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Research Article

Optimal Control Model for Alcohol–Related Risk Behaviors and Beliefs in Tanzania

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An optimal control theory is applied to a system of ordinary differential equations representing the dynamics of health-related risks associated with alcoholism in the community with active religious beliefs. Two nonautonomous control variables are proposed to reduce the health risks associated with alcoholism in the community. Consequently, three control strategies are presented using Pontryagin's maximum principle (PMP), and the necessary conditions for the existence of the optimal controls are obtained. The simulation results revealed that the health risks associated with alcoholism behaviors may be effectively eradicated when both controls, $u_1(t)$ and $u_2(t)$, are applied in a combination. On the other hand, the cost–effective analysis of the three different strategies confirmed that the desired cost–effective results may be attained when both controls, $u_1(t)$ and $u_2(t)$, are applied together. Based on these results, this study concludes that, health risks associated with alcoholism behaviors may be efficiently and cost–effectively eradicated from the community when both controls, $u_1(t)$ and $u_2(t)$, are applied together. Whereas application of control option $u_1(t)$ implies increasing the level of protection to the susceptible population by implementing public health education campaign; the control option $u_2(t)$ implies increasing the removal rate of the moderate risky individuals into recovered population. The control strategy in which the two options are featured in a combination is presented in this study as Strategy C exhibiting the least ICER value and more cost–effective than the rest of strategies presented.

1. Introduction

Alcohol drinking behaviors have long been identified as a risk factor for several diseases [1, 2]. It is mentioned as one among the major global risk factors in the Global Burden of Diseases (GBD) [1, 3]. While some literature associate alcoholism with some health conditions including malnutrition, chronic pancreatitis, liver disease, and cancer and damage to the central and peripheral nervous system [4–8], others recommend low level of alcohol consumption for some health benefits such as prevention of thrombosis [9]. However, recent studies brought on board a different feelings among drinking communities. According to Griswold et al. [1], alcohol drinking may be harmful to human health regardless of the volume and frequency in which it is taken. Based on this context, alcohol consumption at any drinking level subjects drinkers' health at stake. However, the studies about health drinking maintain that both volume and frequency of alcohol uptake are the key determinant of risk levels in which drinkers are exposed into. It is an undeniable fact that some social cultural beliefs practiced in different communities act as useful control agents in installing behavioral changes among its members. For instance, studies consistently reveal the negative association between religiosity and alcoholism behaviors [10–12]. This makes religious belief an important change agent for promoting the health and molding behaviors of its members [13, 14].

Epidemiological studies consider the transmission of infectious agents in the host population as a key process that requires descriptive analysis when the model compartments are used to study a particular infectious disease [15]. These models may be extended to describe behavioral dynamics and transmission, where people already in the behavior act as transmission agents in the host population provided that a reasonable amount of interactions between them is allowed. When a behavior associated with health risk factors emerges in the community, a total population can be partitioned into a number of categories depending on the risk levels or defined patterns individuals exhibit.

Mathematical modeling of alcohol drinking epidemic and its consequences on human health has been an interesting topic for many researchers. An increased interests in the use of mathematical modeling as an essential tool for simulating alcoholism (and similar) behaviors and provide valuable control analysis may be stimulated by similarity between the spreading nature of alcoholism (and other similar) behaviors and that of infectious diseases. The common alcoholism (and its consequences) models available fall in the category of the basic SIR with or without significant modification [4, 5, 16, 17]. In some cases, mathematical control theorem has been very useful approach in solving control problems for nonautonomous system models [18, 19]. This approach is aimed at determining a control and state values for the dynamical system in a specified period of time in order to find the optimal values of a given objective function [20]. As opposed to autonomous systems where the constant controls are used, nonautonomous system occurs when a continuous variable control is employed to the system as a function of time [20]. The usefulness of mathematical modeling of contagious conditions in the designing of health policies may not be overemphasized [18].

In this paper, we use a nonautonomous approach to modify crisp model developed in Mayengo et al. [21] and Mayengo et al. [22] to accommodate control functions and analyze the mathematical model for optimal control strategies to improve the understanding of influencing dynamics in health-related risks associated with alcoholism. We do this in response to the recommendations made by [23] by considering three main distinguishing aspects of the model formulation. These are social cultural beliefs as an integral part of the society, the staged process in which alcoholic behaviors take in the spread of health risks, and the optimal control options. Later, the study compares different control strategies obtained from the combinations of the control variables and recommends the best control strategy in terms of cost–effectiveness in a relatively short period.

2. Model Formulation

Adopting the crisp model framework of Mayengo et al. [21] and Mayengo et al. [23], in which an alcohol-related risk models are formulated and analyzed with reference to Tanzanian population with active religious beliefs. The epidemiological set–up includes six model compartments, namely, risk susceptible population $S(t)$, comprising individuals at risk of engaging in alcohol drinking; protected population $P(t)$, comprising individuals who have virtually gained protection from alcohol drinking through religious/cultural practices; low risk $L(t)$, comprising individuals who drink responsibly on occasional basis; medium/moderate risk $M(t)$, comprising individuals who regularly consume alcohol; alcohol dependent–high risk $A(t)$, comprising individuals who have developed high dependence in alcohol; and recovered population $R(t)$, comprising of individuals who have voluntarily quit drinking on health related challenges.

New recruits enter the population at a constant rate π where a proportion of $\phi \in (0, 1)$ of the new recruits are subjected religious beliefs and enter the system through $P(t)$, and the remaining portion $(1-\phi)$ goes to $S(t)$. The susceptible class is further increased by individuals backsliding from $P(t)$ at a constant rate γ_2 and a constant relapse rate ω. The susceptible class is decreased at the rate λ, and the effective religious conversion occurs at a rate of γ_1. The protected class is increased by successful conversion to religious beliefs of individuals from $S(t), L(t), M(t)$, and $A(t)$ classes and decreased by backsliding at constant rate γ_2. Low-risk drinkers $L(t)$ progress to moderate-risk drinkers $M(t)$ at a constant rate σ. Moderate risk drinkers $M(t)$ progress to alcohol addiction $A(t)$ at a constant rate δ and recover at a constant rate ξ and are protected at a constant rate τ. Alcohol addicts either recover at the rate η or get protected at the rate ψ. The parameter α defines alcohol-induced fatality rate. The recovered class becomes susceptible again at the rate ω. All the classes are subjected to reduction due to natural causes at a constant rate μ.

