Hindawi
Journal of Applied Mathematics
Volume 2023, Article ID 8699882, 14 pages
https://doi.org/10.1155/2023/8699882

Research Article

Mathematical Model for Crimes in Developing Countries with Some Control Strategies

Bilali Mataru[1,2] **Okelo Jeconiah Abonyo**[3] **and David Malonza**[4]

[1]*Pan African University Institute for Basic Science, Technology and Innovation, Kenya*
[2]*Muslim University of Morogoro, Tanzania*
[3]*Jomo Kenyatta University of Agriculture and Technology, Kenya*
[4]*South Eastern Kenya University, Kenya*

Correspondence should be addressed to Bilali Mataru; bilalimataru@mum.ac.tz

Received 17 October 2022; Revised 12 December 2022; Accepted 20 January 2023; Published 7 February 2023

Academic Editor: Anum Shafiq

Crime is one among the most challenging problems in most developing countries in which unemployment is among the causes. Not all kind of crimes can be eradicated indeed; this paper is intended to contribute on eradication of unemployment-related crimes in the developing countries by proposing a deterministic mathematical model of unemployment-crime dynamics including vocational training and employment as control measures for crime. The study adopts the epidemiological model concepts on model formulation and model analysis while considering unemployment as main driver of crime. The basic properties of the model are analyzed, and well-posed of the model is established by using the Lipschitz condition. The next-generation matrix is used to obtain the criminal reproduction number which help to derive the conditions for local and global stability of the model. Moreover, the existence of backward and forward bifurcation when the crime reproduction number is equal to one was analyzed by center manifold theory. Simulations of the model are carried out to validate the theoretical part of the model and demonstrate vocational training, and employment strategies are more effective in combating crime when applied simultaneously. The findings suggest that unemployment problem should be addressed in order to reduce the number of unemployed individuals in joining the criminal activities.

1. Introduction

Developing countries are facing a lot of socioeconomic challenges, and one among them is a high level of crime which is a big threat to social development and prosperity. The main reason of this paper is to understand the dynamic of crime, to take a protective and control plan against crime, to make sure that we reduce the prevalence of new criminals, and, if possible, to completely eradicate crimes that are caused by unemployment issues, that is, reducing the risk factors that influence the good civilians to be involved in criminal activities and other illegal activities. Edge et al. [1] defined crime as intentional committing an act which is socially harmful and is precisely defined, forbidden under the law, and also punishable. Crime is a major global challenge that occurs in various categories, but property crimes, financial crimes, violence crime, and moral crime are some of the most common types of crimes that affect many societies in developing countries especially Africa and Latin America [2]. Despite a crime to have many variables, unemployment has been mentioned in many social and criminology literatures as one of the common risk factors to the increase in crime especially in the urban societies [3–9]. According to UN report on youth and unemployment report, an average of 40% of young population who are joining in label forces, militant groups, prostitution, armed robbery, terrorist activities, and other criminal gangs is because of unemployment and losing the hope of getting a decent job. Yaacoub [10] mentioned that most crimes are a result of high level of poverty, unemployment, instability of family, political instability, and demographic aspects as other factors associated with crime growth, while Jonathan et al. [11] also identified income inequality, influence of peer groups, heredity from family, poor parenting, unemployment, and poverty as causative factors of crime. Most of developing countries face economic challenges particularly mass unemployment to young population, which causes economic hardship and inequality within a society and therefore increases the risk of youth to be influenced and then recruited in illegal activities compared to developed countries. According to social learning theories, social interaction is one of the means in which criminality behavior can spread between individuals through peer influences, learning, recruitment, and imitation from criminally involved people with experience [12–16].

Effects of crimes can be observed in many aspects, socially, politically, and economically, depending on its level whether a petty crime or serious crime. A report of United Nations Office on Drugs and Crime in 2017 mentioned crimes as the source of many political and social problems including corruption; insecurity and miserable life cause deaths among civilians, destabilize social development and economic prosperity, and undermine governments [11]. According to Olsson [17], crimes are associated with various kinds of losses: firstly, loss of productivity which involves time and the man power who spend in criminal activities in steady economic production; secondly, victim costs that include loss of property and other valuables, loss of capitals, injuries, and deaths; and lastly, loss of other nontangible properties, for instance, suffering and body pain, destruction of life quality, and other health and psychological problems. A recent study of Jeke et al. [18] considered funds that are used to repair and restore the damages caused by crimes, costs of maintaining prisons, and costs of payment of prosecutors and law enforcement officers which are some of the negative impact of crime in which any governments with good governance cannot ignore.

Mathematical modeling has become one among the important tools in enhancing an understanding of the dynamics to social problems including crime, which is useful for political and economic decision-making regarding the intervention program design for crime eradication and control. The use of compartmental models in connecting mathematics in solving social challenges currently has been greatly useful, for instance, climate change, diseases, unemployment, and crime. Crime model is a new area of research connecting social studies and mathematics. Short et al. [19] developed a first statistical model for burglary crime which was motivated by criminology and social for general understanding. From that point, the series of linear and nonlinear mathematical models for crime has been developed while proposing the transmission of criminality behavior through social interaction with criminals. Soemarsono et al. [20] develop a crime model extending mathematical model for unemployment developed by Munoli and Gani [21] by introducing a crime compartment as the effect of unemployment. Sundar et al. [22] developed a nonlinear mathematical model of crime induced by unemployment. González-Parra et al. [23] develop a nonlinear mathematical model for crime and considered the transmission of criminal behavior as social epidemic while including behavior change as an important parameter, and reducing the contact rate between good civilians and active criminals can help to reduce the number of criminals. Srivastav et al. [24] investigated on crime prediction and prevention while taking into account the severity of corruption and crime in developing countries. The study considered social interaction as the main factor of changes in human behavior through imitation of what is happening in the surroundings and further emphasizes on the law enforcement for crime eradication. Some more studies that considered criminal behavior as the social epidemic which spread though social interactions can be found in [25–28].

This paper is devoted to nonlinear mathematical model for crime due to unemployment focusing the situation in developing countries. The main purpose is to understand the dynamics and the control of crimes that are caused by unemployment problem, for instance property crime, violence crime, and moral crime such as drug abuse and prostitutions in developing countries especially Africa. The model formulation is adopted from Opoku et al. [29] in which they considered crimes committed during festive periods only and where crimes spreading though social interactions. We incorporate vocational training and employment opportunities as control or protective measures for crime instead of law enforcements like jail which are suggested in most existing crime models, for example, González-Parra et al. [23] and Mebratie and Dawed [28]. The law enforcement strategy (imprisonment and detention of criminals) is regarded to be a temporary solution because studies found that the average 65% of released prisoners were arrested again in few years with new offences [30]. Therefore, we propose the solution that will solve the root courses of crime problems which are mostly committed by jobless individuals. We also regard an employment class as the recovery class for criminals which is considered as a social epidemic.

