Hindawi
Journal of Applied Mathematics
Volume 2023, Article ID 9132285, 13 pages
https://doi.org/10.1155/2023/9132285

Research Article

Study on the Lateral Dynamic Impedance of Pile Groups in Transversely Isotropic Soil Using Novak's Plane Model

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Received 5 December 2022; Revised 4 February 2023; Accepted 7 March 2023; Published 16 March 2023

Academic Editor: Fernando Simoes

In order to consider the effect of anisotropy of soil around the pile on the lateral vibration of pile groups, the soil around the pile is regarded as a transversely isotropic medium, and a lateral dynamic interaction model of pile-pile in transversely isotropic soil is established. According to Novak's plane assumption and wave propagation theory, the lateral vibration of transversely isotropic soil layer is solved by introducing potential function and using mathematical and physical means, and the attenuation function of lateral displacement of free field is given. The Dobry and Dazetas simplified solution of attenuation function is different from that of solution of plane model. The pile-pile horizontal dynamic interaction factor in transversely isotropic soil is obtained by using the initial parameter method and Krylov function. The horizontal dynamic impedance of pile groups is obtained by using the pile-pile superposition principle. The change rule of the lateral displacement attenuation function of transversely isotropic soil with frequency is related to the direction and frequency. The ratio G_{hv} of the shear modulus in the lateral plane to the shear modulus in the vertical plane and the pile spacing S/d have a great impact on the lateral vibration of pile groups, and when the pile spacing is large, the curves of attenuation function varying with frequency fluctuate greatly. The ratio of elastic modulus of pile to vertical plane shear modulus of soil E_p/G_v has an effect on the lateral stiffness of pile groups, which is related to frequency, while the effect on dynamic damping is not affected by frequency. The difference of mechanical properties on different surfaces of soil around the pile has a great influence on the lateral vibration of pile groups in transversely isotropic soil, and the influence of the anisotropy on the attenuation function of the lateral displacement and the dynamic impedance cannot be ignored.

1. Introduction

The research on the dynamic characteristics of pile foundation is related to the safety and stability of building structures. Therefore, since the 1960s, Novak [1], Novak and Nogami [2], Nogami and Novák [3], El Naggar and Novak [4], Wu et al. [5], Zheng et al. [6], Cui et al. [7], Zhang et al. [8], and others have conducted systematic theoretical research on the vibration of single pile. In practical engineering, pile foundation usually appears in the form of pile groups. Under the dynamic excitation of wind, waves, and earthquake, the dynamic response of pile groups is very complex, so the research on the dynamic characteristics of pile groups is relatively late. The research on the dynamic response of pile groups under various dynamic loads has important theoretical value and engineering significance for ensuring the stability and safety of the structure.

At present, the main research methods on the dynamic characteristics of pile groups are direct method and superposition method based on interaction factors. The direct method mainly adopts continuum mechanics method, finite element method, or boundary element method. Tajimi [9] and Koo et al. [10] studied the vibration of pile groups and pile group-structure by using the continuum mechanics method and the wave propagation theory. Sen et al. [11] proposed a boundary element formula for dynamic analysis of single pile and pile groups under axial and lateral loads. Maeso et al. [12] proposed a three-dimensional boundary element model for calculating the dynamic stiffness coefficient of pile groups in two-phase saturated elastic soil. Zhou

and Wang [13] used the three-dimensional wave principle of saturated soil proposed by Biot and the Muki and Sternberg's methods to study the dynamic response of pile groups in saturated soil under horizontal harmonic loads. Fattah et al. [14] studied the dynamic behavior of pile group model in two-layer sandy soil subjected to lateral earthquake excitation. Wang and Ai [15] studied the vertical vibration of pile groups with rigid caps in multilayer porous-saturated soil based on the finite element method, and the pile is regarded as a one-dimensional rod. Ma et al. [16] considered the soil as a three-dimensional axisymmetric continuum, considered the effects of radial and vertical displacement and saturation on the dynamic shear modulus of soil, and studied the vertical dynamic impedance of end-bearing pile groups in homogeneous unsaturated soil. Finite element method and boundary element method need special analysis programs, and the amount of calculation is huge, so they are difficult to be applied to large-scale problems. At present, the interaction factor method is the most effective method to calculate the dynamic impedance of pile groups. Poulos [17] first put forward the concept of interaction factor and carried out the static calculation of pile groups on this basis. Kaynia [18] extended the principle of static interaction factor proposed by Poulos to the problem of dynamic interaction of pile groups and put forward the concept of dynamic interaction for the first time. Its solution is generally considered as an exact solution. Based on the simplified plane strain model, Gazetas and Dobry [19] gave an approximate expression of the wave displacement attenuation function of soil layer. Gazetas and Makris [20] and Makris and Gazetas [21] used the dynamic Winkler model to put forward a simple analysis method for calculating the axial and lateral dynamic steady-state responses of frictional pile groups in homogeneous foundation. This method further remedied the defect of Gazetas and Dobry in the calculation of dynamic interaction factors and gave the three-step method of pile-soil-pile interaction to specifically calculate and analyze the dynamic characteristics of pile groups. Liu et al. [22] used a simple Winkler model to derive the pile-soil-pile dynamic interaction factor and studied the dynamic response of partially buried pile groups in layered saturated soil under horizontal simple harmonic loads. Zhang et al. [23] adopted the three-phase porous elastic medium motion theory of unsaturated soil and the superposition method based on dynamic interaction factors to give the dynamic impedance of pile groups in unsaturated soil under horizontal vibration load in the frequency domain. In most of the previous studies on the dynamic properties of single pile and pile groups, the soil around the pile is regarded as an isotropic medium.

