Hindawi
Journal of Applied Mathematics
Volume 2023, Article ID 8103614, 12 pages
https://doi.org/10.1155/2023/8103614

Research Article

Horizontal Vibration of Pile in Transversely Isotropic Saturated Soil Based on Saturated Porous Medium Theory

Linchao Liu and Qifang Yan

College of Architecture and Civil Engineering, Xinyang Normal University, Xinyang, Henan 464000, China

Correspondence should be addressed to Linchao Liu; llc109@126.com

Received 31 August 2023; Revised 10 October 2023; Accepted 2 November 2023; Published 15 November 2023

Academic Editor: Fernando Simoes

In order to consider the influence of liquid phase and soil anisotropy, the soil around the pile is considered a transversely isotropic saturated porous medium, and a horizontal dynamic model of transversely isotropic saturated soil-pile is established. Based on Beor's saturated porous medium theory and the constitutive equation of transversely isotropic media, the horizontal dynamic control equation of transversely isotropic saturated soil is obtained without considering the vertical displacement. The horizontal vibration of transversely isotropic soil layers was solved using the potential function and separation of the variable method, and the horizontal and radial displacements of the solid soil skeleton of transversely isotropic saturated soil were obtained. The horizontal force of transversely isotropic saturated soil on a single pile was also obtained. On this basis, the horizontal dynamic equation of a single pile in transversely isotropic saturated soil was established, and the horizontal vibration of the pile foundation was solved using the initial parameter method, and the horizontal dynamic impedance of a single pile in transversely isotropic saturated soil is obtained. The influence of soil anisotropy parameter, liquid-solid coupling coefficient, diameter-length ratio, and modulus ratio on the horizontal dynamic impedance of a single pile in transversely isotropic saturated soil was analyzed through numerical examples. The analysis results show that the anisotropy parameters δ_1, diameter-length ratio r_0/H, and modulus ratio E_p/C_{66} have a significant impact on the horizontal dynamic impedance of pile foundation in transversely isotropic saturated soil, and the influence of anisotropy on the horizontal vibration of pile foundation should not be ignored. The influence of the liquid-solid coupling coefficient on the horizontal dynamic impedance factor is related to frequency to a certain extent.

1. Introduction

Since the study of pile foundation vibration is critical to the safety and stability of pile foundation and whole building structure, the study of pile foundation dynamic characteristics has not stopped since the 1960s. Due to the significant influence of the properties of the soil around the pile on the dynamic interaction between the pile and soil, studying the dynamic characteristics of pile foundations under various complex geological conditions has significant engineering application value. Novak et al. [1, 2], Nogami and Novak [3], Koo et al. [4], Ding et al. [5], Wu et al. [6], Luan et al. [7], Zhao et al. [8], Meng et al. [9] and others have conducted systematic theoretical research on the vibration of pile foundations in single-phase homogeneous elastic and viscoelastic soil. For coastal areas and river basins, the soil is usually saturated soil, and the influence of liquid phase needs to be considered. Therefore, Maeso et al. [10], Zhou and Wang [11], Zheng et al. [12], Cui et al. [13], and Wang and Ai [14] studied the dynamic problem of pile foundations in saturated soil. It should be noted that the current research on the dynamic interaction between saturated soil and pile is mostly based on Biot's saturated porous medium theory. Although Biot saturated porous medium theory has been successfully applied to many engineering fields, the research shows that its theoretical model has some defects [15]. Beor's saturated porous medium theory uses the concepts of continuum mixture axiom and volume

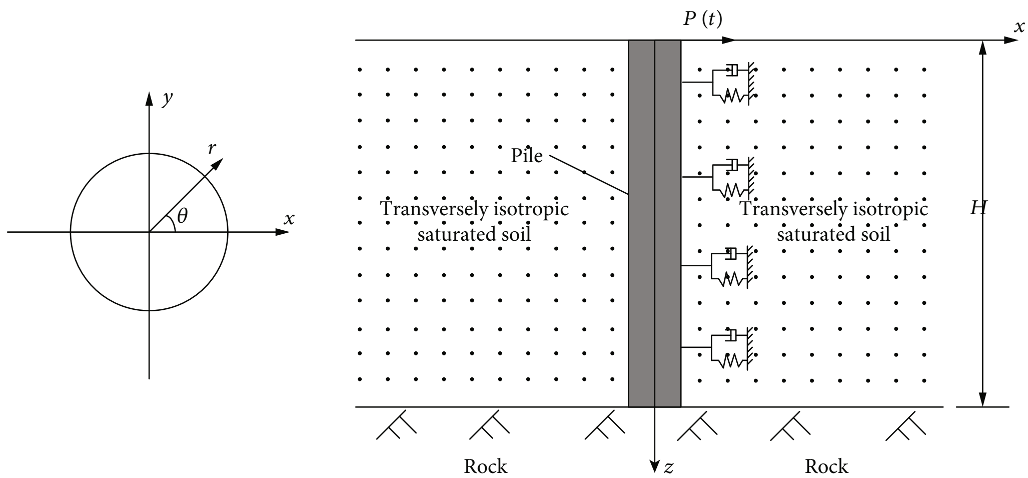


Figure 1: Horizontal dynamic model of transversely isotropic saturated soil-pile.

fraction, and several microscopic properties of porous medium can be directly described by macroscopic properties, avoiding the complicated formula of hybrid mixture theory. Without additional assumptions, some effects such as dynamic, material, and geometric nonlinearity can be easily reflected in its mathematical model. Therefore, Beor's saturated porous medium theory is used to describe the mechanical properties of the soil around the pile, and the pile-saturated soil dynamic interaction model is more reasonable and accurate. In addition, in the process of soil deposition, the orientation and directionality of the arrangement of flat medium particles lead to the difference in the properties (elastic modulus, shear modulus, and Poisson's ratio) of the soil in the vertical and horizontal directions, and the soil shows various characteristics. Usually, the vertical modulus of soil is smaller than the horizontal modulus, resulting in the horizontal modulus exhibiting isotropic characteristics. At this point, treating the soil as a transversely isotropic medium is more in line with engineering practice. Due to the complexity of the mathematical solution, it is difficult to obtain analytical expression for the dynamic interaction between transversely isotropic soil and pile. Therefore, the current research on the vibration of pile foundations in transversely isotropic soil mainly focuses on torsional vibration. For example, Zheng et al. [16] studied the torsional vibration of pipe pile in transversely isotropic saturated soil based on Biot's porous medium theory, and the influence of anisotropy of internal and external soil on the torsional dynamic response of pipe pile was discussed. Chen et al. [17] studied the dynamic response of pile in transversely isotropic saturated soil under transient torsional load by means of the Laplace transform. Ma et al. [18] studied the torsional vibration of pile foundations in transversely isotropic saturated soil considering construction disturbances. In terms of vertical and horizontal vibrations of pile foundations in transversely isotropic soil, Li and Ai [19] proposed a finite element-boundary element coupling method to study the dynamic response of end-bearing pile groups in layered transversely isotropic media under transient horizontal loads. Based on Boer's porous medium theory, Zhang et al. [20] gave a simplified analytical expression for the vertical dynamic impedance of pile groups in layered transversely isotropic saturated viscoelastic soil. Yan and Liu [21] gave the analytical solution expression of soil horizontal damping factor and horizontal dynamic impedance of a single pile in transversely isotropic soil by considering the influence of soil anisotropy and three-dimensional wave effect. Liu and Yan [22] regarded the soil around the pile as a single-phase transversely isotropic medium, studied the lateral vibration of pile groups in transversely isotropic soil using Novak's plane model, and obtained the analytical solution of the problem. In this paper, based on Boer's porous medium theory, the horizontal vibration of a single pile in transversely isotropic saturated soil is studied by means of mathematical physics, and the analytical expression of the horizontal dynamic impedance at the pile top will be obtained and will explore the influence of relevant parameters on the horizontal vibration of pile foundations.