2. Model Formulation

The model formulation is adopted from Opoku et al. [29] by making some modification by suggesting employment and vocational training compartments for controlling crime; it makes a total of five subpopulations, denoted by $N(t)$ with the following subpopulations: unemployed population denoted by $U(t)$, exposed to crime population denoted by $S(t)$, active criminal population denoted by $C(t)$, population of those who are in vocational training $V(t)$, and lastly employed population denoted by $E(t)$. So $N(t) = U(t) + S(t) + C(t) + V(t) + E(t)$.

It is assumed that unemployed population is recruited at the constant rate τ for both educated and uneducated individuals; the natural exit (migration and natural death) from every subclass is given by the rate μ. Individuals move from unemployment class to class exposed with criminal activities at a rate ϕ; we assume that before making the decision of being an active criminal, an individual spends some time to make this decision. It is also assumed that crime is caused by unemployment only; unemployed individuals U and class of those who are exposed to crime/criminal activities S are assumed to become active criminals at a constant rate of β and $\beta(1-\theta)$, respectively, where $\beta = kp$; k is the interaction or contact rate between criminals and unemployed individuals, and p is the probability of every interaction that will lead to criminality behavior to unemployed individual. θ is the efficacy rate (guidance and counselling, religious teachings, and other behavior restoration programs) where $0 \le \theta \le 1$. The rate of unemployed due to criminal activities is given by π, and φ is the death rate due to criminality. The rate of individual joining vocational training is given by ω, and σ is the rate of unemployed individuals joining the vocational training. It is also assumed that employed person cannot commit crime and employment includes both regular and self-employment, where ρ is the rate of unemployed individuals becoming self-employed after obtaining skills from vocational training. The rate of unemployed individuals becoming employed isρ, and the rate of some criminals joining self-employment after finishing the rehabilitation programs (jail, detention, counselling, etc.) is given byε. The associate state variables and list of parameter values are described in Table 1, and Figure 1 shows the flowchart diagram of the aforementioned model formulation.

With the model assumptions, parameters, and formulation, we end up having the system of nonlinear governing equations as follows:

$$\begin{aligned}
\frac{dU}{dt} &= \tau - \beta UC + \pi C - (\mu + \alpha + \sigma + \phi)U, \\
\frac{dS}{dt} &= \phi U - \beta(1-\theta)SC - (\omega + \mu)S, \\
\frac{dC}{dt} &= \beta UC + \beta(1-\theta)SC - (\pi + \mu + \varepsilon + \delta + \varphi)C, \\
\frac{dV}{dt} &= \omega S + \sigma U + \delta C - (\rho + \mu)V, \\
\frac{dE}{dt} &= \varepsilon C + \alpha U + \rho V - \mu E,
\end{aligned} \tag{1}$$

where the transmission rate $\beta = kp$ and $0 \le \theta \le 1$, $U(0) \ge 0$, $S(0) \ge 0$, $C(0) \ge 0$, $V(0) \ge 0$, and $E(0) \ge 0$.

3. Model Properties

In this section, we check if the model is socially and mathematically meaningful by proving positivity and invariant region and the existence and uniqueness of a solution by considering the Lipschitz condition.

3.1. Positivity and Invariant Region of the Model

Lemma 1 (positivity). *Let $\Omega = \{(U, S, C, V, E) \in R_+^5 : U_0 > 0, S_0 > 0, C_0 > 0, V_0 > 0, E_0 > 0\}$; then, the solutions of $\{U(t), S(t), C(t), V(t), E(t)\}$ are all positive for $t \ge 0$.*

Proof. We consider first equation of (1)

$$\frac{dU}{dt} = \tau - \beta UC + \pi C - (\mu + \alpha + \sigma + \phi)U. \tag{2}$$

Equation (2) gives

$$\frac{dU}{dt} \ge -(\mu + \alpha + \sigma + \phi)U. \tag{3}$$

By separating variables in (3), we get

$$\frac{dU}{U} \ge -(\mu + \alpha + \sigma + \phi)dt. \tag{4}$$

Integrating both sides of equation (4) from 0 to t_0,

$$\int \frac{dU}{U} \ge -\int (\mu + \alpha + \sigma + \phi)dt. \tag{5}$$

Equation (5) gives

$$\ln U \ge -(\mu + \alpha + \sigma + \phi)c. \tag{6}$$

Applying e both sides of (6), we get

$$U(t) \ge U_0 e^{-(\mu+\alpha+\sigma+\phi)t} \ge 0. \tag{7}$$

Similar technique is applied to all other state variables S, C, V, and E to have $S(t) \ge S_0 e^{-(\omega+\mu)} \ge 0$, $C(t) \ge C_0 e^{-(\pi+\mu+\varepsilon+\delta+\varphi)t} \ge 0$, $V(t) \ge V_0 e^{-(\rho+\mu)t} \ge 0$, and $E \ge E_0 e^{-\mu t} \ge 0$, which prove the theorem. □

Table 1: Description of state variables and parameters in the crime model.

State variables	Descriptions
U	Unemployed individuals
S	Individuals who are exposed to crime/criminal activities
C	Active criminals
V	Individuals who receive vocational training
E	Employed population
Parameters	**Descriptions**
τ	Recruitment rate to unemployment class
μ	Rate at which individual exists from a population (migration and death)
$\beta = kp$	Transmission rate
k	Rate of interaction/contact rate
p	Probability at which every contact leads to criminality behavior
π	Rate at which criminals cause unemployment
ϕ	Rate at which unemployed individuals join exposed class
φ	Rate death due to criminal behavior
θ	Efficacy rate (factor prevents individual from being a criminal)
δ	Rate of criminals joining vocational training after rehabilitation
α	Rate of unemployed individuals getting a job
ε	Rate in which criminals get job after rehabilitation
ρ	Rate of skilled individual getting employed
σ	Rate of unemployed individual joining vocational training

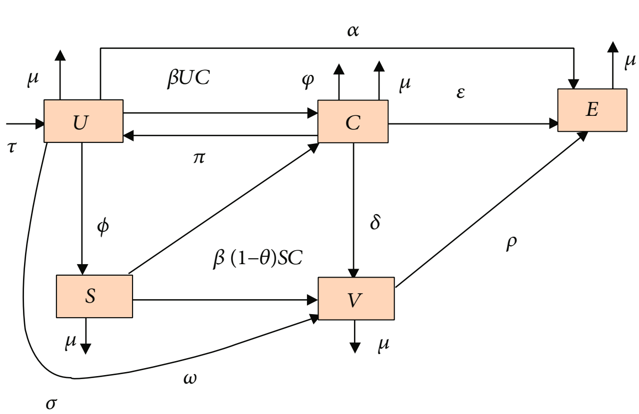

Figure 1: Schematic figure of crime model.

Lemma 2 (invariant region). *All solutions* $(U(t)\,; S(t)\,; C(t)\,; V(t)\,; E(t))$ *of model system (1) for any initial conditions in domain are bounded in the region* Ω.