A nondrinker acquires alcohol drinking habits through social contacts [4, 23, 24] at the force of peer influence λ defines by

$$\lambda = \beta c \left(\frac{L + \theta_1 M + \theta_2 A}{N} \right). \tag{1}$$

where β defines the chances that a susceptible individual will drink alcohol after prolonged contact with a drinking individuals; c is the contact rate between a susceptible member and a drinker necessary to cause effect on a susceptible member. The parameter θ_1 is the chances of becoming an alcoholic after successful influence of a moderate-risk drinker, and θ_2 is the chances of becoming an alcoholic after successful influence of a high-risk drinker. The proportion $\rho \in (0, 1)$ of susceptible individuals is recruited via peer influence into low-risk drinking class, while $(1-\rho)$ enters the moderate-risk class.

Formulation of the model is guided by the following set of assumptions: the mixing of individuals in each population is homogeneous; there is no direct progression from susceptible to high-risk compartment. Also, it is assumed that the natural mortality rate in each of the population states is constant; the recruitment rate for each population is greater than natural mortality/removal rate; individuals in the protected compartment remains protected from alcohol drinking for their entire life in the compartment; virtual protection for alcohol drinking acquired by the protected population is not permanent; and a nonalcoholic drinker acquires alcohol drinking habits through social contacts with drinking individuals.

We then introduce the following control functions as follows: $u_1(t)$, and $u_2(t)$ to extend the crisp model for alcohol-related health risk presented by Mayengo et al. [21] and Mayengo et al. [22], whereby the time dependent variable $u_1(t)$ is introduced as a control variable describing public health education campaigns aiming at increasing public awareness on health risks in connection to alcoholism

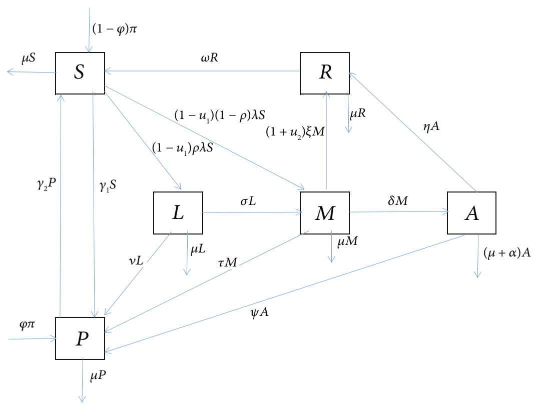

Figure 1: The model compartment for the population dynamics of health risks associated with alcoholism.

behaviors. It helps to reduce the number of risk-susceptible population who joins any of the drinking compartments. On the other hand, the time dependent variable $u_2(t)$ describes organized treatment and rehabilitation programs aiming at speeding up the rate of recovery from the two higher risk compartments. This information is summarized in the model compartment diagram presented in Figure 1. We therefore have the following optimal control system

$$\begin{cases} \dot{S} = -(\mu + \gamma_1 + (1-u_1)\lambda)S + \gamma_2 P + \omega R + (1-\phi)\pi, \\ \dot{P} = \gamma_1 S - (\mu + \gamma_2)P + (\nu L + \tau M + \psi A) + \phi\pi, \\ \dot{L} = (1-u_1)\lambda\rho S - (\nu + \mu + \sigma)L, \\ \dot{M} = (1-u_1)(1-\rho)\lambda S + \sigma L - (\tau + \mu + \delta + (1+u_2)\xi)M, \\ \dot{A} = \delta M - (\mu + \alpha + \eta + \psi)A, \\ \dot{R} = (1+u_2)\xi M + \eta A - (\omega + \mu)R, \end{cases} \tag{2}$$

subject to nonnegative initial conditions $N > 0, S > 0, P \geq 0, L \geq 0, M \geq 0, A \geq 0$ and $R \geq 0$.

3. Model Analysis

3.1. Equilibrium Points. The model system (2) has the risk-free equilibrium point

$$\mathscr{E}_0 = \left(\frac{\pi((1-\phi)\mu + \gamma_2)}{\mu(\mu + \gamma_1 + \gamma_2)}, \frac{\pi(\phi\mu + \gamma_2)}{\mu(\mu + \gamma_1 + \gamma_2)}, 0, 0, 0, 0 \right), \tag{3}$$

showing the total population with only susceptible and protected individuals. Examining the impact generated by the introduction of a small number of risky individuals in the community, we computed the effective risk reproduction number of system (2) by means of Next Generation Matrix van den Driessche and Watmough [25]; Diekmann et al. [26]; Mayengo et al. [21]. The effective risk reproduction number, $\mathscr{R}_e$, is given by

$$\mathscr{R}_e = \frac{1}{2}\left(a_{11} + a_{22} + \sqrt{\left((a_{11} - a_{22})^2 + 4a_{12}a_{21} \right)} \right), \tag{4}$$

