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Research Article

Mathematical Optimization on Hybrid Channel Pricing Digital Products in Two-Sided Market with Network Effect

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In the two-sided market where the third-party platforms connect the providers and consumers, the online platforms become the significant distribution channel of digital products; therefore, the digital product firms face the hybrid channel pricing problem in the two-sided market in which the products are launched through the platform channel and the existing direct channel. Because the network externality effect is the significant economic characteristic of digital products and services, the current work presents the models of consumers' utility obtained by adopting digital products from the direct and platform channels, and the utility models use the network effect in the direct and platform channels as the parameters. The optimization model on pricing is derived from the utility models and solved mathematically. The closed-form solutions show that the price in the direct channel is supposed to be lower than that in the platform channel, while the prices of digital products would be affected by the network effect only when the products are distributed through the direct channel. The comparative statics analysis on the network effect illustrates that the network effect in the direct channel and the platform channel would, respectively, have the positive and negative impact on the products' prices and the firms' profit. The current work explores the hybrid channel pricing problem and provides insights for the digital product firms on the optimal pricing decision in the context of the emerging platform economy.

1. Introduction

Digital product suppliers often distribute their output through the hybrid channel in the two-sided market where a third-party platform connects two or more user groups to facilitate transactions or relationships between them. The emerging third-party online platforms have become a significant channel for the digital product vendors in the two-sided market; in addition, the vendors often maintain their original channels. For instance, Microsoft participates in the App Store (the App Store provided by Apple Inc. is a typical third-party platform connecting two groups that are mobile applications' publishers and the end users of Mac devices) to sell the MS OfficeSuite (Mac Edition) to the users of Apple's Mac devices; meanwhile, Microsoft also provides OfficeSuite (Mac Edition) through the official website. It follows that the digital product firms may have a hybrid channel structure to distribute their products to the users in the context of two-sided market. However, these multiple types of channels would compete with each other, and the products sold in one channel might cannibalize the demand of the same products in another channels; therefore, the digital product firms need to regulate the competition between the channels. In reality, pricing strategy is the significant way to determine a trade-off between the channels, and it would be a vital challenge for the digital product firms to address the hybrid channel pricing issue in the emerging and complex two-sided market. Hence, it is necessary to investigate the optimal pricing decisions for the digital products firms when they are facing the hybrid channel in a two-sided market.

The decision on the prices of digital products is supposed be basing on the value of the products because of the special cost structure of digital products. When the suppliers develop digital products, the fixed cost is huge, but the marginal cost is extremely low, even negligible. For example, the providers of mobile applications for iPhone or iPad often invest plenty of resources to develop the first copy. However, after developing the first copy, the marginal production process is just that the providers publish the applications into App Store and allow the consumers to download. Thus, the marginal cost paid by the digital product firms is rather low. In this case, the marginal cost-based pricing strategy that is often used by the physical product firm is inappropriate to the digital products, and another pricing strategy, i.e., the value-based pricing, makes more sense [1].

In reality, the value of digital products is greatly affected by the network effect, which means that the products' value would increase with the growing user base [2]. Taking YouTube as the example, if there are more consumers log in and use YouTube, each user would enjoy more videos and then obtain more utility, and YouTube would definitely become more valuable to consumers. Due to the prominent impact of the network effect on the value of digital products, the astute managers of digital product firms are supposed to take the network effect into account when they decide the optimal prices of the products.

Therefore, the current work intends to explore the optimal prices of digital products distributed through the hybrid channel in the context of two-sided market considering network effect. The current work develops the consumers' utility models in the situations of the consumers purchasing digital products from the direct and platform channels, and the consumers' utility is positively affected by the network effect in the two channels. Basing on the utility models, it is derived that the pricing optimization model in the context of two-sided market where the digital products are distributed through the direct channel and platform channel simultaneously. The optimization model is solved to investigate the closed-form equilibrium solutions, in which the optimal price of digital products in the direct channel is supposed to be lower than that in the platform channel, and the network externality effect would impact the prices of digital products iff (if and only if) they are launched in the direct channel. In addition, the comparative statics analysis on the solutions shows that the network effect in the direct channel (or the platform channel) would positively (or negatively) affect the prices of digital products and the benefits of the providers.