In the process of soil deposition, due to the orientation relationship of flat particles and the directionality of their arrangement, the properties (elastic modulus, shear modulus, and Poisson's ratio) of soil in the vertical and lateral directions are different, and the soil shows different characteristics in all directions. The vertical modulus of soil is often smaller than the lateral modulus, but the lateral modulus is isotropic. Therefore, regarding this kind of soil as transversely isotropic soil is more in line with the engineering practice. Chen et al. [24], Shahmohamadi et al. [25], Zheng et al. [6], Li and Ai [26], and Ye and Yong Ai [27] studied the torsional vibration of a single pile in transversely isotropic soil. Due to the complexity of lateral and vertical vibration of transversely isotropic soil and the difficulty of mathematical solution, there are few studies on lateral and vertical vibration of transversely isotropic soil and less on pile-pile dynamic interaction and vibration of pile groups in transversely isotropic soil. Cui et al. [28] provided the analytical solution for horizontal vibration of end-bearing single pile in radially heterogeneous saturated soil based on Biot's dynamic equations and Novak's thin-layer theory. The existing attenuation function of displacement of homogeneous soil and the pile-pile dynamic interaction factor are not suitable for the study of pile-pile dynamic interaction and dynamic characteristics of pile groups in transversely isotropic soil. In this paper, based on the wave propagation theory and Novak's plane assumption, the attenuation function of displacement of homogeneous soil and the pile-pile dynamic interaction factor are extended to the transversely isotropic soil, the lateral vibration of pile groups in transversely isotropic soil is studied, and the lateral dynamic impedance of pile groups is given.

2. Basic Equations

According to the elastic theory, the lateral dynamic control equation of the soil around the pile is (Luan et al. [29])

$$\begin{cases} \dfrac{\partial \sigma_{rr}}{\partial r} + \dfrac{1}{r}\dfrac{\partial \sigma_{r\theta}}{\partial \theta} + \dfrac{\partial \sigma_{rz}}{\partial z} + \dfrac{\sigma_{rr} - \sigma_{\theta\theta}}{r} = \rho \dfrac{\partial^2 u_r}{\partial t^2} \\ \dfrac{\partial \sigma_{r\theta}}{\partial r} + \dfrac{1}{r}\dfrac{\partial \sigma_{\theta\theta}}{\partial \theta} + \dfrac{\partial \sigma_{\theta z}}{\partial z} + 2\dfrac{\sigma_{r\theta}}{r} = \rho \dfrac{\partial^2 u_\theta}{\partial t^2} \end{cases}. \tag{1}$$

In order to consider the anisotropy of the soil around the pile, the three-dimensional constitutive relationship of transversely isotropic elastic medium proposed by Ding et al. [30] is used to describe the stress-strain relationship of the soil, i.e.,

$$\begin{cases} \sigma_{rr} = C_{11}\varepsilon_{rr} + C_{12}\varepsilon_{\theta\theta} + C_{13}\varepsilon_{zz} \\ \sigma_{\theta\theta} = C_{12}\varepsilon_{rr} + C_{22}\varepsilon_{\theta\theta} + C_{23}\varepsilon_{zz} \\ \sigma_{zz} = C_{13}\varepsilon_{rr} + C_{23}\varepsilon_{\theta\theta} + C_{33}\varepsilon_{zz} \\ \sigma_{\theta z} = C_{44}\varepsilon_{\theta z}, \sigma_{rz} = C_{55}\varepsilon_{rz}, \sigma_{r\theta} = C_{66}\varepsilon_{r\theta} \end{cases}, \tag{2}$$

where $\sigma_{rr}, \sigma_{\theta\theta}, \sigma_{zz}, \sigma_{r\theta}, \sigma_{rz}, \sigma_{\theta z}$ are the radial stress, circumferential stress, vertical stress, and shear stress of transversely isotropic soil; $\varepsilon_{rr}, \varepsilon_{\theta\theta}, \varepsilon_{zz}, \varepsilon_{r\theta}, \varepsilon_{rz}, \varepsilon_{\theta z}$ are the radial strain, circumferential strain, vertical strain, and shear strain of transversely isotropic soil mass. C_{11}, C_{22}, C_{33}, C_{12}, C_{13}, C_{23}, C_{44}, C_{55}, and C_{66} are the elastic constants of transversely isotropic soil mass, which meet $E_v = 2(1 + \mu_{vh})G_v$, $E_h = 2(1 + \mu_{hh})G_h$, $C_{11} = C_{22} = E_h(1 - \mu_{hv}\mu_{vh})/(1 + \mu_{hh})(1 - \mu_{hh} - 2\mu_{hv}\mu_{vh})$, $C_{13} = C_{23} = E_h\mu_{hh}/1 - \mu_{hh} - 2\mu_{hv}\mu_{vh}$, $C_{44} = C_{55} = G_v$, $C_{33} = E_h(1 - \mu_{hh})/1 - \mu_{hh} - 2\mu_{hv}\mu_{vh}$, $C_{12} =$

$E_h(\mu_{hh}+\mu_{hv}\mu_{vh})/(1+\mu_{hh})(1-\mu_{hh}-2\mu_{hv}\mu_{vh})$, and $C_{66}=1/2(C_{11}-C_{12})=G_h$. E_h and E_v are the lateral and vertical elastic modulus, respectively, G_h and G_v are the shear modulus in the lateral and vertical planes, respectively, μ_{hh} is Poisson's ratio of orthogonal lateral strain caused by lateral stress, μ_{hv} is the Poisson's ratio of vertical strain caused by lateral stress, μ_{vh} is the Poisson's ratio of lateral strain caused by vertical stress, and $\mu_{vh}/E_v=\mu_{hv}/E_h$.