where,

$$\begin{aligned} a_{11} &= c\beta(1-u_1)\rho\left(\frac{1}{(\mu+\nu+\sigma)} + \frac{\theta_1\sigma}{(\mu+\nu+\sigma)(\delta+\mu+\tau+(1+u_2)\xi)} \right. \\ &\quad \left. + \frac{\theta_2\sigma\delta}{(\mu+\nu+\sigma)(\delta+\mu+\tau+(1+u_2)\xi)(\mu+\eta+\alpha+\psi)} \right)\frac{S_0}{N_0}, \\ a_{12} &= c\beta(1-u_1)\rho\left(\frac{\theta_1}{(\delta+\mu+\tau+(1+u_2)\xi)} \right. \\ &\quad \left. + \frac{\theta_2\delta}{(\delta+\mu+\tau+(1+u_2)\xi)(\mu+\eta+\alpha+\psi)} \right)\frac{S_0}{N_0}, \\ a_{13} &= c\beta(1-u_1)\rho\left(\frac{\theta_2}{\mu+\eta+\alpha+\psi} \right)\frac{S_0}{N_0}, \\ a_{21} &= c\beta(1-u_1)(1-\rho)\left(\frac{1}{(\mu+\nu+\sigma)} + \frac{\theta_1\sigma}{(\mu+\nu+\sigma)(\delta+\mu+\tau+(1+u_2)\xi)} \right. \\ &\quad \left. + \frac{\theta_2\sigma\delta}{(\mu+\nu+\sigma)(\delta+\mu+\tau+(1+u_2)\xi)(\mu+\eta+\alpha+\psi)} \right)\frac{S_0}{N_0}, \\ a_{22} &= c\beta(1-u_1)(1-\rho)\left(\frac{\theta_1}{(\delta+\mu+\tau+(1+u_2)\xi)} \right. \\ &\quad \left. + \frac{\theta_2\delta}{(\delta+\mu+\tau+(1+u_2)\xi)(\mu+\eta+\alpha+\psi)} \right)\frac{S_0}{N_0}. \end{aligned} \tag{5}$$

3.2. Formulation of the Optimal Control Problem. Consider the time-varying control function set $\boldsymbol{U}(t)$ whose components represent the deliberate efforts geared to reduce the level of health risks associated with alcoholism behavior in the community targeting different population compartments. Suppose that $u_i(t)$ are Lebesque measurable and the components of the control function set $\boldsymbol{U}(t)$, we define

$$\boldsymbol{U}(t) = \{u_i(t) \forall i \in \{1,2\} : u_i(t) \in [0,1]\, ; 0 \leq t \leq \boldsymbol{T}. \tag{6}$$

To investigate the optimal level of efforts that would be required to control the spread of health risks in the community, following Lee et al. [27], Mushayabasa [16], Khajanchi and Ghosh [28], Hugo et al. [29], Berhe et al. [20], Khajanchi and Banerjee [30], and Nyerere et al. [31], the objective function J is formulated as follows:

$$\boldsymbol{J} = \min_{u_i \in U} \int_0^{\boldsymbol{T}} \left(\boldsymbol{B}_1\boldsymbol{L} + \boldsymbol{B}_2\boldsymbol{M} + \boldsymbol{B}_3\mathrm{A} + \frac{1}{2}\left(\boldsymbol{C}_1 u_1^2 + \boldsymbol{C}_2 u_2^2 \right) \right) dt \tag{7}$$

whereby $\boldsymbol{B}_j$ and $\boldsymbol{C}_i$ are, respectively, positive balancing constants of the risky population and cost factors associated with control strategies $u_i(t)$ for $i \in \{1,2\}$ and $j \in \{1,2,3\}$. Assuming nonlinearity in the cost of each control strategy, we use quadratic form, that is, $(\mathrm{C}_1 u_1^2)/2$ is the cost of control strategy associated with public health education campaign, and $(\mathrm{C}_1 u_2^2)/2$ is the cost associated with running rehabilitation and sober houses. Our goal is to minimize both the total number of the population at risk and the cost of controls,

$u_1(t)$ and $u_2(t)$. By choosing appropriate positive balancing constants $\boldsymbol{B}_j$'s and $\boldsymbol{C}_i$'s, we aim to minimize the risky population at the minimum cost of the control.

3.3. Characterization of the Optimal Control. By using Pontryagin's maximum principle, we derive necessary conditions for our optimal control and corresponding states [32], Lee et al. [27], Joshi et al. [33], Hugo et al. [29], Khajanchi and Banerjee [30], and Nyerere et al. [31]. The Hamiltonian function in (8) is formulated.

$$\begin{aligned}\mathscr{H} &= \mathrm{B}_1\mathrm{L} + \mathrm{B}_2\mathrm{M} + \mathrm{B}_3\mathrm{A} + \frac{1}{2}\left(\mathrm{C}_1u_1^2 + \mathrm{C}_2u_2^2\right)\\ &+ \lambda_1(-(\mu+\gamma_1+(1-u_1)\lambda)\mathrm{S} + \gamma_2\mathrm{P} + \omega\mathrm{R} + (1-\phi)\pi)\\ &+ \lambda_2(\gamma_1\mathrm{S} - (\mu+\gamma_2)\mathrm{P} + \nu\mathrm{L} + \tau\mathrm{M} + \psi\mathrm{A} + \phi\pi)\\ &+ \lambda_3((1-u_1)\lambda\rho\mathrm{S} - (\nu+\mu+\sigma)\mathrm{L})\\ &+ \lambda_4((1-u_1)(1-\rho)\lambda\mathrm{S} + \sigma\mathrm{L} - (\tau+\mu+\delta+(1+u_2)\xi)\mathrm{M})\\ &+ \lambda_5(\delta\mathrm{M} - (\mu+\alpha+\eta+\psi)\mathrm{A}) + \lambda_6((1+u_2)\xi\mathrm{M} + \eta\mathrm{A} - (\omega+\mu)\mathrm{R}).\end{aligned} \tag{8}$$

