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Research Article

Mathematical Modeling for Optimal Management of Human Resources in Banking Sector of Bangladesh

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A new mathematical model on human resources divided employees into two compartments, namely, fresher and expert employees, has been designed and analyzed. A system of ordinary nonlinear differential equations has three state variables including vacancies. This model describes the dynamics of the number of fresher employees and expert employees as well as vacancies and shows the impacts of training programs and benefits of provided facilities for employees. The equilibria of this proposed model are determined, and its stability at these points is checked. Moreover, characteristics of state variables with respect to parameters have been discussed. Using two optimal control variables, this study finds the maximum number of experts including the minimum cost of provided facilities as well as the training program based on Pontryagin's maximum principle.

1. Introduction

Bangladesh's banking sector is one of the country's most disciplined and biggest, employing around one million people. Bangladesh's banking industry has played a critical role in the country's economic and social growth. In contemporary terms, our country's banking history spans more than half a century.

With experienced and competent bankers, the banking sector must operate in the current economy. In comparison to other companies, managing a bank has become incredibly hard during the last two decades [1]. Banking is a highly regulated industry in any organization. To survive and expand in the future, it will require specialized and qualified staff. Bangladesh now has 57 scheduled banks operating under the full authority and supervision of the Bangladesh Bank. This industry includes a variety of banks in several categories. The industry may be divided into four groups based on ownership: (i) state-owned commercial banks (SOCBs), (ii) state-owned development financial institutions (DFIs), (iii) private commercial banks (PCBs), and (iv) foreign commercial banks (FCBs). Bangladesh has 56 banks as of December 2015, and the number of bank branches has climbed to 9397 from 9040 in December 2014, owing to the banks' establishment of new branches during the year. The total number of bank branches climbed to 9453 at the end of June 2016 [1, 2]. So it is time to work on resource management in this sector because resource management is one of the most important aspects of every project or organization since it entails the effective planning, allocation, and utilization of resources. Resources can refer to money, human capital (workers and skills), material possessions, technological advancements, time, and more. Thus, effective resource management aims to make sure that resources are used well, with as little waste as possible and as much output as possible [3]. In this fast-paced and very competitive industry, a bank's success and longevity depend on how well it allocates and uses its resources and how well it manages its human capital. In parallel, human resource management is also very important in the banking industry because workers are key to providing good services, building relationships with customers, and coming up with new ideas. However, banks with skilled and driven employees are better able to handle complicated financial deals, adapt to changing market conditions, and execute profitable development strategies [4]. Therefore, the close relationship between managing resources and controlling humans is the key to success in the dynamic and profitable banking industry [5]. Consequently, by choosing a time-required, all-encompassing strategy, banks not only strengthen their position in the industry but also build stronger relationships with their most valued customers which will help to build trust, loyalty, and future success. For this reason, banks expect more talented, expert, and dedicated bankers.

To make expert employees, these banks should train their employees. In 1983, the Government of the People's Republic of Bangladesh formed the National Training Council to design a training and development policy because banking is a major driver of a country's economic and social development conducting a banking growth research study is critical to its success. In Bangladesh, private sector commercial banks should have a well-defined strategy, policy, methodologies, and objectives that guide all choices, particularly when it comes to successful training. Private commercial banks should make training a priority in order to outperform those who do not. Most private banks want to succeed, but they need to invest in quality training that will increase their chances of success.

Training that is effective aids in the achievement of banking objectives. Employees are able to achieve their objectives through training. The study [6] built on prior successful training literature by offering a full review of effective training systems, training needs assessment, significance, and evaluation procedures, as well as correlating those practices to perceived organizational effectiveness levels. The assessment procedure answers the question of how much training is learned and how much instruction is effective. As a result, the goal of assessing training needs is to create value for a company. As a result, assessment tracks progress toward this aim by enhancing and evaluating training programs. Training needs assessments determine the evaluation approach and training program.

Training is the systematic development of an individual's knowledge, skills, and attitudes necessary to execute satisfactorily a particular activity or employment [7–10]. Employees benefit from training since it allows them to improve their existing skills while also learning new ones. Training has various positive impacts, including improved morale, productivity, work satisfaction, and employee dedication to corporate goals. Since job satisfaction is highly impacted by training and development, recruiting and selection, and selection, whereas employee performance is significantly affected by evaluations of performance [11], it aids employees in balancing their professional and personal lives, as well as improving communication at all levels of the organization [12, 13].

Training programs have been shown to improve managerial efficiency and train future leaders. Due to the availability of a pool of talented and skilled executives, succession planning becomes easier. Employee's creativity and innovation talents can be unlocked through specially designed training programs [14, 15].

In the banking sector, Mitu et al. [16] investigated the impact of training and development programs on employer performance. Because the number of respondents was unknown, a suitable sampling approach was utilized, along with SPSS and PLS software to analyze demographic data and SEM, which revealed that employers perform better when training and development programs were in place.

Maaly et al. [17] demonstrated a questionnaire-based study that was conducted by a Malaysian small and medium enterprise to determine the attitudes and performance skills of administrative leaders and employers after providing training and to determine the relationship between them as well as modern methods of delivery. The content of the training was recommended.

Employee development is the process of encouraging each employee's professional and personal development inside a firm. To survive the severe competition, it is critical for staff to stay up with the newest advances in the sector. Personal and professional developments are two aspects of development programs. Training in soft skills, computer skills, exercise sessions, lectures, and workshops are all forms of growth [14].

Employees acquire a sense of belonging and camaraderie, which helps to lower employee turnover in the long run. Employees' knowledge and production levels rise, lowering stress levels. According to Alsomaidaee [18], employee engagement is largely influenced by nonwork-related stress. Appropriate training allows employees to understand and use the most up-to-date technologies that are required in today's world [19]. It is abundantly obvious that training benefits both people and organizations and that it assists employees in aligning their own aspirations with the organization's aims. Training assists employees throughout their careers, assisting them in professional progression by preparing them to take on more responsibilities in the future [20, 21].

Many writers have researched employee training in order to increase their quality [22–27]. Wagonhurst stated [19] that "Inadequate needs assessment can result in inappropriate and ineffectual interventions which could either have no impact or have a deleterious impact on the actual performance problem". According to Ferdousi and Razzak [14] as training expenditures rise despite limited resources, there is a greater need to assess the effectiveness or failure of training programs. As a result, the goal of training needs assessment is to offer value to a company. Therefore, assessment tracks progress toward this aim by systematically developing training programs and assessing their value.

Denby [28] and Vijayalakshmi and Vaidhyasubramaniam [21] note that training serves to broaden the available skills within the workforce, boost competency, and enhance the team's capabilities, which encourages retention. In the banking industry, training is a key instrument for improving employee performance and allowing financial institutions to contribute to economic growth. Banks all across the world are investing billions of dollars in training programs. As a result, good training may assure and develop a more capable and professional workforce, which may turn out to be financial advisers who can lead the community in the right direction [29]. According to Rahman et al. [30], effective training enables employees to learn essential abilities, resulting in organizational effectiveness. Debnath [31] indicates that the production of manpower in Bangladesh's banking industry would need to be boosted by proper on-the-job and off-the-job training. Informal training needs assessment processes are still used by certain banks.

To face the increased competition of organizations as a result of globalization, changes in technology, political, and economic environments [32], their employees will be prepared to implement training programs. Not only growing knowledge in the business corporate world and progress in technology in the last decade but also increasing efforts towards the development of human resources play a vital role in improvements of the banking sector.

Employees play an important role in the success of any organization. As a result, maximizing work performance in today's competitive business climate requires boosting these assets through effective training, motivation, and the offer of special facilities [23, 33–39]. According to Tung and Phong [40], compared to high-commitment HRM practices, high-involvement HRM practices have greater impacts on innovation. Though there has been some study in the area of human research management, mathematical modeling is rare. In order to strive for success, organizations have to utilize human resources efficiently because functional conflict and organizational success are closely linked [41]. So organizations need to be aware of the face and more realistic towards keeping their human resources up-to-date. This study, therefore, goes on to discuss the effects of training and offering special facilities on employees including optimal cost.

(A) Mathematical Analysis of a Model on Human Resources in Banking Sector. Experts are able to play a vital role in developing the banking sector. So increasing experts is with proper training and increasing collaboration between freshers and experts. On the other hand, high-quality employees demand more salary and facilities from banks. However, management would like to continue their work with a minimum number of employees. But more pressure on experts generates demotivate them as a result they may be switched to other banks. To illustrate the flow from fresher compartment to experts in the banking sector, we will have formulated a mathematical model which is discussed below.

2. Formulation of the Mathematical Model

For the formulation of a mathematical model, we have taken three state variables $F(t)$, $E(t)$, and $V(t)$ that represent fresher employees of the bank, expert employees, and vacancies of a bank, respectively.

Recruitment of new employees depends on the total number of vacancies in the bank. But freshers may switch to another job or may die. Even freshers can become experts with experienced staff. They also can move to another job or may die. By proper training, freshers may be members of the expert category. The aforementioned information can be written as follows:

$$\frac{dF}{dt} = kV - \beta F - \gamma FE - \delta F - \tau F, \quad (1)$$

where k is the rate at which freshers are employed against vacancies, the rate at which freshers switch their jobs is denoted by β, γ represents the rate at which freshers are becoming experts in touch with expert employees, and death rate of the new employees are expressed by δ.

If there are huge vacancies in the bank, then the pressure of work will be exposed on the experts. As a result, they may leave the bank and join another job. The number of knowledgeable staff will increase as new professionals come into contact with knowledgeable staff, and the specialists may retire or die. So we can write these in the following form:

$$\frac{dE}{dt} = -\xi V + \gamma FE - \psi E - \delta E + \tau F, \quad (2)$$

where ξ denotes the demotivated rate of experts for more vacancies. Due to demotivation, they are encouraged to change their working place. Here, ψ represents a rate of job switching for unsatisfactory made from insufficient facilities, and δ represents the retirement rate as well as the death rate of professional employees. The rate of training that empowers a fresher to become an expert is denoted by τ.

From time to time, new bank vacancies may be created when freshers are becoming experts. Also, vacancies will be decreased when there are a lot of experts because the experts can do more work than freshers. The number of vacancies will increase when newcomers move or die and when experienced employees retire or die. In addition, freshers are recruited according to the number of vacancies. Vacancies will also be increased when experts leave the bank. So the following differential equation can be formed:

$$\frac{dV}{dt} = -\phi E + \gamma FE - kV + \beta F + \delta F + \psi E + \delta E + \xi V, \quad (3)$$

where a decrease in vacancies when there are many experts in the bank is denoted by ϕ.

3. Mathematical Analysis

3.1. Positivity and Boundedness. We must establish that the solutions to the system of differential equations are positive and bounded for all values of t to make the model biologically feasible. It is not practically possible to conclude that a population is negative.

Lemma 1. *Considering $F(0) > 0$, $E(0) > 0$, and $V(0) > 0$, it must be proved that $F(t)$, $E(t)$, $V(t)$ will be positive for all $t \in [0, T]$ in R_3^+ where $T > 0$.*

Proof. Taking all parameters of the system and all initial values to be positive, we have to prove that $F(t)$, $E(t)$, and

$V(t)$ will be positive for all $t \in [0, T]$ in R_3^+. From the equation (1), we can write the inequality as follows:

$$\frac{dF}{dt} > -\beta F - \gamma FE - \delta F - \tau F. \tag{4}$$

Integrating both sides of the equation with respect to time t, we obtain

$$F(t) > e^{-\beta t - \gamma \int E dt - \delta t - \tau t} > 0. \tag{5}$$

Adding equations (1) and (2), we get

$$\frac{d(F+E)}{dt} > (k-\xi)V + (-\nu - \delta)(F+E) \text{where} \nu = \max(\beta, \psi) \tag{6}$$

If $k > \xi$, then $E(t) > e^{(-\nu-\delta)t} > 0$.

