evolution dynamics [16], etc.

On the other hand, the susceptible-infected-removed (SIR) model introduced in [17] is one of the most widely used numerical models for modeling different issues concerning different areas. This model is a deterministic compartment model that separates a fixed population of individuals, into three possible states: susceptible (S), infected (I), and removed (or recovered) (R). The transition from one state to another is managed by three coupled differential equations.

In the field of financial applications, and to the best of our knowledge, there are only five references, namely, refs. [12, 18–21], which have used the SIR model for modeling the spread of liquidity risk contagion. In what follows, we describe each of these references in order to identify its advantages and disadvantages. The major advantage of references [12, 19–21] is that they have used real data for simulating the liquidity risk contagion based on SIR model. However, there are three drawbacks with these references. The first drawback is that they have used only two parameters: the contagion rate and recovery rate in order to ensure the transition between the states: susceptible (S), infected (I), and recovered (R) of the SIR model. The second drawback is that there is no proof of the positivity and boundedness concerning the results of the model SIR. The third drawback is that there is no mathematical analysis of the SIR system proposed.

On the other hand, the major advantage of reference [18] is that the dynamical flow of the SIR model is managed by three parameters which are the contagion rate, the recovery rate, and the bankruptcy rate. However, the SIR model proposed in reference [18] suffers from two limitations. The first one is that there is no proof of the positivity and

boundedness concerning the results of the SIR model. The second limitation is that there are no simulations results to validate the proposed SIR model.

Based on this literature review, in this paper, we model the financial contagion linked to liquidity in interbank networks through the SIR model similar to the system used in [18]. The main contributions of this paper can be summarized as follows:

- (i) We have introduced the SIR system to model the liquidity risk contagion
- (ii) We have used three parameters, the contagion rate, the recovery rate, and the bankruptcy rate, for ensuring the transition between different states of the SIR model
- (iii) We have proven the positivity and boundedness concerning the results of the SIR model
- (iv) We have provided mathematical analysis of the SIR system
- (v) We have evaluated the performance of our SIR model by using real data

2. Liquidity Risk

2.1. Short-Term Liquidity Ratio. This standard was established to ensure that the bank has an adequate level of high liquid asset unencumbered quality that can be converted into liquidity to cover its need over 30 calendar days in the event of serious financing difficulties [22].

These serious financing difficulties refer to the scenario of the financial crisis of 2007/2008. The banking system world has experienced strong tensions following significant downgrades of the ratings of several credits. These downgrades have damaged the confidence of the public and the markets in these establishments and resulted in partial withdrawals of deposits and loss of funding.

The ratio will thus require banks to have a certain amount of liquid assets enabling them to cover the net cash outflow for at least 30 days. According to reference [23], it is defined as follows:

$$\frac{\text{Out standing high-quality liquid assets}}{\text{Net cash outflow over the next 30 days}} \geq 100\%. \quad (1)$$

According to this standard, the stock of high-quality liquid assets must at least be equal to the net cash outflows during the 30 days following the cut-off date for calculating the ratio. By respecting this ratio, the establishment should thus have sufficient liquidity despite the difficulties of refinancing on the markets.

2.2. Long-Term Structural Liquidity Ratio. The long-term structural liquidity ratio complements the liquidity ratio of the short term. Its goal is to provide any financial institution with “stable funding that allows it to continue to sound its activities for one year in a scenario of prolonged tensions.” Based on reference [23], it is defined as follow:

$$\frac{\text{Stable funding amount available}}{\text{Amount of stable funding required}} \geq 100\%. \quad (2)$$

It corresponds to the amount of stable funding available (liabilities) compared to the required amount (active). This ratio must be at least equal to 100%. In other words, the amount of stable funding available must at least be equivalent to the amount of stable funding required.

The NSFR implies that the liability items making up the stable resources are weighted in a declining fashion depending on their stability over a one-year horizon, the weighting depending on the type of resource and the counterpart. Thus, the texts understand by available stable funding of a financial institution the sum of its equity, its preferred shares, and all its other liabilities with a term of more than one year as well as its deposits without maturity or with a duration of less than one year from individuals or SMEs or large companies. The required stable funding comprises assets to which variable weightings are assigned depending on their degree of liquidity.

2.3. Cash Ratios. Banks must select a set of risk indicators that are most relevant to their situation and strategy [24]. Banks specializing in lending should consider the net lending, deposits, size, and capitalizations of banks. At the same time, studies of the causes of liquidity risk have also pointed out that the determinants of liquidity risk can be measured with different balance sheet indices [25].