The contributions of the current work are threefold. First, the current work expands the knowledge on dual channel management strategy of digital product suppliers. Previous studies commonly analyze the dual channel management issues under a one-sided market, in which characteristics of the sellers interact with the buyers without intermediary organizations. However, the economic mechanism of a two-sided market where the third-party platforms play a significant role in the interactions between the sellers and buyers is quite different with that of a one-sided market, and this may result in that the theory on the dual channel management of digital products in the one-sided market would not be reasonable in the two-sided market. Therefore, investigating the dilemma of the hybrid channel pricing problem faced by the digital product firms in the context of the emerging and dramatic two-sided market is definitely necessary.

Second, the current work also determines the complex affect (especially the drawbacks) of the network effect, which is scarce in previous studies. The prior researches commonly focus on the one-sided market and argue that the network effect positively impacts the prices of digital products and the profit of their providers. However, the current work deems that the network effect may have more complex impact when the research context tends to the two-sided market; explicitly, it may positively or negatively impact the products' prices and the firms' profit in the different channel. The findings of this study could impel the digital product vendors to make pricing decision prudently when they are facing the complicated two-sided market.

Third, this study explores the mathematical mechanism of the dual channel pricing on digital products in the context of two-sided market. Previous studies discussing the dual channel management problem under one-sided market often adopt the mathematical optimization methods and attempt to obtain the closed-form solutions of the decision variables (e.g., the products' prices) and the target functions (e.g., the firms' profits). The closed-form solutions allow the researchers to minutely and deeply examine the effect of each parameter. Inspired by the previous studies, the current work also chooses the mathematical optimization method to investigate the closed-form equilibrium of hybrid channel pricing problem. Those closed-form solutions obtained from mathematical optimization methods are helpful to explicitly understand the pricing principles and the effect of network externality in the two-sided market.

The remainders of the current work are organized as follows: Section 2 reviews the previous studies on the pricing and channel management of digital products; Section 3 models the consumers' utility and the provider's profit in the monopoly setting, and the models take the network effect into account; then, the current work develops the optimization model on the pricing decision of the monopoly digital product firm; Section 4 solves the optimization models mathematically and conducts the comparative statics analysis on the network effect to explore its impact on the optimality; then, the current work provides the managerial insights of the findings; and Section 5 concludes the findings of current study and discusses the possible directions of the future work.

2. Literature

This study focuses on the dual channel pricing problem faced by the digital product firms, and researchers have been addressing this problem by exploring the channel management of digital products. This study reviews two streams of the existing literature: (1) pricing in the online and offline channels and (2) pricing in the direct and retail channels.

There exists a stream of literature that investigates the pricing strategy of digital product vendors in the online and offline channel distribution channels and explores the trade-off between these two channels. The past decades have witnessed the rapid development of the Internet, and during this decade-long process, the development of the Internet made the consumers familiar with the websites, smart devices, and other technologies related to the Internet [3], and in this situation, there are plenty of digital product suppliers launch the products online. For example, the music copy providers [4], choose to distribute their copies digitally through the Internet. Therefore, the digital products sold through online channel would compete with their offline counterparts. Considering the economic characteristics of the online and offline channels, many factors, such as the network effect [5], the customization cost of the products [6], the security externality [7], and the market capacity [8] would impact the pricing decision of the digital product providers. In recent years, the mobile Internet subscriptions have grown much more quickly than fixed line broadband subscriptions [9], which makes the mobile Internet a significant online channel to distribute digital products. Comparing with the fixed line channel, consumers in the mobile Internet channel tend to choose "head" products, which are the most popular products or the most sold products [10]. This behavior of the consumers has affected the vendors' pricing strategy in the distribution channel; for instance, the digital product firms might provide the versions with the higher prices in the mobile Internet channel to obtain more profit.