Implying the equation (3)

$$\begin{gathered}\frac{dV}{dt} + (k-\xi)V = q(x) \text{ where } q(x) = \gamma FE + \beta F + \delta F + \psi E + \delta E - \phi E, \\ \therefore V(t) = e^{-k+\xi} \int e^{k-\xi} q(x) > 0.\end{gathered} \tag{7}$$

□

Lemma 2. *The solution trajectories $(F(t), E(t), V(t))$ of the model are bounded.*

Proof. Firstly, we add the equations (1) and (2), then we obtain the following equation:

$$\begin{aligned}\frac{d(F+E)}{dt} &= kV - \beta F - \delta F - \xi V - \psi E - \delta E \\ &\Rightarrow \frac{d(F+E)}{dt} + (\beta+\delta)F + (\psi+\delta)E = (k-\xi)V \\ &\Rightarrow \frac{d(F+E)}{dt} + (\nu+\delta)(F+E) \\ &\le (k-\xi)V,\end{aligned} \tag{8}$$

choosing $\nu = \min(\psi, \beta)$ for maximization of total populations.

Taking the limit supremum, we obtain

$$\lim_{t \longrightarrow \infty} Sup\{F(t) + E(t)\} \le \frac{(k-\xi)}{\nu+\delta} \lim_{t \longrightarrow \infty} Sup\{V(t)\}. \tag{9}$$

Now,

$$\begin{gathered}\lim_{t \longrightarrow \infty} \mathrm{Sup}\{F(t)+E(t)+V(t)\} \\ \le \lim_{t \longrightarrow \infty} \mathrm{Sup}\{F(t)+E(t)\} + \lim_{t \longrightarrow \infty} \mathrm{Sup}\{V(t)\} \\ \lim_{t \longrightarrow \infty} \mathrm{Sup}\{F(t)+E(t)+V(t)\} \le \left(1 + \frac{(k-\xi)}{\nu+\delta}\right) \lim_{t \longrightarrow \infty} \mathrm{Sup}\{V(t)\}.\end{gathered} \tag{10}$$

So all solutions $(F(t), E(t), V(t))$ of the system are bounded. □

3.2. Equilibrium Analysis. By solving the following equations below, we can obtain the equilibrium point

$$kV - \beta F - \gamma FE - \delta F - \tau F = 0, \tag{11}$$

$$-\xi V + \gamma FE - \psi E - \delta E + \tau F = 0, \tag{12}$$

$$-\phi E + \gamma FE - kV + \beta F + \delta F + \psi E + \delta E + \xi V = 0. \tag{13}$$

Solving the equations (11)–(13) we get two equilibrium points $E_0(0, 0, 0)$ and $E_1(F^*, E^*, V^*)$. Where

$$\begin{aligned}F^* &= \frac{\phi}{\gamma}, \\ E^* &= \frac{\phi(\beta\xi + \delta\xi + \tau\xi - k\tau)}{\delta\gamma k - \gamma k\phi + \gamma k\psi + \gamma\phi\xi}, \\ V^* &= \frac{\delta^2\phi - \delta\phi^2 - \beta\phi^2 + \beta\delta\phi + \beta\phi\psi + \delta\phi\psi + \delta\phi\tau + \phi\psi\tau}{\delta\gamma k - \gamma k\phi + \gamma k\psi + \gamma\phi\xi}.\end{aligned} \tag{14}$$

For positiveness of the equilibrium point $E_1(F^*, E^*, V^*)$, we consider some following conditions.

$$\begin{gathered}\beta\xi + \delta\xi + \tau\xi > k\tau, \\ \delta k + k\psi + \phi\xi > k\phi.\end{gathered} \tag{15}$$

3.3. Future Status of Expert Employees. Using the next generation matrix method [42], we will find the basic reproduction number R_0. To examine the behaviour of dynamics of the expert's compartment, this class is considered for the discussion. We take

$$\frac{dE}{dt} = -\xi V + \gamma FE - \psi E - \delta E + \tau F. \tag{16}$$

Therefore, two matrices M and N which represent the gain and loss of expert employees are $M = \gamma F^*$ and $N = \psi + \delta$. Then, the next generation number is the largest eigenvalue of MN^{-1}. Therefore, we have

$$R_0 = \frac{\gamma F^*}{\psi + \delta}. \tag{17}$$

Experts will be extinct in the bank if $R_0 < 1$ and that will exist if $R_0 > 1$.

3.4. Stability Analysis of the System at the Equilibrium Point. Jacobian matrix will be used and then the characteristic equation will be analyzed investigating the behavior of the stability.

Theorem 3. *The system is unstable at the quilibrium point E_0*
.

Proof. Firstly, we consider

$$f = kV - \beta F - \gamma FE - \delta F - \tau F, \tag{18}$$

$$g = -\xi V + \gamma FE - \psi E - \delta E + \tau F, \tag{19}$$

$$\begin{aligned} h = {} & -\phi E + \gamma FE - kV + \beta F + \delta F \\ & + \psi E + \delta E + \xi V. \end{aligned} \tag{20}$$

For the equations (18)–(20), the Jacobian matrix is

$$\begin{aligned} J = \frac{\partial(f,g,h)}{\partial(F,E,V)} &= \begin{bmatrix} \frac{\partial f}{\partial F} & \frac{\partial f}{\partial E} & \frac{\partial f}{\partial V} \\ \frac{\partial g}{\partial F} & \frac{\partial g}{\partial E} & \frac{\partial g}{\partial V} \\ \frac{\partial h}{\partial F} & \frac{\partial h}{\partial E} & \frac{\partial h}{\partial V} \end{bmatrix} \Rightarrow J \\ &= \begin{bmatrix} -\beta - \delta - E\gamma - \tau & -F\gamma & k \\ E\gamma + \tau & F\gamma - \delta - \psi & -\xi \\ \beta + \delta + E\gamma & \delta - \phi + F\gamma + \psi & \xi - k \end{bmatrix}. \end{aligned} \tag{21}$$

At the equilibrium point $E_0(0,0,0)$ equation (21) becomes

$$J = \begin{bmatrix} -\beta - \delta - \tau & 0 & k \\ \tau & -\delta - \psi & -\xi \\ \beta + \delta & \delta - \phi + \psi & \xi - k \end{bmatrix}. \tag{22}$$

The characteristic equation is

$$\begin{aligned} & \lambda^3 + (\beta + 2\delta + k + \psi + \tau - \xi)\lambda^2 \\ & \quad + (\beta\delta + \delta k + \beta\psi + \delta\psi + \delta\tau - \beta\xi \\ & \quad + k\psi - \delta\xi + k\tau + \psi\tau - \phi\xi - \tau\xi + \delta^2)\lambda \\ & \quad + k\phi\tau - \delta\phi\xi - \beta\phi\xi - \phi\tau\xi = 0. \end{aligned} \tag{23}$$

At the equilibrium point $E_0(F^*, E^*, V^*)$, equation (23) becomes

$$\Rightarrow \beta_0\lambda^3 + \beta_1\lambda^2 + \beta_2\lambda + \beta_3 = 0, \tag{24}$$

where

$$\begin{aligned} \beta_0 &= 1, \\ \beta_1 &= \beta + 2\delta + k + \psi + \tau - \xi, \\ \beta_2 &= \beta\delta + \delta k + \beta\psi + \delta\psi + \delta\tau - \beta\xi \\ & \quad + k\psi - \delta\xi + k\tau + \psi\tau - \phi\xi - \tau\xi + \delta^2, \\ \beta_3 &= k\phi\tau - \delta\phi\xi - \beta\phi\xi - \phi\tau\xi. \end{aligned} \tag{25}$$

Since $\beta_3 < 0$ according to the conditions (15), therefore, the equation (23) has at least one positive root. So, the system is unstable at the critical point E_0. □

Theorem 4. *The interior equilibrium point E_1 is unstable based on the value of parameters which is in Table 1.*

Proof. Using the value of parameters from Table 1, the interior equilibrium point is (300,778,321). At the interior equilibrium point E_1, the characteristic equation (21) becomes $\lambda^3 + 0.3187\lambda^2 + 0.0142\lambda - 0.000050388 = 0$ □

The roots of the above equation are 0.0033, -0.0578, and -0.2642. Therefore, the system is unstable at the interior point E_1.

3.5. Characteristics of Equilibrium Points with respect to $\boldsymbol{\psi}$. The behaviour of critical points with respect to ψ will be discussed below. From the equations (11)–(13), we can obtain two functions of F^*, E^*, ψ as follows:

$$\begin{aligned} & f(F^*, E^*, \psi) \\ & \quad = \frac{\gamma E^* F^* + \tau F^* - \psi E^* - \delta E^* - \xi(\beta F^* + \delta F^* + \tau F^* + \gamma E^* F^*)}{k}, \\ & g(F^*, E^*, \psi) \\ & \quad = \frac{\delta E^* - \phi E^* + \psi E^* - \tau F^* + \xi(\beta F^* + \delta F^* + \tau F^* + \gamma E^* F^*)}{k}, \\ & \frac{dF^*}{d\psi} = \frac{\begin{vmatrix} \partial f/\partial E^* & \partial f/\partial \psi \\ \partial g/\partial E^* & \partial g/\partial \psi \end{vmatrix}}{\begin{vmatrix} \partial f/\partial F^* & \partial f/\partial E^* \\ \partial g/\partial F^* & \partial g/\partial E^* \end{vmatrix}} = \frac{(\partial f/\partial E^* \partial g/\partial \psi) - (\partial f/\partial \psi \partial g/\partial E^*)}{(\partial f/\partial F^* \partial g/\partial E^*) - (\partial f/\partial E^* \partial g/\partial F^*)}. \end{aligned} \tag{26}$$

Applying $F^* = \phi/\gamma$, we obtain

$$\frac{dF^*}{d\psi} = \frac{0}{(\gamma E^*(\delta k - k\phi + k\psi + \phi\xi))/k}. \tag{27}$$

Table 1: Parameter values and their descriptions.

Descriptions of the parameter	Notations	Values (year^{-1})
Rate of freshers who are joining against vacancies	k	0.16
Job switching rate of freshers	β	0.07
Rate of collaboration of freshers and experts employee	γ	2.103×10^{-5}
Retire and death rate	δ	0.0053
Rate of newly expert employees for proper training	τ	0.08
Reduction rate of experts due to workload	ξ	0.031
Job switching rate of experts	ψ	1.9×10^{-2}
Rate of deletion of vacancies for adequate experts	ϕ	0.0063

Therefore, the job switching rate ψ does not affect on 1st components of equilibrium points directly.

$$\frac{dE^*}{d\psi} = \frac{\begin{vmatrix} \partial f/\partial\psi & \partial f/\partial F^* \\ \partial g/\partial\psi & \partial g/\partial F^* \end{vmatrix}}{\begin{vmatrix} \partial f/\partial F^* & \partial f/\partial E^* \\ \partial g/\partial F^* & \partial g/\partial E^* \end{vmatrix}} = \frac{(\partial f/\partial\psi \partial g/\partial F^*) - (\partial f/\partial F^* \partial g/\partial\psi)}{(\partial f/\partial F^* \partial g/\partial E^*) - (\partial f/\partial E^* \partial g/\partial F^*)},$$
$$\frac{dE^*}{d\psi} = \frac{-\gamma E^{*2}}{(\gamma E^*(\delta k - k\phi + k\psi + \phi\xi))/k}. \quad (28)$$

Applying the conditions (15), the denominator of the above fraction is positive, but it is clear that the numerator is negative. So, for an increase in the job switching rate ψ, the number of experts is decreasing.