Since our model is lending-driven, we need to use lending-related factors like total loans, total deposits, volatile deposits, liquid assets, short-term investments, and central bank lending. These elements can be converted into ratios with specific thresholds determined by experts. These ratios can be used, in turn, as indicators of liquidity risk and, therefore, as input variables of the model. Based on reference [26], the indicators/input variables we have chosen for the model are listed below:

$$\begin{aligned} R_1 &= \frac{\text{Liquid assets}}{\text{Total assets}}, \\ R_2 &= \frac{\text{Liquid assets}}{\text{Current liabilities}}, \\ R_3 &= \frac{\text{Loans}}{\text{Total assets}}, \\ R_4 &= \frac{\text{Liquid assets}}{\text{Deposit}}, \\ R_5 &= \frac{\text{Loans}}{\text{Deposit} + \text{Current liabilities}}, \\ R_6 &= \frac{\text{Loans} - \text{Deposits}}{\text{Total assets}}. \end{aligned} \quad (3)$$

Results refer to the first economic scenario (LC-LD) and a density of initially distressed banks equal 1% Figure 1 [12]. In this work, we are interested in the following main three ratios:

- (i) General liquidity ratio: the general liquidity ratio is calculated by dividing the number of assets at less than one year by liabilities of equivalent duration. According to reference [27], its expression is as follows:

$$\frac{\text{Short-term assets}}{\text{Short-term liabilities}} = \text{General liquidity ratio} \quad (4)$$

Like working capital, this ratio makes it possible to know whether a company is capable (or not) of its short-term debts by comparing them with its short-term assets. This ratio must always be greater than 1. The higher it is, the greater the latitude a company can develop its activities. In the circumstances, a ratio less than one means that the working capital is negative.

- (ii) Restricted liquidity ratio: the restricted liquidity ratio is a liquidity indicator equal to the ratio of current assets (the most liquid assets on a balance sheet) from which stocks are removed over short-term liabilities (debts within 12 months). Based on reference [26], its calculation formula is the following:

$$\frac{\text{Current assets} - \text{inventories}}{\text{Short-term debts}} = \text{Restricted liquidity ratio} \quad (5)$$

A company with a ratio of less than 1 will find itself in difficulty if its creditors demand to be paid immediately.

- (iii) Immediate liquidity ratio: a company with a ratio of less than 1 will find itself in difficulty if its creditors demand to be paid immediately. Although there is no strict standard for setting it, this ratio is usually established from the following formula [28]:

$$\frac{\text{Cash}}{\text{Short-term debts}} = \text{Immediate liquidity ratio} \quad (6)$$

3. Mathematics Model

The modeling of this problem is based on the research work in [18].

We consider the following differential system:

$$\begin{cases} \frac{df_1(t)}{dt} = -\alpha_1 f_1(t) f_2(t) + \alpha_2 f_2(t), \\ \frac{df_2(t)}{dt} = -\alpha_2 f_2(t) - \alpha_3 f_2(t) + \alpha_1 f_1(t) f_2(t), \\ \frac{df_3(t)}{dt} = \alpha_3 f_2(t). \end{cases} \quad (7)$$

with

$f_1(t)$: the set of banks at instant t , which have the general ratio >1

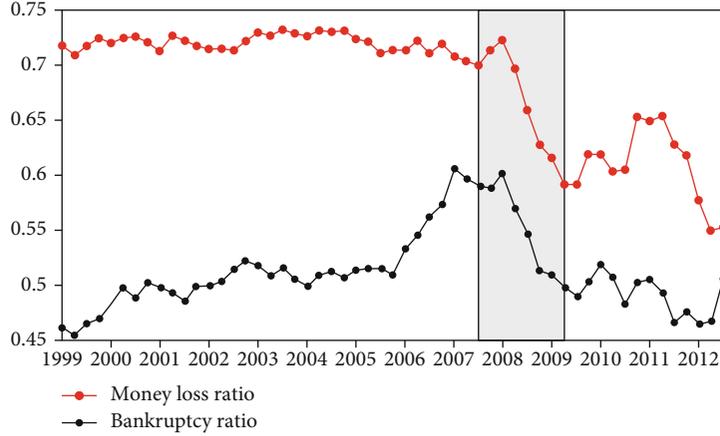


FIGURE 1: Relative liquidity loss compared to systemic risk given by the ratio of bankrupted banks [12].

$f_2(t)$: the set of banks at instant t , which have the general ratio <1

$f_3(t)$: all the banks at the instant t , which have gone bankrupt

α_1 : spread rate. Parameter α_1 indicates the speed at which contagion spreads. It is a measure of the contagiousness of an infected bank, where a susceptible bank in contact with a single infected bank will become infected with probability α_1

α_2 : solution rate. This second parameter represents the speed of recovery. It is a measure of the resistance to contagion. Accordingly, an infected bank will recover with probability α_2

α_3 : bankruptcy rate. This third parameter represents the rate at which an infected bank will go bankrupt. Therefore, a distressed bank will go bankrupt with a probability α_3 , with the initial conditions

$$\begin{aligned}
 t &= 0, \\
 f_1(0) &= a \neq 0, \\
 f_2(0) &= 0, \\
 f_3(0) &= 0
 \end{aligned} \tag{8}$$

and $f_1(t) + f_2(t) + f_3(t) = N$, where N is the total number of banks

$df_1(t)/dt$: breakdown of two terms. The first term means that the set of banks at the moment t , who have the general ratio >1 , decreased by $\alpha_1 f_1(t) f_2(t)$, and the second term means that the set of banks at the moment t , who have the general ratio <1 , increases by $\alpha_2 f_2(t)$

$df_2(t)/dt$: breakdown of 3 terms. The first term means that the set of banks at the moment t , who have the general ratio <1 , decreased by $\alpha_2 f_2(t)$ and $\alpha_3 f_2(t)$, and the last term means that the set of banks at the moment t , who have the general ratio <1 , increases by $\alpha_1 f_1(t) f_2(t)$

$df_3(t)/dt$: means that all banks at the moment t , who have reached bankruptcy, increased by $\alpha_3 f_2(t)$

4. Bornitude and Positivity of Solutions

In this section, we will establish the positivity and the Bornitude of the solutions of the model (1).