Both the online and offline channels discussed in the aforementioned papers are the direct channels enabling suppliers to distribute the products to their end users directly. However, the digital products, such as the medical information systems discussed by Modak et al. [11] and Moheimani et al. [12, 13], can also be distributed through the retail channel in which the retailers sell their products to end users [14]. Such suppliers who distribute products through the direct channel become the competitor of their reseller partners [15]. Therefore, a stream of previous studies explores the firms' pricing decision in the direct channel and the retail channel. The researchers argue that the network effect [16], the risk attitude of retailers [17], the channels' operation cost, the consumers' preference, the governments' policies such as subsidy [18–20], and the competitors' channel strategies [21] would deeply impact the digital product firms' pricing strategy when they are exerting the effort to determine the trade-off between the direct channel and retail channel.

In reality, the digital product suppliers often find that the retailers are inclined to create an artificially low price, which is harmful for the suppliers' benefits. Hence, the suppliers would adopt a strategy that the suppliers set the retail price for the products and the retailers receive a proportion of the revenue. Zhu and Yao [22] compared this strategy with the traditional wholesale model of the e-book, a typical digital product, and demonstrate that the strategy that the providers set price for the retailors is often suboptimal to the traditional wholesale model.

To summarize the previous studies above, all previous studies explored the dual channel pricing issue of digital product firms in the context of one-sided market. The researchers have investigated the dilemma faced by the digital product suppliers when they are making pricing decision to find trade-off between the direct channel and retail channel, as well as between the online channel and offline channel. All distribution channels investigated by previous studies are those that connect providers (or retailers) to consumers directly, and there are no third-party organizations in the transaction process, which is the typical business model in the one-sided market. In recent years, with the development of the two-sided market, online platforms have been playing an increasingly important role in the distribution of digital products. For example, airlines often distribute electronic tickets through online travel agency platforms [23]. Therefore, the astute managers would trade-off between selling products through the emerging platform channel and other traditional channels [24]. However, little is known about the optimality of pricing digital products in the two-sided market, in which the platform has become the significant distribution channel, and this knowledge gap might restrict the development of digital product firms in the two-sided market. Thus, the current work extends the story by exploring the pricing decision of digital product vendors facing the new emerging platform channels and their existing channels and examines the optimality of the vendors' hybrid channel pricing and their benefits in the context of the two-sided market.

3. Model Settings

The current work assumes that a firm with a market monopoly position develops and launches digital products in the two-sided market where the third-party platform becomes the distribution channel after the firm participates in it, and the consumers could purchase the products from it. In reality, the consumers are also able to purchase digital products, such as Adobe Acrobat and Oracle Database, from the providers' direct channels (e.g., their official website); hence, the current work assumes that the monopoly digital product firm still maintains its direct channel. Therefore, the monopolist distributes the digital products through the direct and platform channels. The monopolist needs to decide the price of products in both channels to maximize its profit. This section presents the utility model of the consumers and the optimization model of the monopolist in the situations that the digital products are launched in the hybrid channel. For the convenience of readers to understand the model settings below, the notations and their definitions of the symbols in current work are shown in Table 1.

3.1. Consumers' Utility. The consumer type is denoted as v_i, which is uniformly distributed between 0 and 1 (i.e., $v_i \sim U$ [0,1], and the subscript i represents that the consumers are heterogeneous). While the digital products whose features are denoted as s are simultaneously released through the direct channel and the platform channel, the consumers need to decide from which channel to purchase the digital products: the direct channel or the online platform whose feature is denoted as s^T.

If a consumer purchases the digital products from the direct channel, the consumers would obtain utility U_1 defined in Equation (1) in which p_1 and Q_1 are, respectively, the price and install base of digital products in the direct channel; the parameter λ is the intensity of network externality effect in the direct channel, and the parameter

Table 1: Notations and definitions in current work.

