Research Article


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In this article, the effect of electromagnetic force with the chemical and thermal radiation effect on the Oldroyd-B fluid past an exponentially stretched sheet with a heat sink and porous medium was studied. The governing system of nonlinear partial differential equations has transformed into a system of ordinary differential equations using similarity transformations. The system is then solved numerically using a successive linearization method. The numerical results of velocity, temperature, and concentration profiles are represented graphically. Several parameters’ effects are investigated and examined. Local Nusselt number, skin friction coefficient, porosity, Deborah numbers, and local Sherwood number numerical values are listed and analyzed. The results reveal that many parameters have a significant impact on the fluid flow profiles. The concentration profiles were considerably affected by the reaction rate parameter, and the concentration thickness of the boundary layer decreased as the reaction rate parameter increased. The results of the analysis were compared to the results of existing works and found to be in excellent agreement.

1. Introduction

Extrusion processes, biological fluid flow, hot rolling, lubricant and paint performance, wire drawing, melt-spinning, plastic manufacturing, polymer extrusion, and aerodynamic plastic sheet extrusion, among others, are required and have attracted significant attention in recent years to study flow over a stretching sheet. Many academics are looking into how fluid moves across a stretched surface [1–6].

Many industrial processes rely on chemical reactions, including hot rolling, chemical coating of flat plates, polymer extrusion, and heat exchange [7, 8]. Pure water or air cannot occur naturally since there may be distant mass in the air or water [9]. As a result, the existence of certain mixes may result in chemical reactions within the substances. Sinha [10] investigated the impacts of chemical reaction on unsteady MHD free convective flow via a permeable plate under slope temperature and discovered that increasing the chemical reaction parameter leads the reaction rate to increase. For the MHD flow through an exponentially stretched sheet, Chaudhary et al. [11] and Ishak [12] investigate the radiation consequences. The heat transfer rate increases as the Prandtl number rises but decreases when the radiation and magnetic factors rise. The temperature rises as the radiation parameter is increased. In many industries, particularly manufacturing, research that combines the effects of radiation and chemical reactions on MHD flow is crucial. Seini and Makinde [13] studied the movement of the MHD boundary layer across an exponentially stretched sheet under the influence of chemical reaction and radiation and found that as the reaction rate parameter was increased, the concentration of the boundary layer rose. Rasool et al. [14] explored the Darcy-Forchheimer relation in magneto-hydrodynamic Jeffrey nanofluid flow over stretched surface. The findings show that heat generation and response rate components have a significant impact on mass and heat transfer rates. Our research is focused on the effect of Oldroyd-B fluid dissipation and chemical reaction on
Table 1: Nusselt number $-\theta'(0)$.

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Table 2: Values of skin friction coefficient in comparison between current study and previous study ref. [32].

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Table 4: Values of local Sherwood number in comparison between current study and previous study ref. [32].

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MHD free convection flow over an exponentially extending sheet via a porous media. The governing equation for the Oldroyd-B fluid is of fourth order in general. When higher order nonlinearities are neglected, the order of equation in the Oldroyd-B fluid is reduced [15, 16]. Traditional numerical approaches, such as the finite difference method, shooting method, Keller box method [17], Runge-Kutta, and artificial neural networks (ANNs) [18–23] are used to solve some of these problems. A new method called successive linearization method (SLM) has just been proposed in various papers. This approach contains the following key features, which are justified. It is a strong approach to solving these types of problems since it converts the original linear differential equation into a system of linear algebraic equations. This method has been successfully applied to a variety of nonlinear issues in science and engineering, including [24–27]. As a result, all of these successful applications attest to the SLM’s usefulness, validity, accuracy, and flexibility.

As a result, the purpose of this study is to apply the findings of refs [1, 13] to a broader problem, such as the influence of Oldroyd-B fluid dissipation and chemical reaction on MHD in porous media. This work visually depicts and tabulates the impacts of various flow parameters encountered in the governing equations. The SLM technique is used to solve the issue numerically, which is more computationally efficient. The relevant results are graphed and quantitatively analyzed.