According to the plane assumption proposed by Novak and Sachs [31], the soil layer is regarded as composed of many infinite thin soil layers with a circular hole, and the diameter of the hole is d. These thin layers are independent of each other, and it is considered that the soil layer displacement is independent of the coordinate z, that is, $\partial u_r/\partial z=0$ and $\partial u_\theta/\partial z=0$. The pile-soil vibration is small deformation and assumed complete contact between pile and soil and without relative sliding and separation. Because the pile-soil system is small deformation, this assumption is rationality. Although Novak plane model ignores the interaction between soil layers, it has been widely used in engineering because of its simplicity, practicality, and high accuracy. If the vertical displacement of transversely isotropic soil is neglected, the corresponding strain displacement relationship is

$$\begin{cases}\varepsilon_{rr}=\dfrac{\partial u_r}{\partial r},\varepsilon_{\theta\theta}=\dfrac{1}{r}\dfrac{\partial u_\theta}{\partial\theta}+\dfrac{u_r}{r},\varepsilon_{zz}=0\\ \varepsilon_{rz}=0,\varepsilon_{r\theta}=\dfrac{1}{r}\dfrac{\partial u_r}{\partial\theta}+\dfrac{\partial u_\theta}{\partial r}-\dfrac{u_\theta}{r},\varepsilon_{\theta z}=\dfrac{\partial u_\theta}{\partial r}\end{cases}. \tag{3}$$

Here, u_r and u_θ are the radial and circumferential displacements of transversely isotropic soil. From formulas (1)–(3), the lateral dynamic equation of transversely isotropic soil expressed by radial and circumferential displacements is

$$\left[C_{11}\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}-\frac{1}{r^2}\right)+C_{66}\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}\right]u_r+\left[(C_{11}-C_{66})\frac{1}{r}\frac{\partial^2}{\partial r\partial\theta}-(C_{11}+C_{66})\frac{1}{r^2}\frac{\partial}{\partial\theta}\right]u_\theta=\rho\frac{\partial u_r}{\partial t^2}, \tag{4}$$

$$\left[(C_{11}-C_{66})\frac{1}{r}\frac{\partial^2}{\partial r\partial\theta}+(C_{11}+C_{66})\frac{1}{r^2}\frac{\partial}{\partial\theta}\right]u_r+\left[C_{66}\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}-\frac{1}{r^2}\right)+C_{11}\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}\right]u_\theta=\rho\frac{\partial u_\theta}{\partial t^2}. \tag{5}$$

3. Lateral Displacement Attenuation Function of Transversely Isotropic Free Field

3.1. Solution of Attenuation Function of Lateral Displacement. When the transversely isotropic soil-pile system vibrates in a steady state under the simple harmonic load at pile top, the radial displacement and circumferential displacement of the soil meet $u_r=\tilde{u}_r e^{i\omega t}$, $u_\theta=\tilde{u}_\theta e^{i\omega t}$, $\tilde{u}_r$ and $\tilde{u}_\theta$ are the amplitudes of the radial and circumferential displacement of the transversely isotropic soil, respectively; i is imaginary unit, and ω is load frequency. Following formulas can be obtained by substituting them into Equations (4) and (5) and performing dimensionless operation:

$$\left[c_{11}G_{hv}\left(\frac{\partial^2}{\partial\bar{r}^2}+\frac{1}{\bar{r}}\frac{\partial}{\partial\bar{r}}-\frac{1}{\bar{r}^2}\right)+G_{hv}\frac{1}{\bar{r}^2}\frac{\partial^2}{\partial\theta^2}\right]\bar{u}_r+\left[G_{hv}(c_{11}-1)\frac{1}{\bar{r}}\frac{\partial^2}{\partial\bar{r}\partial\theta}-G_{hv}(c_{11}+1)\frac{1}{\bar{r}^2}\frac{\partial}{\partial\theta}\right]\bar{u}_\theta=-\bar{\omega}^2\bar{u}_r, \tag{6}$$

$$\left[G_{hv}(c_{11}-1)\frac{1}{\bar{r}}\frac{\partial^2}{\partial r\partial\theta}+G_{hv}(c_{11}+1)\frac{1}{\bar{r}^2}\frac{\partial}{\partial\theta}\right]\bar{u}_r+\left[G_{hv}\left(\frac{\partial^2}{\partial\bar{r}^2}+\frac{1}{\bar{r}}\frac{\partial}{\partial r}-\frac{1}{\bar{r}^2}\right)+c_{11}G_{hv}\frac{1}{\bar{r}^2}\frac{\partial^2}{\partial\theta^2}\right]u_\theta=-\bar{\omega}^2\bar{u}_\theta, \tag{7}$$

where $c_{11}=2[(1+\mu_{vh})-(1+\mu_{hh})G_{hv}\mu_{vh}^2]/[(1-\mu_{hh})(1+\mu_{vh})-2(1+\mu_{hh})G_{hv}\mu_{vh}^2]$, $c_{12}=2G_h[\mu_{hh}(1+\mu_{vh})+(1+\mu_{hh})G_{hv}\mu_{vh}^2]/[(1-\mu_{hh})(1+\mu_{vh})-2(1+\mu_{hh})G_{hv}\mu_{vh}^2]$, $\bar{r}=r/d$, $\bar{z}=r/d$, $\bar{u}_r=\tilde{u}_r/d$, $\bar{u}_\theta=\tilde{u}_\theta/d$, $\bar{\omega}=d\omega/v_v$, $G_{hv}=G_h/G_v$, and $v_v=\sqrt{G_v/\rho}$ represent the vertical shear wave velocity of soil mass. The following potential function is introduced to decouple Equations (6) and (7):

$$\bar{u}_r=\frac{\partial\varphi}{\partial\bar{r}}+\frac{1}{\bar{r}}\frac{\partial\psi}{\partial\theta},\bar{u}_\theta=\frac{1}{\bar{r}}\frac{\partial\varphi}{\partial\theta}-\frac{\partial\psi}{\partial\bar{r}}. \tag{8}$$

We obtained

$$c_{11}G_{hv}\nabla^2\varphi+c_{11}G_{hv}\frac{1}{\bar{r}^2}\frac{\partial^2\varphi}{\partial^2\theta}=-\bar{\omega}^2\varphi, \tag{9}$$

$$G_{hv}\nabla^2\psi+G_{hv}\frac{1}{\bar{r}^2}\frac{\partial^2\psi}{\partial\theta^2}=-\bar{\omega}^2\psi. \tag{10}$$