4.1. Positivity of Solutions

Proposition 1. For any positive initial conditions $f_1(0), f_2(0)$, and $f_3(0)$, model variables (1) $f_1(t), f_2(t)$, and $f_3(t)$ will remain positive for everything $t > 0$.

Proof. Is

$$T = \{ \tau \geq 0 : \forall t, 0 \leq t \leq \tau : f_1(t) \geq 0, f_2(t) \geq 0, f_3(t) \geq 0 \}. \tag{9}$$

Let us show that $T = +\infty$. Suppose that $0 < T < +\infty$; by continuity of solutions, we have $f_1(T) = 0$ or $f_2(T) = 0$ or $f_3(T) = 0$.

If $f_1(T) = 0$, before variables f_2 and f_3 equal to zero, therefore

$$\frac{df_1(T)}{dt} = \lim_{t \rightarrow T^-} \frac{f_1(T) - f_1(t)}{T - t} = \lim_{t \rightarrow T^-} \frac{-f_1(t)}{T - t} \leq 0. \tag{10}$$

From the first equation of system (7), we have

$$\frac{df_1(T)}{dt} = \alpha_2 f_2(T) > 0. \tag{11}$$

If $f_2(T) = 0$, before f_1 and f_3 equal to zero, therefore

$$\frac{df_2(T)}{dt} = \lim_{t \rightarrow T^-} \frac{f_2(T) - f_2(t)}{T - t} = \lim_{t \rightarrow T^-} \frac{-f_2(t)}{T - t} \leq 0. \tag{12}$$

From the second equation of system (7), we have

$$\frac{df_2(T)}{dt} = 0 \geq 0. \tag{13}$$

If $f_3(T) = 0$, before f_1 and f_2 equal to zero, therefore

$$\frac{df_3(T)}{dt} = \lim_{t \rightarrow T} \frac{f_3(T) - f_3(t)}{T - t} = \lim_{t \rightarrow T} \frac{-f_3(t)}{T - t} \leq 0. \quad (14)$$

From the third equation of system (7), we have

$$\frac{df_3(T)}{dt} = \alpha_3 f_2(T) > 0. \quad (15)$$

Hence, we conclude that $T = +\infty$. \square

4.2. Boundedness of Solutions

Proposition 2. *The solutions $f_1, f_2,$ and f_3 of system (7) are bounded.*

Proof. From the first and second equation of (7), we have

$$\begin{aligned} \dot{f}_1(t) + \dot{f}_2(t) &= -\alpha_3 f_2(t), \\ \dot{f}_1(t) + \dot{f}_2(t) + \alpha_3 f_2(t) &= 0. \end{aligned} \quad (16)$$

Since

$$f_1(t) + f_2(t) + f_3(t) = N, \quad (17)$$

hence

$$\dot{f}_1(t) + \dot{f}_2(t) + \alpha_3 f_2(t) + f_1(t) \leq N, \Rightarrow \dot{g}(t) + \gamma g(t) \leq N. \quad (18)$$

With

$$\begin{aligned} g(t) &= f_1(t) + \alpha_3 f_2(t), \\ \gamma &= \min(\alpha_3, 1), \end{aligned} \quad (19)$$

hence

$$\begin{aligned} \frac{d}{d\xi} \left(e^{\gamma\xi} g(\xi) \right) &\leq N e^{\gamma\xi}, \Rightarrow e^{\gamma t} g(t) + g(0) \leq \frac{N}{\gamma} (e^{\gamma t} - 1), \\ g(t) &\leq e^{-\gamma t} g(0) + \frac{N}{\gamma} (1 - e^{-\gamma t}). \end{aligned} \quad (20)$$

Gold

$$0 \leq e^{-\gamma t} \leq 1 \text{ et } 0 \leq 1 - e^{-\gamma t} \leq 1, \quad (21)$$

hence

$$g(t) \leq g(0) + \frac{N}{\gamma}. \quad (22)$$

According to the third equation of (7), we have

$$\begin{aligned} \dot{f}_3(t) &\leq \alpha_2 f_2(t), \Rightarrow \int_0^t \dot{f}_3(\xi) d\xi \\ &\leq \int_0^t \alpha_2 f_2(\xi) d\xi, \Rightarrow f_3(t) \\ &\leq f_3(0) + \alpha_3 \|f_2\|_\infty. \end{aligned} \quad (23)$$

\square

5. Mathematical Analysis of the System

This part concerns the study of variation of f_2 in terms of f_1 .

