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Research Article

Symmetry Analysis and Wave Solutions of the Fisher Equation Using Conformal Fractional Derivatives

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In the present article, the time fractional Fisher equation is considered in conformal form to study the application of the Lie classical method and quantitative analysis. The Lie symmetry method has been applied to find the infinitesimal generators and symmetry reductions of the fractional Fisher equation. The obtained reduced form of the equation is solved by the method of G'/G, which gives different forms of solutions. The theory of bifurcation has been utilized in the reduced form to check the stability and nature of critical points by transforming the equations into an autonomous system. Some phase portraits have been drawn at different critical points by the use of maple.

1. Introduction

Nonlinear fractional partial differential equations play a crucial role in science and engineering such as fluid mechanics, electrochemistry, viscoelasticity, optics, and signals processing. Finding exact solutions can be a challenging task, so many researchers have developed numerous methods [1–3] for it. Some of them are the first integral method [4, 5], the new extended G'/G-expansion method [6], the tanh method [7], and the hyperbolic B-spline differential quadrature method [8]. The concept of fractional-order derivatives was first introduced by Leibniz in 1695. An effective theory of fractional calculus was developed after progression.

The significant method to examine the differential equations is the Lie symmetry analysis [9]. Many researchers also utilize the theory of Lie symmetries. It was given by Sophus Lie in the 19th century to find the solution of a differential equation. The limitation of the Riemann-Liouville theory is that, by using this theory, traveling wave solutions cannot be obtained. The theory of fractional derivatives has been modified by the use of conformable fractional derivatives which were utilized by Khalil et al. [10] to find the Lie symmetries of some differential equations.

The motivation of this paper is to find the Lie symmetries using the conformable fractional derivatives [11–16], and to find the fractional symmetries, we consider the Fisher equation which exists in different forms.

The Fisher equation is a NLPDE that is part of the reaction-diffusion equation [17].

$$\bar{q}_{\bar{t}} = r\bar{q}\left(\frac{1-\bar{q}}{K}\right). \tag{1}$$

This equation is very useful in modeling phenomena in neurophysiology, population dynamics, and chemical kinetics, and in economics, it determines the relationship between nominal and real interest rates under the effect of inflation. It is named after Irving Fisher, who was famous for his works on the theory of interest. Different forms of the Fisher equations [17] exist such as

$$\begin{aligned} &\frac{\partial \bar{q}}{\partial \bar{t}} - \frac{\partial^2 \bar{q}}{\partial^2 \bar{x}} = \omega \bar{q}\left(\frac{1-\bar{q}}{K}\right), \\ &\bar{q}_{\bar{t}} = \bar{q}_{\bar{x}\bar{x}} + \bar{q}(1-(\bar{q})(\bar{q}-a), 0 < a < 1, \\ &\bar{q}_{\bar{t}} = +b\bar{q}_{\bar{x}\bar{x}} + c\bar{q}^2(1-(\bar{q})). \end{aligned} \tag{2}$$

Fisher introduced a PDE

$$\bar{q}_{\bar{t}}(\bar{x},\bar{t}) = a\bar{q}_{\bar{x}\bar{x}}(\bar{x},\bar{t}) + f(\bar{q}) \tag{3}$$

describing a mutation occurring in a population distributed in a linear habitat. A particular case of this equation is [8]

$$\bar{q}_{\bar{t}}(\bar{x},\bar{t}) = a\bar{q}_{\bar{x}\bar{x}}(\bar{x},\bar{t}) + b\bar{q}(\bar{x},\bar{t})(1-\bar{q}(\bar{x},\bar{t})), \tag{4}$$

where b is known as the real parameter and the reaction term is given by $b\bar{q}(1-\bar{q})\{b>0\}$. The focus of this paper is to acquire the Lie symmetries of the fractional Fisher partial differential equation.

$$\frac{\partial^{\bar{\alpha}}\bar{q}}{\partial t^{\bar{\alpha}}} - \bar{q}_{\bar{x}\bar{x}} - \bar{q} + (\bar{q})^2 = 0, \tag{5}$$

where $\partial^{\bar{\alpha}}\bar{q}/\partial t^{\bar{\alpha}}$ denotes the conformal fractional derivative [18–20].

This paper is divided into 8 parts. The definitions of conformable fractional derivatives and their properties are described in Section 2. In Section 3, the Lie symmetry analysis of time fractional partial differentiable equations is discussed briefly. The algorithm of the G'/G-expansion method is described in Section 4. The description of the fractional Fisher equation is discussed in Section 5. In Section 6, the exact traveling wave solution of the fractional Fisher equation is provided in trigonometric form. The bifurcation with phase portrait of the fractional Fisher equation is explained in Section 7. At last, in Section 8, the concluding statement for this work is recorded.

2. The Conformable Fractional Derivative

The definition of the conformable fractional derivative is defined as in [21].

Let us consider $\alpha \in (0,1]$ and $f:[0,\infty) \longrightarrow R$, then the conformable fractional derivative [22] of $\bar{f}$ of order α is defined as

$$D_t^{\alpha}\bar{f}(\bar{t}) = \lim_{\bar{\varepsilon}\longrightarrow 0}\frac{\bar{f}\left(\bar{t}+\bar{\varepsilon}\,(\bar{t})^{(1-\alpha)}\right)-\bar{f}(\bar{t})}{\bar{\varepsilon}}, \forall \bar{t} \in (0,+\infty). \tag{6}$$

Here, the function $\bar{f}$ if α-conformably differentiable at a point $\bar{t}$ if the limit in Equation (6) exists.

2.1. Properties of the Conformable Fractional Derivative. The conformable fractional derivative has many important properties [23, 24]. Some of them are

$$\begin{gathered} D_t^{\alpha}(\bar{t}) = (\bar{t})^{(1-\alpha)}\frac{\partial\bar{q}}{\partial\bar{t}}. \\ D_t^{\alpha}\left(a\bar{f}(\bar{t}+b\,\bar{g}(\bar{t})\right) = a\,D_t^{\alpha}\bar{f}(\bar{t}) + b\,D_t^{\alpha}\bar{g}(\bar{t}), \forall a,b \in R. \end{gathered} \tag{7}$$

3. Methods

There are two methodologies used in this paper:

(i) The Lie symmetry analysis method

(ii) The (G'/G)-expansion method

3.1. Lie Symmetry Analysis of Time Fractional Partial Differentiable Equations. Initially, we explain some key points related to the Lie symmetry analysis [25, 26] of fractional PDEs concisely. The time-fractional PDE with two independent variables is

$$\frac{\partial^{\bar{\alpha}}\bar{q}}{\partial\bar{t}^{\bar{\alpha}}} = G[\bar{q}], 0 < \bar{\alpha} \le 1, \tag{8}$$

where $\bar{q} = \bar{q}(\bar{x},\bar{t})$, $\bar{G}[\bar{q}]$ is a nonlinear function, and $\partial^{\bar{\alpha}}/\partial\bar{t}^{\bar{\alpha}}$ is the conformal fractional derivative. Consider the one-parameter lie group of infinitesimal transformations

$$\begin{aligned} \hat{\bar{x}} &= \bar{x} + \bar{\varepsilon}\chi(\bar{t},\bar{x},\bar{q}) + o\left(\bar{\varepsilon}^2\right), \\ \hat{\bar{t}} &= \bar{t} + \bar{\varepsilon}\sigma(\bar{t},\bar{x},\bar{q}) + o\left(\bar{\varepsilon}^2\right), \\ \hat{\bar{q}} &= \bar{w} + \bar{\varepsilon}\kappa(\bar{t},\bar{x},\bar{q}) + o\left(\bar{\varepsilon}^2\right). \end{aligned} \tag{9}$$