In which

$$\nabla^2=\frac{\partial^2}{\partial\bar{r}^2}+\frac{1}{\bar{r}}\frac{\partial}{\partial\bar{r}}. \tag{11}$$

Considering that the soil displacement at infinity is zero, and the parity of radial displacement and circumferential displacement, it is assumed that the pile bottom is bedrock, and the pile and bedrock are completely fixed. Solving

Equations (9) and (10) with the method of separating variables will have

$$\varphi = AH_1^{(2)}(q\bar{r})\cos\theta, \tag{12}$$

$$\psi = BH_1^{(2)}(g\bar{r})\sin\theta, \tag{13}$$

where $q^2 = \bar{\omega}^2/c_{11}G_{hv}$, and $g^2 = \bar{\omega}^2/G_{hv}$. $H_1^{(2)}(\cdot)$ is the Hankel function of the second kind of order 1, and A and B are undetermined coefficients. Substituting Equations (12) and (13) into Equation (8), it can be obtained that the radial displacement and circumferential displacement of transversely isotropic soil layer are

$$\bar{u}_r = \left[qH_0^{(2)}(q\bar{r}) - \frac{1}{\bar{r}}H_1^{(2)}(q\bar{r})\right]A\cos\theta + \frac{1}{\bar{r}}H_1^{(2)}(g\bar{r})B\cos\theta, \tag{14}$$

$$\bar{u}_\theta = -\frac{1}{\bar{r}}H_1^{(2)}(q\bar{r})A\sin\theta - \left[gH_0^{(2)}(g\bar{r}) - \frac{1}{\bar{r}}H_1^{(2)}(g\bar{r})\right]B\sin\theta. \tag{15}$$

In which, $H_0^{(2)}(\cdot)$ is the Hankel function of the second kind of order 0. It is assumed that the dimensionless lateral displacement of pile foundation at the pile-soil contact surface is $\bar{u}_p$; then, there are boundary conditions:

$$\begin{cases} \bar{u}_r(1/2,\theta,t) = \bar{u}_p, & \theta = 0 \\ \bar{u}_\theta(1/2,\theta,t) = -\bar{u}_p, & \theta = \dfrac{\pi}{2}. \end{cases} \tag{16}$$

From Equations (14)–(16), the undetermined coefficient A and B can be determined, and then, the radial displacement and circumferential displacement of the transversely isotropic soil layer can be determined as

$$\begin{aligned} \bar{u}_r = {} & \psi_1\left[H_0^{(2)}(q\bar{r}) - \frac{1}{q\bar{r}}H_1^{(2)}(q\bar{r})\right]\bar{u}_{\mathrm{p}}\cos\theta \\ & + \psi_2\frac{1}{q\bar{r}}H_1^{(2)}(g\bar{r})\bar{u}_{\mathrm{p}}\cos\theta, \end{aligned} \tag{17}$$

$$\begin{aligned} \bar{u}_\theta = {} & -\psi_1\frac{1}{q\bar{r}}H_1^{(2)}(q\bar{r})\bar{u}_p\sin\theta \\ & - \psi_2\left[\frac{g}{q}H_0^{(2)}(g\bar{r}) - \frac{1}{q\bar{r}}H_1^{(2)}(g\bar{r})\right]\bar{u}_p\sin\theta. \end{aligned} \tag{18}$$

In which

$$\psi_1 = \frac{4H_1^{(2)}(g/2) - gH_0^{(2)}(g/2)}{2H_0^{(2)}(q/2)H_1^{(2)}(g/2) - gH_0^{(2)}(q/2)H_0^{(2)}(g/2) + 2(g/q)H_1^{(2)}(q/2)H_0^{(2)}(g/2)}, \tag{19}$$

$$\psi_2 = \frac{4H_1^{(2)}(q/2) - qH_0^{(2)}(q/2)}{2H_0^{(2)}(q/2)H_1^{(2)}(g/2) - gH_0^{(2)}(q/2)H_0^{(2)}(g/2) + 2(g/q)H_1^{(2)}(q/2)H_0^{(2)}(g/2)}. \tag{20}$$

Under the action of simple harmonic load on the top of the active pile, the vibration of the active pile will cause the vibration of the soil around the pile and then produce the outward spreading cylindrical wave. It is assumed that the radiation wave follows the plane strain assumption, that is, the radiation wave only propagates along the lateral direction in each soil layer, but the attenuation function delays with the propagation distance S and the load direction angle θ in the propagation process. The displacement attenuation function of soil layer at $\theta = 0^0$ and $\theta = 90^0$ can be defined by Equations (17) and (18), that is

$$\begin{aligned} \psi(\bar{S},0^0) &= \frac{\bar{u}_r(\bar{S})}{\bar{u}_r(1/2)} = \frac{\psi_1\left[H_0^{(2)}(q\bar{S}) - (1/q\bar{S})H_1^{(2)}(q\bar{S})\right] + \psi_2\left[(1/q\bar{S})H_1^{(2)}(g\bar{S})\right]}{\psi_1\left[H_0^{(2)}(q/2) - (2/q)H_1^{(2)}(g/2)\right] + \psi_2\left[(2/q)H_1^{(2)}(g/2)\right]}, \\ \psi(\bar{S},90^0) &= \frac{\bar{u}_\theta(\bar{S})}{\bar{u}_\theta(1/2)} = \frac{\psi_1\left[(1/q\bar{S})H_1^{(2)}(q\bar{S})\right] + \psi_2\left[(g/q)H_0^{(2)}(g\bar{S}) - (1/q\bar{S})H_1^{(2)}(g\bar{S})\right]}{\psi_1\left[(2/q)H_1^{(2)}(q/2)\right] + \psi_2\left[(g/q)H_0^{(2)}(g/2) - (2/q)H_1^{(2)}(g/2)\right]}. \end{aligned} \tag{21}$$