2. Mathematical Formulation

Consider the movement of heat and mass down a semi-infinite vertical plate contained in a doubly stratified, electrically conducting Oldroyd-B fluid that is stable, laminar, and incompressible in two dimensions. Select a coordinate system with the $x$-axis parallel to the vertical plate and the $y$-axis perpendicular to the plate. Figure 1 illustrates the physical model and coordinate system. A uniform magnetic field of magnitude is applied to the plate in a normal direction. Due to the low magnetic Reynolds number, the induced magnetic field can be neglected in comparison to the applied magnetic field.

This research also made assumptions $T = T_w = T_{\infty} + T_0 e^{2xL}$ and $C = C_w = C_{\infty} + C_0 e^{2xL}$ where $T_w > T_{\infty}$ and $C_w > C_{\infty}$. The governing equations for the Oldroyd-B fluid as given by Fetteuc et al. [28] and Hayat et al. [29] using the Boussinesq and boundary layer approximations as described by Sparrow and Abraham [30] are given by

$$ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, $$

$$ u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + \lambda \left( u^2 \frac{\partial^2 u}{\partial x^2} + v^2 \frac{\partial^2 u}{\partial y^2} + 2uv \frac{\partial^2 u}{\partial x \partial y} \right) + \frac{\partial^2 u}{\partial y^2} = \nu \frac{\partial^2 u}{\partial x^2} + \nu \lambda_2 \left( \frac{\partial^3 u}{\partial x^2 \partial y} - \frac{\partial u}{\partial x} \frac{\partial^2 u}{\partial y^2} + \nu \frac{\partial^3 u}{\partial x^2 \partial y} - \frac{\partial u}{\partial x} \frac{\partial^2 u}{\partial y^2} \right) $$

$$ - \frac{u}{k} - \frac{B^2}{\rho \beta} \left( u + \lambda_1 \frac{\partial u}{\partial y} \right) + g \beta_2 (C - C_\infty), $$

$$ \frac{\partial T}{\partial x} + \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p} \frac{\partial q}{\partial y} + \frac{Q}{\rho c_p} (T - T_{\infty}), $$

$$ \frac{\partial C}{\partial x} + \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} - k(x)(C - C_{\infty}). $$
In the equations above, \((u, v)\) denote the components of velocity in \((x, y)\) directions, \(v = \mu/\rho\) is the kinematic viscosity, \(\mu\) is the dynamic viscosity, \(\lambda_1, \lambda_2\) are the relaxation and retardation times, respectively, \(\rho\) is the density of fluid, \(\sigma\) is the electric conductivity, \(B_0\) is the uniform magnetic fluid, \(k\) is the permeability of the porous medium, \(C\) is the consternation field, \(g\) is the gravitational acceleration, \(\beta_c\) is the coefficient of thermal expansion of concentration, \(T\) is temperature of fluid, \(\alpha = k/\rho\) is the thermal diffusivity, and \(\lambda_f\) is the thermal conductivity of the fluid.

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Table 5: Different values of skin friction coefficient, local Nusselt number, and local Sherwood number using SLM for different material parameters.
is the thermal diffusivity, \( q_r \) is the radiative heat flux in \( y \)-direction, \( Q \) is the dimensional of heat source (sink), \( \rho c \) is the fluid capacity heat, \( c_p \) is the specific heat, \( D \) is the diffusion coefficient, and \( k(x) \) is the chemical reaction rate.