3.6. Characteristics of Equilibrium Points with respect to γ. We will discuss the characterization of the equilibrium values of freshers and experts in banks with respect to γ.

Using equations (11)–(13), we get the functions of F^*, E^*, γ as follows:

$$f(F^*, E^*, \gamma) = \frac{\gamma E^* F^* + \tau F^* - \psi E^* - \delta E^* - \xi(\beta F^* + \delta F^* + \tau F^* + \gamma E^* F^*)}{k},$$
$$g(F^*, E^*, \gamma) = \frac{E^* - \phi E^* + \psi E^* - \tau F^* + \xi(\beta F^* + \delta F^* + \tau F^* + \gamma E^* F^*)}{k},$$
$$\frac{dF^*}{d\gamma} = \frac{\begin{vmatrix} \partial f/\partial E^* & \partial f/\partial\gamma \\ \partial g/\partial E^* & \partial g/\partial\gamma \end{vmatrix}}{\begin{vmatrix} \partial f/\partial F^* & \partial f/\partial E^* \\ \partial g/\partial F^* & \partial g/\partial E^* \end{vmatrix}} = \frac{\partial f/\partial E^* \partial g/\partial\gamma - \partial f/\partial\gamma \partial g/\partial E^*}{\partial f/\partial F^* \partial g/\partial E^* - \partial f/\partial E^* \partial g/\partial F^*}. \quad (29)$$

Using $F^* = \phi/\gamma$, we get

$$= \frac{-EF(k\delta - k\phi + k\psi + \phi\xi)/k}{(\gamma E^*(\delta k - k\phi + k\psi + \phi\xi))/k}. \quad (30)$$

Applying the conditions (15), the denominator of the above fraction is positive but the numerator is negative. So, for an increase in collaboration rate γ, the number of freshers is decreasing.

$$\frac{dE^*}{d\gamma} = \frac{\begin{vmatrix} \partial f/\partial\gamma & \partial f/\partial F^* \\ \partial g/\partial\gamma & \partial g/\partial F^* \end{vmatrix}}{\begin{vmatrix} \partial f/\partial F^* & \partial f/\partial E^* \\ \partial g/\partial F^* & \partial g/\partial E^* \end{vmatrix}} = \frac{\partial f/\partial\gamma \partial g/\partial F^* - \partial f/\partial F^* \partial g/\partial\gamma}{\partial f/\partial F^* \partial g/\partial E^* - \partial f/\partial E^* \partial g/\partial F^*}$$
$$= \frac{EF(\beta\xi + \delta\xi - k\tau + \tau\xi)/k}{(\gamma E^*(\delta k - k\phi + k\psi + \phi\xi))/k}, \quad (31)$$

Applying the conditions (15), both the denominator and numerator of the above fraction are positive. So, for the increase of collaboration rate γ, the number of experts is increasing.

3.7. Characteristics of States for Equilibrium Values with respect to ϕ. Characterization of the equilibrium values of freshers and experts with respect to ϕ will be discussed in below.

Two functions of F^*, E^*, ϕ from (11)–(13) can be obtained as follows:

$$f(F^*, E^*, \phi) = \frac{\gamma E^* F^* + \tau F^* - \psi E^* - \delta E^* - \xi(\beta F^* + \delta F^* + \tau F^* + \gamma E^* F^*)}{k},$$
$$g(F^*, E^*, \phi) = \frac{\delta E^* - \phi E^* + \psi E^* - \tau F^* + \xi(\beta F^* + \delta F^* + \tau F^* + \gamma E^* F^*)}{k},$$
$$\frac{dF^*}{d\phi} = \frac{\begin{vmatrix} \partial f/\partial E^* & \partial f/\partial\phi \\ \partial g/\partial E^* & \partial g/\partial\phi \end{vmatrix}}{\begin{vmatrix} \partial f/\partial F^* & \partial f/\partial E^* \\ \partial g/\partial F^* & \partial g/\partial E^* \end{vmatrix}} = \frac{\partial f/\partial E^* \partial g/\partial\phi - \partial f/\partial\phi \partial g/\partial E^*}{\partial f/\partial F^* \partial g/\partial E^* - \partial f/\partial E^* \partial g/\partial F^*}. \quad (32)$$

Putting the values $F^* = \phi/\gamma$ in above calculations, we can write

$$\frac{dF^*}{d\phi} = \frac{E(k\delta + k\psi - \phi k + \phi\xi)/k}{(\gamma E^*(\delta k - k\phi + k\psi + \phi\xi))/k}. \quad (33)$$

Applying the conditions (15), both the denominator and numerator of the above fraction are positive. So, for the increase of ϕ, the number of freshers is increasing.

$$\frac{dE^*}{d\phi} = \frac{\begin{vmatrix} \partial f/\partial \phi & \partial f/\partial F^* \\ \partial g/\partial \phi & \partial g/\partial F^* \end{vmatrix}}{\begin{vmatrix} \partial f/\partial F^* & \partial f/\partial E^* \\ \partial g/\partial F^* & \partial g/\partial E^* \end{vmatrix}} = \frac{\partial f/\partial \phi \partial g/\partial F^* - \partial f/\partial F^* \partial g/\partial \phi}{\partial f/\partial F^* \partial g/\partial E^* - \partial f/\partial E^* \partial g/\partial F^*}$$
$$= \frac{-(E(\beta\xi + \delta\xi - k\tau + \tau\xi - E\gamma k + E\gamma\xi))/k}{(\gamma E^* (\delta k - k\phi + k\psi + \phi\xi))/k}. \tag{34}$$

Applying the conditions (15), the denominator and numerator of the above fraction are opposite. So, for the increase of ϕ, the number of experts is dropping.

3.8. Nature of Dynamics of Experts Employees when the Number of Freshers Are Nondecreasing and Vacancies Are opposite to Freshers. Considering the monotonic increase of freshers and deletion of vacancies, the dynamics of expert employees will be analyzed. That means new employees are recruited continuously such that present freshers must be higher than previous ones. Taking the equation (2), we differentiate both sides with respect to time t.

$$\begin{aligned} \frac{dE}{dt} &= -\xi V + \gamma FE - \phi E - \delta E + \tau F \\ \therefore \frac{d^2E}{dt^2} &= -\xi \frac{dV}{dt} + \delta \frac{dF}{dt} E + \gamma F \frac{dE}{dt} - \psi \frac{dE}{dt} - \delta \frac{dE}{dt} + \tau \frac{dF}{dt} \\ &> \gamma F \frac{dE}{dt} - \psi \frac{dE}{dt} - \delta \frac{dE}{dt}. \end{aligned} \tag{35}$$

At equilibrium point,

$$\frac{d^2E}{dt^2} > 0, \tag{36}$$

Therefore, the expert employees are minimal at critical points.

3.9. Convergence of Expert Employees when Freshers and Vacancies Are Constant. Taking an interesting recruitment policy of jobs such that the number of vacancies is always constant. That means no new opportunities are opened to reduce unemployment. Therefore, we shall consider the average vacancy to be V_c. Moreover, the number of freshers remains constant as many freshers have become experts, these posts in the bank will be filled by recruitment policies. So taking the fixed number of freshers F_C for the above system, we analyze the mathematical model of nonlinear differential equations to investigate the converging point of expert employees in the banking sector. From the equation (2), we get

$$\frac{dE}{dt} + (\psi + \delta - \gamma F_c)E = \tau F_c - \xi V_c. \tag{37}$$

After the solution of the above differential equations, taking limit $t \longrightarrow \infty$, we get

$$\lim_{t\longrightarrow\infty} \mathrm{Sup} E(t) = \frac{\tau F_c - \xi V_c}{(\psi + \delta - \gamma F_c)E}. \tag{38}$$

Therefore, the sequence of expert staffs $E_n(t)$ converges to $(\tau F_c - \xi V_c / \psi + \delta - \gamma F_c)E$.

4. Numerical Analysis

Using MATLAB (2021b), the aforementioned model has been numerically analyzed. For numerical simulations, the Runge-Kutta 4th-ordered technique has been applied to illustrate the dependency of state variables on various parameters of the model. The systems of nonlinear differential equations have been solved by using the values of parameters from Table 1 which have been calculated from the information of staff of Pubali Bank, Bangladesh.

According to their opinions and experience, we have divided the employees into two compartments: fresher and expert employees. Taking the initial values of fresher employees $F(0) = 3389$, expert employees $E(0) = 4756$, and vacancies $V(0) = 970$, we will observe the effect of new faces in the fresher compartment, mutual work, training, and motivation as well as deletion of vacancies for toxicity of experts.

Figure 1 illustrates the effects of variation of k on the state variables. k is the rate at which new employees are hired to fill vacancies. When the bank has adequate vacancies, freshers will be hired to fill those openings. As a consequence, vacancies will be reduced as a result of the newcomers. Again, the number of experts will increase in parallel with the number of newcomers. Because after a certain period of time, a novice will become an expert.

Therefore, the number of freshers and experts in the bank is growing, and the overall number of workers in the bank will grow as well. As a result, as the value of k rises, the number of new workers rises also. Because of the surge of k, the number of experts and total workforces is growing, while the number of vacancies is dropping.

In Figure 2, the behavior of state variables for different γ values is illustrated. Where γ is the rate at which new employees become proficient in the presence of experienced employees. When a large number of new employees become experts in contact with skilled employees, the overall number of new employees decreases while the number of expert employees increases. Since freshers are becoming experts, several jobs at the bank have arisen.