The infinitesimal operator can be written as

$$\bar{V} = \chi(\bar{t},\bar{x},\bar{q})\frac{\partial}{\partial\bar{x}} + \sigma(\bar{t},\bar{x},\bar{q})\frac{\partial}{\partial\bar{t}} + \kappa(\bar{t},\bar{x},\bar{q})\frac{\partial}{\partial\bar{q}}. \tag{10}$$

By finding the solutions of the Lie equations, the group transformation equivalent to the infinitesimal operator can be achieved

$$\begin{aligned} \frac{d\hat{\bar{x}}}{d\varepsilon} &= \chi\left(\hat{\bar{t}},\hat{\bar{x}},\hat{\bar{q}}\right), \\ \frac{d\hat{\bar{t}}}{d\varepsilon} &= \sigma\left(\hat{\bar{t}},\hat{\bar{x}},\hat{\bar{q}}\right), \\ \frac{d\hat{\bar{q}}}{d\varepsilon} &= \kappa\left(\hat{\bar{t}},\hat{\bar{x}},\hat{\bar{q}}\right). \end{aligned} \tag{11}$$

subject to initial conditions $\hat{\bar{x}}|_{\varepsilon=0} = \bar{x}$, $\hat{\bar{t}}|_{\varepsilon=0} = \bar{t}$, and $\hat{\bar{q}}|_{\varepsilon=0} = \bar{q}$.

Extending transformation (9) to the fractional differentiation operator $\partial^{\bar{\alpha}}\hat{q}/\partial\hat{t}^{\bar{\alpha}}$, $r = 1, 2, 3, \cdots\cdots$, one can obtain

$$\begin{aligned} \frac{\partial^{\bar{\alpha}}\hat{\bar{q}}}{\partial\hat{\bar{t}}^{\bar{\alpha}}} &= \frac{\partial^{\bar{\alpha}}\bar{q}}{\partial\bar{t}^{\bar{\alpha}}} + \varepsilon\kappa_{\bar{\alpha}}^{\bar{t}}(\bar{t},\bar{x},\bar{q}) + o\left(\varepsilon^2\right), \\ \frac{\partial\hat{\bar{q}}}{\partial\hat{\bar{x}}} &= \frac{\partial\bar{q}}{\partial\bar{x}} + \varepsilon\kappa^{\bar{x}}(\bar{t},\bar{x},\bar{q}) + o\left(\varepsilon^2\right), \\ \frac{\partial^2\hat{\bar{q}}}{\partial\hat{\bar{x}}^2} &= \frac{\partial^2\bar{q}}{\partial\bar{x}^2} + \varepsilon\kappa^{\bar{x}\bar{x}}(\bar{t},\bar{x},\bar{q}) + o\left(\varepsilon^2\right), \\ \frac{\partial^3\hat{\bar{q}}}{\partial\hat{\bar{x}}^3} &= \frac{\partial^3\bar{q}}{\partial\bar{x}^3} + \varepsilon\kappa^{\bar{x}\bar{x}\bar{x}}(\bar{t},\bar{x},\bar{q}) + o\left(\varepsilon^2\right). \end{aligned} \tag{12}$$

where

$$\begin{aligned}\kappa^{\bar{x}} &= D_{\bar{x}}(\kappa) - \bar{q}_{\bar{t}}D_{\bar{x}}(\sigma) - \bar{q}_{\bar{x}}D_{\bar{x}}(\chi),\\ \kappa^{\bar{x}\bar{x}} &= D_{\bar{x}}\left(\kappa^{\bar{x}}\right) - \bar{q}_{\bar{x}\bar{t}}D_{\bar{x}}(\sigma) - \bar{q}_{\bar{x}\bar{x}}D_{\bar{x}}(\chi),\\ \kappa^{\bar{x}\bar{x}\bar{x}} &= D_{\bar{x}}\left(\kappa^{\bar{x}\bar{x}}\right) - q_{\bar{x}\bar{x}\bar{t}}D_{\bar{x}}(\sigma) - q_{\bar{x}\bar{x}\bar{x}}D_{\bar{x}}(\chi),\\ &\vdots,\end{aligned} \tag{13}$$

where $D_{\bar{x}}$ is the total derivative operator and is given by

$$D_{\bar{x}} = \frac{\partial}{\partial \bar{x}} + \bar{q}_{\bar{x}}\frac{\partial}{\partial \bar{q}} + \bar{q}_{\bar{x}\bar{x}}\frac{\partial}{\partial \bar{q}_{\bar{x}}} + \bar{q}_{\bar{t}\bar{x}}\frac{\partial}{\partial \bar{q}_{\bar{t}}}, \tag{14}$$

*The prolongation of point transformation to the $\bar{\alpha}$th derivative for some $\bar{\alpha} \in (0, 1]$ is given by $\partial^{\bar{\alpha}}\hat{\bar{q}}/\partial\hat{\bar{t}}^{\bar{\alpha}} = \partial^{\bar{\alpha}}\bar{q}/\partial\bar{t}^{\bar{\alpha}} + \varepsilon\kappa_{\bar{\alpha}}^{\bar{t}}(\bar{t}, \bar{x}, \bar{q}) + o(\varepsilon^2)$,where $\bar{\alpha}$th extended infinitesimal is denoted by $\kappa_{\bar{\alpha}}^{\bar{t}}$ and is given as

$$\kappa_{\bar{\alpha}}^{\bar{t}} = D_{\bar{t}}^{\bar{\alpha}}\kappa - \bar{q}_{\bar{x}}D_{\bar{t}}^{\bar{\alpha}}\chi - \bar{q}_{\bar{t}}D_{\bar{t}}^{\bar{\alpha}}\sigma + (1-\bar{\alpha})\sigma(\bar{t})^{(-\bar{\alpha})}\bar{q}_{\bar{t}}, \tag{15}$$

where $D_{\bar{t}}^{\bar{\alpha}} = \bar{t}^{(1-\bar{\alpha})}D_{\bar{t}}$ is the total fractional derivative operator and $D_{\bar{t}}$ is the total derivative operator given by

$$D_{\bar{t}} = \frac{\partial}{\partial \bar{t}} + \bar{q}_{\bar{t}}\frac{\partial}{\partial \bar{q}} +_{\bar{x}\bar{t}}\frac{\partial}{\partial \bar{q}_{\bar{x}}} + \bar{q}_{\bar{t}\bar{t}}\frac{\partial}{\partial \bar{q}_{\bar{t}}} \tag{16}$$

is the total derivative operator.

3.2. Algorithm of the (G'/G)-Expansion Method. The (G'/G)-expansion method is a method presented by the Chinese Mathematics Wang et al. [27] to compute the exact traveling solutions of NLPDEs. This method is very beneficial as it is uncomplicated and derives traveling wave solutions.