From the first and second equations of system (7), we have:

$$\begin{aligned} \frac{df_2(t)}{df_1(t)} &= \frac{-\alpha_2 f_2(t) - \alpha_3 f_2(t) + \alpha_1 f_1(t) f_2(t)}{-\alpha_1 f_1(t) f_2(t) + \alpha_2 f_2(t)} \\ &= \frac{\alpha_1 f_1(t) f_2(t) - \alpha_2 f_2(t)}{-\alpha_1 f_1(t) f_2(t) + \alpha_2 f_2(t)} \\ &\quad + \frac{-\alpha_3 f_2(t)}{-\alpha_1 f_1(t) f_2(t) + \alpha_2 f_2(t)} \\ &= -1 + \frac{-\alpha_3 f_2(t)}{f_2(t)(-\alpha_1 f_1(t) + \alpha_2)}, \end{aligned} \quad (24)$$

$$\frac{df_2(t)}{df_1(t)} = -1 + \frac{\alpha_3}{\alpha_1 f_1(t) - \alpha_2}. \quad (25)$$

His solution is

$$\begin{aligned} \frac{f_2'(t)}{f_1'(t)} &= -1 + \frac{\alpha_3}{\alpha_1 f_1(t) - \alpha_2}, \Rightarrow \int f_2'(t) = \int -f_1'(t) + \int \frac{\alpha_3 f_1'(t)}{\alpha_1 f_1(t) - \alpha_2}, \Rightarrow f_2(t) \\ &= -f_1(t) + \frac{\alpha_3}{\alpha_1} \int \frac{\alpha_1 f_1'(t)}{\alpha_1 f_1(t) - \alpha_2}, \Rightarrow \boxed{f_2(t) = -f_1(t) + \frac{\alpha_3}{\alpha_1} \ln(\alpha_1 f_1(t) - \alpha_2) + C}. \end{aligned} \quad (26)$$

According to the initial conditions (8), we have

$$f_2(0) = -f_1(0) + \frac{\alpha_3}{\alpha_1} \ln(\alpha_1 f_1(0) - \alpha_2) + C, \Rightarrow \boxed{C = a - \frac{\alpha_3}{\alpha_1} \ln(\alpha a \alpha_1 - \alpha_2)}. \quad (27)$$

Drifting

$$f_2'(t) = -f_1'(t) + \frac{\alpha_3}{\alpha_1} \times \frac{\alpha_1 f_1'(t)}{\alpha_1 f_1(t) - \alpha_2} = -f_1'(t) + \frac{\alpha_3 f_1'(t)}{\alpha_1 f_1(t) - \alpha_2}, \boxed{f_2'(t) = f_1'(t) \left[\frac{\alpha_3}{\alpha_1 f_1(t) - \alpha_2} - 1 \right]}. \quad (28)$$

We have

$$f_1'(t) = -\alpha_1 f_1(t) f_2(t) + \alpha_2 f_2(t) = f_2(t) [\alpha_2 - \alpha_1 f_1(t)]. \quad (29)$$

$\exists t_0 \in [0, +\infty[$, $\forall t \in [0, t_0]$; we have $f_1(t)$ decreasing and $f_2(t)$ increasing that is to say that $f_1'(t) < 0, f_2'(t) > 0$ for

$$\begin{aligned} f_2'(t) &= f_1'(t) \left[\frac{\alpha_3}{\alpha_1 f_1(t) - \alpha_2} - 1 \right] > 0, \\ f_1'(t) &= f_2(t) [\alpha_2 - \alpha_1 f_1(t)] < 0. \end{aligned} \quad (30)$$

The solution exists if

$$t > t_0, f_1(t) > \frac{\alpha_2 + \alpha_3}{\alpha_1}. \tag{31}$$

Accordingly, there is a possible solution, when $t \in [t_0, +\infty)$, $f_1(t) < (\alpha_2/\alpha_1)$.

More if

$$f_1'(t) \left[\frac{\alpha_3}{\alpha_1 f_1(t) - \alpha_2} - 1 \right] > 0. \tag{32}$$

We have 2 cases to discuss:

(1) If $f_1'(t) > 0$,

$$\frac{\alpha_3}{\alpha_1 f_1(t) - \alpha_2} - 1 > 0, \tag{33}$$

so there is no solution

(2) If $f_1'(t) < 0$,

$$\frac{\alpha_3}{\alpha_1 f_1(t) - \alpha_2} - 1 < 0. \tag{34}$$

The solution is

$$f_1(t) > \frac{\alpha_2 + \alpha_3}{\alpha_1} \tag{35}$$

When $f_1(t) > ((\alpha_2 + \alpha_3)/\alpha_1)$,

$$f_2'(t) > 0, \tag{36}$$

which means that the number of banks at the moment t , who have the general ratio >1 , increased.

If we have

$$f_1'(t) \left[\frac{\alpha_3}{\alpha_1 f_1(t) - \alpha_2} - 1 \right] < 0, \tag{37}$$

we have two cases to be discussed:

(1) If $f_1'(t) > 0$,

$$\frac{\alpha_3}{\alpha_1 f_1(t) - \alpha_2} - 1 < 0. \tag{38}$$

TABLE 1: The values of initial parameters conditions of the model (1) in Spain.

Parameters	Figure 2(a)	Figure 3(a)	Figure 4(a)
α_1	0.005	0.005	0.005
α_2	0.013	0.013	0.013
α_3	1.0	0.1	0.0001
$f_1(0)$	158.0	158.0	158.0
$f_2(0)$	10.0	10.0	10.0
$f_3(0)$	0.0	0.0	0.0

TABLE 2: The values of initial parameters conditions of the model (1) in France.