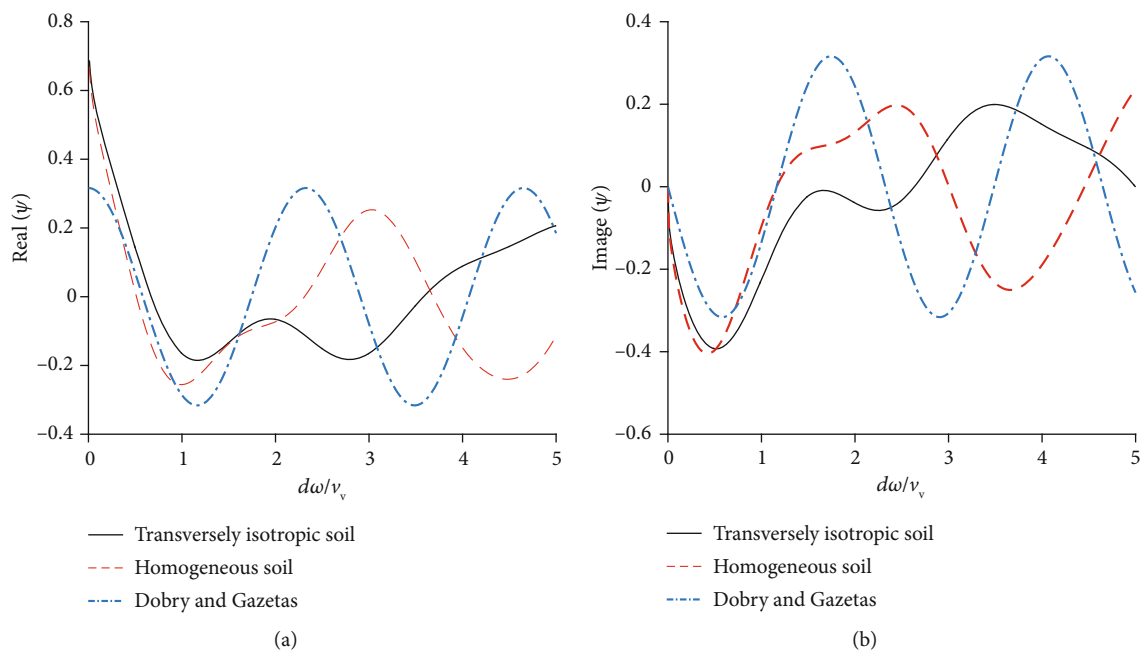

FIGURE 1: (a) Curves of real part of lateral displacement attenuation function varying with frequency ($S/d = 5, \theta = 0^0$). (b) Curves of image part of lateral displacement attenuation function varying with frequency ($S/d = 5, \theta = 0^0$).

In which, $\bar{S} = S/d$. Considering the lateral displacement, radial displacement, and circumferential displacement, the attenuation function of lateral displacement amplitude of free field can be obtained as

$$\psi(\bar{S}, \theta) = \psi(\bar{S}, 0^0)\cos^2\theta + \psi(\bar{S}, 90^0)\sin^2\theta. \tag{22}$$

In practical engineering, sometimes, for the convenience of research, the simplified formula given by Gazetas and Dobry [19] is used:

$$\begin{aligned} \psi(S, 0^0) &= \left(\frac{d}{2S}\right)^{1/2} \exp\left[\frac{-(i+\xi)\omega(S-d/2)}{v_{La}}\right], \\ \psi(S, 90^0) &= \left(\frac{d}{2S}\right)^{1/2} \exp\left[\frac{-(i+\xi)\omega(S-d/2)}{v_s}\right], \end{aligned} \tag{23}$$

where $v_{La} = 3.4v_s/\pi(1-\nu)$. ξ is the viscosity coefficient of soil mass, and v_s is the shear wave velocity of soil mass. The simplified formula given by Dobry and Gazetas is applicable to homogeneous soil, and the plane strain model solution is required for transversely isotropic soil.

3.2. Numerical Examples and Comparative Analysis. Figures 1–4 show the variation curve of attenuation function of the lateral displacement of the free field with frequency. The value of relevant parameters is $G_{hv} = 1.5$, $\mu_{hh} = 0.4$, $\mu_{vh} = 0.35$, and $\nu = 0.35$. Figures 1 and 2 show the comparison of plane model solution of transversely isotropic soil and homogeneous soil, and Dobry and Gazetas simplified solution of homogeneous soil when $\theta = 0^0, \theta = 90^0$ and $S/d = 5$. It can be seen that the real and imaginary part of the lateral displacement attenuation function of transversely isotropic soil with frequency is the smallest in the direction $\theta = 0^0$. At low frequency ($d\omega/v_v < 1.0$), and the solutions of the three cases are relatively close regardless of in the direction $\theta = 0^0$ or $\theta = 90^0$. In high frequency, when direction angle $\theta = 0^0$, the peak values of the real and imaginary parts of the attenuation function of lateral displacement of transversely isotropic soil with frequency are the smallest, while the Dobry and Gazetas simplified solutions of homogeneous soil are the largest. When direction angle $\theta = 90^0$, the peak value of the real part and imaginary part of the attenuation function of transversely isotropic soil varying with frequency is the largest, and the frequency corresponding to the peak value is the largest, while the peak value of the curve of Dobry and Gazetas simplified solution of homogeneous soil is the smallest. It can be seen that the anisotropy of the soil around the pile has a great influence on the propagation of the radiation wave, and the influence of the anisotropy on the attenuation function of the lateral displacement cannot be ignored. The result of the Dobry and Gazetas simplified solutions is different from solution of the plane model. Figures 3 and 4 show the variation curves of real and imaginary parts of attenuation function of lateral displacement of transversely isotropic soil with frequency when $\theta = 0^0, \theta = 90^0$ and $S/d = 2, 5, 10$. When pile spacing S/d is small, the real and imaginary parts of attenuation function of the soil displacement change slowly and stably with frequency. When it is large, the curve fluctuates greatly with frequency, and curves fluctuate more greatly and regularly when $\theta = 90^0$ than that when $\theta = 0^0$. For homogeneous soil, the difference between the displacement attenuation function obtained by Novak plane strain model and the simplified formula of Dobry and Gazetas should be caused by the model difference. The difference between the displacement attenuation