Notation	Definition
v_i	The heterogeneous consumers
v_1/v_2	The consumer who is indifferent between buying and not buying product from direct/platform channel
v_T	The consumer who is indifferent between adopting and not adopting platform service
s/s^T	Features of the digital products/platform service
p_1/p_2	Price in the direct/platform channel (decision variable)
π_1/π_2	Profit in the direct/platform channel (objective function)
λ/λ^T	Intensity of network externality in the direct/platform channel
Q_1/Q_2	Installed base of the digital products in direct/platform channel
Q^T	Installed base of the platform service
D_1/D_2	Demand of the digital products in direct/platform channel
D^T	Demand of the platform service
c/c^T	Learning cost of the consumers in adopting digital products/platform service

$c < s$ is the learning cost of the consumers, that is, the effort devoted by the consumers to be skilled in the digital products.

$$U_1 = s \cdot v_i + \lambda \cdot Q_1 - p_1 - c. \tag{1}$$

Let us turn to the platform channel. If a consumer purchases the digital products from the platform, he/she is supposed to adopt the platform service first. In reality, the platform service is often free to the consumers, for example, Amazon and eBay. Therefore, the consumers' utility obtained from the platform service is denoted as U^T which is shown in Equation (2), where $c^T < s^T$ represents the learning cost that the consumers paid for understanding and using the platform service; Q^T represents the installed base of the platform service, and λ^T represents the intensity of the network externality in the platform channel. After adopting the platform service, the consumer could purchase the digital products from the platform channel and then enjoy the utility U_2 from the products. U_2 is shown in Equation (3), where p_2 and Q_2, respectively, represent the price and installed base of the digital products distributed in the platform channel.

$$U^T = s^T \cdot v_i + \lambda^T \cdot Q^T - c^T, \tag{2}$$

$$U_2 = s \cdot v_i + \lambda^T \cdot Q_2 - p_2 - c. \tag{3}$$

In reality, the value of digital products and the online platform services are mainly derived from their complex and useful functionalities, which means that the functionalities of digital products commonly create more value than the network effect. Therefore, the current work assumes that $s \geq \lambda, \lambda^T$ and $s^T \geq \lambda, \lambda^T$. Moreover, consumers often need to pay more learning costs to conquer the online platform service. For example, the users of the App Store need to learn how to search and pay for the applications and how to distinguish the best products from the applications with similar functionalities. Therefore, the current work assumes that the quality–cost ratio of the digital products dominates that of the online platform services; that is, $((s-\lambda)/(c-\lambda)) > ((s^T-\lambda^T)/(c^T-\lambda^T))$.

3.2. Firm's Profit. There would be an indifferent consumer (denoted as v_T) whose utility from adopting the online platform is zero, while the consumers $v_i \geq v_T$ would adopt the platform service. Within the consumers $v_i \geq v_T$ who have adopted the platform, it would exist an indifferent consumer (denoted as v_2) whose utility from adopting the digital products is zero, and the consumers $v_i \geq v_2$ would purchase the products through the platform channel. Within the other consumers $v_i < v_T$ who do not participate in the platform, an indifferent consumer (denoted as v_1) whose utility from adopting the digital products is zero would emerge, and the consumers $v_1 \leq v_i < v_T$ would purchase the products from the direct channel. The market segmentation for the hybrid channel strategy is shown in Figure 1.

Therefore, the demand of the online platform $D^T = 1 - v_T = 1 - ((c^T - \lambda^T \cdot Q^T)/s^T)$, the demand of the digital products in the platform channel $D_2 = 1 - v_2 = 1 - ((p_2 + c - \lambda^T \cdot Q_2)/s)$, and the demand of the digital products in the direct channel $D_1 = v_T - v_1 = ((c^T - \lambda^T \cdot Q^T)/s^T) - ((p_1 + c - \lambda \cdot Q_1)/s)$. Inspired by Cheng et al. [25], the demand of products is just the products' installed base; thus, $D^T = Q^T$, $D_1 = Q_1$, and $D_2 = Q_2$, and it could be obtained that $D^T = (s^T - c)/(s^T - \lambda^T)$, $D_1 = ((c^T - \lambda^T)/(s^T - \lambda^T)) - (((p_1 + c) - \lambda)/(s - \lambda))$, and $D_2 = (s - (p_2 + c))/(s - \lambda^T)$. Therefore, the optimization model for the hybrid strategy is shown in Equations (4) and (5) in which the prices p_1 and p_2 are the decision variables and the profit π is the objective function.