The overall number of workers in the bank will expand due to an increase in the total number of expert employees and the total number of vacancies. As a result, as γ rises, more and more newcomers become experts. As a

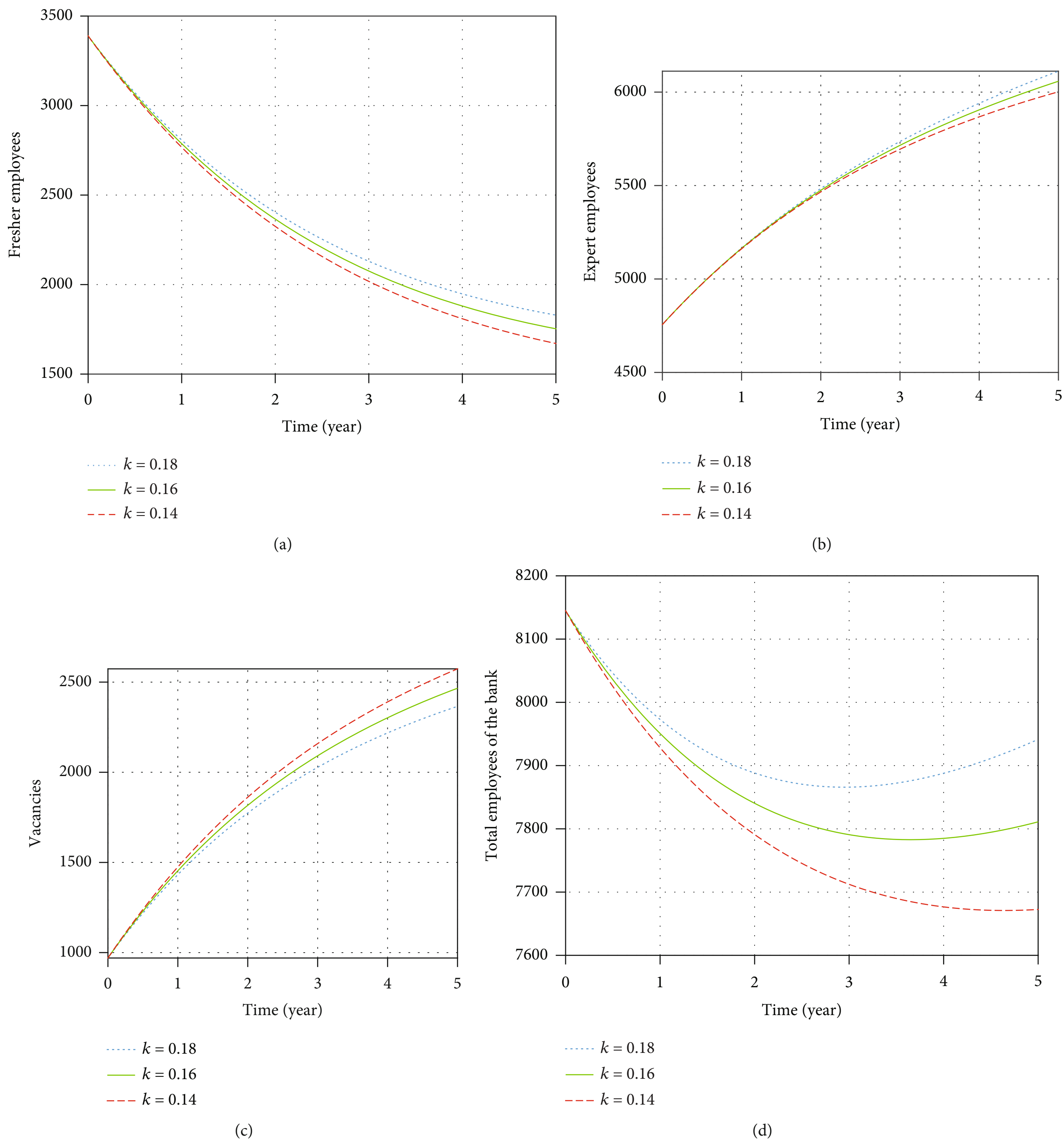
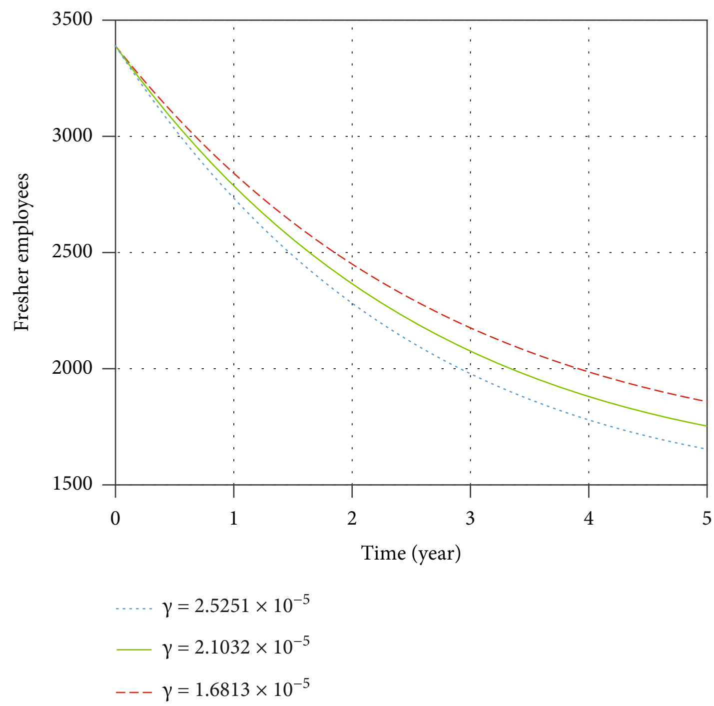


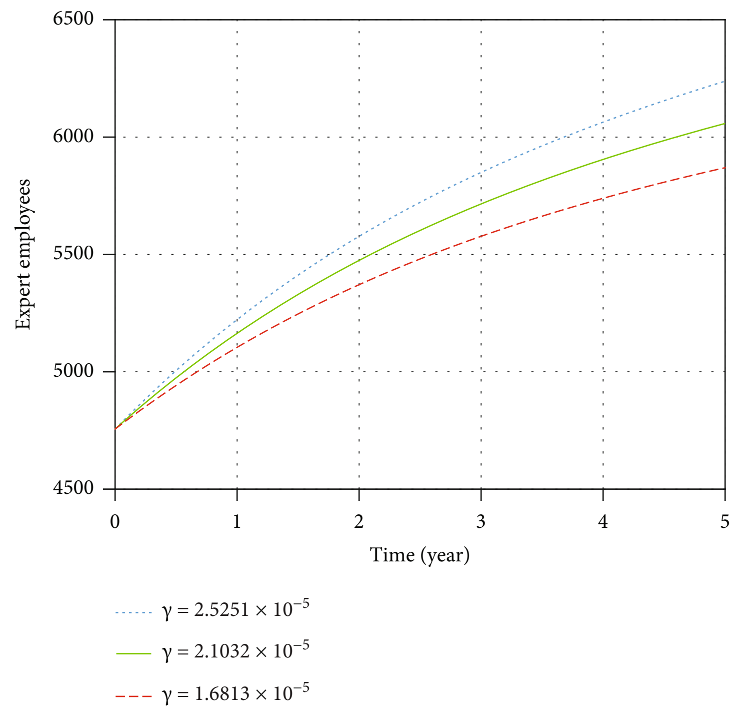
Figure 1: (a) Newly appointed employees enlarge the size of freshers. (b) The number of experts is rising due to meeting more newcomers. (c) Some vacancies are diminished due to joining new faces. (d) The number of staff rises as more join.

consequence, the number of fresher employees begins to fall as γ rises, while the overall number of employees, expert employees, and vacancies rise.

The behavior of state variables changes as ψ varies, as seen in Figure 3. The rate at which expert employees retire is represented by ψ. When expert staff retire, the number of expert employees decreases, resulting in some vacancies. As a result, the number of freshers for these positions will grow, but the total number of employees will fall. As a result, the number of expert employees will be reduced as the value of ψ rises. As a consequence, there will be more vacancies and fresher. Finally, the overall number of staff will be reduced in order to raise the ψ value.

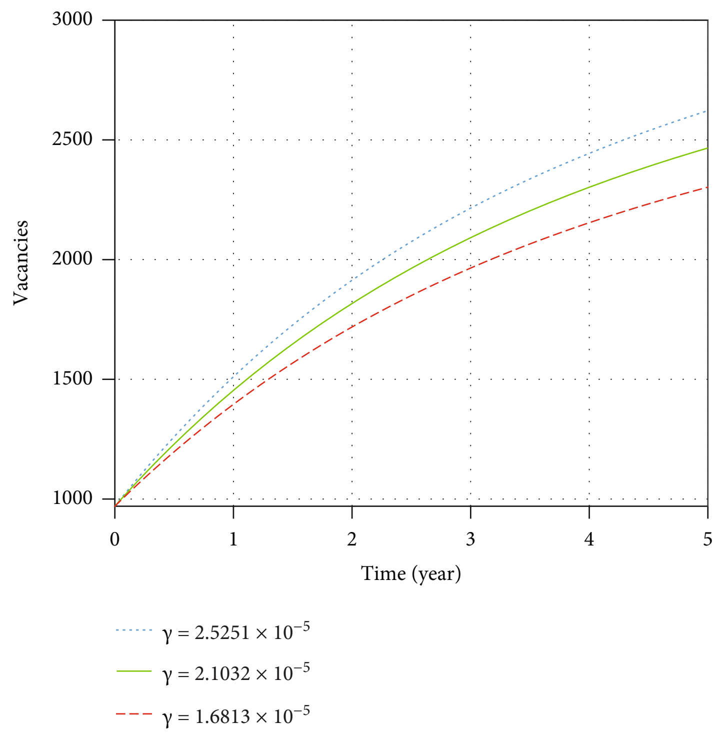


(a)

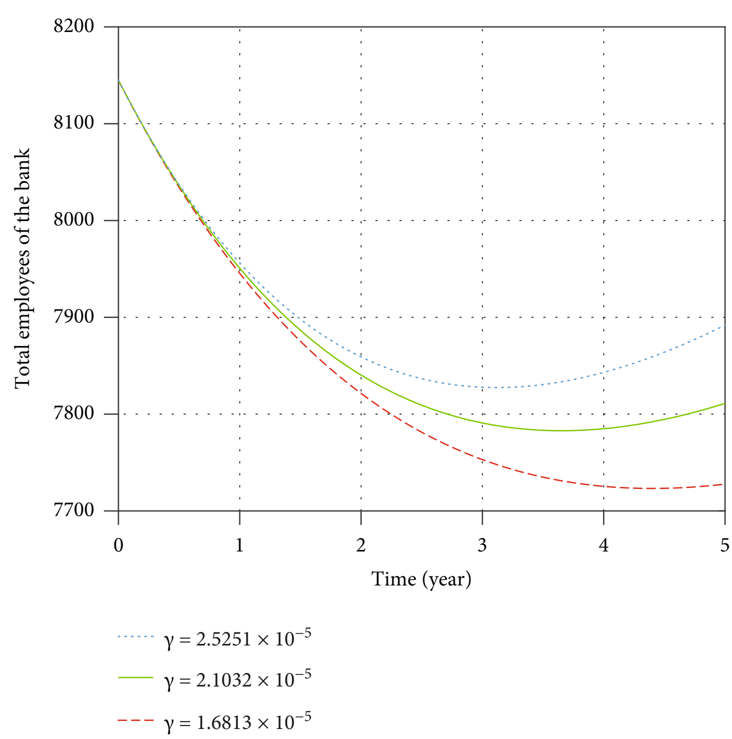


(b)

Figure 2: Continued.



(c)



(d)

Figure 2: (a) The less contact between newcomers and specialists, the more newcomers there will be. (b) The more interaction between freshers and experts, the more experts. (c) To increase manpower in business expansion due to more experienced employees, vacancies are raised, though management have another tendency to serve with minimum employees. (d) The number of staff has expanded as a result of increasing contact.