The temperature, velocity, and concentration profiles have corresponding boundary conditions.

\[
\begin{align*}
  u &= U_w = U_0 e^{\eta L}, & v &= 0, \\
  T &= T_w = T_\infty + T_0 e^{\eta L}, \\
  C &= C_w = C_\infty + C_0 e^{\eta L} \text{ at } y = 0, \\
  u &\to 0, T &\to T_\infty, C &\to C_\infty \text{ at } y &\to \infty.
\end{align*}
\]

(5)

Here, \( U_w \) is the uniform velocity of the sheet and \( L \) is the reference length. The stream function \( \psi \) is introduced here to satisfy the continuity equation (1):

\[
\begin{align*}
  u &= \frac{\partial \psi}{\partial y}, \\
  v &= \frac{\partial \psi}{\partial x}.
\end{align*}
\]

(7)

2.1. Similarity Conversion. The similarity transformation is used to convert the system’s partial differential equations into ordinary differential equations. The following nondimensional variables developed by Mukhopadhyay [31] are used to simplify the resulting equations. For radiation, we have used the
Rosseland approximation.

\[ \eta = \sqrt{\frac{U_0}{2L}} e^{\sigma \psi}, \]
\[ u = U_0 e^{\sigma \psi} f(\eta), \]
\[ v = -\sqrt{\frac{U_0}{2L}} \left[ f(\eta) + \eta f'(\eta) \right], \]
\[ T = T_\infty + T_0 e^{\sigma \psi} \theta(\eta), \]
\[ C = C_\infty + C_0 e^{\sigma \psi} \phi((n)), \]
\[ B = B_0 e^{\sigma \psi}. \]

By substituting (7) and (8) in (2)–(6), we discover that similarity exists, and therefore, we get

\[ f'' - 2f'f'' + A_1 f f'' - \beta_1 \left( 4f'^3 - \eta f'^2 f'' + f^2 f'' - 6f' f'' \right) \]
\[ + \beta_2 \left( 3f''^2 + 2f' f'' - f f'' \right) - \left( \frac{1}{K_p} + M \right) f' + N \eta f' f'' + \lambda \phi(\eta) = 0, \]

\[ \left( 1 + \frac{4}{3} R \right) \theta'' + P \left( f \theta' - f' \theta + S \theta \right) = 0, \]

\[ \phi'' + S \left( f \phi' - f' \phi - \gamma \phi \right) = 0, \]
where

\[ A_1 = \left( 1 + \frac{\lambda_1 \sigma B_2^2}{\rho} \right) \]

\[ \beta_1 = \frac{\lambda_1 U_0 e^{\alpha \eta}}{2L} \]

\[ \beta_2 = \frac{\lambda_2 U_0 e^{\alpha \eta}}{2L} \]

\[ M = \frac{2 \sigma B_2^2 L}{U_0 \rho e^{\alpha \eta}} \]

\[ \frac{1}{K_p} = \frac{2a L}{U_0 K_p e^{\alpha \eta}} \]

\[ N = \frac{\beta_2 \sigma B_2^2}{\rho} \]

\[ \lambda = \frac{2L C_0 \sigma \beta_1}{U_0 e^{\alpha \eta}} \]

where primes represent differentiation in relation to the similarity variable \( \eta \), \( P_r = \nu/\alpha \) is the Prandtl number, \( S_c = \nu/D \) is the Schmidt number, \( M \) is the magnetic field parameter, and \( 1/K_p \) is the porosity parameter. \( R = 4\sigma T_\infty^3 K \) is the radiation parameter and \( S = 2LQ/\rho U_0 e^{\alpha \eta} C_p \) is the heat generation parameter.

Boundary conditions (5) and (6) in terms of \( f, g, \theta, \) and \( \phi \) become

\begin{align*}
  f(0) &= 0, \\
  f'(0) &= 1, \\
  f'(\infty) &= 0, \\
  \theta(0) &= 1, \\
  \psi(0) &= 1,
\end{align*}

Figure 4: (a) Different values of thermal radiation \( R \) on the temperature profile. (b) Different values of Prandtl number \( P_r \) on the temperature profile.
\[ f'(\eta) = 0, \theta(\eta) = 0, \phi(\eta) = 0 \quad \text{as} \quad \eta \to \infty. \]  