Given the optimal control $U^* = (u_1^*, u_2^*)$, there exist adjoint functions $\lambda_i \forall i \in \{1, 2, \cdots, 6\}$ corresponding to the states x_i such that $\lambda_{i'} = -\partial\mathscr{H}/\partial x_i, \forall x_i \in \{\mathrm{S}, \mathrm{P}, \mathrm{L}, \mathrm{M}, \mathrm{A}, \mathrm{R}\}$. It follows that the adjoint system in (9) is established

$$\begin{aligned}\frac{d\lambda_1}{dt} &= \left((1-u_1)\left(\frac{\mathrm{S}}{\mathrm{N}}-1\right)\lambda - (\mu+\gamma_1)\right)\lambda_1 - \gamma_1\lambda_2\\ &+ (1-u_1)\left(\rho\frac{\mathrm{S}}{\mathrm{N}} - \lambda\right)\lambda_3 + (1-u_1)(1-\rho)\lambda\left(\frac{\mathrm{S}}{\mathrm{N}}-1\right)\lambda_4,\end{aligned} \tag{9}$$

$$\begin{aligned}\frac{d\lambda_2}{dt} &= -\left((1-u_1)\lambda\frac{\mathrm{S}}{\mathrm{N}} + \gamma_2\right)\lambda_1 + (\mu+\gamma_2)\lambda_2\\ &+ (1-u_1)\rho\lambda\frac{\mathrm{S}}{\mathrm{N}}\lambda_3 + (1-u_1)(1-\rho)\lambda\frac{\mathrm{S}}{\mathrm{N}}\lambda_4,\end{aligned}$$

$$\begin{aligned}\frac{d\lambda_3}{dt} &= -\mathrm{B}_1 + (1-u_1)(c\beta-\lambda)\frac{\mathrm{S}}{\mathrm{N}}\lambda_1 - \nu\lambda_2\\ &- \left((1-u_1)\rho(c\beta-\lambda)\frac{\mathrm{S}}{\mathrm{N}} + (\mu+\nu+\sigma)\right)\lambda_3\\ &- \left((1-u_1)(1-\rho)(c\beta-\lambda)\frac{\mathrm{S}}{\mathrm{N}} + \sigma\right)\lambda_4,\end{aligned}$$

$$\begin{aligned}\frac{d\lambda_4}{dt} &= -\mathrm{B}_2 + (1-u_1)(c\beta\theta_1-\lambda)\frac{\mathrm{S}}{\mathrm{N}}\lambda_1 - \tau\lambda_2\\ &- (1-u_1)\rho(c\beta\theta_1-\lambda)\frac{\mathrm{S}}{\mathrm{N}}\lambda_3\\ &- \left((1-u_1)(1-\rho)(c\beta\theta_1-\lambda)\frac{\mathrm{S}}{\mathrm{N}} - \mu - \tau - \delta - (1+u_2)\xi\right)\lambda_4\\ &- \delta\lambda_5 - (1+u_2)\xi\lambda_6,\end{aligned}$$

$$\begin{aligned}\frac{d\lambda_5}{dt} &= -\mathrm{B}_3 + (1-u_1)(c\beta\theta_2-\lambda)\frac{\mathrm{S}}{\mathrm{N}}\lambda_1 - \psi\lambda_2\\ &- (1-u_1)\rho(c\beta\theta_2-\lambda)\frac{\mathrm{S}}{\mathrm{N}}\lambda_3\\ &- (1-u_1)(1-\rho)(c\beta\theta_2-\lambda)\frac{\mathrm{S}}{\mathrm{N}}\lambda_4\\ &+ (\mu+\alpha+\nu+\psi)\lambda_5 - \eta\lambda_6,\end{aligned}$$

$$\begin{aligned}\frac{d\lambda_6}{dt} &= -\left((1-u_1)\lambda\frac{\mathrm{S}}{\mathrm{N}} + \omega\right)\lambda_1 + (1-u_1)\rho\frac{\mathrm{S}}{\mathrm{N}}\lambda\lambda_3\\ &+ (1-u_1)(1-\rho)\frac{\mathrm{S}}{\mathrm{N}}\lambda\lambda_4 + (\mu+\omega)\lambda_6,\end{aligned} \tag{10}$$

with transversality conditions

$$\lambda_i(\mathrm{T}) = 0; \forall i \in \{1, 2, \cdots, 6\}. \tag{11}$$

Now, we need to minimize the Hamiltonian function $\mathscr{H}$ with respect to the control function U in order to obtain an optimal control $\boldsymbol{U}^*$ such that

$$\mathrm{J}(u_1^*, u_2^*) = \min_{\Omega}\mathrm{J}(u_1, u_2), \tag{12}$$

where $\Omega = \{(u_1(t), u_2(t)) \in \mathrm{U} \mid 0 \le u_1(t), u_2(t) \le 1, t \in [0, \mathrm{T}]\}$.

3.4. Existence of Optimal Controls

Theorem 1. *Given $\boldsymbol{J}(\boldsymbol{U})$ subject to system (2) with nonnegative initial conditions of the state variables, that is, $\boldsymbol{S}(0) \ge 0$, $\boldsymbol{P}(0) \ge 0, \boldsymbol{L}(0) \ge 0, \boldsymbol{M}(0) \ge 0, \boldsymbol{A}(0) \ge 0$ and $\boldsymbol{R}(0) \ge 0$, there exist an optimal control $\boldsymbol{U}^* = (u_1^*, u_2^*)$ that minimizes $\boldsymbol{J}(\boldsymbol{U})$ over $\boldsymbol{U}$, and the corresponding adjoint variables $\lambda_i \forall i \in \{1, 2, \cdots, 6\}$ satisfying the following equations*

$$\begin{cases}\dfrac{\partial\mathscr{H}}{\partial u_1} = C_1u_1 - \lambda(\lambda_1 - \rho\lambda_3 - (1-\rho)\lambda_4)S,\\ \dfrac{\partial\mathscr{H}}{\partial u_2} = C_2u_2 + \xi(\lambda_6 - \lambda_4)M.\end{cases} \tag{13}$$

The control set $U^* = (u_1^*, u_2^*)$ gives $\partial\mathscr{H}/\partial\mathrm{U} = 0$, in particular, at $u_1 = u_1^*$ and $u_2 = u_2^*$ we have, respectively, $\partial\mathscr{H}/\partial u_1 = 0$ and $\partial\mathscr{H}/\partial u_2 = 0$. Thus, solving the system (13) for u_1^* and u_2^* gives

$$\begin{aligned}u_1^* &= \frac{\lambda}{\mathrm{C}_1}(-\lambda_1 + \rho\lambda_3 + (1-\rho)\lambda_4)\mathrm{S},\\ u_2^* &= \frac{\xi}{\mathrm{C}_2}(\lambda_4 - \lambda_6)\mathrm{M}.\end{aligned} \tag{14}$$