$$\max_{p_1, p_2} \quad \pi = p_1 \cdot \left(\frac{c^T - \lambda^T}{s^T - \lambda^T} - \frac{p_1 + c - \lambda}{s - \lambda} \right) + p_2 \cdot \frac{s - (p_2 + c)}{s - \lambda^T}, \tag{4}$$

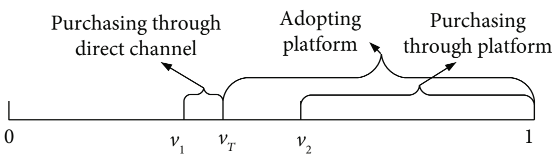

Figure 1: Market segmentation for the hybrid strategy.

$$\text{s.t.} \quad p_1, p_2 \geq 0. \tag{5}$$

4. Results and Analysis

This section solves the optimization model (4), explores the optimal pricing of digital products in the direct and platform channels, and investigates how the intensity of network effect affects the optimal price and profit when the digital products are distributed in the hybrid channel. In this section, the closed-form solutions of the optimality are formulated and illustrated by the numerical examples. In reality, the parameters s and s^T, respectively, represent the number of functionalities that the digital products and the platform services have; c and c^T, respectively, represent the effort conducted by the consumers to use the products and platform service; λ and λ^T, respectively, represent the value created by the individual user in the direct channel and platform channel. What it follows is that the units of those parameters are various. Therefore, in order to avoid the problem causing by the unit of parameters, these parameters are normalized into [0,1] in the following numerical examples, and their values are set according to the definitions and assumptions in Section 3.

4.1. Solutions of the Optimization Model. The optimization model (4) is solved through the Lagrangian method. The Lagrangian function of the optimization model and the corresponding Kuhn-Tucker conditions are as follows:

$$L(p_1, p_2) = p_1 \cdot \left(\frac{c^T - \lambda^T}{s^T - \lambda^T} - \frac{p_1 + c - \lambda}{s - \lambda} \right) + p_2 \cdot \frac{s - (p_2 + c)}{s - \lambda^T} + \varepsilon p_1 + \varphi p_2, \tag{6}$$

$$\begin{cases} \dfrac{\partial L}{\partial p_1} = \dfrac{c^T - \lambda^T}{s^T - \lambda^T} - \dfrac{p_1 + c - \lambda}{s - \lambda} - \dfrac{p_1}{s - \lambda} + \varepsilon = 0, \\ \dfrac{\partial L}{\partial p_2} = \dfrac{s - (p_2 + c)}{s - \lambda^T} - \dfrac{p_2}{s - \lambda^T} + \varphi = 0, \\ \dfrac{\partial L}{\partial \varepsilon} = p_1 \geq 0, \\ \dfrac{\partial L}{\partial \varphi} = p_2 \geq 0, \\ \varepsilon p_1 = 0, \\ \varphi p_2 = 0. \end{cases} \tag{7}$$

The Hessian matrix of $L(p_1, p_2)$ is denoted as H, and it could be obtained from Equation (6) that

$$H = \begin{vmatrix} -\dfrac{2}{s - \lambda} & 0 \\ 0 & -\dfrac{2}{s - \lambda^T} \end{vmatrix}. \tag{8}$$

Its 1×1 leading minor $H_1 - (2/(s - \lambda)) < 0$, and 2×2 leading minor $H_2 = (4/(s - \lambda)(s - \lambda^T)) > 0$; hence, it could be known that H is negative definite, and $L(p_1, p_2)$ is concave function; then, the optimization model has inner-point solution. The optimality of the price of digital products and the profit of the monopolist are able to be obtained from Equations (6) and (7), while the optimality is shown as follows:

$$p_1^* = \frac{1}{2\left(s^T - \lambda^T\right)} \left[\begin{array}{c} (s - \lambda)\left(c^T - \lambda^T\right) - \\ \left(s^T - \lambda^T\right)(c - \lambda) \end{array} \right], \tag{9}$$

$$p_2^* = \frac{s - c}{2}, \tag{10}$$

$$\pi^* = \frac{1}{4(s - \lambda)\left(s^T - \lambda^T\right)^2} \left[\begin{array}{c} (s - \lambda)\left(c^T - \lambda^T\right) - \\ \left(s^T - \lambda^T\right)(c - \lambda) \end{array} \right]^2. \tag{11}$$