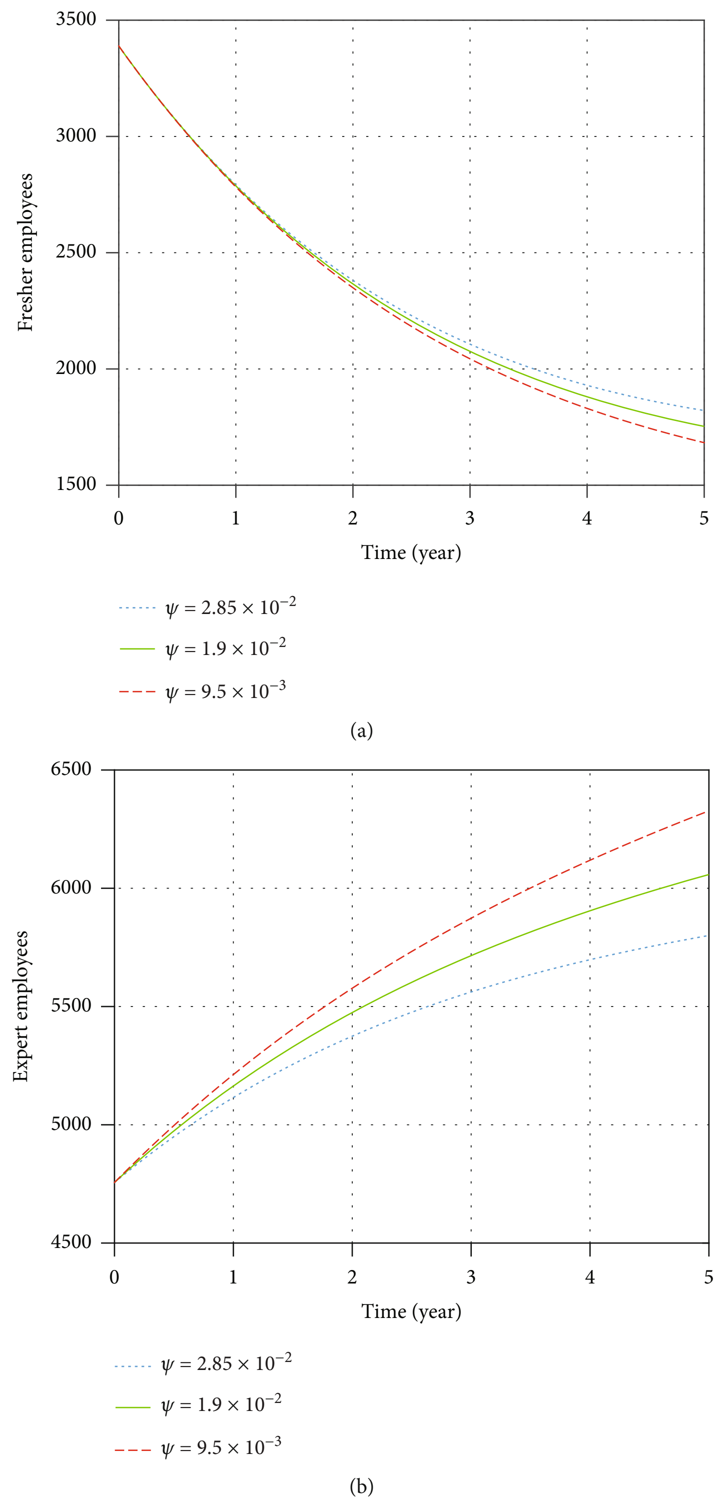
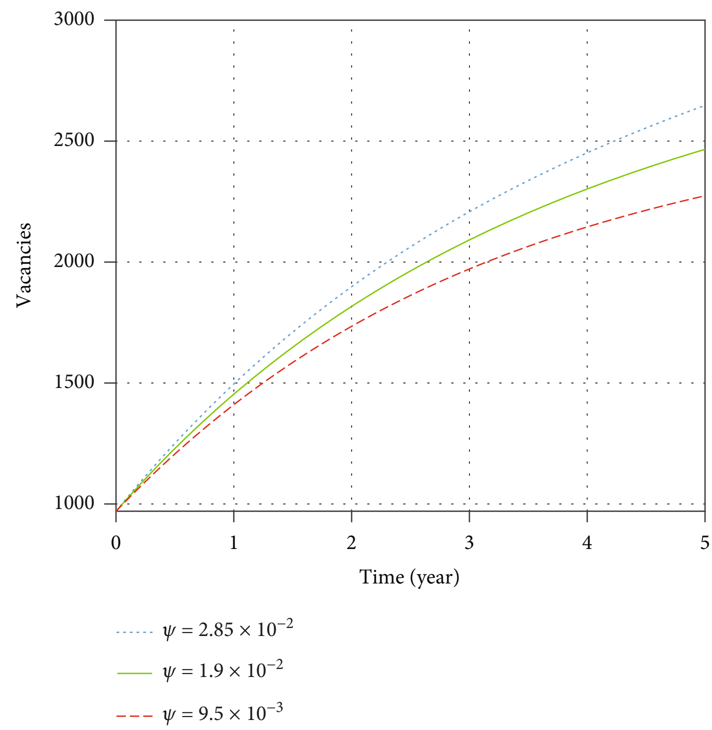
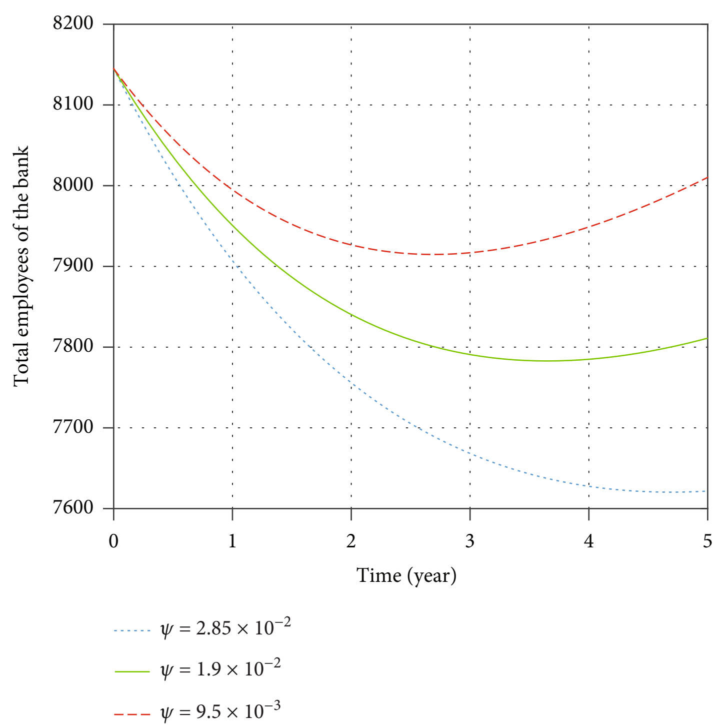


Figure 3: Continued.

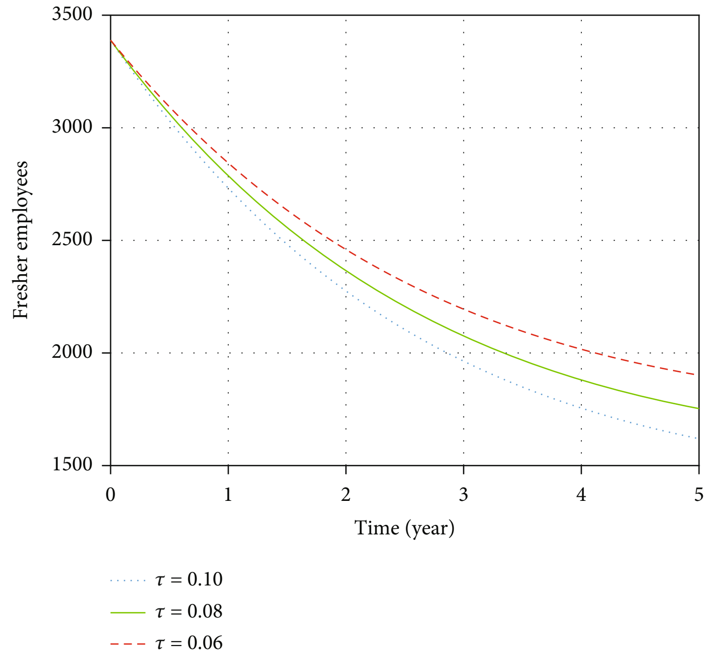


(c)

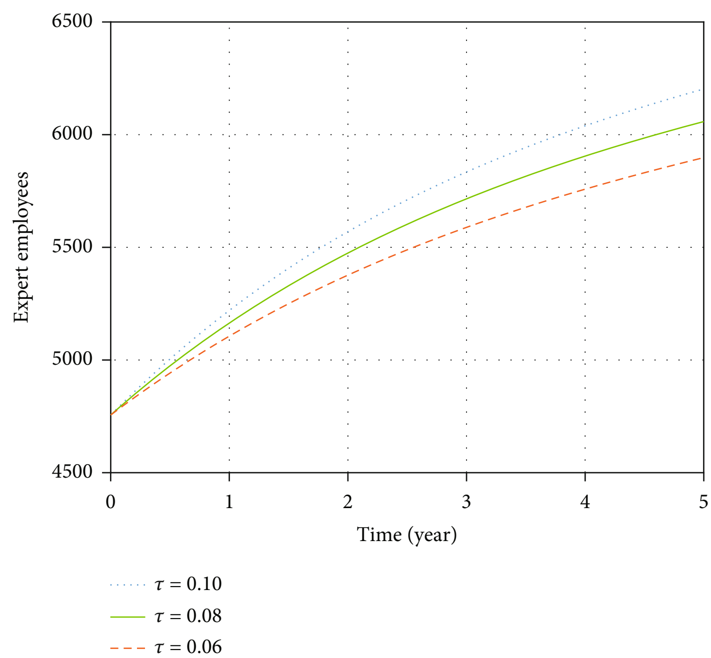


(d)

Figure 3: (a) The rate of retirement and death creates more fresher due to vacancies. (b) As a result of giving up work after an age, the number of specialists is gradually reducing. (c) Vacancies are continuously growing due to retiring of experts. (d) A large number of employees retire from professions results in a shortage of human resources.

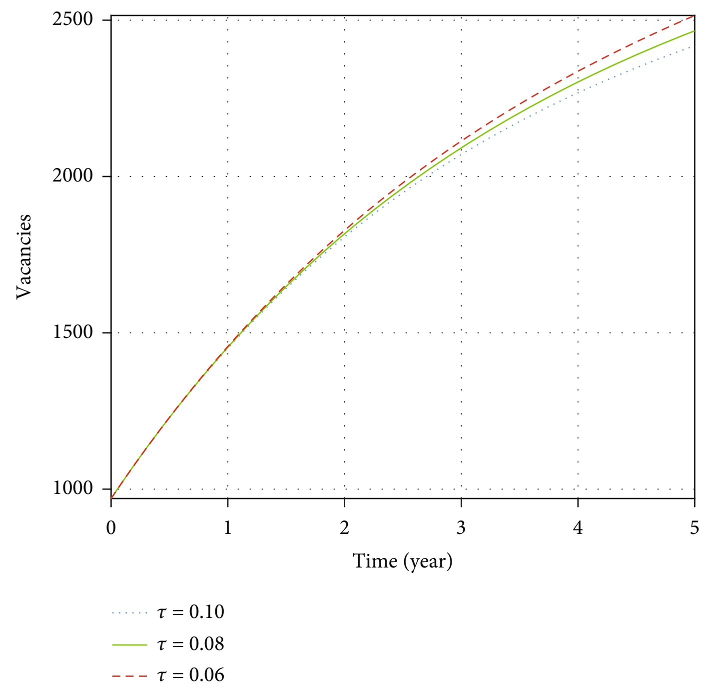


(a)



(b)

Figure 4: Continued.