The total population considered at any time $N(t) = U(t) + S(t) + C(t) + V(t) + E(t)$ gives

$$\frac{dN}{dt} = \tau - N\mu - \varphi C. \tag{8}$$

For the initial condition in $\mathbb{R}_+^5$ for $t \geq 0$, from equation (8), we get

$$\frac{dN}{dt} \leq \tau - N\mu. \tag{9}$$

Separating the variables in (9), we obtain

$$\frac{dN}{\tau - N\mu} \leq dt. \tag{10}$$

Integrating both side of (10) gives

$$\tau - N\mu \geq Ae^{-\mu t}. \tag{11}$$

Obtaining the constant A by applying the initial condition $N(0) = N_0$ to equation (11), we get $A = \tau - \mu N_0$; then,

$$N \leq \frac{\tau}{\mu} - \left(\frac{\tau - \mu N_0}{\mu}\right)e^{-\mu t}. \tag{12}$$

From (12) as $t \longrightarrow \infty$, the total population $N \longrightarrow \tau/\mu$, which imply that $0 \leq N \leq \tau/\mu$. Thus, we conclude that the solution set of system (1) always remains in the domain $\Omega = \{(\mathrm{U}, \mathrm{S}, \mathrm{C}, \mathrm{V}, \mathrm{E}) \in \mathbb{R}_+^5 : 0 \leq N \leq \tau/\mu\}$.

3.2. Existence and Uniqueness of Solution

Lemma 3. *Consider the system of differential equation, for real $f_i \in C$ given in the form of*

$$x' = f(x, t), t > 0, x(0) > 0 \text{ where } x = (x_1 \cdots x_n) \text{ and } f = (f_1 \cdots f_n). \tag{13}$$

A unique solution of (13) on domain $(0, \infty)$ exists if the Lipschitz condition holds for vector function $f(x, t) \in (0,\infty)$ [31].

Theorem 4 (existence and uniqueness). *If model (1) is bounded in $\Omega = \{(U, S, C, V, E) \in \mathbb{R}_+^5 : 0 \le N \le \tau/\mu\}$ with initial conditions $U(0) = U_0$,$S(0) = S_0$,$C(0) = C_0$,$V(0) = V_0$,$E(0) = E_0$, then there exists a unique solution of system (1).*

Proof. To prove the theorem, we show that a family of vectors $X(x, t)$ satisfies the Lipschitz condition in the given domain Ω.

Let X and $\bar{X}$ be the vector functions such that $X = (U, S, C, V, E)$ and $\bar{X} = (\bar{U}, \bar{S}, \bar{C}, \bar{V}, \bar{E})$; we prove that $|F(X) - F(\bar{X})| \le K|X - \bar{X}|$ where K is the Lipschitz constant and the functions $F(X) = (F_1(X), F_2(X), F_3(X), F_4(X), F_5(X))$ and $F(\bar{X}) = F_1(\bar{X}), F_2(\bar{X}), F_3(\bar{X}), F_4(\bar{X}), F_5(\bar{X})$where

$$\begin{aligned}
F_1(\bar{X}) &= \frac{d\bar{U}}{dt} = \tau - \beta\bar{U}\bar{C} + \pi\bar{C} - (\mu + \alpha + \sigma + \phi)\bar{U}, \\
F_2(\bar{X}) &= \frac{d\bar{S}}{dt} = \phi\bar{U} - \beta(1-\theta)\bar{S}\bar{C} - (\omega + \mu)\bar{S}, \\
F_3(\bar{X}) &= \frac{d\bar{C}}{dt} = \beta\bar{U}\bar{C} + \beta(1-\theta)\bar{S}\bar{C} - (\pi + \mu + \varepsilon + \delta + \varphi)\bar{C}, \\
F_4(\bar{X}) &= \frac{d\bar{V}}{dt} = \omega\bar{S} + \sigma\bar{U} + \delta\bar{C} - (\rho + \mu)\bar{V}, \\
F_5(\bar{X}) &= \frac{d\bar{E}}{dt} = \varepsilon\bar{C} + \alpha\bar{U} + \rho\bar{V} - \mu\bar{E}.
\end{aligned} \tag{14}$$

Then,

$$\begin{aligned}
\left|F(X) - F(\bar{X})\right| = {} & \left|F_1(X) - F_1(\bar{X})\right| + \left|F_2(X) - F_2(\bar{X})\right| \\
& + \left|F_3(X) - F_3(\bar{X})\right| + \left|F_4(X) - F_4(\bar{X})\right| \\
& + \left|F_5(X) - F_5(\bar{X})\right|.
\end{aligned} \tag{15}$$

Applying the triangular inequality to (15), we get

$$\begin{aligned}
\left|F(X) - F(\bar{X})\right| \le {} & [(\beta k_1 + \pi) + \beta(1-\theta)k_3 + (\beta k_5 + \beta(1-\theta)k_3 \\
& + (\pi + \mu + \varepsilon + \delta + \varphi)) + \varepsilon]\left|C - \bar{C}\right| \\
& + [\beta k_2(\mu + \alpha + \sigma + \phi) + \phi + \beta k_2 + \sigma]\left|U - \bar{U}\right| \\
& + [(\beta(1-\theta)k_2 + (\omega + \mu)) + \beta(1-\theta)k_2 + \omega]\left|(S - \bar{S})\right| \\
& + [(\rho + \mu) + \rho]\left|V - \bar{V}\right| + \mu\left|E - \bar{E}\right|,
\end{aligned} \tag{16}$$

which gives

$$\begin{aligned}
\left|F(X) - F(\bar{X})\right| \le {} & K_1\left|C - \bar{C}\right| + K_2\left|U - \bar{U}\right| + K_3\left|(S - \bar{S})\right| \\
& + K_4\left|V - \bar{V}\right| + K_5\left|E - \bar{E}\right|.
\end{aligned} \tag{17}$$

$K = \max\ \{K_1, K_2, K_3, K_4, K_5\}$ is the Lipschitz constant; then, equation (17) gives

$$\left|F(X) - F(\bar{X})\right| \le K\left|X - \bar{K}\right|. \tag{18}$$

The Lipschitz condition is satisfied by the function $F(X)$; therefore, by Lemma 3, the system of ordinary differential equation (1) with given initial conditions $U_0 \ge 0$, $S_0 \ge 0$, $C_0 \ge 0$, $V_0 \ge 0$, and $E_0 \ge 0$ exists and is unique. □

4. Model Analysis

4.1. Crime-Free Equilibrium Point. The crime-free equilibrium (CFE) point, the point where the society is free from crime, exists and is given by

$$C_0 = \left(\frac{\tau}{Q}, \frac{\phi\tau}{Q(\omega + \mu)}, 0, \frac{\omega\phi\tau + \sigma(\omega + \mu)\tau}{Q(\omega + \mu)(\rho + \mu)}, \frac{\alpha\tau(\omega + \mu)(\rho + \mu) + \rho(\omega\phi\tau + \sigma\tau(\omega + \mu))}{\mu Q(\omega + \mu)(\rho + \mu)}\right), \tag{19}$$

where $Q = \mu + \alpha + \sigma + \phi$.