2. Dynamic Control Equations of Transversely Isotropic Saturated Soil

The horizontal dynamic problem of a single pile in transversely isotropic saturated soil as shown in Figure 1 will be investigated, the radius of the pile is r_0, the pile length of the pile is H, E_p is the elastic modulus of the pile, and ρ_p is the density of the pile. A horizontal harmonic load $P(t) = P_0 e^{i\omega t}$ acts on the pile top, where P_0 is the amplitude of the load, ω is the harmonic load frequency, and i is the virtual unit. In order to consider the influence of the liquid phase and anisotropy of soil around the pile, the soil around the pile is regarded as transversely isotropic saturated porous medium. Here, Boer's saturated porous medium theory and the constitutive equation of transversely isotropic elastic media proposed by Ding et al. [23] are used to build the motion equations of transversely isotropic saturated porous medium and the pile-soil dynamic interaction model. When ignoring the mass exchange and energy exchange between liquid and solid phases, according to Boer's saturated porous medium theory, it can be known that the momentum equation of fluid solid mixture, pore fluid momentum equation, and volume fraction equation of saturated soil around the pile is [24, 25]

$$\begin{gathered}\operatorname{div}\sigma^S+\rho^S\left(b^S-\ddot{u}^S\right)+\widehat{p}^S=0,\\ \operatorname{div}\sigma^F+\rho^F\left(b^F-\ddot{u}^F\right)+\widehat{p}^F=0,\\ \operatorname{div}\left(n^S\dot{u}^S+n^F\dot{u}^F\right)=0.\end{gathered}\tag{1}$$

In the formula, σ^S and σ^F, respectively, represent the macroscopic stress tensors of the solid soil skeleton and the liquid phase, $\dot{u}^S$ and $\ddot{u}^S$ represent the velocity and acceleration of the solid soil skeleton, $\dot{u}^F$ and $\ddot{u}^F$ represent the velocity and acceleration of the liquid phase, ρ^S and ρ^F represent the volume density of the solid soil skeleton and the liquid phase, b^S and b^F represent the volume force per unit mass of the solid phase and liquid phase, and $\widehat{p}^S$ and $\widehat{p}^F$ represent the effective interaction force between the solid soil skeleton and the liquid phase and meet the requirement $\widehat{p}^S+\widehat{p}^F=0$. Considering the incompressible condition, using the volume fraction theory, in the relationship between the stress tensors σ^S and σ^F and the interaction force p^S, p^F can be obtained as [26]

$$\begin{aligned}\sigma^S&=-n^S pI+\sigma^{\mathrm{SE}},\\ \sigma^F&=-n^F pI,\\ \widehat{p}^F&=p\ \operatorname{grad}\ n^F+\widehat{p}^{\mathrm{LE}},\\ \widehat{p}^{\mathrm{LE}}&=-S_v\left(\dot{u}^F-\dot{u}^F\right),\end{aligned}\tag{2}$$

where σ^{SE} is the effective stress tensor of the solid soil skeleton, $\widehat{p}^{\mathrm{LE}}$ is the additional quantity, p is the effective pore water pressure of the incompressible fluid in saturated soil, S_v is the coupling coefficient describing the coupling effect between the solid phase and liquid phase, and n^S and n^F are the volume fraction and satisfy $n^S+n^F=1$.

In order to consider the anisotropy of saturated soil around the pile, the constitutive equation of transversely isotropic elastic medium proposed by Ding et al. [23] is used here to describe the stress and strain relationship of the solid skeleton of saturated soil around the pile, that is

$$\begin{aligned}\sigma_{\mathrm{rr}}^{\mathrm{SE}}&=C_{11}\varepsilon_{\mathrm{rr}}^S+C_{12}\varepsilon_{\theta\theta}^S+C_{13}\varepsilon_{\mathrm{zz}}^S,\\ \sigma_{\theta\theta}^{\mathrm{SE}}&=C_{12}\varepsilon_{\mathrm{rr}}^S+C_{11}\varepsilon_{\theta\theta}^S+C_{13}\varepsilon_{\mathrm{zz}}^S,\\ \sigma_{\mathrm{zz}}^{\mathrm{SE}}&=C_{13}\varepsilon_{\mathrm{rr}}^S+C_{13}\varepsilon_{\theta\theta}^S+C_{33}\varepsilon_{\mathrm{zz}}^S,\\ \sigma_{\theta z}^{\mathrm{SE}}&=C_{44}\varepsilon_{\theta z}^S,\sigma_{rz}^{\mathrm{SE}}=C_{44}\varepsilon_{rz}^S,\sigma_{r\theta}^{\mathrm{SE}}=C_{66}\varepsilon_{r\theta}^S,\end{aligned}\tag{3}$$

where $\sigma_{\mathrm{rr}}^{\mathrm{SE}}$, $\sigma_{\theta\theta}^{\mathrm{SE}}$, $\sigma_{\mathrm{zz}}^{\mathrm{SE}}$, $\sigma_{\theta z}^{\mathrm{SE}}$, $\sigma_{rz}^{\mathrm{SE}}$, and $\sigma_{r\theta}^{\mathrm{SE}}$ are, respectively, the radial, circumferential, vertical, and shear effective stresses of the solid skeleton of transversely isotropic saturated soil; ε_{rr}^S, $\varepsilon_{\theta\theta}^S$, ε_{zz}^S, $\varepsilon_{\theta z}^S$, ε_{rz}^S, and $\varepsilon_{r\theta}^S$ are, respectively, the radial strain, circumferential strain, vertical strain, and shear strain of the solid skeleton of transversely isotropic saturated soil; C_{11}, C_{12}, C_{13}, C_{33}, C_{44}, and C_{66} are the elastic constants of transversely isotropic saturated soil and meet $C_{66}=1/2(C_{11}-C_{12})=G_h$, and G_h and G_v are, respectively, the shear modulus on the horizontal plane and vertical plane.

The strain-displacement relationship of transversely isotropic saturated soil is

$$\begin{aligned}\varepsilon_{\mathrm{rr}}^S&=\frac{\partial u_r^S}{\partial r},\\ \varepsilon_{\theta\theta}^S&=\frac{1}{r}\frac{\partial u_\theta^S}{\partial\theta}+\frac{u_r^S}{r},\\ \varepsilon_{\mathrm{zz}}^S&=\frac{\partial u_z^S}{\partial z},\\ \varepsilon_{rz}^S&=\frac{\partial u_r^S}{\partial z}+\frac{\partial u_z^S}{\partial r},\\ \varepsilon_{r\theta}^S&=\frac{1}{r}\frac{\partial u_r^S}{\partial\theta}+\frac{\partial u_\theta^S}{\partial r}-\frac{u_\theta^S}{r},\\ \varepsilon_{\theta z}^S&=\frac{\partial u_z^S}{r\partial\theta}+\frac{\partial u_\theta^S}{\partial z}.\end{aligned}\tag{4}$$

In which u_r^S, u_z^S, and u_θ^S are the radial, vertical, and circumferential displacements of the solid skeleton of transversely isotropic saturated soil. Ignoring the volumetric force ($b^S=b^F=0$), the motion control equation of transversely isotropic saturated soil represented by displacement can be obtained from Eq. (1) to Eq. (4) as follows:

$$\begin{aligned}&\left[C_{11}\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}-\frac{1}{r^2}\right)+\frac{C_{11}-C_{12}}{2r^2}\frac{\partial^2}{\partial\theta^2}+C_{44}\frac{\partial^2}{\partial z^2}\right]u_r^S\\ &+\left[\frac{C_{11}-C_{66}}{r}\frac{\partial^2}{\partial r\partial\theta}-\frac{C_{11}+C_{66}}{r^2}\frac{\partial}{\partial\theta}\right]u_\theta^S+(C_{13}+C_{44})\frac{\partial^2 u_z^S}{\partial r\partial z}\\ &-n^S\frac{\partial p}{\partial r}-\rho^S\frac{\partial^2 u_r^S}{\partial t^2}-\rho^F\frac{\partial^2 u_r^F}{\partial t^2}=0,\end{aligned}\tag{5}$$

$$\begin{aligned}&\left[\frac{C_{11}-C_{12}}{2}\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}-\frac{1}{r^2}\right)+\frac{C_{11}}{r^2}\frac{\partial^2}{\partial\theta^2}+C_{44}\frac{\partial^2}{\partial z^2}\right]u_\theta^S\\ &+\left[\frac{C_{11}-C_{66}}{r}\frac{\partial^2}{\partial r\partial\theta}+\frac{C_{11}+C_{66}}{r^2}\frac{\partial}{\partial\theta}\right]u_r^S+(C_{13}+C_{44})\frac{1}{r}\frac{\partial^2 u_z^S}{\partial\theta\partial z}\\ &-n^S\frac{\partial p}{r\partial\theta}-\rho^S\frac{\partial^2 u_\theta^S}{\partial t^2}-\rho^F\frac{\partial^2 u_\theta^F}{\partial t^2}=0,\end{aligned}\tag{6}$$

$$\left[C_{44}\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}\right)+C_{33}\frac{\partial^2}{\partial z^2}\right]u_z^S + (C_{13}+C_{44})\left(\frac{\partial^2}{\partial r\partial z}+\frac{1}{r}\frac{\partial}{\partial z}\right)u_r^S+(C_{13}+C_{44})\frac{\partial^2 u_\theta^S}{r\partial\theta\partial z} - n^S\frac{\partial p}{\partial z}-\rho^S\frac{\partial^2 u_z^S}{\partial t^2}-\rho^F\frac{\partial^2 u_z^F}{\partial t^2}=0, \quad (7)$$