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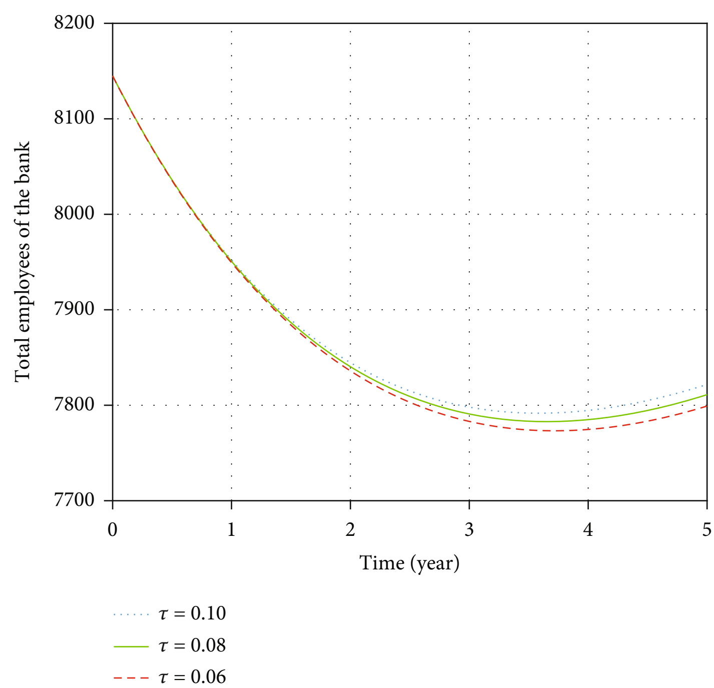


(d)

Figure 4: (a) More training facilities assist in reducing the number of fresher employees. (b) Increasing the number of skilled employees requires more training facilities. (c) Monotonically increased the number of vacancies is affected by the rate of trained staffs. (d) The impacts/effects of training programs on total employees in bank.

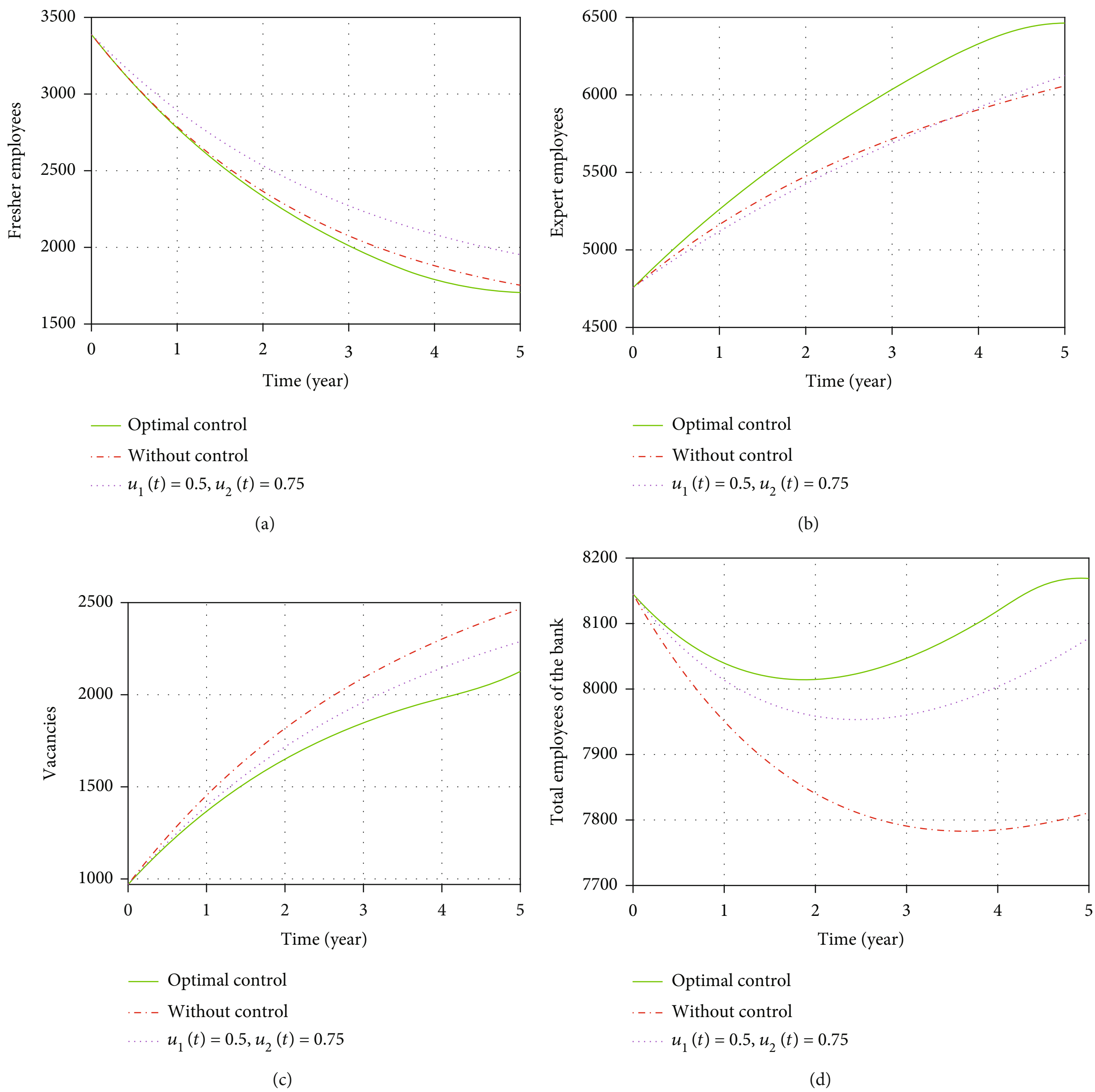
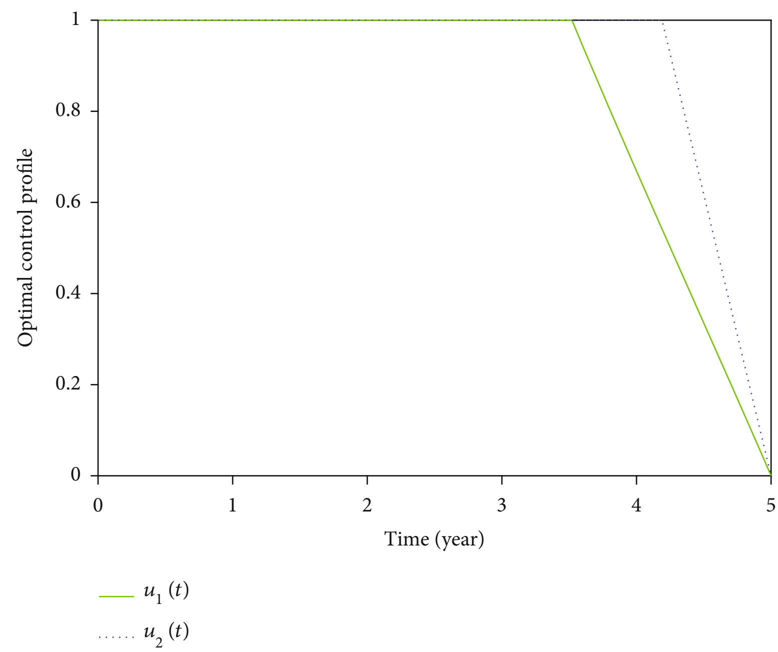


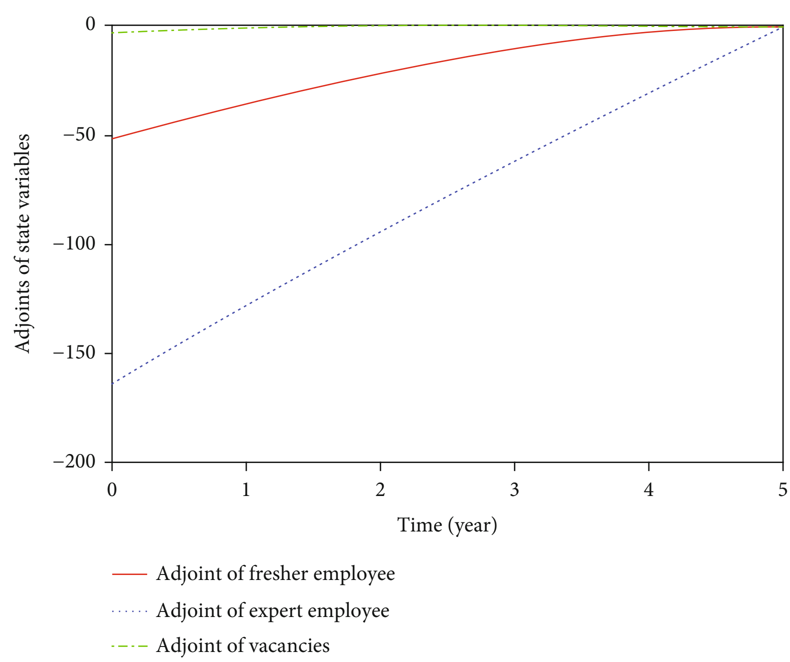
FIGURE 5: Expert employees as well as total employees are growing when both controls are optimal.

TABLE 2: Summary of objective functional.

Sl no.	Status of controls	Objective functional	Fresher employees	Expert employees	Vacancies
1	$u_1(t) = 1,\ u_2(t) = 0$	815770	1753	6058	2466
2	$0 \le u_1(t) \le 1 u_2(t) = 0$	817200	1847	5962	2472
3	$u_1(t) = 0.5 u_2(t) = 0.75$	820330	1953	6125	2289
4	$u_1(t) = 1 0 \le u_2(t) \le 1$	848690	1614	6558	2119
5	$0 \le u_1(t) \le 1 0 \le u_2(t) \le 1$	850210	1706	6463	2126



(a)



(b)

FIGURE 6: (a) Optimal control $u_1(t)$ of training or motivation is less than 1 (maximum capacity of training infrastructure) after three years and six months, and it is zero at the end of period, and $u_2(t)$ for serving special facilities of employees is less than 1 (maximum investment) after four years, add two and half months, and it is zero at the final time. (b) At the ending time, the adjoints are zero which satisfies the optimality condition.

For more training facilities, the size of expert staff grows (see Figure 4(b)), while the quantity of fresher staff declines in Figure 4(a). If we observe Figure 4(c), as a result of more training, employment opportunities are shrinking. Now, there is a concern that has been illuminated. Firstly, adding equations (1) and (2), the getting equation (39) have absence of τ.

$$\frac{d(F+E)}{dt} = V(k-\xi) - (E+F)\delta - F\beta - E\psi. \tag{39}$$

However, the overall number of staff is higher for training. It is clear that the second term of the right side of the equation (39) does not affect the number of total staff. It is already shown that the number of nonexperts and more experienced staff are cycled for training purposes. Therefore, the number of vacancies, $F\delta$, and $E\psi$ play a vital role in the total number of bankers. So, for better training sessions enlarge not only experts but also the total human resources.

(B) Optimal Control Analysis for Optimal Expert Employees in Banking Sector. A mathematical model of human resource management was designed using nonlinear differential equations to represent the dynamics of freshers, experts, and vacancies in the previous part, as well as examined the model with the characterization of state variables for various parameters. In this study, we establish a five-year project for expert development and also simulate this model in the present utilizing two optimal controls of a bank's staff management model using an optimal control approach. Several researchers utilize the optimal control technique to analyze and implement optimal controls for biological [43–45] and social problems. Recently, we have numerically and analytically analyzed a mathematical model of the unemployment problem. Here, two optimal control strategies are applied to find the minimum cost function [46]. Retirement and death of employed people are considered vacancies alternatively vacancies are reduced for recruitment in that research. This chapter will construct and evaluate an optimal controlled model for bank staff analysis based on these.