4.1.1. The Crime Basic Reproduction Number. The crime basic reproduction number (R_0) is obtained by using the next-generation matrix and is given by

$$R_0 = \frac{\beta\tau(\omega+\mu) + \phi\tau\beta(1-\theta)}{(\omega+\mu)(\mu+\alpha+\sigma+\phi)(\pi+\mu+\varepsilon+\delta+\varphi)}. \tag{20}$$

4.2. Local Stability of CFE

Theorem 5. *The crime-free equilibrium point is locally asymptotically stable provided $R_0 < 1$ and unstable when $R_0 > 0$.*

Proof. The Jacobian matrix of the model is given by

$$J(U, S, C, V, E) = \begin{bmatrix} -Q & 0 & -\beta U + \pi & 0 & 0 \\ \phi & -(\omega+\mu) & -\dfrac{\beta(1-\theta)\phi\tau}{Q(\omega+\mu)} & 0 & 0 \\ 0 & 0 & \dfrac{\beta\tau(\omega+\mu) + \beta\phi\tau(1-\theta) - PQ(\omega+\mu)}{Q(\omega+\mu)} & 0 & 0 \\ 0 & \omega & \delta & -(\rho+\mu) & 0 \\ \alpha & 0 & \varepsilon & \rho & -\mu \end{bmatrix}. \tag{21}$$

The eigenvalues of a nonlinear crime model (1) are $\lambda_1 = -\mu$, $\lambda_2 = -(\rho+\mu)$, $\lambda_3 = -(\omega+\mu)$, and $\lambda_4 = -(\mu+\alpha+\sigma+\phi)$, and the last fifth eigenvalue is $(\beta\tau(\omega+\mu) + \beta\phi\tau(1-\theta) - PQ(\omega+\mu))/(Q(\omega+\mu))$. It can be seen that first to fourth eigenvalues are all negative, and if the fifth eigenvalue is negative, the following should hold:

$$\frac{\beta\tau(\omega+\mu) + \beta\phi\tau(1-\theta) - PQ(\omega+\mu)}{Q(\omega+\mu)} < 0. \tag{22}$$

(22) implies that

$$R_0 = \frac{\beta\tau(\omega+\mu) + \phi\tau\beta(1-\theta)}{(\omega+\mu)(\mu+\alpha+\sigma+\phi)(\pi+\mu+\varepsilon+\delta+\varphi)} < 1. \tag{23}$$

From equation (23), we conclude that the crime-free equilibrium point is asymptotically stable when $R_0 < 1$. □

4.3. Crime Equilibrium Point. The crime equilibrium (CE) point is the point where crime persists in the population and is given by

$$\begin{aligned} U^* &= \frac{\tau + \pi C^*}{(\mu+\alpha+\sigma+\phi) + \beta C^*}, \\ S^* &= \frac{\phi(\pi C^* + \tau)}{B}, \\ V^* &= \frac{A + B\delta C^*}{B(\rho+\mu)}, \\ E^* &= \frac{\rho(A + B\delta C^*) + \alpha(\tau+\pi C^*)[(\rho+\mu)(\beta(1-\theta)C^* - (\omega+\mu))] + B\varepsilon(\rho+\mu)C^*}{B\mu(\rho+\mu)}, \end{aligned} \tag{24}$$

where

$$A=\omega\phi(\pi C^*+\tau)+(\beta(1-\theta)C^*+(\omega+\mu))\sigma(\pi C^*+\tau),$$
$$B=(\beta(1-\theta)C^*+(\omega+\mu))(\beta C^*+\mu+\alpha+\sigma+\phi),$$
$$\beta^2(1-\theta)PC^{*2}+n(\omega+\mu)(m-R_0)C^*+PQ(\omega+\mu)(1-R_0)=0,$$
$$P=\pi+\mu+\varepsilon+\delta+\varphi,$$
$$Q=\mu+\alpha+\sigma+\phi,$$
$$m=\frac{\beta\tau}{\pi Q},$$
$$n=\frac{SQ\pi}{\tau}. \tag{25}$$

Proposition 6.

(1) If $m\geq 1$, then the system crime model (1) is exhibiting transcritical bifurcation

(2) If $m<1$, then the system crime model (1) is exhibiting backward bifurcation

Proof.

(1) When $m\geq 1$, the following are existing possibilities:

(i) If the value of $R_0>1$, then the constant $PQ(\omega+\mu)(1-R_0)$ is negative, and then, the polynomial has a unique $+ve$ solution

(ii) If the value of $R_0\leq 1$, then the coefficient $n(\omega+\mu)(m-R_0)\geq 0$ and $PQ(\omega+\mu)(1-R_0)\geq 0$, and since the coefficient $[\beta^2(1-\theta)P]$ is always positive because $0<\theta<1$, then the polynomial has no positive solution

(2) When $m<1$, the following are existing possibilities:

(i) If the value of $R_0\geq 1$, then the constant $PQ(\omega+\mu)(1-R_0)\leq 0$ and then the polynomial has a unique $+ve$ solution

(ii) If the value of $R_0\leq m$, then the constant $PQ(\omega+\mu)(1-R_0)$ is positive and the coefficient $n(\omega+\mu)(m-R_0)\geq 0$. This implies that the polynomial has no positive solution

(iii) If $m<R_0$, we determine the roots by considering the discriminant of quadratic polynomial $D(R_0)=b^2-4ac$. We can note that $D(m)=-4ac$ and also $D(1)=b^2$. Hence, there exist a number R_e between m and 1 such that $D(R_e)=0$ and also $D<0$ for $R_0\in(m,R_e)$ and $D>0$ for $R_0\in(R_e,1)$ which imply that if $m<R_0<R_e$, then the polynomial possesses no $+ve$ solution; if $R_0=R_e$, then $D=0$ and $b<0$ and then the polynomial has only one positive solution; and when $R_e<R_0<1$, it implied that the polynomials in the two distinct solutions are real and positive since $c>0$ and $b<0$

From Proposition (6), there exist two crime steady states for$R_0\in(R_e,1)$; hence, we use bifurcation analysis to study the stability of the points as proposed in [32]. □

4.4. Bifurcation Analysis at $R_0=1$. The backward bifurcation at R_0 for model (1) is analyzed by using center manifold theory, and the bifurcation parameter is the transmission rate $\beta=\beta^*$ at $R_0=1$. Therefore, the bifurcation parameter is given by

$$\beta=\beta^*=\frac{(\omega+\mu)(\mu+\alpha+\sigma+\phi)(\pi+\mu+\varepsilon+\delta+\varphi)}{\tau(\omega+\mu)+\phi\tau(1-\theta)}. \tag{26}$$