$$n^F\frac{\partial p}{\partial r}+\rho^F\frac{\partial^2 u_r^F}{\partial t^2}+S_v\left(\frac{\partial u_r^F}{\partial t}-\frac{\partial u_r^S}{\partial t}\right)=0, \quad (8)$$

$$n^F\frac{1}{r}\frac{\partial p}{\partial\theta}+\rho^F\frac{\partial^2 u_\theta^F}{\partial t^2}+S_v\left(\frac{\partial u_\theta^F}{\partial t}-\frac{\partial u_\theta^S}{\partial t}\right)=0, \quad (9)$$

$$\frac{\partial}{\partial r}\left\{\frac{1}{r}\frac{\partial}{\partial r}\left[r\left(n^S\frac{\partial u_r^S}{\partial t}+n^F\frac{\partial u_r^F}{\partial t}\right)\right]+\frac{1}{r}\frac{\partial}{\partial\theta}\left(n^S\frac{\partial u_r^S}{\partial t}+n^F\frac{\partial u_r^F}{\partial t}\right)\right\} + \frac{\partial^2}{\partial z^2}\left(n^S\frac{\partial u_r^S}{\partial t}+n^F\frac{\partial u_r^F}{\partial t}\right)=0, \quad (10)$$

where u_r^F, u_z^F, and u_θ^F are the radial, vertical, and circumferential displacements of the liquid phase of transversely isotropic saturated soil. Eq. (5)–Eq. (10) are the motion control equations of transversely isotropic saturated soil represented by displacement.

3. Solution to Horizontal Vibration of Transversely Isotropic Saturated Soil

Here, we only study the horizontal vibration problem of pile foundation in transversely isotropic saturated soil under the action of the horizontal harmonic load $P(t)=P_0e^{i\omega t}$ at the pile top (as shown in Figure 1). We ignored the influence of vertical displacement and only considered radial and circumferential displacements, and radial and circumferential displacements are independent of coordinate z. At this point, Eqs. (5), (6), and (10) are simplified as follows:

$$\left[C_{11}\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}-\frac{1}{r^2}\right)+\frac{C_{66}}{r^2}\frac{\partial^2}{\partial\theta^2}\right]u_r^S + \left[\frac{C_{11}-C_{66}}{r}\frac{\partial^2}{\partial r\partial\theta}-\frac{C_{11}+C_{66}}{r^2}\frac{\partial}{\partial\theta}\right]u_\theta^S - n^S\frac{\partial p}{\partial r}-\rho^S\frac{\partial^2 u_r^S}{\partial t^2}-\rho^F\frac{\partial^2 u_r^F}{\partial t^2}=0, \quad (11)$$

$$\left[C_{66}\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}-\frac{1}{r^2}\right)+\frac{C_{11}}{r^2}\frac{\partial^2}{\partial\theta^2}\right]u_\theta^S + \left[\frac{C_{11}-C_{66}}{r}\frac{\partial^2}{\partial r\partial\theta}+\frac{C_{11}+C_{66}}{r^2}\frac{\partial}{\partial\theta}\right]u_r^S - n^S\frac{\partial p}{r\partial\theta}-\rho^S\frac{\partial^2 u_\theta^S}{\partial t^2}-\rho^F\frac{\partial^2 u_\theta^F}{\partial t^2}=0, \quad (12)$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\left(n^S\frac{\partial u_r^S}{\partial t}+n^F\frac{\partial u_r^F}{\partial t}\right)\right]+\frac{1}{r}\frac{\partial}{\partial\theta}\left(n^S\frac{\partial u_r^S}{\partial t}+n^F\frac{\partial u_r^F}{\partial t}\right)=0. \quad (13)$$

Eqs. (8), (9), and (11) to (13) are the horizontal vibration control equations for transversely isotropic saturated soil.

The whole system makes a steady simple harmonic vibration under the action of horizontal harmonic load $P(t)=P_0 e^{i\omega t}$ at the pile top. Considering the harmonic nature of the problem, the form of each parameter is $f=\tilde{f}e^{i\omega t}$ and is substituted into the horizontal movement control equations of transversely isotropic saturated soil (Eq. (8), Eq. (9), and Eqs. (11)–(13)), and dimensionless quantities $\bar{r}=r/H$, $\bar{z}=r/H$, $\bar{u}_r^S=\tilde{u}_r^S/H$, $\bar{u}_\theta^S=\tilde{u}_\theta^S/H$, $\bar{u}_r^F=\tilde{u}_r^F/H$, $\bar{u}_\theta^F=\tilde{u}_\theta^F/H$, $\bar{\omega}=H\omega/v$, $\bar{p}=p/v^2\rho^S$, $s_v=\mathrm{HS}_v/v\rho^S$, $\rho=\rho^S/\rho^F$, and $v=\sqrt{C_{66}/\rho^S}$ are introduced, $\tilde{u}_r^S$ and $\tilde{u}_\theta^S$ are the amplitudes of radial and circumferential displacement of the solid phase skeleton of the transversely isotropic saturated soil, and $\tilde{u}_r^F$ and $\tilde{u}_\theta^F$ are the amplitudes of radial and circumferential displacement of the liquid phase fluid of the transversely isotropic saturated soil; $v=\sqrt{C_{66}/\rho^S}$ is the vertical shear wave velocity of soil. By performing dimensionless operations on the horizontal movement control equations of transversely isotropic saturated soil (Eq. (8), Eq. (9), and Eq. (11)–Eq. (13)), it can be obtained that

$$\delta_1\left[\left(\frac{\partial^2}{\partial\bar{r}^2}+\frac{1}{\bar{r}}\frac{\partial}{\partial\bar{r}}-\frac{1}{\bar{r}^2}\right)+\frac{1}{\bar{r}^2}\frac{\partial^2}{\partial\theta^2}\right]\bar{u}_r^S + \left[\frac{\delta_1-1}{\bar{r}}\frac{\partial^2}{\partial\bar{r}\partial\theta}-\frac{\delta_1+1}{\bar{r}^2}\frac{\partial}{\partial\theta}\right]\bar{u}_\theta^S-n^S\frac{\partial p}{\partial\bar{r}}+\bar{\omega}^2\bar{u}_r^S-\frac{\bar{\omega}^2}{\rho}\bar{u}_r^F=0, \quad (14)$$

$$\left[\left(\frac{\partial^2}{\partial\bar{r}^2}+\frac{1}{\bar{r}}\frac{\partial}{\partial\bar{r}}-\frac{1}{\bar{r}^2}\right)+\frac{\delta_1}{\bar{r}^2}\frac{\partial^2}{\partial\theta^2}\right]\bar{u}_\theta^S + \left[\frac{\delta_1-1}{\bar{r}}\frac{\partial^2}{\partial\bar{r}\partial\theta}+\frac{\delta_1+1}{\bar{r}^2}\frac{\partial}{\partial\theta}\right]\bar{u}_r^S-n^S\frac{\partial\bar{p}}{\bar{r}\partial\theta}+\rho^S\omega^2\bar{u}_\theta^S+\frac{\bar{\omega}^2}{\rho}\bar{u}_\theta^F=0, \quad (15)$$

$$n^F\frac{\partial\bar{p}}{\partial\bar{r}}-\frac{\bar{\omega}^2}{\rho}\bar{u}_r^F+i\bar{\omega}s_v\left(\bar{u}_r^F-\bar{u}_r^S\right)=0, \quad (16)$$

$$n^F\frac{1}{\bar{r}}\frac{\partial\bar{p}}{\partial\theta}-\frac{\bar{\omega}^2}{\rho}\bar{u}_\theta^F+i\bar{\omega}s_v\left(u_\theta^F-u_\theta^S\right)=0, \quad (17)$$

$$\frac{1}{\bar{r}}\frac{\partial}{\partial r}\left[\bar{r}\left(n^S\bar{u}_r^S+n^F\bar{u}_r^F\right)\right]+\frac{1}{\bar{r}}\frac{\partial}{\partial\theta}\left(n^S\bar{u}_r^S+n^F\bar{u}_r^F\right)=0. \quad (18)$$

In which, $\delta_1=C_{11}/C_{66}$ is the anisotropic parameter that reflects the degree of anisotropy of the saturated soil.