From Equation (9), it could be obtained that $p_1^* = 1/2 [(((s - \lambda)(c^T - \lambda^T)) / (s^T - \lambda^T)) - (c - \lambda)]$; hence, $p_1^* < 1/2 [(((s - \lambda)(c^T - \lambda^T))/(c^T - \lambda^T)) - (c - \lambda)] = (s - c)/2 = p_2^*$; then, the following proposition is obtained.

Proposition 1. *(a) The network externality affects the price of digital products iff the products are distributed through the direct channel. (b) The price in the direct channel is supposed to be lower than that in the platform channel.*

The optimality shows that the λ and λ^T are the parameter of the optimal price only when the digital products are distributed through the direct channel; it means that the network externality affects the pricing decision only in the direct channel. This is definitely rational. When the firm determines the price of digital products, those products are not yet released to the market, and the firm did not know the possible installed base. Therefore, it is suggested that the provider decides the price in the platform channel depending on the products' features rather than the products' network effect. However, the provider is familiar with the characteristics of its own direct channel, including the network effect that might generate in the direct channel; hence, the pricing decision in the direct channel is supposed to take into account the network effect.

In addition, a lower price in the direct channel is beneficial for the provider to penetrate in the market and enlarge the demand and grab further profit from the direct channel. Therefore, the provider is inclined to determine a lower price in the direct channel, and in reality, the digital product firms often strategically determine the same price in the both channel but provide more services in the direct channel to make the products in this channel to be more valuable.

4.2. Impacts of λ and λ^T on the Optimality. When the optimality is obtained, the comparative statics analysis could be obtained to explore the impact of parameters λ and λ^T. The comparative statics analysis is shown as follows:

$$
\begin{aligned}
\frac{\partial p_1^*}{\partial \lambda} &= \frac{s^T - c^T}{2\left(s^T - \lambda^T\right)} > 0, \\
\frac{\partial p_1^*}{\partial \lambda^T} &= \frac{(s-\lambda)\left(c^T - s^T\right)}{2\left(s^T - \lambda^T\right)^2} < 0, \\
\frac{\partial \pi^*}{\partial \lambda} &= \frac{1}{4\left(s^T - \lambda^T\right)^2 (s-\lambda)^2} \cdot \begin{bmatrix} \left(s^T - \lambda^T\right)(s-c) + \\ (s-\lambda)\left(s^T - c^T\right) \end{bmatrix} \\
&\quad \cdot \begin{bmatrix} (s-\lambda)\left(c^T - \lambda^T\right) + \\ \left(s^T - \lambda^T\right)(c-\lambda) \end{bmatrix} > 0, \\
\frac{\partial \pi^*}{\partial \lambda^T} &= \frac{\left(c^T - s^T\right)}{2\left(s^T - \lambda^T\right)^3} \cdot \begin{bmatrix} (s-\lambda)\left(c^T - \lambda^T\right) - \\ \left(s^T - \lambda^T\right)(c-\lambda) \end{bmatrix} < 0.
\end{aligned}
\tag{12}
$$

According to the comparative statics analysis above, the following proposition is obtained.

Proposition 2. *The network effect in the direct channel positively affects the prices of products and the profit of the firm, while the network effect in the platform channel has negative impact on the optimality of the firm's pricing decision and profit.*

Equation (1) defined in Section 3.1 shows that the parameter λ positively impact the consumers' utility, and then, the consumers would have higher willingness to pay with the increasing λ. Thus, the optimal price of the digital products p_1^* increases with λ. A higher price is beneficial for the provider to squeeze profit; meanwhile, the increasing utility with λ would lead to more consumers obtaining nonnegative utility; then, the demand would increase with λ. The increasing price and demand would make the firm's profit π^* increase with λ. The impact of the parameter λ on the optimalities p_1^* and π^* is illustrated in Figure 2. Therefore, it is definitely suggested that the digital product providers in reality increase the price if they have more market coverage; for instance, the price of Microsoft Office is higher than that of other word processing software.