The Jacobian matrix of the model at criminal-free equilibrium point calculated at $\beta=\beta^*$ is given as

$$J=\begin{bmatrix}-(\mu+\alpha+\sigma+\phi) & 0 & -\beta^*U+\pi & 0 & 0\\ \phi & -(\omega+\mu) & -\beta^*(1-\theta)S & 0 & 0\\ 0 & 0 & \beta^*U+\beta^*(1-\theta)S-P & 0 & 0\\ 0 & \omega & \delta & -(\rho+\mu) & 0\\ \alpha & 0 & \varepsilon & \rho & -\mu\end{bmatrix}, \tag{27}$$

where $P = \pi + \mu + \varepsilon + \delta + \varphi$. The Jacobian matrix (27) has a zero eigenvalue (simple eigenvalue), and all other eigenvalues have a $-ve$ real part; then, by using the center manifold theory, we can be in a position to decide the local stability of (1) as proposed in [33]. Initially, we determine the left and right eigenvectors corresponding to the eigenvalues which are given by

$$\begin{aligned}
w_3 &= 1, \\
w_1 &= \frac{-\beta^* \tau + \pi(\mu + \alpha + \rho + \phi)}{(\mu + \alpha + \rho + \phi)^2}, \\
w_2 &= \frac{(-\beta^* \tau\phi + \pi\phi(\mu + \alpha + \rho + \phi))(\omega + \mu) - \beta^* \phi\tau(1 - \theta)(\mu + \alpha + \sigma + \phi)}{(\omega + \mu)^2(\mu + \alpha + \sigma + \phi)^2}, \\
w_4 &= \frac{\omega(-\beta^* \tau\phi + \pi\phi(\mu + \alpha + \rho + \phi)(\omega + \mu)(\mu + \alpha + \sigma + \phi) - \beta^* \phi\tau(1 - \theta)) + \delta\left((\omega + \mu)^2(\mu + \alpha + \sigma + \phi)\right)}{(\rho + \mu)(\mu + \alpha + \sigma + \phi)^2(\omega + \mu)^2}, \\
w_5 &= \frac{\alpha w_1 + \varepsilon w_3 + \rho w_4}{\mu}.
\end{aligned} \tag{28}$$

And the left eigenvector corresponding to simple eigenvalue is given by $v = (v_1, v_2, v_3, v_4, v_5) = (0\,0\,1\,0\,0)$ satisfying the condition $w.v = 1$ [33]. The value of a and b is given by the following:

$$\begin{aligned}
a = {} & v_3 w_1 w_3 \frac{\partial^2 f_3}{\partial x_1 \partial x_3} + v_3 w_3 w_1 \frac{\partial^2 f_3}{\partial x_3 \partial x_1} \\
& + v_3 w_2 w_3 \frac{\partial^2 f_3}{\partial x_2 \partial x_3} + v_3 w_3 w_2 \frac{\partial^2 f_3}{\partial x_3 \partial x_2}.
\end{aligned} \tag{29}$$

From (29), we get

$$a = 2\beta^* \left[\frac{\pi Q(\omega + \mu)((\omega + \mu) + \pi\phi(1 - \theta)) - \left[\beta^* \tau(\omega + \mu)^2 + \beta^* \tau\phi(1 - \theta)((\omega + \mu) + (1 - \theta)Q\right]}{(\omega + \mu)^2 Q^2}\right], \tag{30}$$

where $Q = \mu + \alpha + \rho + \phi$ and state variables $(U, S, C, V, E) = (x_1, x_2, x_3, x_4, x_5)$.

Suppose $X = \pi Q(\omega + \mu)((\omega + \mu) + \pi\phi(1 - \theta))$ and $Y = \beta^* \tau(\omega + \mu)^2 + \beta^* \tau\phi(1 - \theta)((\omega + \mu) + (1 - \theta)Q$.

Since all parameter are positive, we conclude that $a > 0$ if $X > Y$ and $a < 0$ if $X < Y$.

$$\begin{aligned}
b = {} & v_3 w_1 \frac{\partial^2 f_3}{\partial x_1 \partial \beta} + v_3 w_2 \frac{\partial^2 f_3}{\partial x_2 \partial \beta} + v_3 w_3 \frac{\partial^2 f_3}{\partial x_3 \partial \beta} \\
& + v_3 w_4 \frac{\partial^2 f_3}{\partial x_4 \partial \beta} + v_3 w_5 \frac{\partial^2 f_3}{\partial x_5 \partial \beta}.
\end{aligned} \tag{31}$$

From (31), we get

$$b = \frac{\tau(\omega + \mu) + \tau\phi(1 - \theta)}{(\mu + \alpha + \rho + \phi)(\omega + \mu)}. \tag{32}$$

Since all parameters are positive and $0 \leq \theta \leq 1$, we conclude that $b > 0$. Thus, the following theorem is established on existence on bifurcation at $R_0 = 1$.

Theorem 7. *Crime model (1) exhibits a backward bifurcation at $R_0 = 0$ if $a > 0$ and $b > 0$.*

Figure 2 illustrates numerically the existence of backward bifurcation at $R_0 = 1$ by using the parameter value in Table 2, where $R_c = 0.2$ is the critical value; the red line in Figure 2 shows unstable state, and the blue line shows the stable state of the equilibrium points; this implies that one of the two crime equilibrium points is stable when $R_0 < 1$ [32]. This implies that in order to eradicate crime, the threshold parameter of crime reproduction number to be less than unity is not sufficient enough to guarantee the crime-free population. Therefore, extra control measures are necessary to reduce the value of R_0 below 0.2 which is

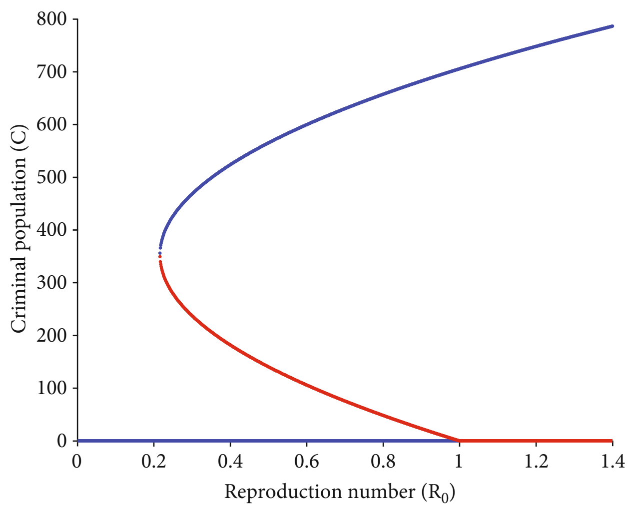

Figure 2: Backward bifurcation.

Table 2: Description of state variables and parameters in the model.

Parameters	Descriptions	Sensitivity index
τ	Recruitment rate to unemployment class	+1
β	Transmission rate	+1
μ	Rate at which individuals exist from a population	-0.8354
δ	Rate of criminal joining vocational training after rehabilitation	-0.2723
ε	Rate in which criminals get job after rehabilitation	-0.2927
θ	Efficacy rate	-0.2125
σ	Rate of unemployed individual joining vocational training	-0.0426
ϕ	Rate at which unemployed individuals joining exposed class	+0.0251
α	Rate of unemployed individuals getting a job	-0.0272
φ	Rate death due to criminal behavior	-0.0173
π	Rate at which criminals cause unemployment	+0.0043

the critical value (R_c); this effort will also avoid the possibility of getting double crime equilibrium points, and therefore, crime will be suppressed by threshold $R_0 < 1$.