Parameters	Figure 2(b)	Figure 3(b)	Figure 4(b)
α_1	0.002	0.002	0.002
α_2	0.02	0.02	0.02
α_3	1.0	0.1	0.0001
$f_1(0)$	158.0	158.0	158.0
$f_2(0)$	10.0	10.0	10.0
$f_3(0)$	0.0	0.0	0.0

TABLE 3: The values of initial parameters conditions of the model (1) in Germany.

Parameters	Figure 2(c)	Figure 3(c)	Figure 4(c)
α_1	0.0012	0.0012	0.0012
α_2	0.008	0.008	0.008
α_3	1.0	0.1	0.0001
$f_1(0)$	158.0	158.0	158.0
$f_2(0)$	10.0	10.0	10.0
$f_3(0)$	0.0	0.0	0.0

His solution is

$$f_1(t) < \frac{\alpha_2}{\alpha_1} \tag{39}$$

(2) If $f_1'(t) < 0$,

$$\frac{\alpha_3}{\alpha_1 f_1(t) - \alpha_2} - 1 > 0. \tag{40}$$

His solution is

$$\frac{\alpha_2}{\alpha_1} < f_1(t) < \frac{\alpha_2 + \alpha_3}{\alpha_1}, \tag{41}$$

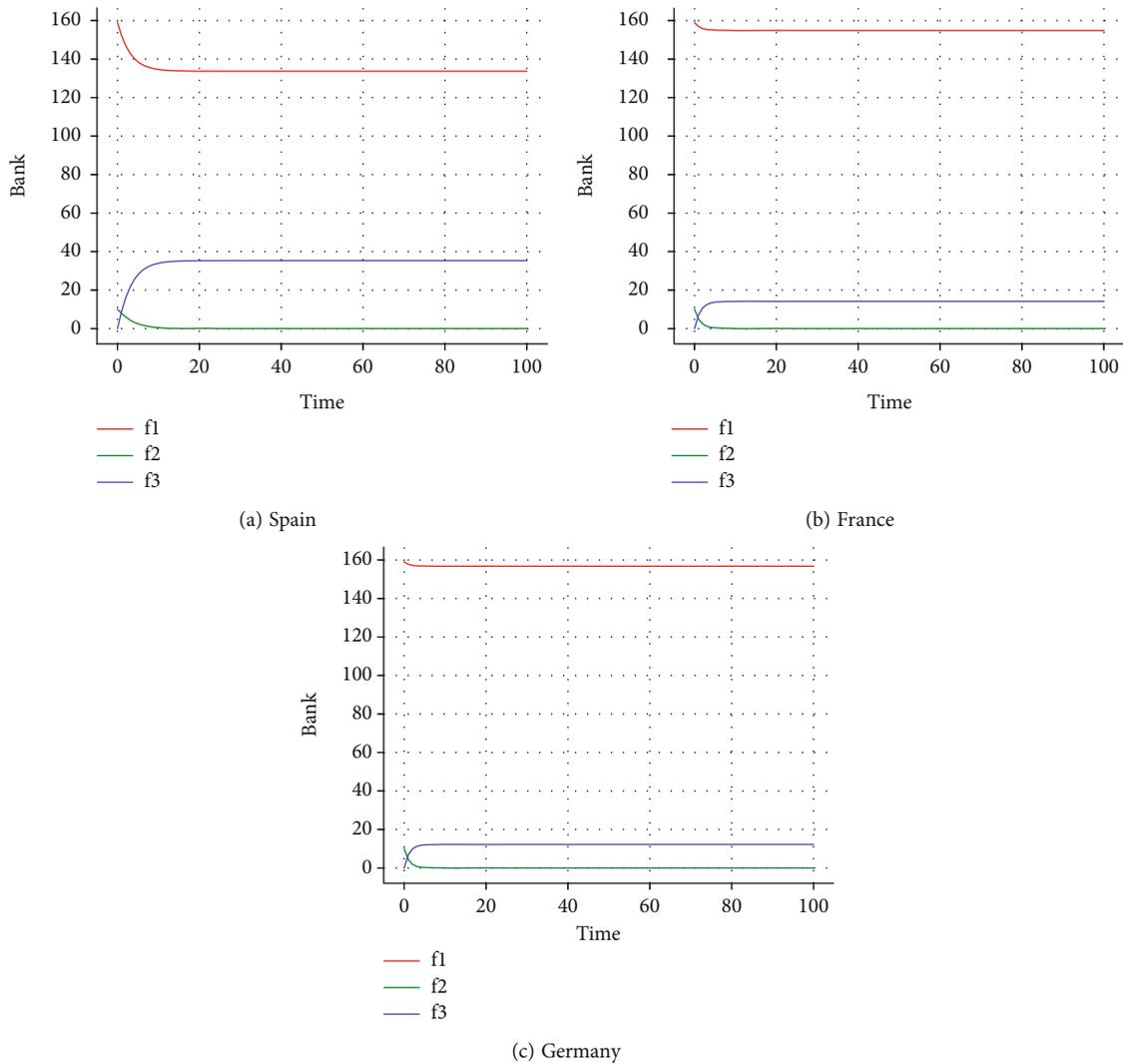


FIGURE 2: Liquidity risk in Spain, France, and Germany behavior over time with $\alpha_3 = 1.0$ and $t = 100$.