To solve the horizontal vibration of transversely isotropic saturated soil layers, Eq. (14)–Eq. (18) need to be decoupled, and the following potential function is introduced for this purpose:

$$\bar{u}_r^S = \frac{\partial \varphi^S}{\partial \bar{r}} + \frac{1}{\bar{r}}\frac{\partial \psi^S}{\partial \theta},\quad \bar{u}_\theta^S = \frac{1}{\bar{r}}\frac{\partial \varphi^S}{\partial \theta} - \frac{\partial \psi^S}{\partial \bar{r}}, \tag{19}$$

$$\bar{u}_r^F = \frac{\partial \varphi^F}{\partial \bar{r}} + \frac{1}{\bar{r}}\frac{\partial \psi^F}{\partial \theta},\quad \bar{u}_\theta^F = \frac{1}{\bar{r}}\frac{\partial \varphi^F}{\partial \theta} - \frac{\partial \psi^F}{\partial \bar{r}}. \tag{20}$$

Here, φ^S, ψ^S and φ^F, ψ^F are the amplitude of the displacement potentials of the solid and liquid phases, respectively.

By decoupling, it can be obtained that

$$\delta_{16}\nabla^2\varphi^S - n^S\bar{p} - \bar{\omega}^2\varphi^S - \frac{\bar{\omega}^2}{\rho}\varphi^F = 0, \tag{21}$$

$$\nabla^2\psi^S - \bar{\omega}^2\psi^S - \frac{\bar{\omega}^2}{\rho}\psi^F = 0, \tag{22}$$

$$n^F\bar{p} + a_1\varphi^F - i\bar{\omega}s_v\varphi^S = 0, \tag{23}$$

$$a_1\psi^F - i\bar{\omega}s_v\psi^S = 0, \tag{24}$$

$$n^S\nabla^2\varphi^S + n^F\nabla^2\varphi^F = 0. \tag{25}$$

In which, $\nabla^2 = (\partial^2/\partial\bar{r}^2) + (1/\bar{r})(\partial/\partial\bar{r}) + (1/\bar{r}^2)(\partial^2/\partial\theta^2)$ and $a_1 = i\bar{\omega}s_v - (\bar{\omega}^2/\rho)$. From Eq. (22) and Eq. (24), it can be concluded that

$$\left(\nabla^2 - q^2\right)\psi^S = 0. \tag{26}$$

In which, $q^2 = \bar{\omega}^2 + (i\bar{\omega}^3 s_v/\rho a_1)$. Using the method of separating variables to solve Bessel's Eq. (26) while considering that the displacement of saturated soil at infinity is zero, it can be obtained that

$$\psi^S = A_1K_1(q\bar{r})\sin\theta, \tag{27}$$

$$\psi^F = \frac{i\bar{\omega}s_v}{a_1}A_1K_1(q\bar{r})\sin\theta, \tag{28}$$

where A_1 is the undetermined coefficient. From Eqs. (21), (23), and (25), it can be concluded that

$$\nabla^2\left(\nabla^2\varphi^S\right) - s^2\nabla^2\varphi^S = 0. \tag{29}$$

In which $a_2 = -(\bar{\omega}^2 n^F + i\bar{\omega}s_v n^S/n^F\delta_{16})a_3 = (a_1\rho n^S - n^F\bar{\omega}^2/\rho n^F\delta_{16})$, $s^2 = (n^S a_3/n^F) - a_2$.

Similarly, using the separation of variable method to solve Bessel's equation (29) and considering that the displacement of saturated soil at infinity is zero, it can be obtained that

$$\nabla^2\varphi^S = A_2K_1(s\bar{r})\cos\theta, \tag{30}$$

where A_2 is the undetermined coefficient. By solving Eq. (30) and considering that the displacement of saturated soil at infinity is zero, it can be obtained that

$$\varphi^S = \left[\frac{K_1(s\bar{r})}{s^2}A_2 + \frac{A_3}{\bar{r}}\right]\cos\theta, \tag{31}$$

where A_3 is the undetermined coefficient; A_1, A_2, and A_3 can be determined by boundary conditions. Further consideration of Eqs. (21), (22), and (31) yields

$$\varphi^F = -\frac{1}{a_3}K_1(s\bar{r})A_2\cos\theta - \frac{a_2}{a_3}\left[\frac{A_3}{\bar{r}} + \frac{K_1(s\bar{r})}{s^2}A_2\right]\cos\theta, \tag{32}$$

$$\bar{p} = \frac{a_1}{n^F a_3}K_1(s\bar{r})A_2\cos\theta + \frac{a_1a_2 + ia_3\bar{\omega}s_v}{n^F a_3}\left[\frac{K_1(s\bar{r})}{s^2}A_2 + \frac{A_3}{\bar{r}}\right]\cos\theta. \tag{33}$$

From Eqs. (19), (20), (27), (28), (31), and (32), it can be obtained that the radial and circumferential horizontal dynamic displacements of the solid skeleton of transversely isotropic saturated soil are

$$\bar{u}_r^S = \left[\frac{K_1(q\bar{r})}{\bar{r}}A_1 - \frac{K_1(s\bar{r})}{s^2\bar{r}}A_2 - \frac{K_0(s\bar{r})}{s}A_2 - \frac{A_3}{\bar{r}^2}\right]\cos\theta, \tag{34}$$

$$\bar{u}_\theta^S = \left[\frac{K_1(q\bar{r})}{\bar{r}}A_1 + qK_0(q\bar{r})A_1 - \frac{K_1(s\bar{r})}{s^2\bar{r}}A_2 - \frac{A_3}{\bar{r}^2}\right]\sin\theta. \tag{35}$$

4. Solution to Horizontal Vibration of a Single Pile in Transversely Isotropic Saturated Soil

In order to solve the horizontal vibration of pile foundations in transversely isotropic saturated soil, the horizontal dynamic interaction between pile and soil is equivalent to spring and damper distributed around the pile; that is, the horizontal dynamic interaction between pile and soil is described using the Winkler spring-damper model. In order to determine the stiffness and damping coefficients of the Winkler spring-damper model, assuming that the dimensionless horizontal displacement of the pile body is 1, the stiffness and damping coefficients are determined by solving the horizontal force of the soil around the pile on the pile body. Assuming that the dimensionless horizontal displacement of the pile body is 1 and the pile-soil contact surface is impermeable, the following boundary conditions can be obtained:

$$\begin{aligned} \bar{u}_{r.}(\bar{r}_0,\theta) &= 1, \\ \theta &= 0, \\ \bar{u}_\theta(\bar{r}_0,\theta) &= -1, \\ \theta &= \frac{\pi}{2}, \\ \left.\frac{\partial \bar{p}}{\partial \bar{r}}\right|_{\bar{r}=\bar{r}_0} &= 0. \end{aligned} \tag{36}$$

In which, $\bar{r}_0 = r_0/H$, r_0 and H are the pile radius and pile length, respectively.

$$\begin{aligned} &\bar{r}_0\left(a_1 s^2 + a_1 a_2 + i a_3 \bar{\omega} s_v\right]\left[s\bar{r}_0 K_0(s\bar{r}_0) + K_1(s\bar{r}_0)\right]A_2 \\ &\quad + \left(a_1 a_2 + i a_3 \bar{\omega} s_v\right)s^2 A_3 = 0, \\ &-\frac{A_3}{\bar{r}_0^2} - \frac{K_1(s\bar{r}_0)}{s^2\bar{r}_0}A_2 - \frac{K_0(s\bar{r}_0)}{s}A_2 + \frac{K_1(q\bar{r}_0)}{\bar{r}_0}A_1 = 1, \\ &-\frac{A_3}{\bar{r}_0^2} - \frac{K_1(s\bar{r}_0)}{s^2\bar{r}_0}A_2 + \frac{K_1(q\bar{r}_0)}{\bar{r}_0}A_1 + qK_0(q\bar{r}_0)A_1 = -1. \end{aligned} \tag{37}$$

In the equations, A_1, A_2, and A_3 are the undetermined coefficients, which can be determined from Eq. (14)–Eq. (18).

$$\begin{aligned} A_1 &= \frac{2a_6 s^2 \bar{r}_0^2}{a_6 a_9 - a_7 a_8}, \\ A_2 &= -\frac{2a_7 s^2 \bar{r}_0^2}{a_6 a_9 - a_{a_7} a_8}, \\ A_3 &= \frac{a_4}{a_5}\frac{2a_7 s^2 \bar{r}_0^2}{a_6 a_9 - a_{a_7} a_8}, \end{aligned} \tag{38}$$

where $a_4 = \bar{r}_0(a_1 s^2 + a_1 a_2 + \mathrm{i} a_3 \bar{\omega} s_v][s\bar{r}_0 K_0(s\bar{r}_0) + K_1(s\bar{r}_0)]$, $a_5 = (a_1 a_2 + i a_3 \bar{\omega} s_v)s^2$, $a_6 = (2a_4/a_5)s^2 - 2\bar{r}_0 K_1(s\bar{r}_0) - s r_0^2 K_0(s\bar{r}_0)$, $a_7 = 2s^2\bar{r}_0 K_1(q\bar{r}_0) + q s^2 \bar{r}_0^2 K_0(q\bar{r}_0)$, $a_8 = -s\bar{r}_0^2 K_0(s\bar{r}_0)$, and $a_9 = -q s^2 \bar{r}_0^2 K_0(q\bar{r}_0)$.