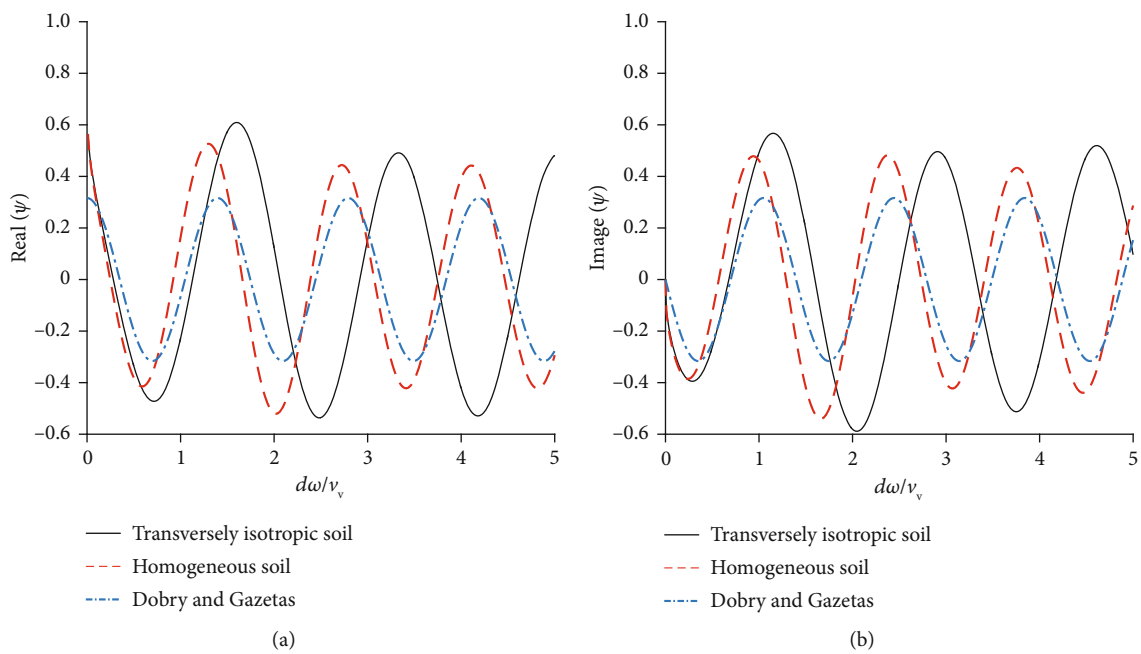

FIGURE 2: (a) Curves of real part of lateral displacement attenuation function varying with frequency ($S/d = 5, \theta = 90^0$). (b) Curves of image part of lateral displacement attenuation function varying with frequency ($S/d = 5, \theta = 90^0$).

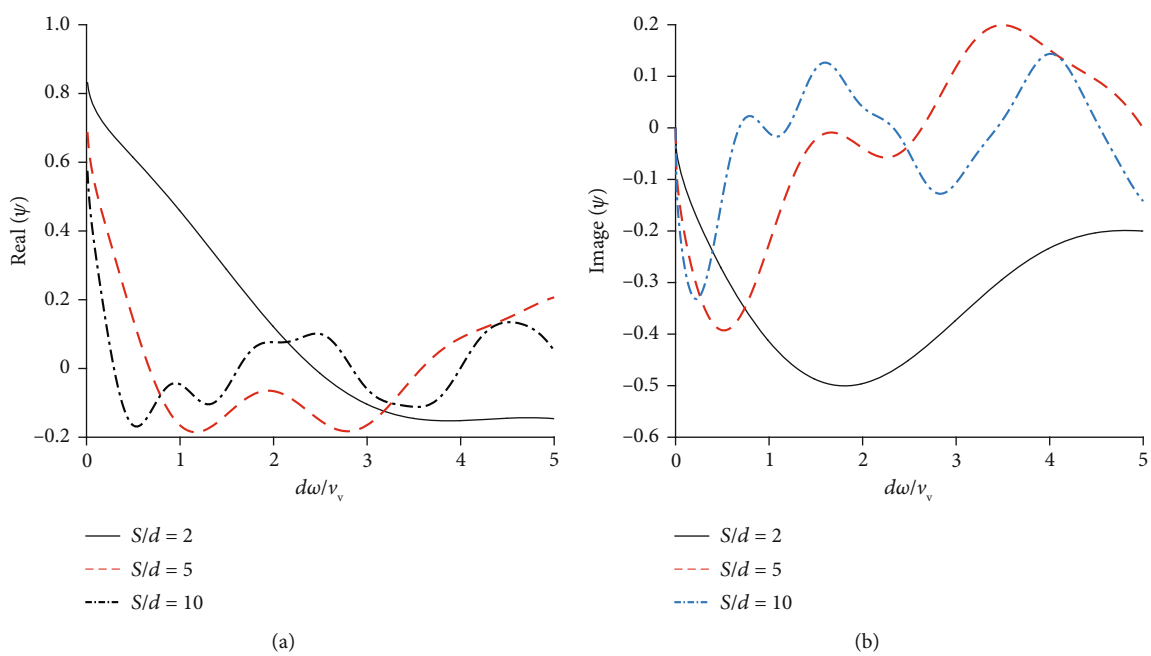

FIGURE 3: (a) Curves of real part of lateral displacement attenuation function varying with frequency ($S/d = 5, \theta = 0^0$). (b) Curves of image part of lateral displacement attenuation function varying with frequency ($S/d = 5, \theta = 0^0$).

function of homogeneous soil and transversely isotropic soil obtained by plane strain model is caused by the anisotropy of soil.