Consider a NLPDE in two independent variables $\bar{x}$ and $\bar{t}$,

$$Q(\bar{q}, \bar{q}_{\bar{t}}, \bar{q}_{\bar{x}}, \bar{q}_{\bar{t}\bar{t}}, \bar{q}_{\bar{x}\bar{t}}, \bar{q}_{\bar{x}\bar{x}}, \bar{q}_{\bar{x}\bar{x}\bar{x}}, \cdots \cdots), \tag{17}$$

where $\bar{q}(\bar{x}, \bar{t})$ is an unknown function and Q is a polynomial in $\bar{q}(\bar{x}, \bar{t})$ and partial derivatives. The key points of the mechanism are

(i) Step 1

Firstly use the wave transformation $\chi = \bar{x} - c\bar{t}$ such that

$$\bar{q}(\bar{x}, \bar{t}) = \bar{F}(\chi), \chi = \bar{x} - c\bar{t}. \tag{18}$$

Equation (17) is reduced to an ODE given by

$$R\left(\bar{F}, -c\bar{F}', \bar{F}', c^2\bar{F}'', -c\bar{F}'', \bar{F}'', \bar{F}''', \cdots \cdots \cdots\right). \tag{19}$$

Integrate the ODE (19) if required, and for simplicity, assume the integration constants to be zero.

(ii) Step 2

Now, consider that the solution is obtained in the form of (G'/G) for the ODE (19) as

$$\bar{F}(\chi) = a_p\left(\frac{G'}{G}\right)^p + a_{p-1}\left(\frac{G'}{G}\right)^{p-1} + \cdots, \tag{20}$$

where $a_p(p = 0, 1, 2, 3, \cdots \cdots)$ are arbitrary constants and As $G = G(\chi)$ satisfies the second-order LDE of the form

$$G'' + \bar{\Lambda}G' + \bar{\Phi}G = 0. \tag{21}$$

where a_p, a_{p-1},..... a_0, $\bar{\Lambda}$, and $\bar{\Phi}$ are constants, $a_p \neq 0$. By making the homogenous balance between the highest-order derivative and nonlinear term appear as in Equation (19), the value of p will be determined easily.

(iii) Step 3

Use Equation (20) into (19) with the help of Equation (21), assemble all the terms of (G'/G) of the same order, and then equate each coefficient to zero; a set of algebraic equations is obtained for a_p, a_{p-1},..... a_0, c, $\bar{\Lambda}$, and $\bar{\Phi}$.

(iv) Step 4

As Equation (21) has well-known general solutions, if we use the values obtained in Step 3 and the general solutions of Equation (21) into (20), we have traveling wave solutions of Equation (17). The general solutions of Equation (21) are given as

$$\left(\frac{G'}{G}\right) = \begin{cases} \dfrac{\sqrt{\bar{\Lambda}^2 - 4\bar{\Phi}}}{2}\left(\dfrac{C_1 \sin h\left(1/2\sqrt{\bar{\Lambda}^2 - 4\bar{\Phi}}\right)\chi + C_2 \cos h\left(1/2\sqrt{\bar{\Lambda}^2 - 4\bar{\Phi}}\right)\chi}{C_1 \cos h\left(1/2\sqrt{\bar{\Lambda}^2 - 4\bar{\Phi}}\right)\chi + C_2 \sin h\left(1/2\sqrt{\bar{\Lambda}^2 - 4\bar{\Phi}}\right)\chi}\right) - \dfrac{\bar{\Lambda}}{2}, \bar{\Lambda}^2 - 4\bar{\Phi} > 0 \\ \dfrac{\sqrt{4\bar{\Phi} - \bar{\Lambda}^2}}{2}\left(\dfrac{-C_1 \sin\left(1/2\sqrt{4\bar{\Phi} - \bar{\Lambda}^2}\right)\chi + C_2 \cos\left(1/2\sqrt{4\bar{\Phi} - \bar{\Lambda}^2}\right)\chi}{C_1 \cos\left(1/2\sqrt{4\bar{\Phi} - \bar{\Lambda}^2}\right)\chi + C_2 \sin\left(1/2\sqrt{4\bar{\Phi} - \bar{\Lambda}^2}\right)\chi}\right), -\dfrac{\bar{\Lambda}}{2}, \bar{\Lambda}^2 - 4\bar{\Phi} < 0 \\ \dfrac{c_2}{c_1 + c_2\chi} - \dfrac{\bar{\Lambda}}{2}, \bar{\Lambda}^2 - 4\bar{\Phi} = 0. \end{cases} \tag{22}$$

4. The Fractional Fisher Equation

Fisher's equation in fractional form is given as follows:

$$\frac{\partial^{\bar{\alpha}}\bar{q}}{\partial t^{\bar{\alpha}}} - \bar{q}_{\bar{x}\bar{x}} - \bar{q} + (\bar{q})^2 = 0, \tag{23}$$

where $0 < \bar{\alpha} \leq 1$ and the parameter $\bar{\alpha}$ expresses the order of conformable fractional derivatives. By using the Lie theory, the infinitesimal equation is given by

$$\kappa_{\bar{\alpha}}^{\bar{t}} - \kappa_{\bar{x}\bar{x}} + 2\bar{q}\kappa - \kappa = 0. \tag{24}$$

Substituting the values of $\kappa_{\bar{x}\bar{x}}$ and $\kappa_{\bar{\alpha}}^{\bar{t}}$ from (13) to (15) into (24). Replace $\bar{q}_{\bar{x}\bar{x}}$ by $\partial^{\bar{\alpha}}\bar{q}/\partial\bar{t}^{\bar{\alpha}} - \bar{q} + (\bar{q})^2$ and equate the coefficients of the various monomials in partial derivatives of $\bar{q}$; the set of determining equations are obtained for the symmetry of Equation (23).

$$\begin{aligned} \sigma_{\bar{x}} &= 0, \\ \sigma_{\bar{q}} &= 0, \\ \chi_{\bar{x}} &= 0, \\ \kappa_{\bar{q}\bar{q}} &= 0, \\ \sigma(\bar{t})^{(-\bar{\alpha})}(1-\bar{\alpha}) - (\bar{t})^{(1-\bar{\alpha})}\sigma_{\bar{t}} + (\bar{t})^{(1-\bar{\alpha})}\kappa_{\bar{q}} &= 0, \\ 2\chi_{\bar{x}}(\bar{q})^2 + 2\chi_{\bar{x}}(\bar{q}_{\bar{t}})^{\bar{\alpha}} - \kappa_{\bar{x}\bar{x}} - \kappa + (\bar{t})^{(1-\bar{\alpha})}\kappa_{\bar{t}} & \\ - \kappa_{\bar{q}}(\bar{q}_{\bar{t}})^{\bar{\alpha}} + 2\bar{q}\kappa - \kappa_{\bar{q}}(\bar{q})^2 + \kappa_{\bar{q}}\bar{q} - 2\chi_{\bar{x}}\bar{q} &= 0. \end{aligned} \tag{25}$$

By solving these determining equations, one can obtain

$$\begin{aligned} \sigma &= \frac{e_1}{(\bar{t})^{(\alpha-1)}}, \\ \chi &= e_2, \\ \kappa &= 0, \end{aligned} \tag{26}$$

where e_1 and e_2 are arbitrary constants. As obtained symmetries are trivial, therefore only wave solutions of Equation (23) exist. The corresponding symmetry group is spanned by the following vector fields:

$$\begin{aligned} \bar{V}_1 &= (\bar{t})^{(1-\bar{\alpha})}\frac{\partial}{\partial\bar{t}}, \\ \bar{V}_2 &= \frac{\partial}{\partial\bar{x}}. \end{aligned} \tag{27}$$