The characterization (15) holds on the interior of the control set $\boldsymbol{U}^*$.

$$u_1^* = \max\left\{0, \min\left(1, \frac{\lambda}{C_1}\lambda(\lambda_1 - \rho\lambda_3 - (1-\rho)\lambda_4)S\right)\right\},$$
$$u_2^* = \max\left\{0, \min\left(1, \frac{\xi}{C_2}(\lambda_4 - \lambda_6)M\right)\right\}. \quad (15)$$

Proof. The adjoint system (9) obtained by taking the negative partial derivatives of Hamiltonian function ($\mathscr{H}$) with respect to each of the state variables under the transversality condition (11) are standard results from Pontryagin maximum principle [32]. Also, the partial derivatives of the Hamiltonian equation (8) with respect to each of the control variables u_i is observed where the optimal solution (14) is obtained by setting $\partial\mathscr{H}/\partial u_i = 0$, and the system is then solved for u_i^* subject to constraints to establish the characterization equation. Hence, using the bonds $0 \le u_1, u_2 \le 1$ the optimality equation (15) is formed.

4. Numerical Methods and Simulations

The optimal control strategies for the transmission of health risks associated with alcoholism are analyzed numerically focusing on public health education campaign and rehabilitation program. For effective reduction of health risks associated with alcoholism in the community, we investigate the impacts of individual control strategies or in combination and simulate it in a period of fifteen years. Choosing arbitrarily the values $B_1 = 10, B_2 = 15, B_3 = 15, C_1 = 10, C_2 = 0.05$ and the parameter values from Table 1 with initial conditions, $S(0) = 800, P(0) = 300, L(0) = 80, M(0) = 10, A(0) = 100$,and $R(0) = 0$. The following strategies are implemented, and their cost–effectiveness is examined. The simulation results showing the effects of variation of control strategies on the risky classes are presented graphically in Figures 2–4 where red dashed lines represent specific population dynamics without control and the blue solid lines represent population dynamic under specific control. However, in the graphs representing the control profiles during the implementation of a particular control strategy, the blue and red lines represent the control options $u_1(t)$ and $u_2(t)$, respectively.

4.1. Strategy A: Implementing Public Health Education Campaign $(u_1(t))$. Implementation of this strategy involves one control option $u_1(t)$, to minimize the objective function ($\boldsymbol{J}$), while keeping $u_2(t) = 0$. Simulation results presented in Figure 2 indicate that while public health education campaign influences significant changes in the low- and medium-risk classes, the strategy has a considerably less effects to the alcoholic population. The positive effects of the application of control Strategy A in the population proportions $\boldsymbol{L}(t)$ and $\boldsymbol{M}(t)$ can be observed in Figures 2(a) and 2(b), respectively. The total risk averted by implementing Strategy A is 257.1488. This was achieved when the control profiles u_1 were implemented at a maximum level.

Table 1: Description of the variables and parameters for model (2).

Parameters	Value	Source
π	0.0310 yr−1	Mayengo et al. [21]
μ	0.0160 yr−1	Assumed
α	0.0350 yr−1	Bhunu [4]
δ	0.0075 yr−1	Bhunu [4]
σ	0.0100 yr−1	Thamchai [34]
ν	0.0020 yr−1	Mayengo et al. [21]
τ	0.0016 yr−1	Mayengo et al. [21]
ψ	0.0100 yr−1	Mayengo et al. [21]
ξ	0.0025 yr−1	Bhunu [4]
η	0.0050 yr−1	Bhunu [4]
ω	0.0010 yr−1	Thamchai [34]
γ_1	0.1300 yr−1	Mushanyu et al. [35]
γ_2	0.2400 yr−1	Mushanyu et al. [35]
θ_1	1.0002	Mayengo et al. [21]
θ_2	1.0005	Bhunu [4]
φ	0.6860	Assumed
ρ	0.6500	Mayengo et al. [21]
β	0.2500	Bhunu [4]
c	0.0500 yr−1	Assumed

4.2. Strategy B: Implementing Rehabilitation Program $(u_2(t))$. Under this strategy, we implement $u_2(t)$ while maintaining that $u_1(t) = 0$ whose simulation results are recorded in Figure 3. In this case, the notable positive effects of implementing rehabilitation program can be seen in medium-risk $\boldsymbol{M}(t)$ and high-risk $\boldsymbol{A}(t)$ drinking populations. However, on the contrary, there is no significant changes of the risk averts in low-risk population, $\boldsymbol{L}(t)$. Again, this was achieved when the control profiles u_2 was implemented at a maximum level.

4.3. Strategy C: Implementing Public Health Education Campaign $(u_1(t))$ *and Rehabilitation Program* $(u_2(t))$. Strategy C is made by implementing a combination of the two control options, that is, $u_1(t)$ and $u_2(t)$ where $u_i(t) \neq 0, \forall i \in \{1, 2\}$. The simulation results presented in Figure 4 indicate that the positive effects of implementation of both public health education campaign and rehabilitation program are distributed in all risky classes of the model. For the period of fifteen years, the total risky averts produced by implementation of Strategy C amounts to 469.2134. The positive effects of the application of control Strategy C in the population proportions $\boldsymbol{L}(t)$, $\boldsymbol{M}(t)$, and $\boldsymbol{A}(t)$ can be observed in Figure 4. This was achieved when the both control profiles $u_1(t)$ and $u_2(t)$ were implemented at a maximum level.