The dimensionless radial and shear stresses of the solid skeleton of transversely isotropic saturated soil can be determined from Eqs. (3), (4), (34), and (35).

$$\begin{aligned} \bar{\sigma}_{\mathrm{rr}}^{\mathrm{SE}} &= (\delta_2 - \delta_1)\left[\frac{2K_1(q\bar{r})}{\bar{r}^2} + \frac{1}{\bar{r}}qK_0(q\bar{r})\right]A_1 \cos\theta \\ &\quad + \left\{(\delta_1 - \delta_2)\left[\frac{2K_1(s\bar{r})}{s^2\bar{r}^2} + \frac{1}{s\bar{r}}K_0(s\bar{r})\right] + \delta_{16}K_1(s\bar{r})\right\}A_2 \cos\theta \\ &\quad + (\delta_1 - \delta_2)\frac{2}{\bar{r}^3}A_3 \cos\theta, \end{aligned} \tag{39}$$

$$\begin{aligned} \bar{\sigma}_{r\theta}^{\mathrm{SE}} &= -\left[\frac{4K_1(q\bar{r})}{\bar{r}^2} + \frac{2}{\bar{r}}qK_0(q\bar{r}) + q^2 K_1(q\bar{r})\right]A_1 \sin\theta \\ &\quad + \left[\frac{4K_1(s\bar{r})}{s^2\bar{r}^2} + \frac{2}{s\bar{r}}K_0(s\bar{r})\right]A_2 \sin\theta + \frac{4A_3}{\bar{r}^3}\sin\theta. \end{aligned} \tag{40}$$

In the equation, $\delta_2 = (C_{12}/C_{66})$. According to the radial stress (Eq. (39)) and shear stress (Eq. (39)) of the solid skeleton of the saturated soil, the dimensionless force per unit thickness of the soil layer on the pile body can be obtained, that is

$$\begin{aligned} F_x &= \int_0^{2\pi}\left[\left(\bar{p} - \bar{\sigma}_{\mathrm{rr}}^{\mathrm{SE}}\right)\cos\theta + \bar{\sigma}_{r\theta}^{\mathrm{SE}}\sin\theta\right]\Big|_{\bar{r}=\bar{r}_0}\bar{r}_0 d\theta \\ &= \pi\bar{r}_0(P + b_1 A_1 + b_2 A_2 + b_3 A_3) = \pi f_x. \end{aligned} \tag{41}$$

In the formula, the real and imaginary parts of F_x are the stiffness and damping coefficients of the Winkler spring-damper model, and

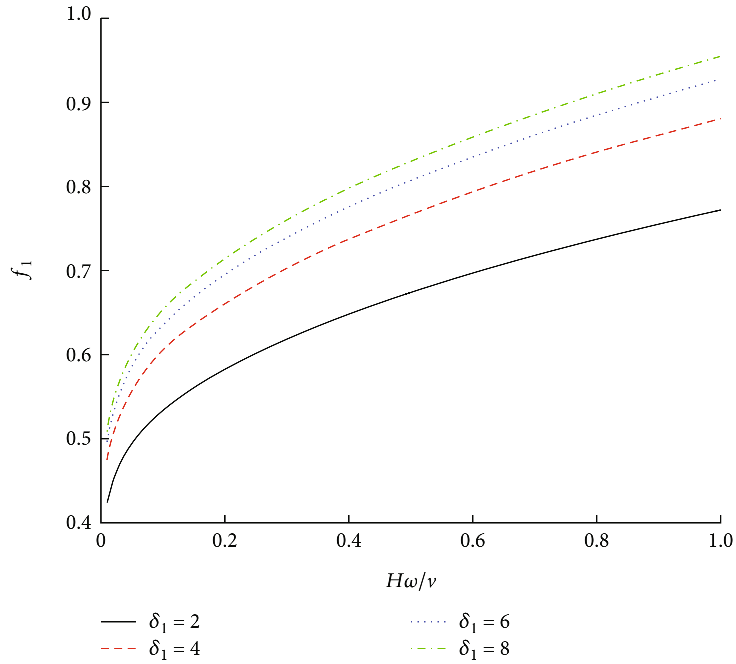
$$\begin{aligned} b_1 &= \frac{2(\delta_1 - \delta_2) - 4 - \bar{r}_0^2 q^2}{\bar{r}_0^2}K_1(q\bar{r}_0) + \frac{\delta_1 - \delta_2 - 2}{\bar{r}_0}qK_0(q\bar{r}_0), \\ b_2 &= \frac{2(\delta_2 - \delta_1) + 4 - \delta_1 s^2\bar{r}_0^2}{s^2\bar{r}_0^2}K_1(s\bar{r}_0) + \frac{(\delta_2 - \delta_1) + 2}{s\bar{r}_0}K_0(s\bar{r}_0), \\ b_3 &= \frac{4 + 2(\delta_2 - \delta_1)}{\bar{r}_0^3}, \\ P &= \frac{a_1}{n^F a_3}K_1(s\bar{r}_0)A_2 + \frac{a_1 a_2 + i a_3 \bar{\omega} s_v}{n^F a_3}\left[\frac{K_1(s\bar{r}_0)}{s^2}A_2 + \frac{A_3}{\bar{r}_0}\right]. \end{aligned} \tag{42}$$

Taking the micro element of the pile body as the research object, while considering the force (equation (41)) of the soil around the pile on the pile body, the dimensionless horizontal vibration equation of the pile foundation in transversely isotropic saturated soil can be obtained as follows:

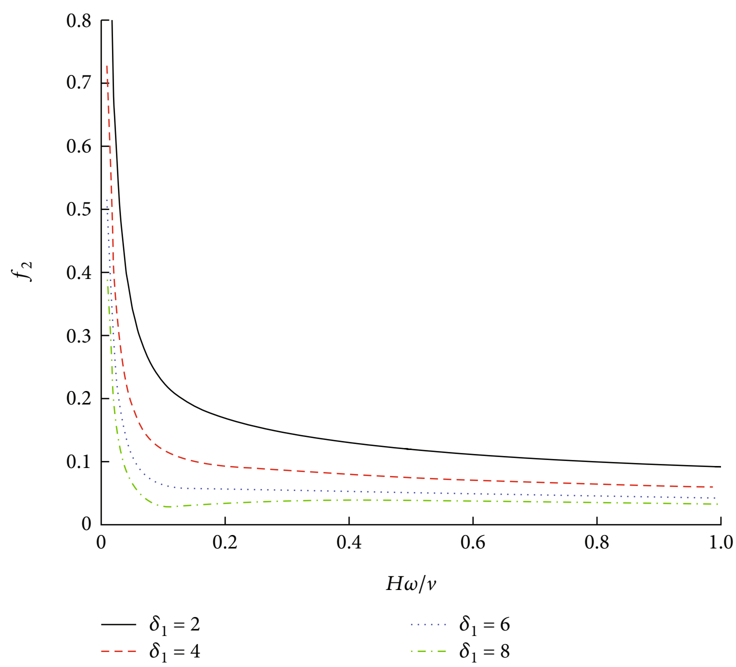
$$\frac{d^4\bar{u}_p(\bar{z})}{d\bar{z}^4} + 4\lambda^4\bar{u}_p(\bar{z}) = 0, \tag{43}$$

where $\bar{u}_p(\bar{z})$ is the dimensionless horizontal displacement amplitude of the pile foundation and $\lambda^4 = \rho f_x - \bar{\rho}_p \bar{r}_0^2 \bar{\omega}^2/\rho\bar{E}_p \bar{r}_0^4$, $\bar{\rho}_p = \rho_p/\rho^F$,, $\bar{E}_p = E_p/C_{66}$, E_p, and ρ_p are the elastic modulus and density of the pile, respectively.