which means that the number of banks at the moment t , who have the general ratio < 1 , decreased when

$$\frac{\alpha_2}{f_1(t)} < \alpha_1 < \frac{\alpha_2 + \alpha_3}{f_1(t)}, \tag{42}$$

if

$$\begin{aligned} f_1'(t) \left[\frac{\alpha_3}{\alpha_1 f_1(t) - \alpha_2} - 1 \right] &= 0, \\ f_1'(t) = 0, \Rightarrow \frac{\alpha_3}{\alpha_1 f_1(t) - \alpha_2} - 1 &= 0, \\ f_1'(t) &= 0. \end{aligned} \tag{43}$$

Hence, according to (25) absurd,

$$f_1(t) = \frac{\alpha_2 + \alpha_3}{\alpha_1} \text{ et } f_2'(t) = 0. \tag{44}$$

For $f_2(t)$ around $f_1(t) = ((\alpha_2 + \alpha_3)/\alpha_1)$, the derivation gives a change of sign from

$$f_1(t) < \frac{\alpha_2 + \alpha_3}{\alpha_1}, f_2'(t) < 0, \tag{45}$$

at

$$f_1(t) > \frac{\alpha_2 + \alpha_3}{\alpha_1}, f_2'(t) > 0, \Rightarrow f_1(t) = \frac{\alpha_2 + \alpha_3}{\alpha_1}. \tag{46}$$

We get a maximum in $f_2(t)$ when

$$\alpha_1 < \frac{\alpha_2 + \alpha_3}{f_1(t)}. \tag{47}$$

Therefore, the liquidity risk will disappear.

From the above, we can see that in liquidity risk, it is necessary to strengthen prevention, making α_2 reduce, and help strengthen α_1 , an increase which makes the maximum