4.4. Cost–Effective Analysis. The method of Cost-effectiveness is used to compare the cost benefits of

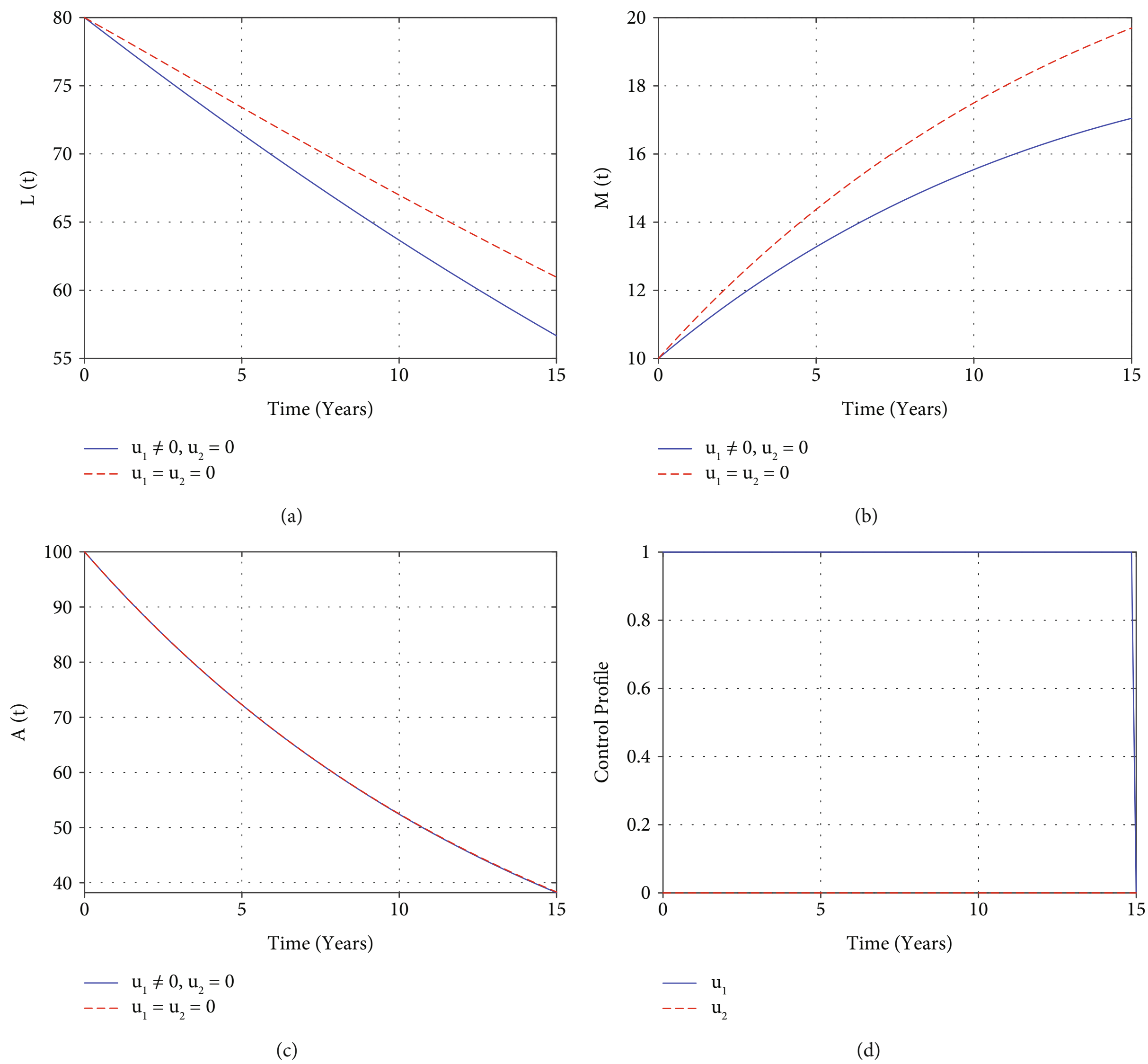

Figure 2: Time series of different risky population proportions with their corresponding control profile for implementation of Strategy A.

implementing the control strategies. During the entire period, the total cost of implementing the control strategies presented in the previous section is given by

$$\Phi(\mathrm{U}) \coloneqq \int_0^T \left(\frac{\mathrm{C}_1 u_1^2}{2} + \frac{\mathrm{C}_2 u_2^2}{2} \right) dt. \quad (16)$$

Following Okosun et al. [36], Hugo et al. [29], Berhe et al. [20], Nyerere et al. [31], and Mayengo [37], we perform the cost-effectiveness analysis by using increamental cost effectiveness ratio (*ICER*) approach. This is the ratio of the difference in cost between two possible interventions to the difference in their outcomes [37]. It is the ratio which describes the comparison results for the competing interventions while incorporating two important features which are cost of implementing the intervention and its outcome (or benefit). For instance, when two interventions $u_i(t)$ and $u_j(t)$, where $\Phi(u_i)$ and $\Phi(u_j)$ represents the cost of interventions $u_i(t)$ and $u_j(t)$, respectively. The *ICER* value can be obtained as follows:

$$ICER = \frac{\Phi(u_i) - \Phi(u_j)}{\mathrm{E}(u_i) - \mathrm{E}(u_j)}, \quad (17)$$

where $\boldsymbol{E}(u_i)$ and $\boldsymbol{E}(u_j)$ stand for the total risks averted for implementing interventions $u_i(t)$ and $u_j(t)$, respectively.

Based on the *ICER* functional equation (17), while the numerator translates the differences in the cost of implementation of intervention strategies, the denominator translates the differences in health outcomes by means of the total number of risk cases averted as the consequences of implemented strategies [20, 29, 31, 37].

The health benefits obtained when intervention strategies (or some combinations) are applied are measured in terms of quality–adjusted life years (QALYs) gained or lost thereafter [37]. The approach requires that more costly