For SLM solution, we select the initial guess functions \( f(\eta), \theta(\eta), \phi(\eta) \) in the form

\[
\begin{align*}
  f(\eta) &= f_i(\eta) + \sum_{m=0}^{i-1} F_m(\eta), \\
  \theta(\eta) &= \theta_i(\eta) + \sum_{m=0}^{i-1} \theta_m(\eta), \\
  \phi(\eta) &= \phi_i(\eta) + \sum_{m=0}^{i-1} \phi_m(\eta).
\end{align*}
\]  

Here, the three functions \( f_i(\eta), \theta_i(\eta), \phi_i(\eta) \) are representative unknown functions. \( F_m(\eta), m \geq 1, \theta_m(\eta), \phi_m(\eta) \) are successive approximations produced by solving the linear component of the problem that arises from substituting equation (15) into the governing equations recursively. The main idea of SLM is that the assumptions of unknown function \( f_i(\eta), \theta_i(\eta), \phi_i(\eta) \) are very small when \( i \) becomes larger; therefore, the nonlinear terms in \( f_i(\eta), \theta_i(\eta), \phi_i(\eta) \) and their derivatives are considered to be smaller and thus neglected. The initial guess functions \( F_0(\eta), \theta_0(\eta), \phi_0(\eta) \) are selected to satisfy the boundary conditions:

\[
\begin{align*}
  &F_0(\eta) = 0, F'_0(\eta) = 1 \quad \text{at} \quad \eta = 0, \\
  &F'_0(\eta) \to 0, F''_0(\eta) \to 0 \quad \text{at} \quad \eta \to \infty, \\
  &\theta_0(0) = 1, \theta_0(\infty) \to 0, \\
  &\phi_0(0) = 1, \phi_0(\infty) \to 0.
\end{align*}
\]

### 3. Numerical Analysis

The transformed system of ordinary differential equations (9)–(11) is numerically solved using the boundary condition (13) and utilized the SLM.
They are considered to be in the form

\[ F_0(\eta) = (1 - e^{-\eta}), \]
\[ \theta_0(\eta) = e^{-\eta}, \]
\[ \phi_0(\eta) = e^{-\eta}. \]

The impacts of various values on \( \theta(\eta), f'(\eta), \) and \( \phi(\eta) \) are investigated and analyzed. The numerical numbers \(-\theta'(0), f''(0), \) and \(-\phi'(0)\) are reported and examined.

4. Results and Discussion

4.1. Study Validation. Table 1 illustrates the Nusselt number \(-\theta'(0), f''(0), \) and \(-\phi'(0)\) are reported and examined.

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Figure 6: Different values of source parameter (sink) \( S \) on the concentration profile.

Figure 7: Different values of chemical reaction \( \gamma \) on the concentration profile.
Figure 8: Continued.
Reddy et al. [1] and Swain et al. [6], and it shows that the results agree with each other.

Tables 2, 3, and 4 illustrate the values of the skin friction coefficient, local Nusselt number, and local Sherwood number for various parameter values. The table compares the results obtained with that of Khalili et al. [32], and it shows that the results are in excellent agreement.

4.2. Results. The current study considers the chemical reaction flow of an incompressible electrically conducting fluid, Oldroyd-B, through a porous material past an exponentially extending sheet in the presence of a transverse magnetic field, as well as thermal radiation in the presence of an even source and sink of heat. The mass transfer analysis is also discussed in this work. The following discussion aims to highlight the impact of medium permeability, plate temperature, and thermal radiation on flow phenomena. In addition, the values of skin friction coefficient, local Nusselt number, and local Sherwood number for various values of parameters are shown in Table 5.

Figure 2(a) depicts the link between velocity and significant magnetic field values. It has been demonstrated that increasing $M$ reduces the velocity profile. The Lorentz force is a resistant force generated when $M$ rises, akin to a drag force. The Lorentz force reduces the intensity of the velocity, causing it to slow down. Figure 2(b) depicts the effect of the magnetic field on temperature profiles. Because the Lorentz force is linked to this magnetic field, the larger the participation of the Lorentz force in the parameters of this most expansive magnet field, the higher the temperature. Figure 2(c) depicts the effect of the magnetic field $M$ on dimensionless concentration. The increase in $M$ is thought to raise the concentration profile.