4. Pile-Pile Lateral Dynamic Interaction Factor in Transversely Isotropic Soil

In order to use the pile-pie superposition principle to solve the dynamic impedance of pile groups, it is necessary to calculate the pile-pile lateral dynamic interaction factor in transversely isotropic soil. It is assumed that the geometric dimensions and material properties of the active pile and the passive pile are the same (as shown in Figure 5), both of which are circular sections; the diameter is d, and the length is H. Since the size of the pile is usually small relative to the wavelength, when the wave generated by the active pile propagates to the passive pile, the displacement caused by each point in the section direction of the passive pile is

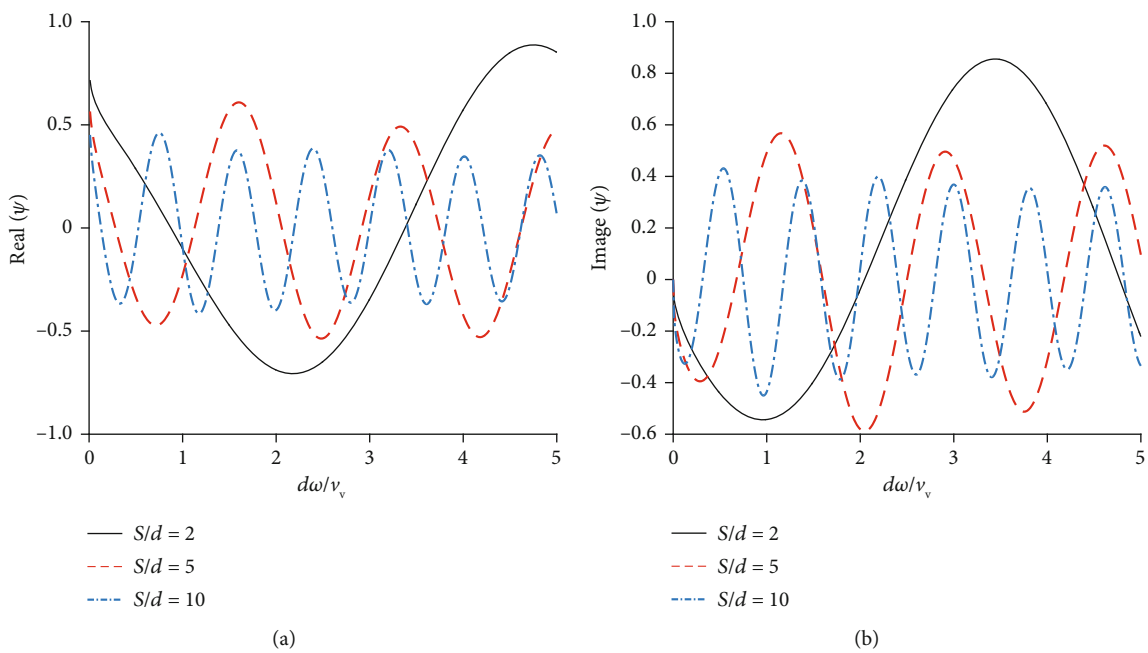

FIGURE 4: (a) Curves of real part of lateral displacement attenuation function varying with frequency ($S/d = 5, \theta = 90^0$). (b) Curves of image part of lateral displacement attenuation function varying with frequency ($S/d = 5, \theta = 90^0$).

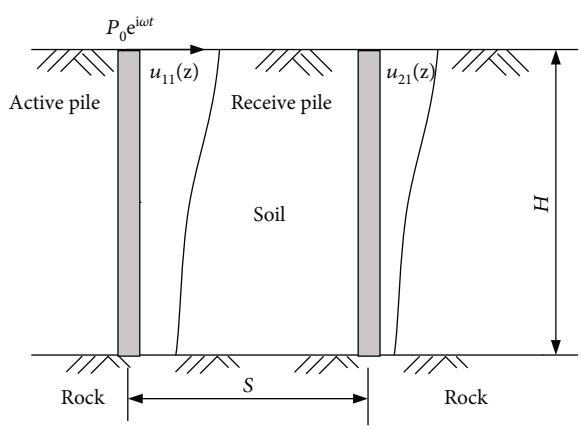

FIGURE 5: Model of pile-pile dynamic interaction in transversely isotropic soil.

almost the same, so the radial size of the passive pile can be ignored and replaced by its axis.

From Equations (1) and (2), it can be obtained that the dimensionless quantities of radial stress and shear stress of transversely isotropic soil are

$$\bar{\sigma}_{rr} = c_{11} G_{\mathrm{hv}} \frac{\partial \bar{u}_r}{\partial \bar{r}} + c_{12} G_{\mathrm{hv}} \left(\frac{1}{\bar{r}} \frac{\partial \bar{u}_\theta}{\partial \theta} + \frac{\bar{u}_r}{\bar{r}} \right), \tag{24}$$

$$\bar{\sigma}_{r\theta} = G_{\mathrm{hv}} \left(\frac{1}{\bar{r}} \frac{\partial \bar{u}_r}{\partial \theta} + \frac{\partial \bar{u}_\theta}{\partial \bar{r}} - \frac{\bar{u}_\theta}{\bar{r}} \right), \tag{25}$$

where $\bar{\sigma}_{rr} = \tilde{\sigma}_{rr}/G_v$, $\bar{\sigma}_{r\theta} = \tilde{\sigma}_{r\theta}/G_v$, $\tilde{\sigma}_{rr}$, and $\tilde{\sigma}_{r\theta}$ are the amplitudes of radial and shear stresses σ_{rr} and $\sigma_{r\theta}$, respectively. From Equations (17), (18), (24), and (25), the radial stress and shear stress of transversely isotropic soil are obtained, respectively.