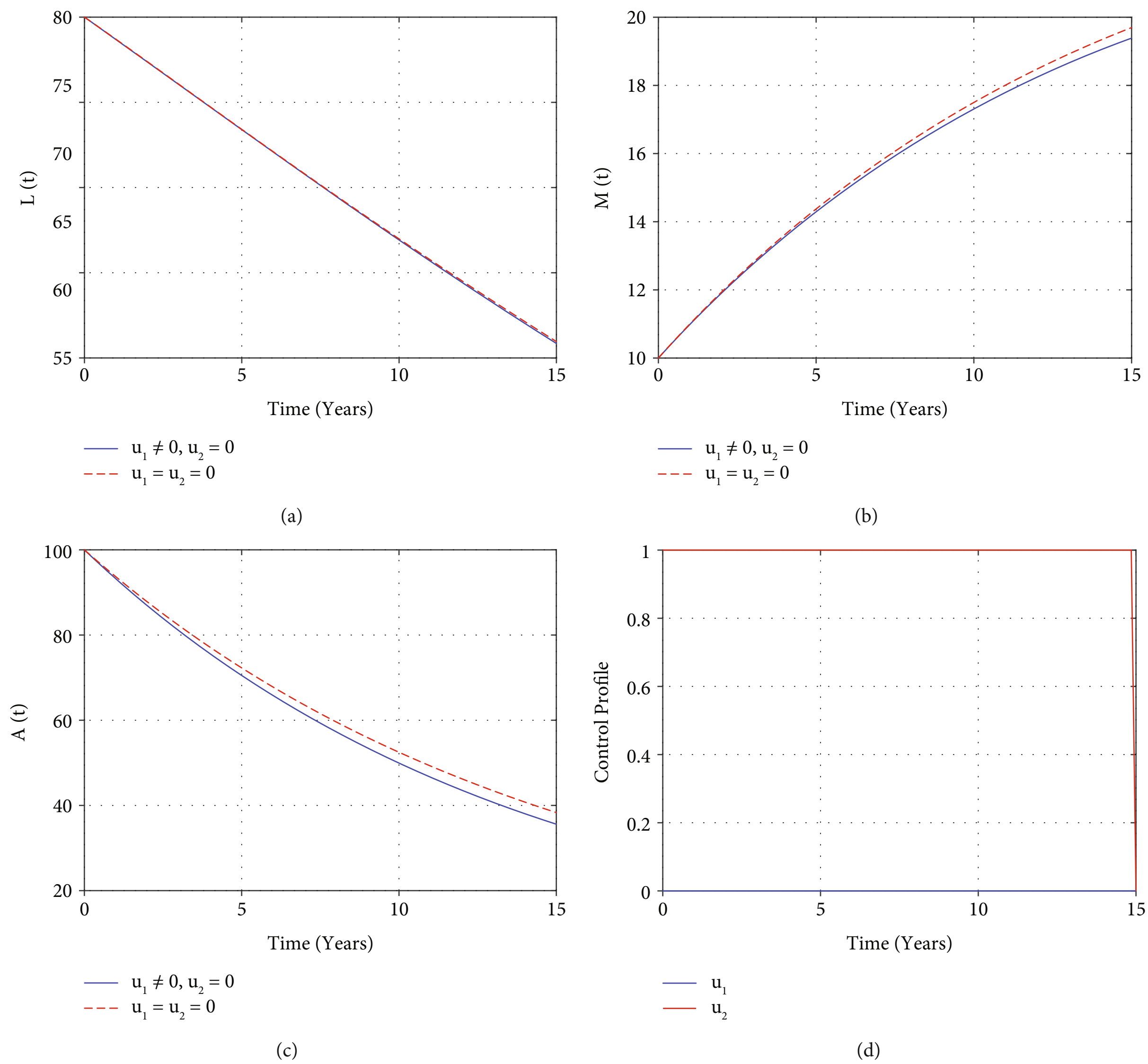

Figure 3: Time series of different risky population proportions with their corresponding control profile for implementation of Strategy B.

alternatives with less effective be excluded while accepting more efficient and less costly strategies.

Simulation results for the implementation of three combinations of intervention strategies are ranked in order of increasing control effectiveness in terms of the number of risk cases averted. These results are presented in Table 2.

The ICER values in Table 2 are calculated as follows:

$$\begin{aligned} \mathrm{ICER_B} &= \frac{1.8501 \times 10^5}{66.6614} = 2775.3693, \\ ICER_A &= \frac{1.8065 \times 10^5}{257.1488 - 66.6614} = -22.8887, \\ \mathrm{ICER_C} &= \frac{1.7723 \times 10^5}{469.2134 - 257.1488} = -16.1272, \end{aligned} \tag{18}$$

where $\boldsymbol{ICER_i}$ is the ICER value corresponding to strategy i. Owing to simulation results presented in Table 2, it is clear that Strategy B is less effective compared to the rest of strategies available. Although it is not the most costly strategy, since it produces the highest ICER value than the rest, we reject it right away and remove it from the list of alternatives.

The *ICER* values are recalculated for the rest of alternatives, and their simulation results are presented in Table 3. In comparison, Strategy C shows lower *ICER* value as compared to Strategy A suggesting strong dominance of Strategy A. This implies that Strategy C is less costly and more effective than Strategy A. Now, since Strategy A is likely to consume limited resources available, we remove it from the set of alternative solutions. Consequently, we accept Strategy C as the best cost–effective combination.

With this results, it can be observed that implementation of both public health education campaign (u_1) and rehabilitation program (u_2) presented as Strategy C in this study has the least ICER value and more cost-effective than the rest of strategies presented.