For the symmetry $\bar{V}_1 + \bar{V}_2$, the corresponding invariant solution takes the form $\chi = \bar{x} + \omega(\bar{t})^{\bar{\alpha}}/\bar{\alpha}$

5. Formation of Solutions of the Fractional Fisher Equation

The Fisher equation is a NLFPDE which is solved by the (G'/G)-expansion method is given as

$$(\bar{q})_{\bar{t}}^{\bar{\alpha}} - \bar{q}_{\bar{x}\bar{x}} - \bar{q} + (\bar{q})^2 = o. \tag{28}$$

By operating the wave transformation

$$\bar{q}(\bar{x},\bar{t}) = \bar{F}(\chi), \chi = \bar{x} + \omega\frac{(\bar{t})^{\bar{\alpha}}}{\bar{\alpha}}, \tag{29}$$

where ω represents nonzero arbitrary functions.

Equation (28) is turned into an ordinary differential equation of the type

$$-\bar{F}'' + \omega\bar{F}' - \bar{F} + \bar{F}^2 = 0. \tag{30}$$

By assuming that the solutions of Equation (30) will be formulated by a polynomial in (G'/G) as

$$\bar{F}(\chi) = a_p\left(\frac{G'}{G}\right)^p + a_{p-1}\left(\frac{G'}{G}\right)^{p-1} +.\cdots\cdots, \tag{31}$$

where $a_p(p = 0, 1, 2, 3, \cdots\cdots)$ are arbitrary constants and $G = G(\chi)$ satisfies the linear differential equation of the second order of the form

$$G'' + \bar{\Lambda}G' + \bar{\Phi}G = 0, \tag{32}$$

where $\bar{\Lambda}$ and $\bar{\Phi}$ are the constants. We obtain $p = 1$ by considering the homogeneous balance between $\bar{F}''$ and $\bar{F}^2$ in Equation (30). So, Equation (31) can be represented as:

$$\bar{F}(\chi) = a_1\left(\frac{G'}{G}\right) + a_0, a_1 \neq 0, \tag{33}$$

$$\bar{F}^2(\chi) = a_1^2\left(\frac{G'}{G}\right)^2 + 2a_0a_1\left(\frac{G'}{G}\right) + a_0^2, \tag{34}$$

$$\bar{F}'(\chi) = -a_1\left(\frac{G'}{G}\right)^2 - a_1\bar{\Lambda}\left(\frac{G'}{G}\right) - \bar{\Phi}a_1, \tag{35}$$

$$\bar{F}''(\chi) = 2a_1\left(\frac{G'}{G}\right)^3 + 3\bar{\Lambda}a_1\left(\frac{G'}{G}\right)^2 + a_1\left(2\bar{\Phi} + \bar{\Lambda}^2\left(\frac{G'}{G}\right) + a_1\bar{\Phi}.\right. \tag{36}$$

Using Equations (33)–(36) in Equation (30), the function in (G'/G) is obtained. Then, by setting the coefficients of identical power of (G'/G) to zero, set of algebric equations is obtained as follows:

$$\left(\frac{G'}{G}\right)^3 : -2a_1 = 0,$$

$$\left(\frac{G'}{G}\right)^2 : -3\bar{\Lambda}a_1 - \omega a_1 + a_1^2 = 0,$$

$$\left(\frac{G'}{G}\right)^1 : -\bar{\Lambda}^2 a_1 - \bar{\Lambda}\omega a_1 - 2\bar{\Phi}a_1 + 2a_0a_1 - a_1 = 0,$$

$$\left(\frac{G'}{G}\right)^0 : a_0^2 - \bar{\Phi}a_1 - \bar{\Phi}\omega a_1 - a_0 = 0. \tag{37}$$

On solving the above algebraic equation with the help of maple, we get

$$\begin{aligned}
a_0 &= \frac{1}{2} + \bar{\Phi} + \frac{\bar{\Lambda}\left(-4\bar{\Phi}\bar{\Lambda} + \bar{\Lambda}^3 - 2\bar{\Phi} - \bar{\Lambda}\right)}{2\left(-\bar{\Lambda}^2 + 2\bar{\Phi}\right)} \\
&\quad + \frac{\bar{\Lambda}\left(\sqrt{12\bar{\Phi}^2\bar{\Lambda}^2 - 8\bar{\Phi}\bar{\Lambda}^4 - 8\bar{\Phi}^2\bar{\Lambda} + 10\bar{\Phi}\bar{\Lambda}^2 + \bar{\Lambda}^6 + 8\bar{\Lambda}^3\bar{\Phi} - 3\bar{\Lambda}^4 + 4\bar{\Phi}^2 + 4\bar{\Phi}\bar{\Lambda} + 2\bar{\Lambda}^2 - 2\bar{\Phi} + 8\bar{\Phi}^3}\right)}{2\left(-\bar{\Lambda}^2 + 2\bar{\Phi}\right)} \\
&\quad + \frac{1}{2}\bar{\Lambda}^2, \\
a_1 &= 3\bar{\Lambda} + \frac{-4\bar{\Phi}\bar{\Lambda} + \bar{\Lambda}^3 - 2\bar{\Phi} - \bar{\Lambda}}{-\bar{\Lambda}^2 + 2\bar{\Phi}} \\
&\quad + \frac{\sqrt{12\bar{\Phi}^2\bar{\Lambda}^2 - 8\bar{\Phi}\bar{\Lambda}^4 - 8\bar{\Phi}^2\bar{\Lambda} + 10\bar{\Phi}\bar{\Lambda}^2 + \bar{\Lambda}^6 + 8\bar{\Lambda}^3\bar{\Phi} - 3\bar{\Lambda}^4 + 4\bar{\Phi}^2 + 4\bar{\Phi}\bar{\Lambda} + 2\bar{\Lambda}^2 - 2\bar{\Phi} + 8\bar{\Phi}^3}}{-\bar{\Lambda}^2 + 2\bar{\Phi}}, \\
\omega &= \frac{-4\bar{\Phi}\bar{\Lambda} + \bar{\Lambda}^3 - 2\bar{\Phi} - \bar{\Lambda}}{-\bar{\Lambda}^2 + 2\bar{\Phi}} \\
&\quad + \frac{\sqrt{12\bar{\Phi}^2\bar{\Lambda}^2 - 8\bar{\Phi}\bar{\Lambda}^4 - 8\bar{\Phi}^2\bar{\Lambda} + 10\bar{\Phi}\bar{\Lambda}^2 + \bar{\Lambda}^6 + 8\bar{\Lambda}^3\bar{\Phi} - 3\bar{\Lambda}^4 + 4\bar{\Phi}^2 + 4\bar{\Phi}\bar{\Lambda} + 2\bar{\Lambda}^2 - 2\bar{\Phi} + 8\bar{\Phi}^3}}{-\bar{\Lambda}^2 + 2\bar{\Phi}}.
\end{aligned} \tag{38}$$