5. Sensitivity Analysis

In this subsection, we perform sensitivity analysis in order to determine and understand the influence of each parameter to the crime reproduction number and the model output in general. Dealing with imperfect information in the environment where a kind of uncertainty arise, we need a depth analysis for every parameter in answering important questions about a model behavior and its output. Hence, understanding the sensitivity index for every parameter has a great contribution in decision-making such as identifying the most effective intervention to be implemented in order to reduce R_0 and therefore eradicating crime problem in a society.

Table 3: Description of parameter estimation in the model.

Parameter	Estimated value	Source
τ	5,000	[35]
μ	0.115	[29]
β	0.58	[29]
π	0.001	Assumed
ϕ	0.1265	[29]
φ	0.004	[20]
θ	0.44	[29]
δ	0.015	Assumed
ε	0.5	Varies
ω	0.015	Assumed
α	9.6%	[37]
ρ	3.5%	Assumed
σ	0.35	Assumed

Definition 8. The parameter that brings a significant change in the dynamical system after small change of it is called sensitive parameter.

Let us consider the crime reproduction number in (20); thus, the sensitivity indices of parameters p with respect to R_0 are obtained by using normalized forward sensitivity method which is given by

$$\Lambda_p^{R_0} = \frac{p}{R_0}\frac{\partial R_0}{\partial p}. \tag{33}$$

The sensitivity indices of crime reproduction number with respect to every parameter of the model are given in Table 2.

From Table 2, we have positive and negative sensitivities of parameters which imply the following: sensitivity analysis shows that τ and β have the sensitivity index of +1, which mean that 1% increase of recruitment rate to unemployment class and transmission rate will produce 1% increase of the criminal reproduction number; regarding the sensitivity index of -0.8354 for μ, these imply that 1% increase to the rate at which individuals exist from a population will produce 0.8354% decrease of the criminal reproduction number. The same interpretation is applied to all other remaining parameters δ, θ,σ, ϕ, α, φ, ε, and π [34].

From Table 2, we can infer that the number of criminals can be reduced from the population by decreasing the recruitment rate to unemployed population, transmission rate, and the rate of unemployed individual joining exposed class. Also, we can control crime by increasing recruitment

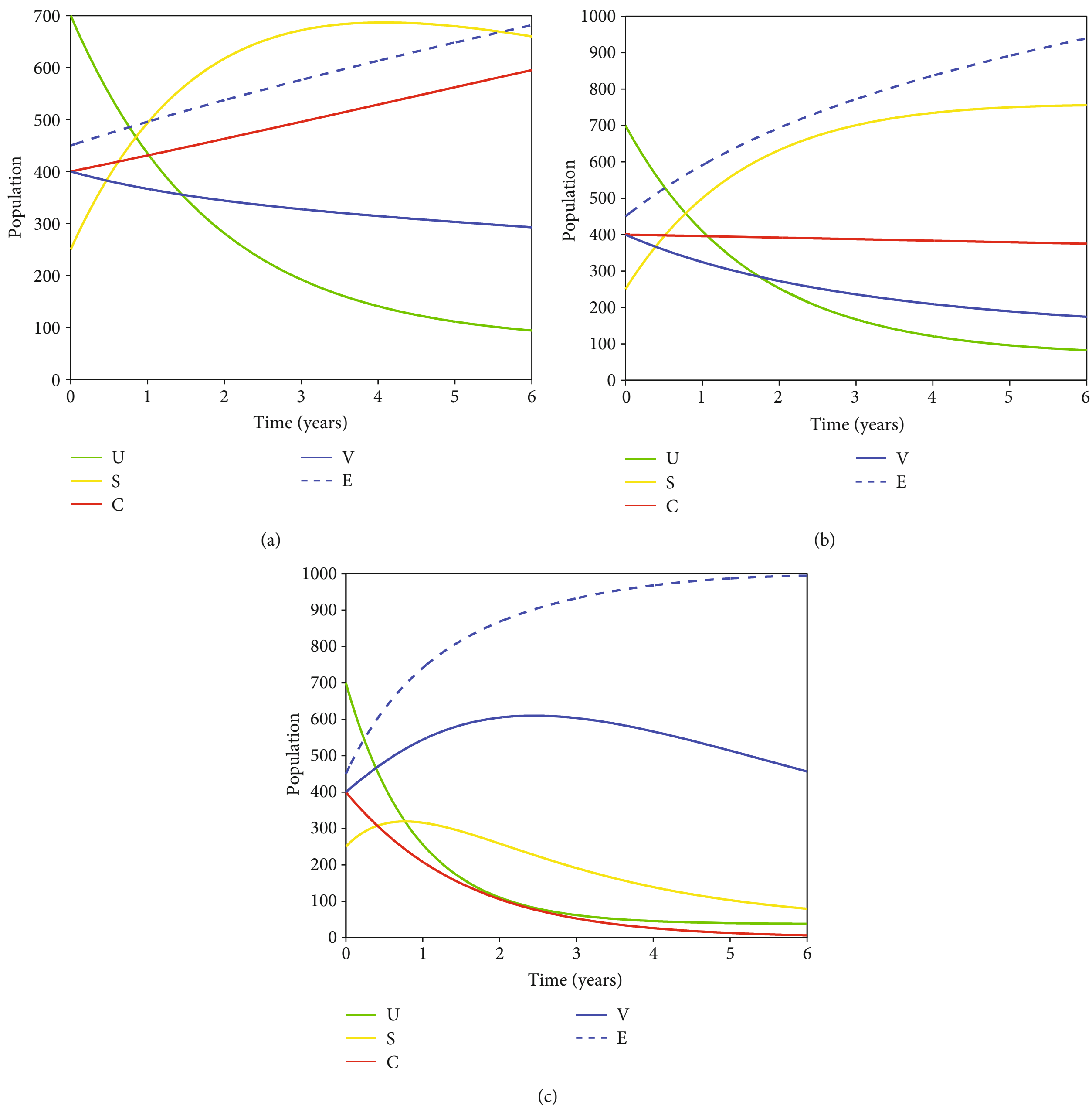

Figure 3: (a) Simulation result showing the population when $R_0 = 1.4045$. (b) Simulation result showing the population when $R_0 = 0.3873$. (c) Simulation result showing the population when $R_0 = 0.0105$.

rate to vocational training, efficacy rate, rate of individuals joining vocational training, and employment rate simply because by doing so the value of crime reproduction number will be decreasing and hence reduce the number of new criminals joining the general population.

6. Simulation

In this section, the numerical simulation of the model is given to establish the validity of the analytical part studied in previous sections using MATLAB R2013b; the ode45 solver is used for model simulation. The parameter values that are used for model simulations are presented in Table 3, and most of parameters are adopted as presented in [29], ILO, and assumptions.