However, when λ^T increases, consumers would obtain more utility from adopting the online platform service (see Equation (2)), and this results that the platform service

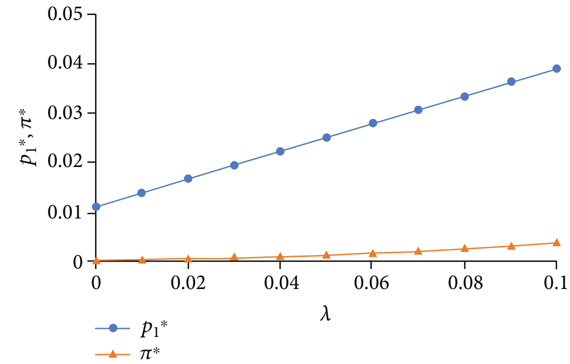

Figure 2: Impact of λ on p_1^* and π^* ($\lambda^T = 0.05$, $s = 0.5$, $s^T = 1$, $c = 0.2$, and $c^T = 0.5$).

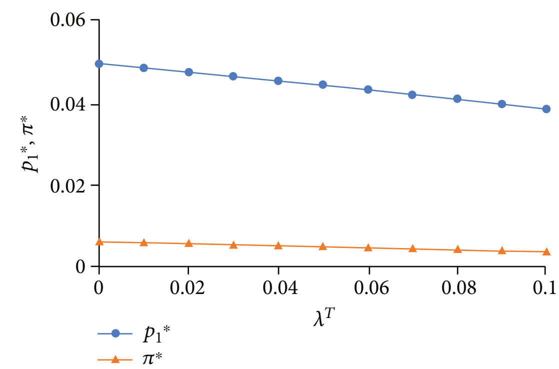

Figure 3: Impact of λ^T on p_1^* and π^* ($\lambda = 0.05$, $s = 0.5$, $s^T = 1$, $c = 0.2$, and $c^T = 0.5$).

becomes more attractive to the consumers than the digital products; therefore, an increasing λ^T may decrease the consumers' willingness to pay for the digital products. This leads to the firm decreasing the products' price p_1^* to appeal more consumers and enlarge the market demand. In this case, the optimal price of the products p_1^* decreases with the parameter λ^T. While a lower price is harmful to the monopolist's profit, therefore, the optimal profit π^* decreases with λ^T. The impact of the parameter λ^T is shown in Figure 3. Therefore, it would be suggested for the digital product firms that they are supposed to adopt pricing or other strategy to decrease the network effect in the platform channel. For example, Microsoft provides more technology details in its official website and then makes the users purchasing Office from its official website could communicate more sufficiently about technologies than that purchasing Office from the platform channel such as eBay, and this might decrease the intensity of network effect in platform channel.

For the parameters λ and λ^T that affect the products' price and the provider's profit, it has been shown in Figures 2 and 3 that λ would have the positive affect on p_1^*/π^* and λ^T would have negative affect on them; therefore, it would be much more beneficial for the firms when their digital products in the direct and platform channels, respectively, have more and less network externality, and then, the optimal situation for the monopolist is the

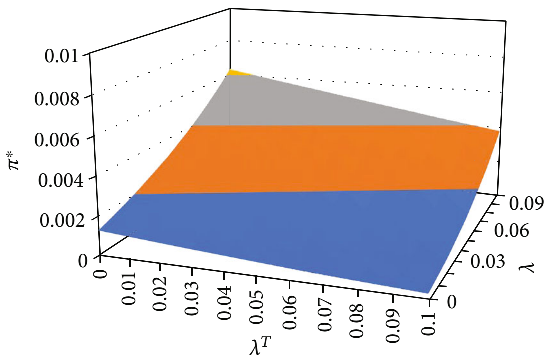

FIGURE 4: Impact of λ and λ^T on π^* ($s = 0.5$, $s^T = 1$, $c = 0.2$, and $c^T = 0.5$).

market where the consumers could obtain lower network externality value from the platform channel, but higher network externality value from the direct channel. This finding has been shown in Figure 4. However, the scale of the platform services' installed base often surpasses the digital products' (e.g., the user of App Store is more than that of an application published in App Store), and this makes the network effect in the platform channel dominates that in the direct channel. Hence, the digital product firms in reality tend to attract more consumers to adopt the products from the direct channel.