The displacement, rotation angle, shear force, and bending moment at the top of the pile are U_0, θ_0, Q_0, and M_0, respectively, using the initial parameter method [27], and the horizontal displacement of the pile body can be obtained from equation (43) as



(a) Curves of horizontal dynamic impedance stiffness factor varying with frequency



(b) Curves of horizontal dynamic impedance damping factor varying with frequency

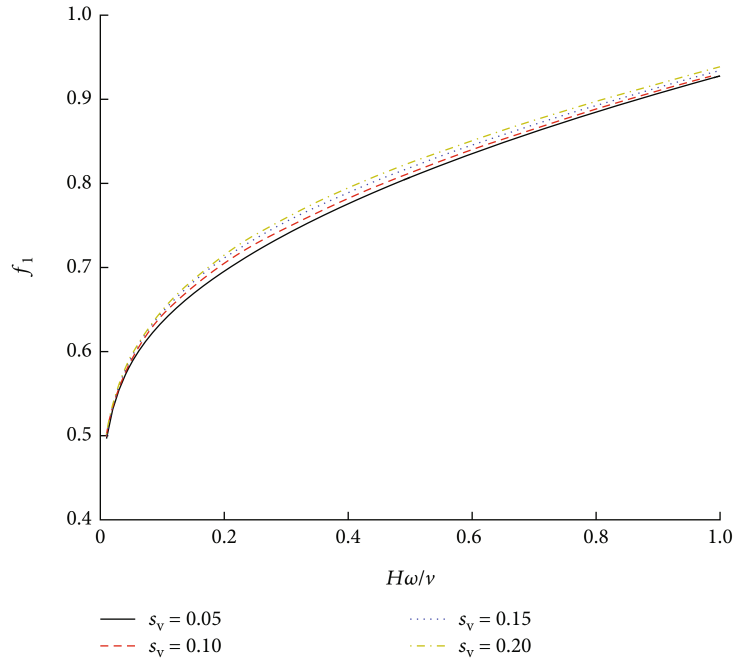
FIGURE 2: Influence of anisotropic parameter δ_1 on horizontal dynamic impedance of a single pile in transversely isotropic saturated soil.

$$u_{\mathrm{p}}(\bar{z}) = -U_0 F_1(\lambda\bar{z}) + \frac{\theta_0}{\lambda} F_2(\lambda\bar{z}) - \frac{M_0}{(\pi/4)\bar{E}_{\mathrm{p}}\lambda^2\bar{r}_0^4} F_3(\lambda\bar{z}) - \frac{Q_0}{(\pi/4)\bar{E}_{\mathrm{p}}\lambda^3\bar{r}_0^4} F_4(\lambda\bar{z}). \quad (44)$$

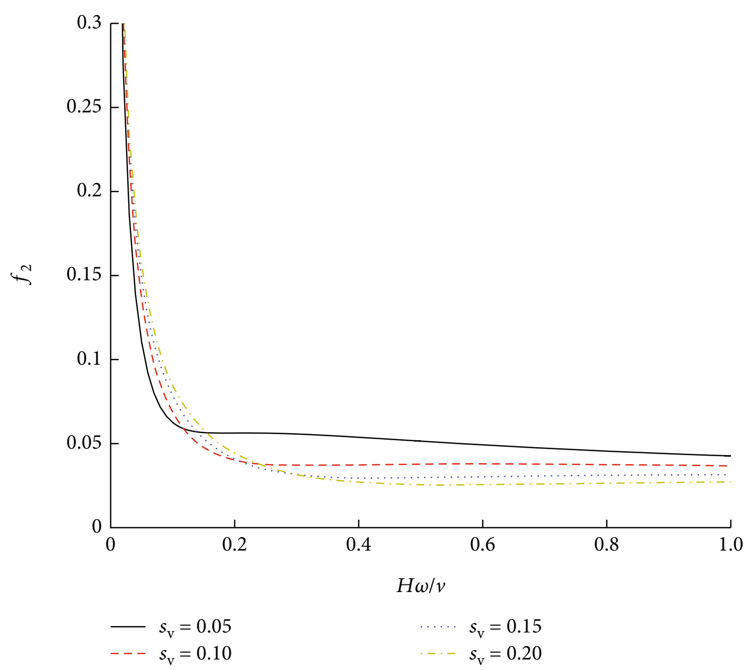
In the formula, F_1, F_2, F_3, and F_4 are the Krylov functions, $F_1(\lambda\bar{z}) = \mathrm{ch}\lambda\bar{z}\cos\lambda\bar{z}$, $F_2(\lambda\bar{z}) = 1/2(\mathrm{ch}\bar{z}\sin\lambda\bar{z} + \mathrm{sh}z\cos\lambda z)$, $F_3(\lambda\bar{z}) = (1/2)\mathrm{sh}\lambda\bar{z}\sin\lambda\bar{z}$, and $F_4(\lambda\bar{z}) = 1/4(\mathrm{ch}\lambda\bar{z}\sin\lambda\bar{z} - \mathrm{sh}\lambda z\cos\lambda z)$.

Taking an end-bearing pile as an example, due to the fixed bottom of the pile, the displacement and rotation angle at the pile bottom are both zero, and there are the following boundary conditions:

$$\bar{u}_p(\bar{z})\big|_{\bar{z}=1} = 0, \qquad \left.\frac{d\bar{u}_p(\bar{z})}{d\bar{z}}\right|_{\bar{z}=1} = 0. \quad (45)$$



(a) Curves of horizontal dynamic impedance stiffness factor varying with frequency



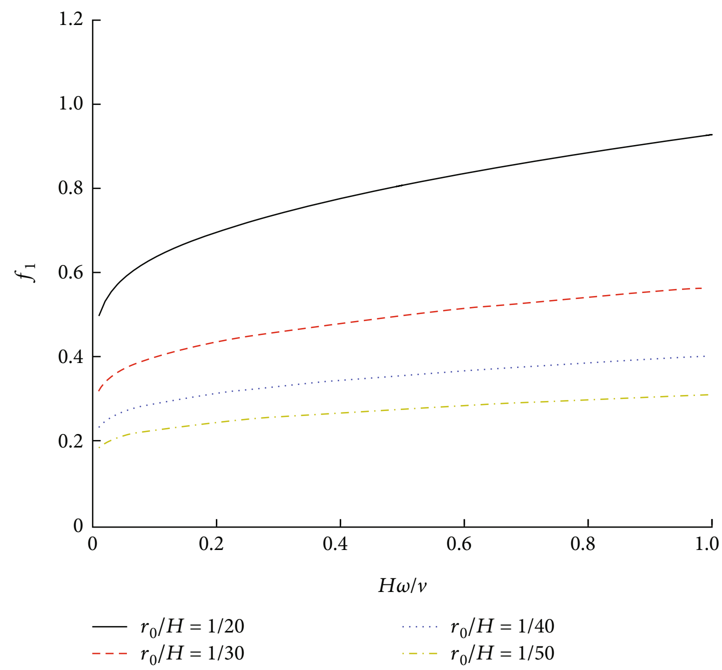
(b) Curves of horizontal dynamic impedance damping factor varying with frequency

FIGURE 3: Influence of liquid-solid coupling coefficient s_v on horizontal dynamic impedance of a single pile in transversely isotropic saturated soil.

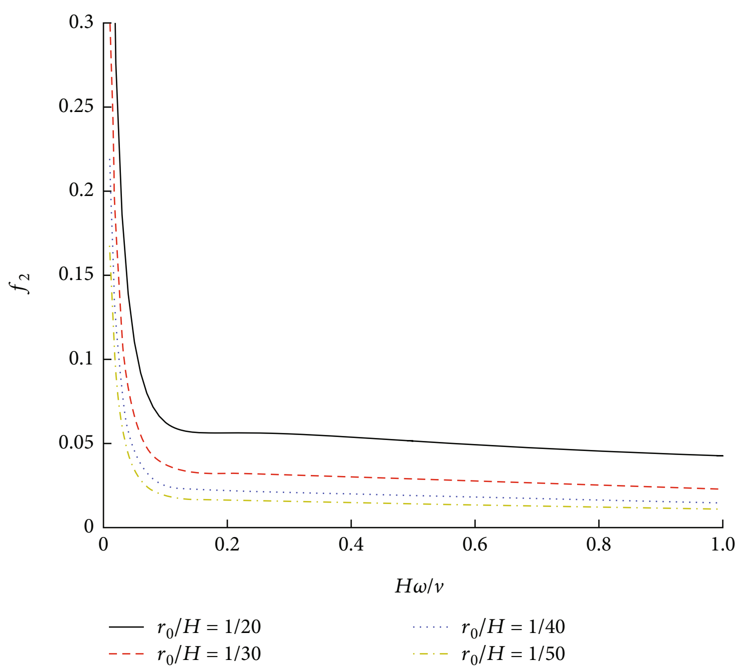
According to equations (44) and (45), the shear force at the top of the pile is

$$Q_0 = \frac{\pi}{4}\bar{E}_p\bar{r}_0^4\left\{\frac{\lambda^3[F_1(\lambda)F_2(\lambda)+4F_3(\lambda)F_4(\lambda)]}{F_3^2(\lambda)-F_2(\lambda)F_4(\lambda)}U_0 - \frac{\lambda^2[F_2^2(\lambda)-F_1(\lambda)F_3(\lambda)]}{F_3^2(\lambda)-F_2(\lambda)F_4(\lambda)}\varphi_0\right\}. \tag{46}$$