Substitute these values into Equation (33), we get $\bar{F}(\chi)$ is given as follows:

$$\begin{aligned}
\bar{F}(\chi) &= \left(3\bar{\Lambda} + \frac{-4\bar{\Phi}\bar{\Lambda} + \bar{\Lambda}^3 - 2\bar{\Phi} - \bar{\Lambda}}{-\bar{\Lambda}^2 + 2\bar{\Phi}}\right. \\
&\quad + \left.\left(\frac{\sqrt{12\bar{\Phi}^2\bar{\Lambda}^2 - 8\bar{\Phi}\bar{\Lambda}^4 - 8\bar{\Phi}^2\bar{\Lambda} + 10\bar{\Phi}\bar{\Lambda}^2 + \bar{\Lambda}^6 + 8\bar{\Lambda}^3\bar{\Phi} - 3\bar{\Lambda}^4 + 4\bar{\Phi}^2 + 4\bar{\Phi}\bar{\Lambda} + 2\bar{\Lambda}^2 - 2\bar{\Phi} + 8\bar{\Phi}^3}}{-\bar{\Lambda}^2 + 2\bar{\Phi}}\right)\right)\frac{G'}{G} \\
&\quad + \frac{1}{2} + \bar{\Phi} + \frac{\bar{\Lambda}\left(-4\bar{\Phi}\bar{\Lambda} + \bar{\Lambda}^3 - 2\bar{\Phi} - \bar{\Lambda}\right)}{2\left(-\bar{\Lambda}^2 + 2\bar{\Phi}\right)} \\
&\quad + \frac{\bar{\Lambda}\left(\sqrt{12\bar{\Phi}^2\bar{\Lambda}^2 - 8\bar{\Phi}\bar{\Lambda}^4 - 8\bar{\Phi}^2\bar{\Lambda} + 10\bar{\Phi}\bar{\Lambda}^2 + \bar{\Lambda}^6 + 8\bar{\Lambda}^3\bar{\Phi} - 3\bar{\Lambda}^4 + 4\bar{\Phi}^2 + 4\bar{\Phi}\bar{\Lambda} + 2\bar{\Lambda}^2 - 2\bar{\Phi} + 8\bar{\Phi}^3}\right)}{2\left(-\bar{\Lambda}^2 + 2\bar{\Phi}\right)} + \frac{1}{2}\bar{\Lambda}^2,
\end{aligned} \tag{39}$$

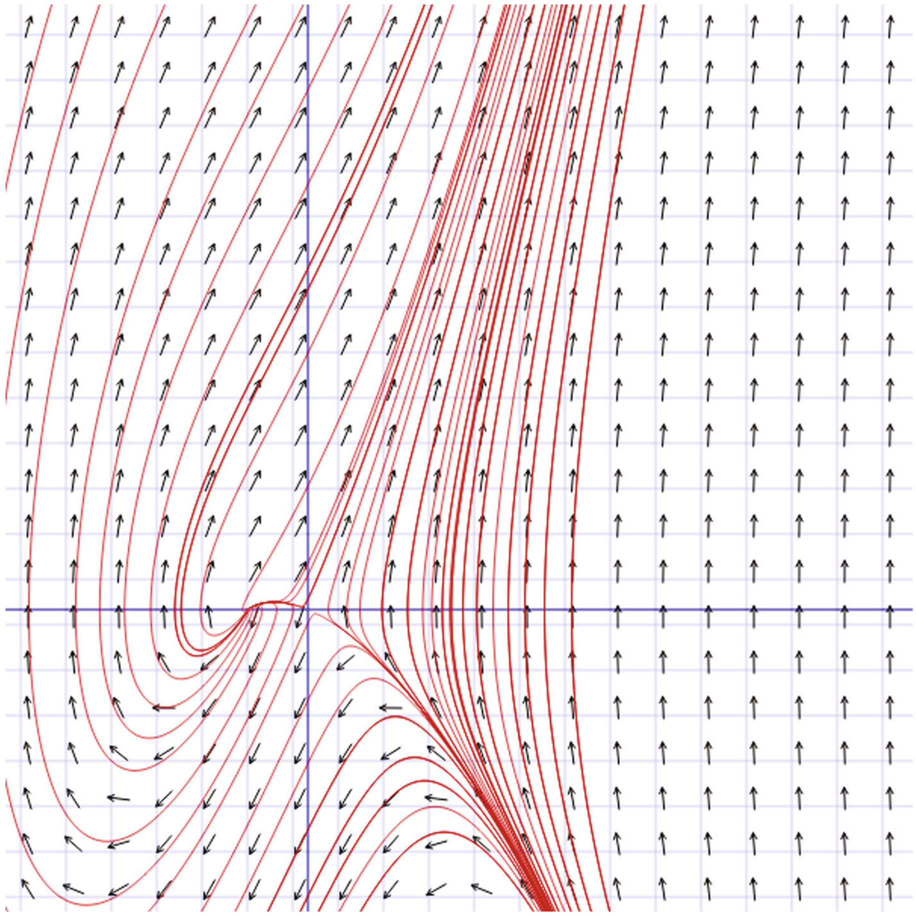

Figure 1: Node.

where $\chi = \bar{x} + \omega \bar{t}^{\bar{\alpha}}/\bar{\alpha}$. Now, the general solution of Equation (32) is well known and is given in Step 4 of the algorithm.

Case 1. When $\bar{\Lambda}^2 - 4\bar{\Phi} > 0$, then

$$\begin{aligned}
\bar{F}(\chi) = {} & \left(3\bar{\Lambda} + \frac{-4\bar{\Phi}\bar{\Lambda} + \bar{\Lambda}^3 - 2\bar{\Phi} - \bar{\Lambda}}{-\bar{\Lambda}^2 + 2\bar{\Phi}}\right) \\
& + \left(\frac{\sqrt{12\bar{\Phi}^2\bar{\Lambda}^2 - 8\bar{\Phi}\bar{\Lambda}^4 - 8\bar{\Phi}^2\bar{\Lambda} + 10\bar{\Phi}\bar{\Lambda}^2 + \bar{\Lambda}^6 + 8\bar{\Lambda}^3\bar{\Phi} - 3\bar{\Lambda}^4 + 4\bar{\Phi}^2 + 4\bar{\Phi}\bar{\Lambda} + 2\bar{\Lambda}^2 - 2\bar{\Phi} + 8\bar{\Phi}^3}}{-\bar{\Lambda}^2 + 2\bar{\Phi}}\right), \\
& \left(\frac{\sqrt{\bar{\Lambda}^2 - 4\bar{\Phi}}}{2}\left(\frac{C_1 \sin h\left(1/2\sqrt{\bar{\Lambda}^2 - 4\bar{\Phi}}\right)\chi + C_2 \cos h\left(1/2\sqrt{\bar{\Lambda}^2 - 4\bar{\Phi}}\right)\chi}{C_1 \cos h\left(1/2\sqrt{\bar{\Lambda}^2 - 4\bar{\Phi}}\right)\chi + C_2 \sin h\left(1/2\sqrt{\bar{\Lambda}^2 - 4\bar{\Phi}}\right)\chi}\right) - \frac{\bar{\Lambda}}{2}\right) \\
& + \frac{1}{2} + \bar{\Phi} + \frac{\bar{\Lambda}\left(-4\bar{\Phi}\bar{\Lambda} + \bar{\Lambda}^3 - 2\bar{\Phi} - \bar{\Lambda}\right)}{2\left(-\bar{\Lambda}^2 + 2\bar{\Phi}\right)} \\
& + \frac{\bar{\Lambda}\left(\sqrt{12\bar{\Phi}^2\bar{\Lambda}^2 - 8\bar{\Phi}\bar{\Lambda}^4 - 8\bar{\Phi}^2\bar{\Lambda} + 10\bar{\Phi}\bar{\Lambda}^2 + \bar{\Lambda}^6 + 8\bar{\Lambda}^3\bar{\Phi} - 3\bar{\Lambda}^4 + 4\bar{\Phi}^2 + 4\bar{\Phi}\bar{\Lambda} + 2\bar{\Lambda}^2 - 2\bar{\Phi} + 8\bar{\Phi}^3}\right)}{2\left(-\bar{\Lambda}^2 + 2\bar{\Phi}\right)} + \frac{1}{2}\bar{\Lambda}^2.
\end{aligned} \tag{40}$$