4.3. Managerial Insights. The findings of the current work revel that, if the firms provide digital products only through the direct channel, the network effect would benefit the firms by improving the consumers' utility; however, if the firms distribute the digital products through the direct and platform channels, the network effect in the platform channel may decrease the consumers' willingness to pay for the products, thus making the demand in the platform channel cannibalize that in the direct channel. These findings provide the significant implications for the digital product firms.

First, the digital product firms need to take the network effect into account seriously, especially in their direct channel. In actual fact, the digital product firms in reality commonly attach importance to the network effect in their direct channel. For instance, Axure Software Solutions Inc. encourages the users to make templates through Axure RP and then upload these templates to its official website. This would help the consumers to enjoy more network externality in the company's official website, which is just its direct channel.

Second, the digital product firms are supposed to increase the attractiveness of the direct channels (comparing with the platform channel) through pricing decisions. In reality, the digital product firms often adopt some pricing strategies to attract the consumers to purchase from the direct channels. For example, Uber provides more coupons in the applications published by itself than in the third-party platforms such as PayPal, and this strategy actually represents a decreasing price in the direct channel.

Third, just because of the positive and negative impact from the network effect, respectively, generated in the direct and platform channels, the providers could attempt to enlarge the network externality effect in the direct channel. For example, some digital product suppliers, such as Microsoft and Adobe, provide more technology support in the direct channel (e.g., the official website of themselves) than in the platform channel (e.g., App Store of Apple Inc.). Through this way, the digital products in direct channel would have more attractiveness, and the providers would increase the demand (i.e., the installed base) in direct channel, and then, the consumers would enjoy more network externality effect from the direct channel.

5. Conclusion

In the two-sided market, the online platforms have brought about opportunities for digital product firms to distribute the products. The platform channel would also compete with the firms' existing direct channel; thus, the digital product suppliers are facing the vital challenge that how to trade-off the pricing in the two channels in order to maximize their profits.

Therefore, the current work investigates the pricing decision problem faced by the digital product suppliers in the two-sided market where the suppliers could provide the products simultaneously through the direct and platform channels. In the current work, the consumers' utility models in the direct and platform channels are presented, while the utility models take into account the network effect as the important factor that affects the consumers' utility; then, the optimization model is developed deriving from the utility functions. This study provides the formulations of the optimal pricing in both channels, demonstrating that the price in the direct channel is supposed to be lower than that in the platform channel. The comparative statics analysis shows that the network effect in the direct channel positively impacts the prices of products and the profits of providers; however, the network effect in the platform channel negatively affects them. The findings of current work imply that the digital product vendors are supposed to be more prudent on the pricing decision when they are facing the hybrid channel, especially, the vendors need to pay more attention to the possible negative effect of network externality in the two-sided market. Those findings are the supplements of the extant literature such as Cheng et al. [25, 26], which investigated the pricing problem of digital products focusing only on the positive network effect in the situation of a one-sided market.

There are several possible directions for future research that follow this paper. First, the future work could explore the impact of some other interesting pricing schemas, such as freemium and pay-per-use, on the hybrid channel management of digital product suppliers. Second, the future work could investigate the other factors that may affect the hybrid channel pricing problem, for example, consumers' switching cost and market competition. Third, the future work could take into account that the competition exists between the direct channel and platform channel and attempt to find the roles of the competition on the hybrid channel pricing. This would definitely be an exciting challenge.

Data Availability

All the data used in the numerical analysis are listed in detail and available in the manuscript.

Conflicts of Interest

The authors have no of conflicts of interest.

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