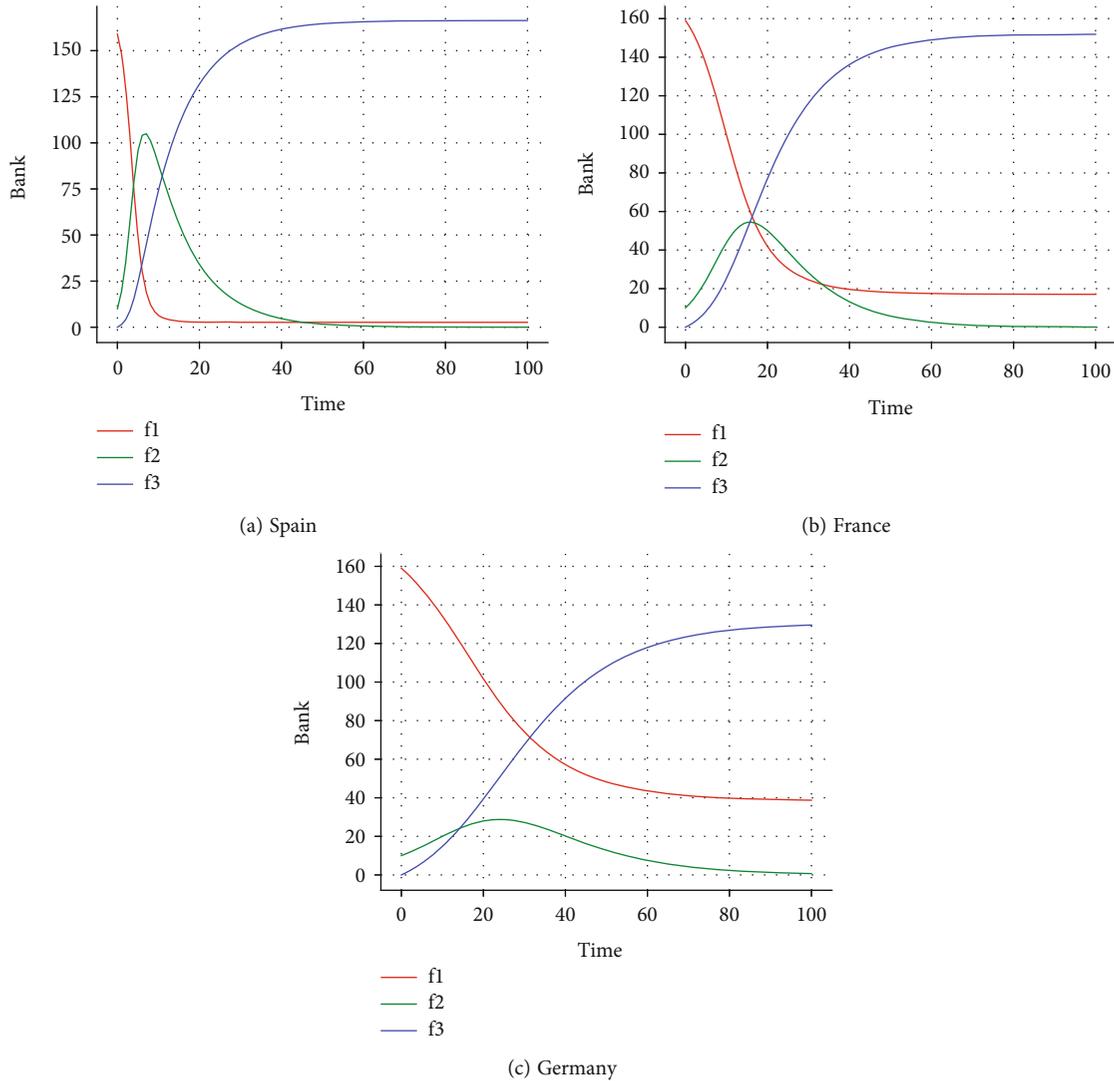


FIGURE 3: Liquidity risk in Spain, France, and Germany behavior over time with $\alpha_3 = 0.1$ and $t = 100$.

$f_2(t)$. Then, we could reduce or avoid the negative impact on the banking group.

6. Numerical Simulation

In our work, in order to visualize the impact of bankruptcy rate, denoted α_3 on system (7), we are going to simulate 3 cases:

- (i) 100% of all distressed banks are going to fail ($\alpha_3 = 1$)
- (ii) The chance of a distressed bank failure is 10% ($\alpha_3 = 0.1$)
- (iii) The probability of a distressed bank failure is almost nil ($\alpha_3 = 0.0001$)

To simulate this cases in a real life scenario, we have extracted the values of the propagation and recovery rates, α_1 and α_2 , from the findings of Philipps et al. [21]. The data is taken in relation to the year 2012, for 169 major European

banks, calculated in [21]. Therefore in our numerical simulation, we assume $N = 169$ as the total number of banks, and that 10 banks are affected by liquidity risk at time $t = 0$.

For each case, we have chosen three liquidity crisis starting scenarios: a crisis emerging from Spain, a crisis emerging from France, and finally a crisis emerging from Germany. The data for each case and scenario are presented in Tables 1, 2, and 3.

Data on the behavior of the system liquidity risk model (1) during the same period is illustrated in Figures 2, 3, and 4.

In the first case, with the highest probability of failure, we find that for the 3 liquidity crisis scenarios, the contagion rate drops rapidly since the failure rate is very high. The banks close quickly, which stops the contagion. Figure 2(a) shows that a crisis that begins in Spain is the most serious, because in a short time, 1/4 of all banks went bankrupt (Figures 2(b) and 2(c)). In France and Germany on the other hand, the failure rate is almost identical, with 1/8 of the banks going bankrupt.