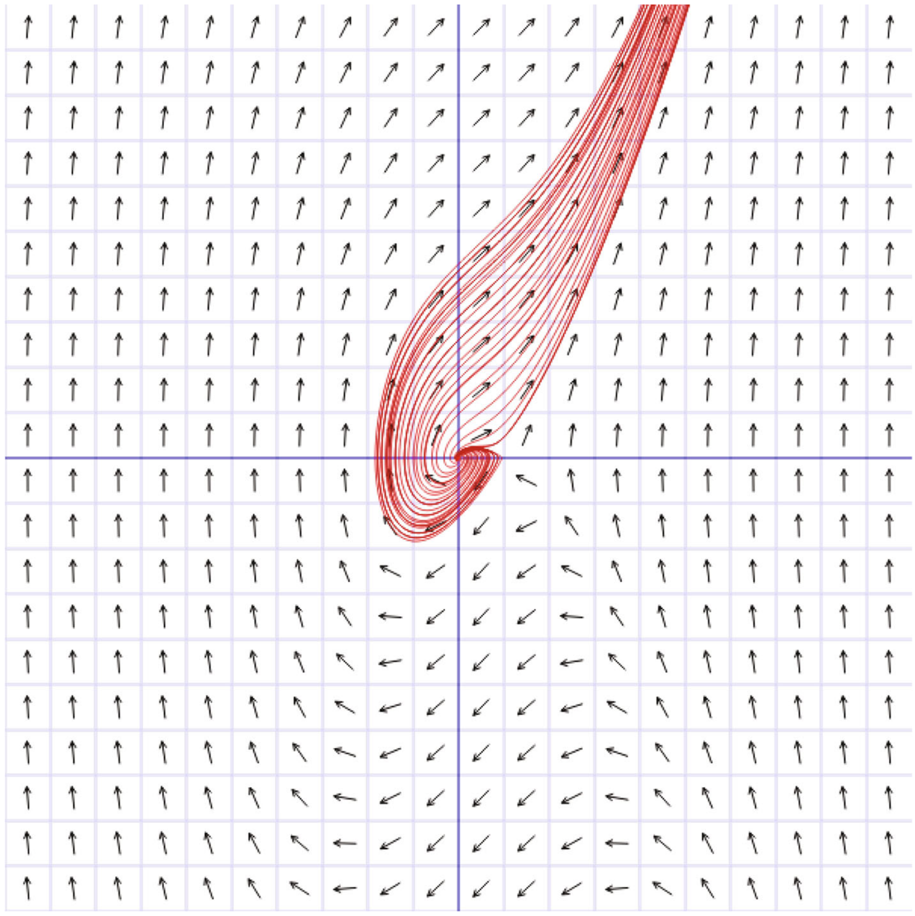

Figure 2: Spiral.

As $\bar{q}(\bar{x},\bar{t}) = \bar{F}(\chi)$, where $\chi = \bar{x} + \omega \bar{t}^{\bar{\alpha}}/\bar{\alpha}$.

$$\begin{aligned}
\bar{q}(\bar{x},\bar{t}) = {} & \left(3\bar{\Lambda} + \frac{-4\bar{\Phi}\bar{\Lambda} + \bar{\Lambda}^3 - 2\bar{\Phi} - \bar{\Lambda}}{-\bar{\Lambda}^2 + 2\bar{\Phi}}\right) \\
& + \left(\frac{\sqrt{12\bar{\Phi}^2\bar{\Lambda}^2 - 8\bar{\Phi}\bar{\Lambda}^4 - 8\bar{\Phi}^2\bar{\Lambda} + 10\bar{\Phi}\bar{\Lambda}^2 + \bar{\Lambda}^6 + 8\bar{\Lambda}^3\bar{\Phi} - 3\bar{\Lambda}^4 + 4\bar{\Phi}^2 + 4\bar{\Phi}\bar{\Lambda} + 2\bar{\Lambda}^2 - 2\bar{\Phi} + 8\bar{\Phi}^3}}{-\bar{\Lambda}^2 + 2\bar{\Phi}}\right), \\
& \left(\frac{\sqrt{\bar{\Lambda}^2 - 4\bar{\Phi}}}{2}\left(\frac{C_1 \sin h\left(1/2\sqrt{\bar{\Lambda}^2 - 4\bar{\Phi}}\right)\chi + C_2 \cos h\left(1/2\sqrt{\bar{\Lambda}^2 - 4\bar{\Phi}}\right)\chi}{C_1 \cos h\left(1/2\sqrt{\bar{\Lambda}^2 - 4\bar{\Phi}}\right)\chi + C_2 \sin h\left(1/2\sqrt{\bar{\Lambda}^2 - 4\bar{\Phi}}\right)\chi}\right) - \frac{\bar{\Lambda}}{2}\right) + \frac{1}{2} + \bar{\Phi} + \frac{\bar{\Lambda}\left(-4\bar{\Phi}\bar{\Lambda} + \bar{\Lambda}^3 - 2\bar{\Phi} - \bar{\Lambda}\right)}{2\left(-\bar{\Lambda}^2 + 2\bar{\Phi}\right)}, \\
& + \frac{\bar{\Lambda}\left(\sqrt{12\bar{\Phi}^2\bar{\Lambda}^2 - 8\bar{\Phi}\bar{\Lambda}^4 - 8\bar{\Phi}^2\bar{\Lambda} + 10\bar{\Phi}\bar{\Lambda}^2 + \bar{\Lambda}^6 + 8\bar{\Lambda}^3\bar{\Phi} - 3\bar{\Lambda}^4 + 4\bar{\Phi}^2 + 4\bar{\Phi}\bar{\Lambda} + 2\bar{\Lambda}^2 - 2\bar{\Phi} + 8\bar{\Phi}^3}\right)}{2\left(-\bar{\Lambda}^2 + 2\bar{\Phi}\right)} + \frac{1}{2}\bar{\Lambda}^2,
\end{aligned} \tag{41}$$

where $\chi = \bar{x} + \omega \bar{t}^{\bar{\alpha}}/\bar{\alpha}$.