We assume two control variables, $u_1(t)$ and $u_2(t)$, to formulate the optimal control problem. Where $u_1(t)$ is used to manage the government's policy of providing qualified staff, and $u_2(t)$ is used to control expert switching. As a result, we expect that the following system of nonlinear differential equations will be used to boost the expertise of bank employees:

$$\dot{F}(t) = kV(t) - \beta F(t) - \gamma F(t)E(t) - \delta F(t) - \tau u_1(t)F(t), \tag{40}$$

$$\begin{aligned}\dot{E}(t) &= -\xi V(t) + \gamma F(t)E(t) - \psi(1 - u_2(t))E(t) \\ &\quad - \delta E(t) + \tau u_1(t)F(t),\end{aligned} \tag{41}$$

$$\begin{aligned}\dot{V}(t) &= -\phi E(t) + \gamma F(t)E(t) - kV(t) + \beta F(t) + \delta F(t) \\ &\quad + \psi(1 - u_2(t))E(t) + \delta E(t) + \xi V(t)A - (kV(t) \\ &\quad + \alpha_1 + \alpha_3 u_1(t))U(t) + \gamma E(t),\end{aligned} \tag{42}$$

with the initial conditions

$$\begin{aligned}F(0) &= U_0, \\ E(0) &= E_0, \\ V(0) &= V_0.\end{aligned} \tag{43}$$

Our goal is to identify methods of control that raise the number of experts at the completion of a bank's implemented policy while reducing the expenditure of policymaking. Because the square of controls helps to stabilize the optimum control condition. Therefore, we are using the square of controls to determine the cost of policymaking because we deploy as many experts as possible. As a consequence, we are using a linear approach to this term. Since we are trying to maximize the difference, the functional goal is chosen as follows:

$$\text{Maximize} J(u_1(t), u_2(t)) = \int_{t_s}^{t_f} \left(B_1 E(t) - B_2 {u_1}^2(t) - B_3 {u_2}^2(t)\right) dt. \tag{44}$$

The cost for policy making is a nonlinear statement of $u_1(t)$ and $u_2(t)$; we use these quadratic cost functions having properties concavity. Since the right hand sides of the equations (40)–(42) are linearly bounded with respect to $u_1(t)$ and $u_2(t)$. These bounds provide the compactness which is needed for the existence of optimal control [47]. So, Pontryagin's maximum principle [48, 49] in our proposed model for the optimal solution will be applied. Also, the parameters A_1, B_1, and $B_2 \geq 0$ represent the desired weights' for finding optimal cost. Our target is to find the control profiles ${u_1}^*(t)$ and ${u_2}^*(t)$ of satisfying

$$J(u_1^*(t), u_2^*(t)) = \max \left\{ J(u_1(t), u_2(t)) | 0 \leq u_1(t), u_2(t) \leq 1 \right\}. \tag{45}$$

5. Optimal Control for Unemployment Problem

Our new model can be reformulated ((40)–(42)) including objective functional (44) in optimal controlled problem as

$$\text{Maximize} \int_{t_s}^{t_f} L(t, x(t), u(t)) dt, \tag{46}$$

subject to

$$\begin{aligned}\dot{x}(t) &= f(x(t)) + g(x(t))u(t), \forall t \in \left[t_s, t_f\right], \\ u(t) &\in \mathscr{U}, \forall t \in \left[t_s, t_f\right], \\ x(0) &= x_0,\end{aligned} \tag{47}$$

where

$$\begin{aligned} x(t) &= (F(t), E(t), V(t)), \\ L(t, x(t), u(t)) &= B_1E(t) - B_2u_1^2(t) - B_3u_2^2(t), \end{aligned} \tag{48}$$

$$\begin{aligned} f(x) &= \begin{bmatrix} kV(t) - \beta F(t) - \gamma F(t)E(t) - \delta F(t) \\ -\xi V(t) + \gamma F(t)E(t) - \psi E(t) - \delta E(t) \\ -\phi E(t) + \gamma F(t)E(t) - kV(t) + \beta F(t) + \delta F(t) + \psi E(t) + \delta E(t) + \xi V(t) \end{bmatrix}, \\ u(t) &= \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}, \\ g(x) &= \begin{bmatrix} -\tau F(t) & 0 \\ \tau F(t) & \psi E(t) \\ 0 & -\psi E(t) \end{bmatrix}. \end{aligned} \tag{49}$$

6. Evaluation of the Maximum Principle

To find the optimal solution, we shall explore using the necessary optimality criteria of maximum principle [48]. Hamiltonian function for maximization of $J(u_1(t), u_2(t))$ has been written as follows:

$$H[x(t), p(t), u(t)] = \lambda L(x(t), u(t)) + \langle p(t), f(x) + g(x)u(t)\rangle, \lambda \in \mathbb{R}, \tag{50}$$

where the adjoint variables are denoted by $p = (p_F, p_E, p_V)$.

Assume that $(x^*(t), u^*(t))$ is an optimal solution. Therefore, from the maximum principle, there appears a scalar $\lambda \geq 0$, an absolutely continuous function $p(t)$, with the time argument $[t]$ denoting the evaluation along the optimal solution:

(i) $\max\{|p(t)|: t \in [t_s, t_f]\} + \lambda > 0$,

(ii) $\dot{p}(t) = -H_x[x] = -\lambda L_x[t] - \langle p(t), f_x[t] + g_x[t]u^*(t)\rangle$,

(iii) $p(t_f) = (0, 0, 0)$,

(iv) $H(x^*(t), p(t), u^*(t)) = \max_u\{H(x^*(t), p(t), u(t)): 0 \leq u \leq 1\}$.

From the adjoint equations (ii) with adjoint variables $p = (p_F, p_E, p_V)$ in normal form (i.e.,$\lambda = 1$) are explicitly given by

$$\begin{aligned} \dot{p}_F(t) &= p_F(t)(\beta + \delta + E(t)\gamma + \tau u_1(t)) - p_E(t)(E(t)\gamma + \tau u_1(t)) \\ &\quad - p_V(t)(\beta + \delta + E(t)\gamma), \\ \dot{p}_E(t) &= B_1 - p_V(t)(\delta - \phi + F(t)\gamma - \psi(u_2(t) - 1)) \\ &\quad - p_E(t)(F(t)\gamma - \delta + \psi(u_2(t) - 1)) + F(t)\gamma p_F(t), \\ \dot{p}_V(t) &= p_E(t)\xi - kp_F(t) + p_V(t)(k - \xi). \end{aligned} \tag{51}$$

We deduce from (iv) and get an explicit characterization of optimal control pair in the normal form i.e., $(\lambda = 1)$ given in terms of the multipliers $p = (p_F, p_E, p_V)$, and we get

$$\begin{aligned} &\langle p, f(x^*(t)) + g(x^*(t))u^*(t)\rangle + L(x^*(t), u^*(t)) \\ &\quad \geq \langle p, f(x^*(t)) + g(x^*(t))u(t)\rangle + L(x^*(t), u(t)). \end{aligned} \tag{52}$$

Simplifying the above inequalities (52) and we get

$$\begin{aligned} u_1^*(t) &= \max\left\{\min\left\{\frac{(F(t)p_F(t)\tau - F(t)p_E(t)\tau)}{2B2}, 1\right\}, 0\right\}, \\ u_2^*(t) &= \max\left\{\min\left\{\frac{(E(t)p_V(t)\psi - E(t)p_E(t)\psi)}{2B_3}, 1\right\}, 0\right\}. \end{aligned} \tag{53}$$

7. Numerical Simulations and Results

To find the optimal solution of the optimally controlled problem (Section 5), it has been characterized (Section 6) and numerically solved using the forward-backward sweep method [50]. According to the opinion of officers of the Pubali Bank of Bangladesh, the numerical optimal solution of the state equations and adjoint equations with objective function can be obtained in MATLAB (R2021b) using the values of parameters from Table 1 with weight parameters $A_1 = 30$, $B_1 = 3000$, and $B_2 = 1500$ and initial conditions $F(0) = 3389$, $E(0) = 4756$, and $V(0) = 970$.

For the iterative process, getting increment of time $\Delta = 0.00434$, 1152 time-grid for the time of five years has been taken. Since our optimal control problem has been solved by the indirect method, we allow the convergence tolerance of cost function at 10^{-8}.

The bank will have an efficient team of employees if adequate training and specialized facilities are provided to bankers. However, in order for the bank to generate more profit, the expenditure of management must be optimum including the maximum number of experts. Figures 5(a)–5(c) demonstrate the states where optimal cost control and facility availability included the maximum expertise comparing the states where unlimited training programs and specialized facilities are available, as well as comparing the states when 50% of available training programs are applied and 75% of available specialized facilities are implied.

Using optimal control techniques, the number of freshers will be reduced more. On the other hand, the number of experts will be increased significantly. Figure 5(a) indicates that a small number of freshers will be reduced, whereas from Figure 5(c), it is easily said that a significant number of vacancies will be reduced for joining of freshers against vacancies. As a result, it is evident that a significant number of freshers will become professionals as a result of good training and interaction between them, as well as the provision of facilities to manage with expert switching. Besides, if we observe Figure 5(d), it is seen that the total number of staff (addition of freshers and experts) will be increased significantly.

In Table 2, $u_1(t) = 1$ and $u_2(t) = 1$ denote the most efficient use of training programs in the absence of staff facilities. On the other hand, $u_1(t) = 1$ and $0 \le u_2(t) \le 1$ describe the maximum use of training and optimal control of facilities. This strategy is better than any previous strategy (see the first three rows in Table 2) to maximize the expert employees as well as the value of the objective function. However, only optimal control of training programs is not better for obtaining more experts. But, applying both optimal controls, we get the maximum value of objective function 850210 with a satisfied number of experts. Therefore, investing minimal cost, we will have to follow the optimal control profiles as shown in Figure 6(a) to get the maximum number of expert employees. According to Figure 6(a), optimal control $u_1(t)$ of training or motivation is active fully before three years and six months. But with the gradual dropping of training facilities, it is zero at the end of the period. Similarly, optimal control $u_2(t)$ for serving special facilities of employees is less than 1 after four years and two and a half months, and it is zero at the end.

The reality that the control's ending value is zero indicates that the policy would not be required at the end of that period. For completeness, we provide the graph of adjoints for the optimal control problem that fulfills the transversality conditions ((iii) of the maximum principle for the optimally controlled problem (see Figure 5). Because the adjoint of states is zero at the final time. As a result, we expect that our proposed model's numerically studied control techniques may be practices that facilitate the bank's expert staff.

Using Pontryagin's maximum principle, an optimal control analysis of the staff management model is performed in this section. To obtain promising efficiency after the bank's policy is successfully implemented, we utilize the two controls to maximize the value of the targeted functional by giving skilled employees with special facilities for five years. This study provides two control strategies in order to maximize the number of qualified professionals at the conclusion of implemented policies while diminishing policy-making expenditures. Therefore, our suggested model's numerically researched control techniques may successfully raise the bank's expert employees.

8. Conclusions

In this study, our new proposed model has shown the dynamics of fresher as well as expert employees of banks including vacancies. To make the model biologically feasible, it has been proven that the solution of the model (a system of differential equations) is positive and bounded. It is also shown theoretically and numerically that the interaction between freshers and experts expands the more experts as well as business based on more experienced employees (Figure 2(b)). For that reason, vacancies are opened. Conversely, a few interactions create a burden of huge inexpert employees. Besides, training programs not only assist in reducing the number of fresher employees but also enlarge the total employees expanding the opportunities of the banks. On the other hand, trained and skilled employees have increased. Hence, it is proven that a large number of freshers will be professionals due to good training and interaction with their peers, as well as the provision of facilities to help them transform into experts. To optimize the objective functional, we have to follow the control profiles (Figure 6), which are clear in Table 2 when both optimal controls are active. Therefore, by managing training and providing some special facilities like health insurance, a safe environment, appreciable activities, festival bonuses, etc., management would enlarge the size of expert employees as well as the total number of employees. Moreover, optimal control on training of employees starts slowly decreasing from three years and six months to the end of the period. Also, optimal control for providing services to employees occurs after four years and two and a half months but drops to zero at the end of the targeted period for scheduling the training programs and ensuring, unlike facilities, management would be more benefited, according to our study. Future works will include state constraints on expert employees to find the optimum cost-effective function.