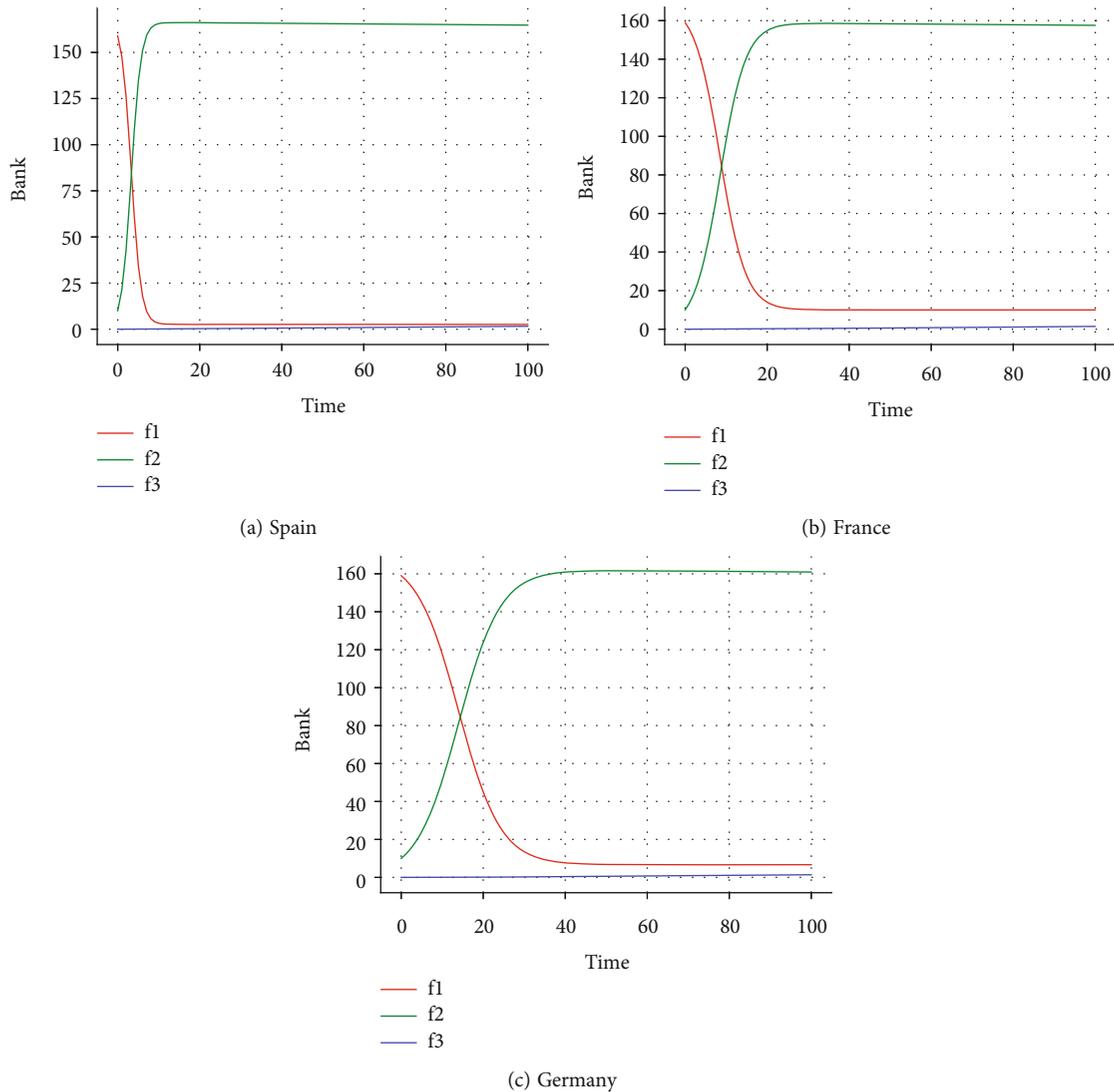


FIGURE 4: Liquidity risk in Spain, France, and Germany behavior over time with $\alpha_3 = 0.0001$ and $t = 100$.

In the second case in Figures 3(a)–3(c), with a realistic probability of failure, the spread of the liquidity crisis is more pronounced. Given that the rate at which banks fail is lower than in the first case, the crisis has more time to spread, which means that more banks are in distress and end up closing their doors. However, the figures show that there is a difference between the crisis scenarios.

The case of Spain is the least favorable; the gap is increasing rapidly leading to a rapid failure of the whole system. For Germany on the other hand, with the strongest economy, the liquidity crisis curve is more flattened resulting in a financial failure occurring in a longer period of time and less bankruptcy rates. In the third case in Figures 4(a)–4(c), we have assumed that the probability of failure after a liquidity crisis is very low. The numerical simulation shows that in the case where the spread of the crisis is the highest, all banks in difficulty continue to operate, which has the consequence of infecting the entire financial system.

The graphs show that the spread of the crisis is different depending on the country of departure. The spread is the fastest from Spain, followed by France and then Germany.

In the 3 cases, the simulation shows that the effect of the severity of the failure rate is closely related to the country's economy. In the case of Spain with the most fragile economy, a high failure rate has more effect on the global banking market but a lower one results in the fastest spread of the liquidity risk across the economy. For Germany, one of the strongest and most influential Europe countries, a high failure rate is less severe and a lower rate results in a more controllable contagion spread overall.

7. Conclusion

This paper studied the modeling of liquidity risk contagion in the banking system, transmission, and control perspective of the dynamic model of infectious diseases. Our results

prove our research's originality and theoretical and practical implications. The theoretical implications consist of our mathematical model and the positivity and the Bornitude of the solutions of the model.

The practical implications are revealed to bank managers and other decision-maker stakeholders by our recommendations that allow them to reduce or avoid the contagion of liquidity risk in the banking system.

The recommendations are based on the following results:

- (i) When the rate spread $(\alpha_2/(f_1(t))) < \alpha_1 < ((\alpha_2 + \alpha_3)/(f_1(t)))$, the spread area is not large; the risk will soon disappear. While the rate spread $\alpha_1 > ((\alpha_2 + \alpha_3)/(f_1(t)))$, the crisis will spread. So, it is necessary to strengthen their awareness of prevention. We must reduce α_2 and at the same time strengthen management and help affected banks to avoid contagion and increase α_1 the maximum point with $f_2(t)$ growing
- (ii) Once the risk has appeared in the banking system, we must disclose information on time and take adequate measures to induce the associated bank to face the risk and strengthen with other banks to manage risk development. Many researchers analyzed risk management in real time in different countries as Holod et al. [29], Amin et al. [30], and Choudhury and Daly [31]
- (iii) The number of initial banks in the household risk in the banking system has a significant impact on the extension of the risk period, and many banks are affected. Our results agree with the research of Machiatti et al. [20], who concluded that their proposed contagion model requires timely, viable, and reliable bank answers to liquidity shocks
- (iv) Once the risk emerges in a bank, it is necessary to isolate the affected banks and take strict control measures. Therefore, X. Zhang et al. [32] underlined the bank's role directly connected with the financial system in quickly propagating systemic risk

Given the horizontal linkages in the interbank market, all banks implement a form of financial control. This control is necessary to prevent global contamination and avoid the serious consequences of spreading contagion between banks. This partly explains why the European Central Bank exists.

Future research will be oriented to modeling new contagion channels because of the COVID-19 crisis and other risk factors, such as the Russian-Ukrainian war. Therefore, the liquidity risk will be analyzed in correlation with the interbank network structure. Finally, we will investigate to use the optimal control of our current system in order to reduce the number of contagious bank.

Data Availability

No data were used.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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