Case 2. If $\bar{\Lambda}^2 - 4\bar{\Phi} < 0$, then we have

$$\bar{F}(\chi)=\left(3\bar{\Lambda}+\frac{-4\bar{\Phi}\bar{\Lambda}+\bar{\Lambda}^{3}-2\bar{\Phi}-\bar{\Lambda}}{-\bar{\Lambda}^{2}+2\bar{\Phi}}\right)+\left(\frac{\sqrt{12\bar{\Phi}^{2}\bar{\Lambda}^{2}-8\bar{\Phi}\bar{\Lambda}^{4}-8\bar{\Phi}^{2}\bar{\Lambda}+10\bar{\Phi}\bar{\Lambda}^{2}+\bar{\Lambda}^{6}+8\bar{\Lambda}^{3}\bar{\Phi}-3\bar{\Lambda}^{4}+4\bar{\Phi}^{2}+4\bar{\Phi}\bar{\Lambda}+2\bar{\Lambda}^{2}-2\bar{\Phi}+8\bar{\Phi}^{3}}}{-\bar{\Lambda}^{2}+2\bar{\Phi}}\right),$$
$$\frac{\sqrt{4\bar{\Phi}-\bar{\Lambda}^{2}}}{2}\left(\frac{-C_{1}\sin\left(1/2\sqrt{4\bar{\Phi}-\bar{\Lambda}^{2}}\right)\chi+C_{2}\cos\left(1/2\sqrt{4\bar{\Phi}-\bar{\Lambda}^{2}}\right)\chi}{C_{1}\cos\left(1/2\sqrt{4\bar{\Phi}-\bar{\Lambda}^{2}}\right)\chi+C_{2}\sin\left(1/2\sqrt{4\bar{\Phi}-\bar{\Lambda}^{2}}\right)\chi}\right)+\frac{1}{2}+\bar{\Phi}+\frac{\bar{\Lambda}\left(-4\bar{\Phi}\bar{\Lambda}+\bar{\Lambda}^{3}-2\bar{\Phi}-\bar{\Lambda}\right)}{2\left(-\bar{\Lambda}^{2}+2\bar{\Phi}\right)}$$
$$+\frac{\bar{\Lambda}\left(\sqrt{12\bar{\Phi}^{2}\bar{\Lambda}^{2}-8\bar{\Phi}\bar{\Lambda}^{4}-8\bar{\Phi}^{2}\bar{\Lambda}+10\bar{\Phi}\bar{\Lambda}^{2}+\bar{\Lambda}^{6}+8\bar{\Lambda}^{3}\bar{\Phi}-3\bar{\Lambda}^{4}+4\bar{\Phi}^{2}+4\bar{\Phi}\bar{\Lambda}+2\bar{\Lambda}^{2}-2\bar{\Phi}+8\bar{\Phi}^{3}}\right)}{2\left(-\bar{\Lambda}^{2}+2\bar{\Phi}\right)}+\frac{1}{2}\bar{\Lambda}^{2}. \tag{42}$$

As $\bar{q}(\bar{x},\bar{t})=\bar{F}(\chi)$, where $\chi=\bar{x}+\omega\bar{t}^{\bar{\alpha}}/\bar{\alpha}$.

$$\bar{q}(\bar{x},\bar{t})=\left(3\bar{\Lambda}+\frac{-4\bar{\Phi}\bar{\Lambda}+\bar{\Lambda}^{3}-2\bar{\Phi}-\bar{\Lambda}}{-\bar{\Lambda}^{2}+2\bar{\Phi}}\right)+\left(\frac{\sqrt{12\bar{\Phi}^{2}\bar{\Lambda}^{2}-8\bar{\Phi}\bar{\Lambda}^{4}-8\bar{\Phi}^{2}\bar{\Lambda}+10\bar{\Phi}\bar{\Lambda}^{2}+\bar{\Lambda}^{6}+8\bar{\Lambda}^{3}\bar{\Phi}-3\bar{\Lambda}^{4}+4\bar{\Phi}^{2}+4\bar{\Phi}\bar{\Lambda}+2\bar{\Lambda}^{2}-2\bar{\Phi}+8\bar{\Phi}^{3}}}{-\bar{\Lambda}^{2}+2\bar{\Phi}}\right),$$
$$\frac{\sqrt{4\bar{\Phi}-\bar{\Lambda}^{2}}}{2}\left(\frac{-C_{1}\sin\left(1/2\sqrt{4\bar{\Phi}-\bar{\Lambda}^{2}}\right)\chi+C_{2}\cos\left(1/2\sqrt{4\bar{\Phi}-\bar{\Lambda}^{2}}\right)\chi}{C_{1}\cos\left(1/2\sqrt{4\bar{\Phi}-\bar{\Lambda}^{2}}\right)\chi+C_{2}\sin\left(1/2\sqrt{4\bar{\Phi}-\bar{\bar{\Lambda}}^{2}}\right)\chi}\right)+\frac{1}{2}+\bar{\Phi}+\frac{\bar{\Lambda}\left(-4\bar{\Phi}\bar{\Lambda}+\bar{\Lambda}^{3}-2\bar{\Phi}-\bar{\bar{\Lambda}}\right)}{2\left(-\bar{\Lambda}^{2}+2\bar{\Phi}\right)}$$
$$+\frac{\bar{\Lambda}\left(\sqrt{12\bar{\Phi}^{2}\bar{\Lambda}^{2}-8\bar{\Phi}\bar{\Lambda}^{4}-8\bar{\Phi}^{2}\bar{\Lambda}+10\bar{\Phi}\bar{\Lambda}^{2}+\bar{\Lambda}^{6}+8\bar{\Lambda}^{3}\bar{\Phi}-3\bar{\Lambda}^{4}+4\bar{\Phi}^{2}+4\bar{\Phi}\bar{\Lambda}+2\bar{\Lambda}^{2}-2\bar{\Phi}+8\bar{\Phi}^{3}}\right)}{2\left(-\bar{\Lambda}^{2}+2\bar{\Phi}\right)}+\frac{1}{2}\bar{\Lambda}^{2}, \tag{43}$$

where $\chi=\bar{x}+\omega\bar{t}^{\bar{\alpha}}/\bar{\alpha}$.

Case 3. If $\bar{\Lambda}^{2}-4\bar{\Phi}=0$, then we have

$$\bar{F}(\chi)=\left(3\bar{\Lambda}+\frac{-4\bar{\Phi}\bar{\Lambda}+\bar{\Lambda}^{3}-2\bar{\Phi}-\bar{\Lambda}}{-\bar{\Lambda}^{2}+2\bar{\Phi}}\right)+\left(\frac{\sqrt{12\bar{\Phi}^{2}\bar{\Lambda}^{2}-8\bar{\Phi}\bar{\Lambda}^{4}-8\bar{\Phi}^{2}\bar{\Lambda}+10\bar{\Phi}\bar{\Lambda}^{2}+\bar{\Lambda}^{6}+8\bar{\Lambda}^{3}\bar{\Phi}-3\bar{\Lambda}^{4}+4\bar{\Phi}^{2}+4\bar{\Phi}\bar{\Lambda}+2\bar{\Lambda}^{2}-2\bar{\Phi}+8\bar{\Phi}^{3}}}{-\bar{\Lambda}^{2}+2\bar{\Phi}}\right)\frac{c_{2}}{c_{1}+c_{2}\chi}-\frac{\bar{\Lambda}}{2}$$
$$+\frac{1}{2}+\bar{\Phi}+\frac{\bar{\Lambda}\left(-4\bar{\Phi}\bar{\Lambda}+\bar{\Lambda}^{3}-2\bar{\Phi}-\bar{\Lambda}\right)}{2\left(-\lambda^{2}+2\bar{\Phi}\right)}+\frac{\bar{\Lambda}\left(\sqrt{12\bar{\Phi}^{2}\bar{\Lambda}^{2}-8\bar{\Phi}\bar{\Lambda}^{4}-8\bar{\Phi}^{2}\bar{\Lambda}+10\bar{\Phi}\bar{\Lambda}^{2}+\bar{\Lambda}^{6}+8\bar{\Lambda}^{3}\bar{\Phi}-3\bar{\Lambda}^{4}+4\bar{\Phi}^{2}+4\bar{\Phi}\bar{\Lambda}+2\bar{\Lambda}^{2}-2\bar{\Phi}+8\bar{\Phi}^{3}}\right)}{2\left(-\bar{\Lambda}^{2}+2\bar{\Phi}\right)}+\frac{1}{2}\bar{\Lambda}^{2}. \tag{44}$$