The simulation of the model illustrated by Figures 3(a)–3(c) shows the effect of the crime reproduction number for $R_0 > 1$, $R_0 < 1$, and $R_0 \leq R_c$, respectively, to the population of criminals as analyzed from bifurcation diagram in Figure 2. Figures 4–6 show the effect of different values of parameters in crime control for model (1) based on their sensitivity. And Figures 7 and 8 are showing the relationship between the population of unemployment and the population of criminals.

Figure 3(a) shows that for $R_0 > 1$ which is $R_0 = 1.4045$, exposed population $S(t)$ and active criminal $C(t)$ populations are increasing, while population in vocational training

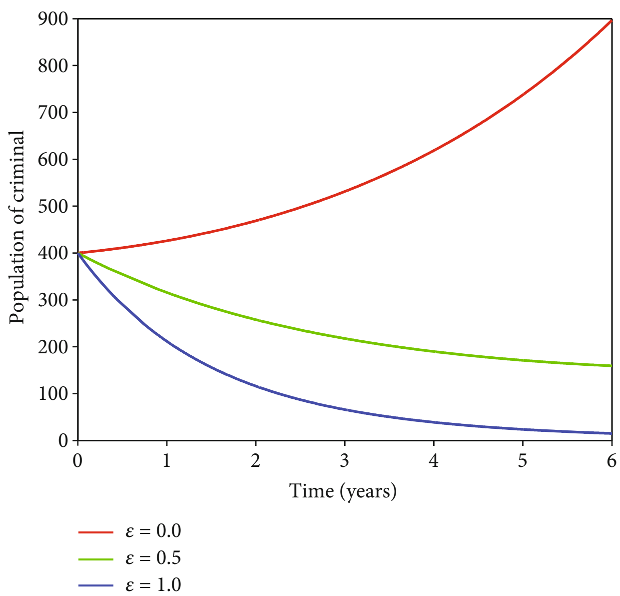

Figure 4: Simulation result showing the impact of vocational training on criminal population.

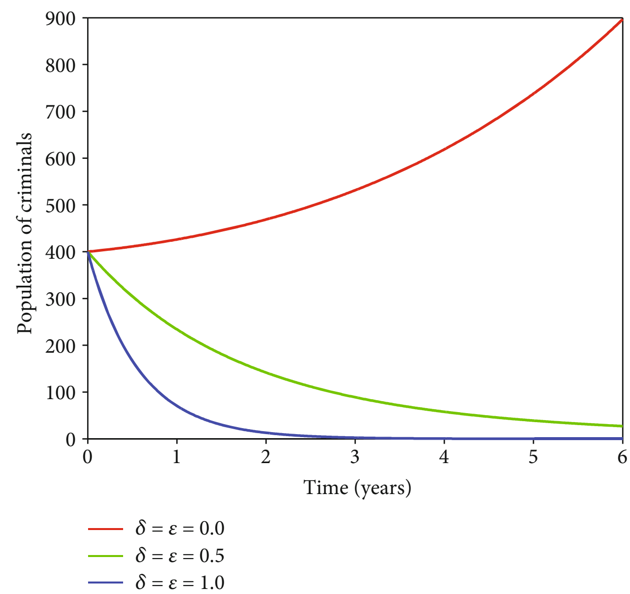

Figure 6: Simulation result showing the impact of vocational training and employment on crime population.

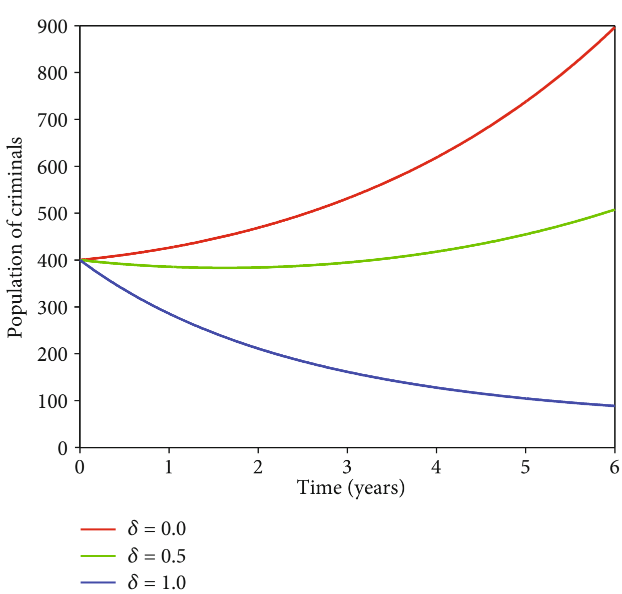

Figure 5: Simulation result showing the impact vocational training on crime population.

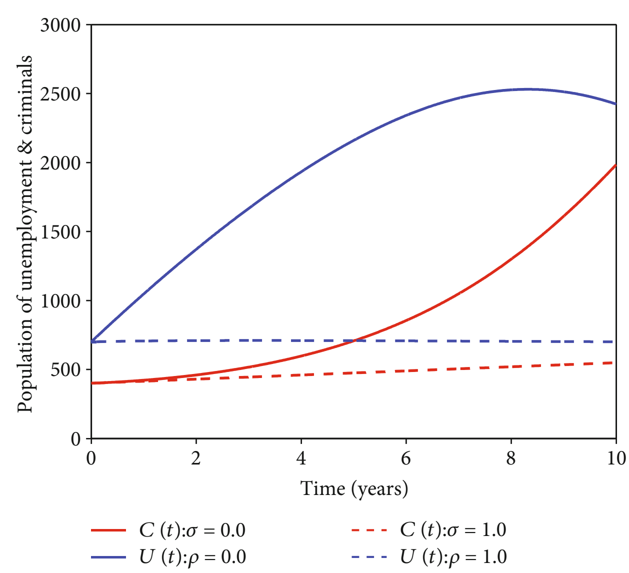

Figure 7: Simulation result showing the population of unemployment $U(t)$ and crime $C(t)$.

$V(t)$ and unemployed population $U(t)$ are decreasing but not sufficient enough to eradicate crime.

Figure 3(b) shows that for a criminal reproduction number $R_0 = 0.3873 < 1$, the population of criminals is decreasing, but this threshold is not sufficient enough to eradicate crime due to the existence of backward bifurcation in the model (1). The figure also shows that the population of unemployed $U(t)$ and exposed population $S(t)$ are decreasing at a slow rate that cannot guarantee to total crime eradication.

Figure 3(c) shows the subpopulations when the crime reproduction number R_0 falls below the critical value $R_0 \leq R_c$ where $R_0 = 0.0105$; it is clear that the population of exposed $S(t)$, active criminals $C(t)$, and unemployed individuals $U(t)$ are decreasing while employed population is also raised rapidly. The value R_0 was reduced after increasing the employment rates α,ρ, and ε and enrollment rates to vocational training σ, ω, and δ. At this point, R_0 is sufficient enough to guarantee the crime eradication in six years.