Considering the definition of the horizontal dynamic impedance at the pile top, the required horizontal shear force is the horizontal dynamic impedance at the pile top for generating unit horizontal displacement when constraining the rotation angle at the pile top. Therefore, the horizontal dynamic impedance of the pile foundation in transversely isotropic saturated soil can be obtained as



(a) Curves of horizontal dynamic impedance stiffness factor varying with frequency



(b) Curves of horizontal dynamic impedance damping factor varying with frequency

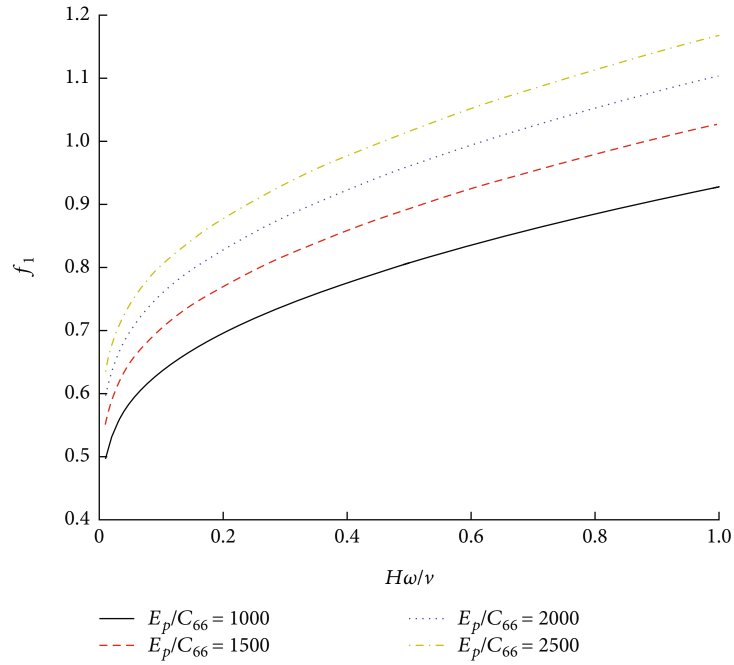
Figure 4: Influence of diameter-length ratio r_0/H on horizontal dynamic impedance of a single pile in transversely isotropic saturated soil.

$$K_{\mathrm{hh}} = \frac{Q_0}{U_0} = \frac{\pi}{4}\bar{E}_p\bar{r}_0^4\frac{\lambda^3[F_1(\lambda)F_2(\lambda)+4F_3(\lambda)F_4(\lambda)]}{F_3^2(\lambda)-F_2(\lambda)F_4(\lambda)} = f_1 + i\bar{\omega}f_2. \tag{47}$$

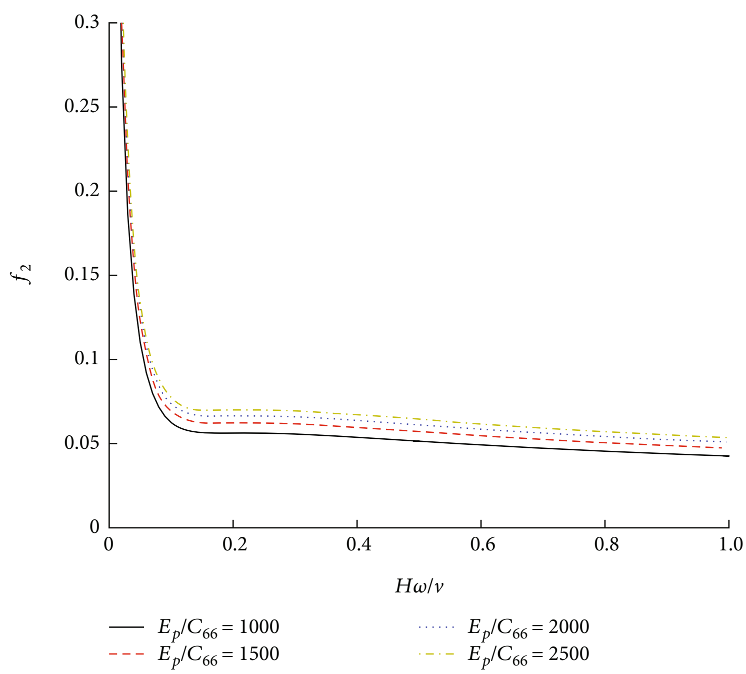
Here, K_{hh} is the horizontal dynamic impedance at the pile top, f_1 is the horizontal dynamic impedance stiffness factor of the pile in transversely isotropic saturated soil, and f_2 is the horizontal dynamic impedance damping factor.

5. Numerical Examples and Discussions

Figures 2–5 show the curves of the horizontal dynamic impedance stiffness factor f_1 and the horizontal dynamic impedance factor f_2 of the pile in transversely isotropic saturated soil varying with dimensionless frequency. The values of the relevant parameters without explanation are $n^S = 0.67$, $n^F = 0.33$, $r_0/H = 1/20$, $E_p/C_{66} = 1000$, $s_v = 0.05$, $\rho = 2.0$, $\rho_p = 5$, $\delta_1 = 6$, and $\delta_2 = 4$.



(a) Curves of horizontal dynamic impedance stiffness factor varying with frequency



(b) Curves of horizontal dynamic impedance damping factor varying with frequency

Figure 5: Influence of modulus ratio E_p/C_{66} on horizontal dynamic impedance of a single pile in transversely isotropic saturated soil.

Overall, as the frequency increases, the horizontal dynamic impedance stiffness factor gradually increases, while the horizontal dynamic impedance factor rapidly decreases at low frequencies and gradually stabilizes. The influence of anisotropic parameter δ_1 of transversely isotropic saturated soil on the horizontal vibration of a single pile is shown in Figure 2. The influence of anisotropic parameters δ_1 on the horizontal dynamic impedance of pile in transversely isotropic saturated soil is significant, whether it is the horizontal dynamic stiffness factor or the horizontal dynamic impedance factor, and the transversely anisotropic parameter δ_1 has a significant impact on them, and with the transverse anisotropy parameter δ_1 increases, its impact gradually decreases. It can be seen that in practical engineering, the influence of anisotropy on the horizontal vibration of pile foundations should not be ignored. The influence of the liquid-solid coupling coefficient s_v of transversely isotropic saturated soil is shown in Figure 3. The influence of the liquid-solid coupling coefficient s_v on the horizontal dynamic impedance damping factor f_2 is greater than that of the horizontal dynamic impedance stiffness factor f_1. The larger the liquid-solid coupling coefficient, the greater the horizontal dynamic impedance stiffness factor. The effect of the liquid-solid coupling coefficient on the horizontal dynamic impedance factor is related to frequency. At high frequencies, the larger the liquid-solid coupling coefficient, the smaller the horizontal dynamic impedance damping factor, while at low frequencies, the opposite is true. At high frequencies, the relative motion between the liquid and solid phases weakens, resulting in a decrease in damping.

The effect of the diameter-length ratio r_0/H of the pile on the horizontal vibration of the pile foundation in transversely isotropic saturated soil is shown in Figure 4. Like homogeneous soil, the diameter-length ratio of the pile has a significant impact on the vibration of the pile foundation. The smaller the diameter-length ratio of the pile, that is, the longer the pile, the smaller the stiffness factor and damping factor of the horizontal dynamic impedance. This is because the longer the pile, the easier it is to deform, the greater the displacement at the pile top, and the smaller the horizontal dynamic impedance at the pile top. At the same time, it can also be seen that when the pile length is longer, the impact of the pile length gradually decreases.

Since $C_{66} = G_h$, G_h is the shear modulus on the water plane of transversely isotropic saturated soil, and the modulus ratio E_p/C_{66} is the ratio of the elastic modulus of the pile body to the shear modulus on the water plane of transversely isotropic saturated soil. From Figure 5, it can be seen that the modulus ratio E_p/C_{66} has a greater impact on the horizontal dynamic impedance of a single pile in transversely isotropic saturated soil, and its impact on the horizontal stiffness factor is much greater than that of the damping factor. The larger the modulus ratio E_p/C_{66}, the greater the horizontal dynamic impedance stiffness factor and damping factor. Because when the modulus ratio E_p/C_{66} is larger, the pile body stiffness is relatively larger, the deformation is smaller, and the displacement at the pile top is smaller, resulting in larger dynamic impedance at the pile top.