As $\bar{q}(\bar{x},\bar{t})=\bar{F}(\chi)$, where $\chi=\bar{x}+\omega\bar{t}^{\bar{\alpha}}/\bar{\alpha}$.

$$\bar{q}(\bar{x},\bar{t})=\left(3\bar{\Lambda}+\frac{-4\bar{\Phi}\bar{\Lambda}+\bar{\Lambda}^{3}-2\bar{\Phi}-\bar{\Lambda}}{-\bar{\Lambda}^{2}+2\bar{\Phi}}\right)+\left(\frac{\sqrt{12\bar{\Phi}^{2}\bar{\Lambda}^{2}-8\bar{\Phi}\bar{\Lambda}^{4}-8\bar{\Phi}^{2}\bar{\Lambda}+10\bar{\Phi}\bar{\Lambda}^{2}+\bar{\Lambda}^{6}+8\lambda^{3}\bar{\Phi}-3\bar{\Lambda}^{4}+4\bar{\Phi}^{2}+4\bar{\Phi}\bar{\Lambda}+2\bar{\Lambda}^{2}-2\bar{\Phi}+8\bar{\Phi}^{3}}}{-\bar{\Lambda}^{2}+2\bar{\Phi}}\right)\frac{c_{2}}{c_{1}+c_{2}\chi}-\frac{\bar{\Lambda}}{2}$$
$$+\frac{1}{2}+\bar{\Phi}+\frac{\bar{\Lambda}\left(-4\bar{\Phi}\bar{\Lambda}+\bar{\Lambda}^{3}-2\bar{\Phi}-\bar{\Lambda}\right)}{2\left(-\bar{\Lambda}^{2}+2\bar{\Phi}\right)}+\frac{\bar{\Lambda}\left(\sqrt{12\bar{\Phi}^{2}\bar{\Lambda}^{2}-8\bar{\Phi}\bar{\Lambda}^{4}-8\bar{\Phi}^{2}\bar{\Lambda}+10\bar{\Phi}\bar{\Lambda}^{2}+\bar{\Lambda}^{6}+8\bar{\Lambda}^{3}\bar{\Phi}-3\bar{\Lambda}^{4}+4\bar{\Phi}^{2}+4\bar{\Phi}\bar{\Lambda}+2\bar{\Lambda}^{2}-2\bar{\Phi}+8\bar{\Phi}^{3}}\right)}{2\left(-\bar{\Lambda}^{2}+2\bar{\Phi}\right)}+\frac{1}{2}\bar{\Lambda}^{2}. \tag{45}$$

where $\chi=\bar{x}+\omega\bar{t}^{\bar{\alpha}}/\bar{\alpha}$.

Note: In a recent paper, the fractional Fisher equation is solved by using the (G'/G)-expansion method, which was never seen in the literature. But, the exact solutions of the Fisher equations were achieved by the generalized Kudryashov approach [28]; the obtained solutions were of hyperbolic nature.

6. Bifurcation and Phase Portrait of the Fractional Fisher Equation

The nonlinear autonomous system (NLAS) corresponding to Equation (30) is given as

$$\bar{X}'(\chi) = \bar{Y}(\chi), \bar{Y}'(\chi) = -\bar{X} + \omega\bar{Y} + \bar{X}^2 \quad (46)$$

This NLAS has critical points $(0, 0)$ and $(1, 0)$. The nature corresponding to each critical point is different.

Case 1. Corresponds to the critical point $(0, 0)$, the given autonomous system takes the form

$$\bar{x}'(\chi) = \bar{y}(\chi), \bar{y}'(\chi) = -\bar{x} + \omega\bar{y} + \bar{x}^2. \quad (47)$$

Corresponding to this nonlinear autonomous system, the characteristic equation is given by

$$\bar{\Lambda}^2 - \omega\bar{\Lambda} + 1 = 0. \quad (48)$$

Therefore, $\bar{\Lambda} = \omega \pm \sqrt{\omega^2 - 4}/2$

At $\omega = 2$, the characteristic of Equation (48) has real and equal roots. Therefore, the nature of the critical point is node and unstable (represented by Figure 1).

At $0 < \omega < 2$, Equation (48) has conjugate complex roots. Therefore, the nature of the critical point is spiral and unstable (represented by Figure 2).

At $\omega > 2$, the characteristic of Equation (48) has values real, unequal, and of the same sign. Therefore, the critical point is node and unstable (represented by Figure 1).

Case 2. Corresponds to the critical point $(1, 0)$, the given autonomous system takes the form

$$\bar{X}'(\chi) = \bar{Y}(\chi), \bar{Y}'(\chi) = \bar{X} + \omega\bar{Y} + \bar{X}^2. \quad (49)$$

Corresponding to this nonlinear autonomous system, the characteristic equation is given by

$$\bar{\Lambda}^2 - \omega\bar{\Lambda} - 1 = 0. \quad (50)$$

Therefore, $\bar{\Lambda} = \omega \pm \sqrt{\omega^2 + 4}/2$

At $\omega = 0$, the characteristic of Equation (50) has real and equal roots. Therefore, the nature of the critical point is node and unstable.

7. Results and Discussion

(1) The Lie symmetry analysis method to find the invariance properties using the conformable fractional derivative has been applied successfully to the nonlinear time fractional Fisher equation

(2) A reduced form of the Fisher equation has been utilized successfully to find the wave using the (G'/G)-expansion method are obtained

(3) The nature of the solutions is in hyperbolic and trigonometric forms and satisfies the equation

(4) The behaviour of reduced ODE at different critical points using qualitative analysis is shown with the help of phase portraits

8. Conclusions

In this paper, trivial symmetries of the time fractional Fisher equation with conformable derivatives are obtained using the Lie symmetry analysis, and three types of wave solutions are obtained using the (G'/G)-expansion method. Then, qualitative behaviour is studied using bifurcation analysis, and corresponding to nature phase portraits are drawn.

The Lie symmetry gives trivial symmetries, and exact solutions are obtained successfully by the use of the (G'/G)-expansion method. Obtained solutions are of hyperbolic and trigonometric forms, which are new in solution.

Data Availability

No any kind of data was collected or produced in this manuscript.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

Conflicts of Interest

The authors declare no competing interests.

Authors' Contributions

Rajeev Kumar, Deeksha, Rishu Arora, and Kamal Kumar contributed equally to this work.

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