Figure 4 shows the impact of employment rate ε to criminal population as a rehabilitation process for criminals and crime controls; in the simulation in Figure 4, we observe that as the employment rate increases, the population of criminals decreases from 900 criminals to around 20 criminals

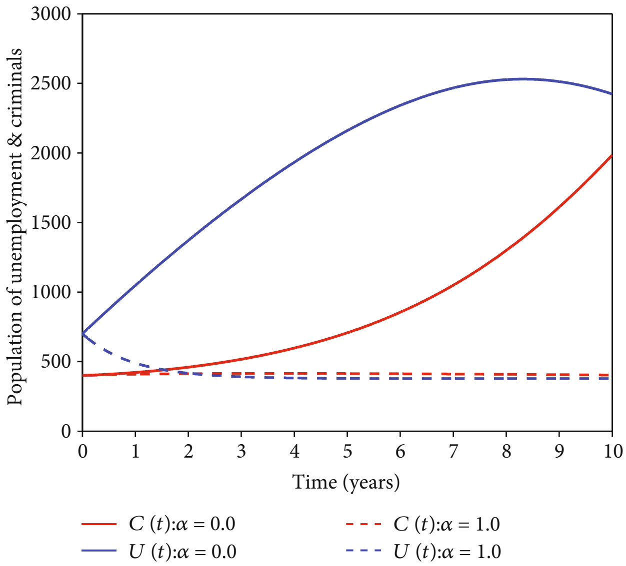

FIGURE 8: Simulation result showing the population of unemployment $U(t)$ and crime $C(t)$.

when the measure is fully implemented; this is because most criminals will find an alternative job which will enable them to afford life expenses and quit from unemployment-related crimes. From the figure, when the employment rate $\varepsilon = 0$ means no control measures were used, then the number of criminals increases exponentially from 400 individuals to 900, which is an increment of 45% of criminals in a society. When the rate employment ε increases from $\varepsilon = 0.5$ to $\varepsilon = 1$, this exhibited the positive results to the population of criminals as successful decline exponentially by 98% which is a lowest level in six years.

Figure 5 shows the effectiveness of vocational training to the population of criminals; we observe that as the enrolment rate to vocational training increases, the population of criminals declines because unemployed individuals will seek to obtain life skills which will enable them for self-employment or regular employment. We can observe in Figure 5 that, without any control measures, the number of population of criminals raised exponentially from 400 individuals to 900 within a period of 6 years, but when the vocational training was employed at the rate of $\delta = 0.5$ to $\delta = 1$, the number of criminals decreases to a tolerance level of 90 individuals from 900 within a period of 6 years, which is equivalent of 90% decrease of criminals in the population.

Figure 6 shows the impact of vocational training δ and employment rate ε when applied to the population of crime simultaneously. It is apparent to the figure that the early intervention by increases of employment rate and enrolment rate to vocational training from $\delta = \varepsilon = 0.5$ to $\delta = \varepsilon = 1$ demonstrates the great impact to the fight against crimes. Within a period of 3 years, the number of criminals was completely eradicated because the population will obtain an optimal vocational skills which will enable them to face unemployment challenge by engaging in self-employment or regular employment, therefore reducing the number of jobless and hence suppressing the number of active criminals. This combination approach of two strategies for crime eradication is more effective in the fight against crime compared to single approach of employment and vocational training strategies when used separately.

These results agree with those results represented in [29]; when education, detention, and sacking were applied to the crimes committed during the period of festivals, the population of criminals was reduced from the general population. The combination of strategies (education, detention, and sacking) also gives the best results compared to the individual control strategy.

Figures 7 and 8 show how unemployed population and population of criminals are related. It can be observed in Figure 7 that the population of unemployed is direct proportional to the population of criminals. $U(t)$ in full line represent the population of unemployed individuals who were initially 700 people, but after 10 years without control, the number raised by 70.8% to 2400 individuals which trigger the exponential increase of criminal population by 80% from 400 to 2000 people without control represented by $C(t)$ in full line. But the fall of unemployed population leads to the proportional decrease to criminals in the population due to the implementation of control measures which includes enrolment rate of unemployed people to vocational training σ and employment rate of skilled individuals ρ. The dashed lines show the decrease of unemployed population by 72% from 2500 individuals to 700 which lead to the fall of criminal population by 75% from 2000 criminals to 500.

Figure 8 shows that unemployed population is increasing from 700 individuals to 2500 in 10 years and the population of criminals is also raised exponentially by 80% from 400 individuals to 2000. When the number of unemployed individuals falls from 2500 individuals to 400 due to the implementation of employment strategy to the unemployed population, the number of criminals also decreases from 2000 to 400 within 10-year period. Hence, from Figures 7 and 8, we can conclude that in order to succeed with the fight against crime, unemployment problem should not be ignored.

7. Conclusion

In this paper, a deterministic mathematical model for crime and unemployment dynamics is formulated and analyzed, incorporating vocational training and employment as the control measures for crimes that are mainly caused or motivated by unemployment which includes property crimes, violence, and moral crime (prostitutions and drug abuse and trafficking), and we focused on the situation of developing countries. The crime reproduction number R_0 of the model was obtained and provides the conditions (threshold) for the crimes to stick in population or to be eradicated. Stability analysis of equilibrium points was considered and CFE was found to be asymptotically stable when $R_0 < 1$, and the existence of bifurcation at $R_0 = 1$ was analyzed where forward and backward bifurcation exists with some conditions. We found that $R_0 < 1$ is necessary but not sufficient enough to eradicate crime and more control measure was needed to reduce the rate of unemployed individuals to join in criminal activities in order the threshold to be falling below critical value R_e. The sensitivity analysis of model parameters revealed that unemployment and the contact rate between criminals and jobless individuals are likely to increase criminals. Also, availability of vocational training opportunities and employment opportunities is likely to prevent people get involved in criminal activities. The simulation of the model revealed that the number of criminals decreases as more of unemployed people get job. Also, various control strategies were tested and show that the combination of vocational training together and creation of more employment opportunities are more efficient in crime eradication. We conclude that, in order to get the best results on the fight against crimes, addressing of unemployment problem should be prioritized.

8. Recommendation

Fighting against crime is among the most expensive and challenging tasks in which most governments incur, because it needs a budget to pay for police officers, judicial sector, maintaining prisons, etc. States could invest on provision of vocational training mostly to young population which will enable them to obtain life skills for self-employment and regular employment. Also, creating more job opportunities will reduce a population most likely to involve in crimes, reducing the costs of fight against crimes, and at the same time, it could improve the wellbeing of the people particularly young population. Optimal control analysis can be considered to extend this work.

Data Availability

The data used in the model simulation were adopted from literatures as summarized in Table 2 (Section 5).

Conflicts of Interest

The authors declare that there is no conflict of interest regarding the publication of this article.

Acknowledgments

The authors are so grateful and appreciate the African Union for providing the financial support through Pan African University Scholarship in every step of this work.

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