The graph in Figure 3(a) demonstrates that the fluid flow resistance force reduces as the parameter of permeability grows; i.e., as the parameter of permeability increases, the fluid flow velocity increases.

Figure 3(b) shows the effect of the parameter of permeability $K_p$ on temperature profiles. Figure 3(b) shows a decrease in the temperature profile due to an increase in the porosity parameter. The effects of the parameter of permeability $K_p$ on dimensionless concentration are shown in Figure 3(c). The concentration profile is shown to decrease as the parameter of permeability $K_p$ advances. The effect of radiation $R$ on the temperature profile is seen in Figure 4(a). The temperature rises as the radiation $R$ increases. This is because the rate of heat transfer over the surface region has decreased. The impact of the Prandtl number on the temperature profile is shown in Figure 4(b). It is worth noting that increasing the Prandtl number reduces the thickness of the thermal boundary layer.

The effect of Schmidt number $S_c$ on dimensionless concentration is shown in Figure 5. It is noted that as the Schmidt number $S_c$ increases, the concentration falls. Figure 6 depicts the effect of the chemical reaction parameter $\gamma$ on the concentration profile. As the $\gamma$ of the chemical reaction increases, the concentration falls. The influence of the source parameter (sink) $S$ on the temperature profile is seen in Figure 7. As the source parameter $S$ is increased, the temperature profile appears to improve. Figure 8(a) depicts the effect of the relaxation time constant $\beta_1$ on the flow field $f'$. The values of $f'$ and the thickness of the boundary layer drop as $\beta_1$ increases. This is due to the fact that the higher the relaxation time, the slower the recovery
Figure 9: Continued.
process is observed, causing the boundary layer thickness to expand at a slower rate.

The influence of $\beta_1$ on $\theta$ can be seen in Figure 8(b). There is a decrease in $\theta$ when $\beta_1$ is increased. Figure 8(c) exhibits the influence of $\beta_1$ on $\phi$. When $\beta_1$ is elevated, it causes a drop in $\phi$. The effects of the retardation time constant $\beta_2$ on the velocity function $f'$ are seen in Figure 9(a). When $\beta_2$ is increased, the fluid flow and the thickness of the boundary layer are both improved. The effects of $\beta_2$ on $\theta$ and $\phi$ are depicted in Figures 9(b) and 9(c), respectively. The temperature and concentration profiles are observed to decline as $\beta_2$ increases. When various values of the parameter involved are studied, Table 2 displays the absolute values of skin friction coefficient $f''(0)$, values of local Nusselt number $-\theta'(0)$, and local Sherwood number $-\phi'(0)$.

5. Concluding Remarks

This study investigates the impact of chemical processes on the radiative MHD flow of Oldroyd-B fluid across stretching sheet in porous material in the presence of viscous dissipation. The SLM method is used to numerically solve the equations governing the flow by solving expressions for velocity, temperature, and concentration distributions. The tables show how various governing parameters influence skin friction, the Nusselt number, and the Sherwood number. When the magnetic field level is increased, the dimensionless velocity falls, while the nanoparticle concentration and temperature increase. When the porosity parameter $K_p$ rises, it indicates an increase in velocity profile, while concentration and temperature fields show the opposite trend. The effects of $\beta_2$ on the velocity profile $f'$ are diametrically opposed to those of $\beta_1$. When the Prandtl number increases, the temperature and thickness of the thermal boundary layer decrease. The temperature profile, $\beta_1$, and $\beta_2$ changes are qualitatively similar. The Nusselt number increases as $\beta_2$ and $Pr$ increase, but it decreases as $\beta_1$ increases and the Schmidt number and the chemical reaction parameter with higher values reduce the nanoparticle concentration.

Data Availability

The data used to support the findings of this study are included within the article. The data generated using MATLAB code are already presented in tables and figures and are included in Results and Discussion of the manuscript.

Conflicts of Interest

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgments

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References