Data Availability

The data has been used here is cited in the manuscript.

Conflicts of Interest

The authors have no conflict of interest.

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References

[1] S. Akther, T. Javed, and N. Islam, “The effectiveness of training need assessment (TNA) practices in private sector banks of Bangladesh,” *International Journal of Modern Trends in Business Research*, vol. 1, no. 4, pp. 82–98, 2018.

[2] N. Islam, E. M. Mridha, J. S. Sharna, M. D. Rahman, M. Mosharraf, and N. Nower, “An evaluation of the effectiveness of employee training in the government banks of Bangladesh,” in *Presented at the 10th International Conference on”Developing Organizations for the Future: Re-Interpreting the Modern Business Landscape” at: ICFAI Business School*, Bangalore, Karnataka, India, 2019.

[3] Planview, “What is resource management and why is it important?,” Retrieved July 29, 2023 from https://www.planview.com/resources/guide/resource-managementsoftware/resource-management-leverage-people-budgets/.

[4] S. Ray, S. Bagchi, S. Alam, and U. S. Luna, “Human resource management practices in banking sector of Bangladesh: a critical review,” *Journal of Business and Management*, vol. 23, no. 7, pp. 1–07, 2021.

[5] J. Kunz and M. Heitz, “Banks’ risk culture and management control systems: a systematic literature review,” *Journal of Management Control*, vol. 32, no. 4, pp. 439–493, 2021.

[6] A. Chahal, “A study of training need analysis based training and development: effect of training on performance by adopting development based strategy,” *International Journal of Business and Management Invention*, vol. 2, no. 4, pp. 41–51, 2013.

[7] R. Athar and F. M. Shah, “Impact of training on employee performance (banking sector Karachi),” *IOSR Journal of Business and Management (IOSR-JBM)*, vol. 17, no. 11, pp. 58–67, 2015.

[8] B. Appiah, *The impact of training on employee performance: a case study of HFC Bank (Ghana) Limited, [Ph.D. thesis]*, Ashesi University College, Accra, Ghana, 2012.

[9] A. J. Adongo, *Examining the Effects of Job Training on Employee Performance in Mobile Telephone Industry. A Case of Telkom Orange Nakuru, [Ph.D. thesis]*, Department of Business Administration, Kenyatta University, Kenya, 2014.

[10] P. Akilandeswari, “A study on effectiveness of training in Indian banks,” *International Journal of Recent Advances in Organizational Behavior and Decision Sciences (IJRAOB)*, vol. 1, no. 1, pp. 85–99, 2014.

[11] M. M. B. Marhil, K. A. R. Masaud, and N. A. Majid, “The mediating role of job satisfaction on the relationship between human resources management strategies and employees performance in Waha oil and gas company in Libya,” *American Journal of Economics and Business Innovation*, vol. 2, no. 1, pp. 63–69, 2023.

[12] P. N. Blanchard, *Effective training: systems, strategies and practices*, Prentice Hall Inc, 3rd edition, 2007.

[13] M. A. Hossain, “An overview on effective employee training systems of private sector banks in Bangladesh,” *International Journal of Ethics in Social Sciences*, vol. 2, no. 1, pp. 33–44, 2014.

[14] T. Ferdous and B. M. Razzak, “Importance of training needs assessment in the banking sector of Bangladesh: a case study on National Bank Limited (NBL),” *International Journal of Business and Management*, vol. 7, no. 10, pp. 63–73, 2012.

[15] M. Kaur, “An empirical study on effectiveness of training programs among women employees in public and private sector banks,” *Ushus Journal of Business Management*, vol. 15, no. 2, pp. 1–22, 2016.

[16] F. Mitu, M. Akter, F. Mita, and M. A. A. Mustafi, “The effectiveness of training and development on employee performance at private commercial banks in Bangladesh: SEM approach,” *International Scholar Journal of Accounting and Finance*, vol. 2, no. 2, pp. 43–54, 2016.

[17] M. M. M. A. Mzary, A. D. A. Al-rifai, and M. O. E. A. Momany, “Training and its impact on the performance of employees at Jordanian universities from the perspective of employees: the case of Yarmouk University,” *Journal of Education and Practice*, vol. 6, no. 32, pp. 1735–2222, 2015.

[18] M. M. Alsomaidaee, “The effect of workplace stress on employee engagement, the mediation role of leadership style,” *American Journal of Economics and Business Innovation*, vol. 2, no. 2, pp. 74–85, 2023.

[19] C. Wagonhurst, “Developing effective training programs,” *Journal of Research Administration*, vol. 33, no. 2/3, pp. 77–81, 2002.

[20] E. Srivastava and N. Agarwal, “Impact of training on bank employee performance: a comparative study of public sector bank and private sector bank in India,” *International Journal*, vol. 2, no. 1, pp. 630–641, 2014.

[21] N. Vijayalakshmi and S. Vaidhyasubramaniam, “Training in banking sector: a study to identify training needs among employees in public sector bank,” in *2012 - International Conference on Management Issues in Emerging Economies (ICMIEE)*, Thanjavur, India, 2012.

[22] A. Ghorbani, M. G. Fard, and I. Buinzahra, “The impact of training on employee performance and customer financial and nonfinancial performance of banks (case study: Tejarat bank),” *Aula Orientalis*, vol. 1, no. 33, pp. 105–118, 2015.

[23] A. R. Karim, “Impact of different training and development programs on employee performance in Bangladesh perspective,” *International Journal of Entrepreneurial Research*, vol. 2, no. 1, pp. 8–14, 2019.

[24] M. I. Khan, “The impact of training and motivation on performance of employees,” *Business Review*, vol. 7, no. 2, pp. 84–95, 2012.

[25] K. Rani and D. Garg, “A study on training and development in public sector banks,” *International Journal of Management and Social Sciences Research (IJMSSR)*, vol. 3, no. 1, pp. 33–37, 2014.

[26] S. Sanyal and M. W. Hisam, “Impact of training and development on employee performance: a comparative study on select banks in Sultanate of Oman,” *Research Journal of Finance and Accounting*, vol. 9, no. 6, pp. 53–58, 2018.

[27] H. Rahman, “Training and job satisfaction for organizational effectiveness: a case study from the banking sector,” *Central European Business Review*, vol. 3, no. 1, pp. 27–34, 2014.

[28] S. Denby, “The importance of training needs analysis,” *Industrial and Commercial Training*, vol. 42, no. 3, pp. 147–150, 2010.

[29] A. Mirza and S. Riaz, “Training needs assessment in Islamic banking sector,” *Qualitative Research in Financial Markets*, vol. 4, no. 2/3, pp. 142–155, 2012.

[30] A. A. Rahman, S. I. Ng, M. Sambasivan, and F. Wong, “Training and organizational effectiveness: moderating role of knowledge management process,” *European Journal of Training and Development*, vol. 37, no. 5, pp. 472–488, 2013.

[31] R. M. Debnath, *Bank and Legal Environment*, Nabajuga Prokashani, Dhaka, 1st edition, 2003.

[32] P. Evans, V. Pucik, and J.-L. Barsoux, *The Global Challenge: Framework for International Human Resource Management*, McGraw-Hill, Boston, 2002.

[33] S. Ahmed and M. N. Uddin, "Job satisfaction of bankers and its impact in banking: a case study of Janata Bank," *ASA University Review*, vol. 6, no. 2, pp. 95–102, 2012.

[34] Z. M. Islam, "Efficiency of training: a comparative study on some selected commercial banks in Bangladesh," *Asian Economic and Social Society*, vol. 5, no. 6, pp. 116–124, 2015.

[35] K. Javaid, N. Ahmad, and N. Iqbal, "Impact of training on employee performance in the context of telecommunication sector of D.G. khan, (Pakistan)," *International Letters of Social and Humanistic Sciences*, vol. 17, no. 6, pp. 60–73, 2014.

[36] K. D. Karthikeyan, R. Karthi, and S. D. Graf, "Impact of training in Indian banking sector–an empirical investigation," *International Journal of Business and Management*, vol. 5, no. 7, pp. 77–83, 2010.

[37] B. M. Megharaja, "The effectiveness of training and development in banking sector- a case study of state bank of India," *EPRA International Journal of Economic and Business Review*, vol. 2, no. 9, 2014http://www.epratrust.com/.

[38] A. M. Mohamud, "The effect of training on employee performance in public sector organizations in Kenya," in *The Case of Chif Machakos County, Master of Arts, Project Planning and Management*, University of Nairobi, Kenya, 2014.

[39] O. Ahmed and B. Rahman, "Employees of commercial banks learning to unlearn: a case study on the impact of electronic banking in the commercial banks of Bangladesh," *International Journal of Business and Technopreneurship*, vol. 10, no. 1, pp. 31–50, 2020.

[40] H. T. Tung and L. B. Phong, "Influence of high-involvement and high-commitment HRM practices on innovation: the mediating role of knowledge sharing," *American Journal of Economics and Business Innovation*, vol. 2, no. 2, pp. 35–44, 2023.

[41] S. I. Siddiqua, T. Fayezin, and R. Hoque, "Intermediating role of human resource management in organizational conflict and its influence on organizational effectiveness–evidence from Bangladesh's banking sector," *American Journal of Economics and Business Innovation*, vol. 3, no. 1, pp. 93–101, 2022.

[42] A. Fentie Bezabih, G. Kenassa Edessa, and P. Rao Koya, "Mathematical eco-epidemiological model on prey-predator system," *Mathematical Modelling and Applications*, vol. 5, no. 3, pp. 183–190, 2020.

[43] M. H. A. Biswas, L. T. Paiva, and M. de Pinho, "A SEIR model for control of infectious diseases with constraints," *Mathematical Biosciences and Engineering*, vol. 11, no. 4, pp. 761–784, 2014.

[44] M. H. A. Biswas, "Optimal control of Nipah virus (NiV) infections: a Bangladesh scenario," *Journal of Pure and Applied Mathematics: Advances and Applications*, vol. 12, no. 1, pp. 77–104, 2014.

[45] M. H. A. Biswas and M. R. de Pinho, "A nonsmooth maximum principle for optimal control problems with state and mixed constraints," *Discrete and Continuous Dynamical Systems*, pp. 174–183, 2013.

[46] U. K. Mallick and M. H. A. Biswas, "Mathematical approach with optimal control: reduction of unemployment problem in Bangladesh," *Journal of Applied Nonlinear Dynamics*, vol. 9, no. 2, pp. 231–246, 2020.

[47] W. H. Fleming and R. W. Rishel, *Deterministic and Stochastic Optimal Control*, Springer, Verlag, New York, 1975.

[48] L. S. Pontryagin, V. G. Boltyankii, R. V. Gamkrelidze, and E. F. Mischenko, *The mathematical theory of optimal processes*, Wiley-Interscience, 4, New York, 1962.

[49] R. Vinter, *Optimal Control*, Birkhauser, Boston, 2000.

[50] S. Lenhart and J. T. Workman, *Optimal Control Applied to Biological Models*, Taylor and Francies Group, New York, 2007.