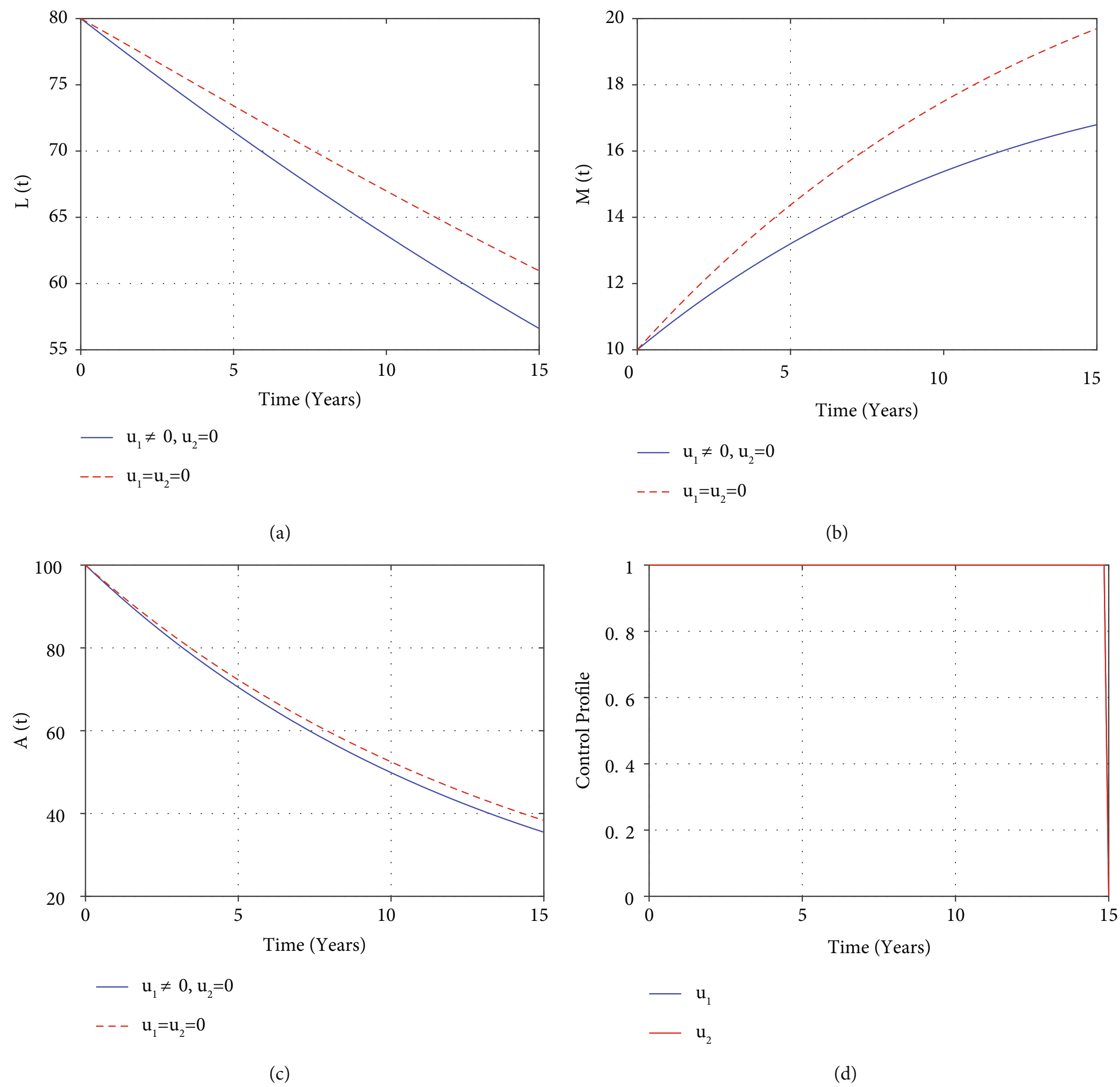

FIGURE 4: Time series of different risky population proportions with their corresponding Control profile for implementation of Strategy C.

TABLE 2: ICER in increasing order of total risks averted.

Strategies	Total cost in \$	Total risk averted	$ICER_i$
Strategy B	1.8501×10^5	66.6614	2775.3693
Strategy A	1.8065×10^5	257.1488	−22.8887
Strategy C	1.7723×10^5	469.2134	−16.1272

TABLE 3: ICER in increasing order of total risks averted.

Strategies	Total cost in \$	Total risk averted	$ICER_i$
Strategy A	1.8065×10^5	257.1488	702.5115
Strategy C	1.7723×10^5	469.2134	−16.1272

5. Discussion and Conclusion

The study presents a deterministic model through which the effects of implementing three different continuous control strategies on health risk epidemic model are examined. An optimal control problem was designed with the aim of minimizing the cost for implementation of the control strategies while keeping total risky individuals over the intervention interval. We first demonstrated the existence of optimal solutions to the optimality system developed and performed the analysis in an attempt to understand how health-related risks associated with alcoholism can be effectively eliminated from the community.

Pontryagin's maximum principle (PMP) is used to find the necessary conditions for the optimal controls, the corresponding states in the minimization of the spread of health risks, and cost of implementing control strategies. Observing from the simulation results, it is clear that there is no direct transition of the population from susceptible to alcoholic compartments. This might influence the positive effects of the low $\boldsymbol{L}(t)$- and medium$\boldsymbol{M}(t)$- risk compartments leaving the high-risk compartment $\boldsymbol{A}(t)$ unaffected upon implementing Strategy A. Although the time dependent control variable $u_2(t)$ was applied to speedup the recovery rate of the medium-risk compartment $\boldsymbol{M}(t)$, the positive effects of Strategy B manifested in two high-risk classes, $\boldsymbol{M}(t)$ and $\boldsymbol{A}(t)$. The effects on $\boldsymbol{A}(t)$ might be influenced by fact that the alcoholic population $\boldsymbol{A}(t)$ is only increased by progression of the population proportion from the medium-risk compartment $\boldsymbol{M}(t)$. Reduction of the population in medium-risk compartment translates into reduction of transition pressure from $\boldsymbol{M}(t)$ into $\boldsymbol{A}(t)$ leaving the low-risk compartment unaffected. Comparatively, implementation of Strategy A appears to be more effective in reduction of medium risk population than Strategy B in quantitative terms. Implementation of Strategy C where both public health education and rehabilitation program were implemented appears to have optimal results than the rest of strategies separately presented, suggesting that Strategy C was the most effective control option. On the other hand, the cost-effectiveness analysis proved that Strategy C was less costly than the rest of strategies.

Based on these results, it is worth concluding that the health risks associated with alcoholism behaviors may be effectively eradicated when both controls $u_1(t)$ and $u_2(t)$ are applied together. Whereas application of control option $u_1(t)$ implies the increase in the level of protection to the susceptible population by implementing public health education campaign; the control option $u_2(t)$ implies the increase of removal rate of the moderate risky individuals into recovered population. The control strategy in which the two control options are featured in a combination is referred to as Strategy C in this study exhibiting the least *ICER* value and more cost-effective than the rest of strategies presented.

Data Availability

No data was used in the manuscript.

Conflicts of Interest

The author declared that there is no conflict of interests.

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