6. Conclusions

Considering the influence of the anisotropy and the liquid phase of the soil around the pile, the horizontal vibration of the transversely isotropic saturated soil layer is solved by means of mathematical and physics, and the analytical expression of the horizontal dynamic impedance of a single pile in transversely isotropic saturated soil is obtained. The numerical example analyzed the influence of the transversely isotropic saturated soil anisotropy parameter, liquid-solid coupling coefficient, etc. on the horizontal vibration of a single pile, and the main conclusions are as follows: (1) Anisotropic parameter δ_1 of soil has a significant impact on the horizontal dynamic impedance of a single pile in transversely isotropic saturated soil. In practical engineering, the influence of soil anisotropy on the horizontal vibration of pile foundations should not be ignored. (2) The influence of the liquid-solid coupling coefficient on the horizontal dynamic impedance damping factor f_2 is greater than that of stiffness factor f_1. The influence of the liquid-solid coupling coefficient on the horizontal dynamic impedance factor is related to frequency to a certain extent. (3) The diameter-length ratio has a significant impact on the horizontal vibration of pile foundations in transversely isotropic saturated soil. The longer the pile, the smaller the horizontal dynamic impedance at the pile top. When the pile length is longer, the influence of the pile length gradually decreases. (4) When the stiffness of the pile body is large, the deformation of the pile body is small, and the horizontal dynamic impedance at the pile top is large. The modulus ratio E_p/C_{66} has a significant impact on the horizontal dynamic impedance stiffness factor of a single pile in transversely isotropic saturated soil, but its impact on the horizontal dynamic impedance damping factor is relatively small.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest to report regarding the present study.

Acknowledgments

This research was supported by the National Natural Science Foundation of China (no. U1504505) and the Key Scientific Research Projects of Colleges and Universities in Henan Province (no. 23B560001).

References

[1] M. Novak, "Dynamic stiffness and damping of piles," *Canadian Geotechnical Journal*, vol. 11, no. 4, pp. 574–598, 1974.

[2] M. Novak and T. Nogami, "Soil-pile interaction in horizontal vibration," *Earthquake Engineering and Structural Dynamics*, vol. 5, no. 3, pp. 263–281, 1977.

[3] T. Nogami and M. Novak, "Soil-pile interaction in vertical vibration," *Earthquake Engineering and Structural Dynamics*, vol. 4, no. 3, pp. 277–293, 1976.

[4] K. K. Koo, K. T. Chau, X. Yang, S. S. Lam, and Y. L. Wong, "Soil–pile–structure interaction under SH wave excitation," *Earthquake Engineering and Structural Dynamics*, vol. 32, no. 3, pp. 395–415, 2003.

[5] X. M. Ding, H. L. Liu, J. Chu, and K. Cheng, "Time-domain solution for transient dynamic response of a large-diameter thin-walled pipe pile," *Earthquake Engineering and Engineering Vibration*, vol. 14, no. 2, pp. 239–251, 2015.

[6] W. B. Wu, G. S. Jiang, S. G. Huang, G. X. Mei, and C. J. Leo, "A new analytical model to study the influence of weld on the vertical dynamic response of prestressed pipe pile," *International Journal for Numerical and Analytical Methods in Geomechanics*, vol. 41, no. 10, pp. 1247–1266, 2017.

[7] L. B. Luan, C. J. Zheng, G. Kouretzis, and X. M. Ding, "Dynamic analysis of pile groups subjected to horizontal loads

considering coupled pile-to-pile interaction," *Computers and Geotechnics*, vol. 117, article 103276, 2020.

[8] M. Zhao, Y. M. Huang, P. G. Wang, Y. H. Cao, and X. L. Du, "An analytical solution for the dynamic response of an end-bearing pile subjected to vertical P-waves considering water-pile-soil interactions," *Soil Dynamics and Earthquake Engineering*, vol. 153, article 107126, 2022.

[9] K. Meng, C. Y. Cui, P. Zhang, B. L. Wang, H. L. Liu, and K. P. Wang, "Analytical solution for vertical vibration of partially embedded large-diameter floating pile," *Soil dynamics and Earthquake Engineering*, vol. 167, article 107799, 2003.

[10] O. Maeso, J. J. Aznarez, and F. García, "Dynamic impedances of piles and groups of piles in saturated soils," *Computers & Structures*, vol. 83, no. 10-11, pp. 769–782, 2005.

[11] X. L. Zhou and J. H. Wang, "Analysis of pile groups in a poroelastic medium subjected to horizontal vibration," *Computers and Geotechnics*, vol. 36, no. 3, pp. 406–418, 2009.

[12] C. J. Zheng, S. S. Gan, G. Kouretzis, X. M. Ding, and L. B. Luan, "Vertical vibration of a large diameter pile partially-embedded in poroelastic soil," *Soil Dynamics and Earthquake Engineering*, vol. 139, article 106211, 2020.

[13] C. Y. Cui, K. Meng, C. S. Xu, Z. M. Liang, H. J. Li, and H. F. Pei, "Analytical solution for longitudinal vibration of a floating pile in saturated porous media based on a fictitious saturated soil pile model," *Computers and Geotechnics*, vol. 131, article 103942, 2021.

[14] L. H. Wang and Z. Y. Ai, "Vertical vibration analysis of a pile group in multilayered poroelastic soils with compressible constituents," *International Journal of Geomechanics*, vol. 21, no. 1, article 04020232, 2021.

[15] I. Edelman and K. Wilmanski, "Asymptotic analysis of surface waves at vacuum/porous medium and liquid/porous medium interfaces," *Continuum Mechanics and Thermodynamics*, vol. 14, no. 1, pp. 25–44, 2002.

[16] C. J. Zheng, J. M. Hua, and X. M. Ding, "Torsional vibration of a pipe pile in transversely isotropic saturated soil," *Earthquake Engineering and Engineering Vibration*, vol. 15, no. 3, pp. 509–517, 2016.

[17] G. Chen, Y. Cai, F. Liu, and H. Sun, "Dynamic response of a pile in a transversely isotropic saturated soil to transient torsional loading," *Computers and Geotechnics*, vol. 35, no. 2, pp. 165–172, 2008.

[18] W. J. Ma, Y. Shan, B. L. Wang, and S. H. Zhou, "Torsional vibration of a pile in transversely isotropic saturated soil considering its construction disturbance effect," *International Journal of Structural Stability and Dynamics*, vol. 22, no. 7, pp. 1–20, 2022.

[19] Y. Li and Z. Y. Ai, "Transient analysis of a fixed-head pile group in multi-layered transversely isotropic media due to horizontal loadings," *Computers and Geotechnics*, vol. 127, article 103772, 2020.

[20] S. P. Zhang, C. Cui, and G. Yang, "Vertical dynamic impedance of pile groups partially embedded in multilayered, transversely isotropic, saturated soils," *Soil Dynamics and Earthquake Engineering*, vol. 117, pp. 106–115, 2019.

[21] Q. F. Yan and L. C. Liu, "Horizontal vibration of pile foundation in transversely isotropic soil considering three-dimensional wave effect," *Geotechnical and Geological Engineering*, vol. 41, no. 2, pp. 819–829, 2023.

[22] L. C. Liu and Q. F. Yan, "Study on the lateral dynamic impedance of pile groups in transversely isotropic soil using Novak's plane model," *Journal of Applied Mathematics*, vol. 2023, Article ID 9132285, 13 pages, 2023.

[23] H. J. Ding, W. Q. Chen, and L. Zhang, *Elasticity of Transversely Isotropic Materials*, 2006, Springer Science & Business Media.

[24] D. R. Boer and Z. Liu, "Propagation of acceleration waves in incompressible saturated porous solids," *Transport in Porous Media*, vol. 21, no. 2, pp. 163–173, 1995.

[25] D. R. Boer and Z. Liu, "Plane waves in a semi-infinite fluid saturated porous medium," *Transport in Porous Media*, vol. 16, no. 2, pp. 147–173, 1994.

[26] D. R. Boer, *Theory of Porous Media: Highlights in the Historical Development and Current State*, Springer-Verlag, Berlin, Heidelberg, 2000.

[27] Q. F. Yan and L. C. Liu, "Study on vertical vibration of partially exposed friction pipe pile groups based on fictitious soil pipe pile model," *Geotechnical and Geological Engineering*, vol. 38, no. 6, pp. 6487–6497, 2020.