$$\begin{aligned}\bar{\sigma}_{rr} = {} & c_{11} G_{hv} \psi_1 \left[-q H_1^{(2)}(q\bar{r}) - \frac{1}{\bar{r}} H_0^{(2)}(q\bar{r}) + \frac{2}{q\bar{r}^2} H_1^{(2)}(q\bar{r}) \right] \bar{u}_p \\ & + c_{11} G_{hv} \psi_2 \left[\frac{g}{q\bar{r}} H_0^{(2)}(g\bar{r}) - \frac{2}{q\bar{r}^2} H_1^{(2)}(g\bar{r}) \right] \bar{u}_p \\ & + c_{12} G_{hv} \psi_2 \left[-\frac{g}{q\bar{r}} H_0^{(2)}(g\bar{r}) + \frac{2}{q\bar{r}^2} H_1^{(2)}(g\bar{r}) \right] \bar{u}_p \\ & + c_{12} G_{hv} \psi_1 \left[\frac{1}{\bar{r}} H_0^{(2)}(q\bar{r}) - \frac{2}{q\bar{r}^2} H_1^{(2)}(q\bar{r}) \right] \bar{u}_p,\end{aligned} \tag{26}$$

$$\begin{aligned}\bar{\sigma}_{r\theta} = {} & G_{hv} \psi_1 \left[\frac{4}{q\bar{r}^2} H_1^{(2)}(q\bar{r}) - \frac{2}{\bar{r}} H_0^{(2)}(q\bar{r}) \right] \bar{u}_p \\ & + G_{hv} \psi_2 \left[\frac{g^2}{q} H_1^{(2)}(g\bar{r}) - \frac{4}{q\bar{r}^2} H_1^{(2)}(g\bar{r}) + \frac{2g}{q\bar{r}} H_0^{(2)}(g\bar{r}) \right] \bar{u}_p.\end{aligned} \tag{27}$$

The dynamic interaction between pile and soil is simulated by Winkler spring-damper model. In order to obtain the stiffness coefficient and damping coefficient of Winkler model, the unit thickness soil layer is taken as the research object, and the force of the transversely isotropic soil around the pile on the pile along the lateral x direction is

$$\bar{P}_x = \int_0^{2\pi} \frac{1}{2} \left[-\bar{\sigma}_r^S \cos\theta + \bar{\tau}_{r\theta} \sin\theta \right] \Big|_{\bar{r}=1/2} d\theta = \pi T \bar{u}_p. \tag{28}$$

In which,

$$T = \frac{1}{2} G_{hv} \psi_1 \left[\frac{8(2 - c_{11} + c_{12})}{q} H_1^{(2)}(q/2) + c_{11} q H_1^{(2)}\left(\frac{q}{2}\right) + 2(c_{11} - c_{12} - 2) H_0^{(2)}\left(\frac{q}{2}\right) \right] + \frac{1}{2} G_{hv} \psi_2 \left[\frac{g^2}{q} H_1^{(2)}\left(\frac{g}{2}\right) + \frac{8(c_{11} - c_{12} - 2)}{q} \cdot H_1^{(2)}\left(\frac{g}{2}\right) + \frac{2g(2 + c_{12} - c_{11})}{q} H_0^{(2)}\left(\frac{g}{2}\right) \right]. \tag{29}$$

When generating unit lateral displacement, the required lateral force πT is the lateral impedance of the soil layer, that is, the stiffness and damping coefficient of the spring meet

$$\pi T = k_h + i\bar{\omega} c_h. \tag{30}$$

Firstly, taking the active pile as the research object and considering the effect of soil around the pile, a dimensionless lateral dynamic equation of active pile in transversely isotropic soil is established as

$$\frac{d^4 U_{11}(\bar{z})}{d\bar{z}^4} + 4\lambda^4 U_{11}(\bar{z}) = 0, \tag{31}$$

where $\lambda^4 = 16T - 4\bar{\rho}_p \bar{\omega}^2 / \bar{E}_p$, $\bar{\rho}_p = \rho_p / \rho$, $\bar{E}_p = E_p / G_v$, $U_{11}(\bar{z}) = \tilde{u}_{11}(z)/d$, and $\tilde{u}_{11}(z)$ are the amplitude of the lateral displacement of the active pile. From the initial parameter method (Yan and Liu [32]), the general solution of Equation (31) is

$$U_{11}(\bar{z}) = -U_{11}(0) F_1(\lambda\bar{z}) + k_1 F_2(\lambda\bar{z}) \varphi_{11}(0) + k_2 F_3(\lambda\bar{z}) M_{11}(0) + k_3 F_4(\lambda z) Q_{11}(0). \tag{32}$$

In which, $k_1 = 1/\lambda$, $k_2 = -64/\pi\bar{E}_p\lambda^2$, $k_3 = -64/\pi\bar{E}_p\lambda^3$, $u_{11}(0)$, $\varphi_{11}(0)$, $Q_{11}(0)$, and$M_{11}(0)$ are the displacement and internal force at the top of the active pile. F_1, F_2, F_3, and F_4 are Krylov functions, and their derivative relations satisfy $F_1' = -4\lambda_i F_4$, $F_2' = \lambda_i F_1$, $F_3' = \lambda_i F_2$, and $F_4' = \lambda_i F_3$.

Taking the passive pile as the research object and considering the influence of the lateral displacement of free soil at the passive pile caused by the active pile, the lateral dynamic equation of the passive pile can be established as

$$\frac{d^4 U_{22}(\bar{z})}{d\bar{z}^4} + 4\lambda^4 U_{22}(\bar{z}) = \frac{64T}{\bar{E}_p} \psi(\bar{S}, \theta) U_{11}(\bar{z}). \tag{33}$$

Similarly, the general solution of Equation (33) can be obtained from the initial parameter method as

$$\begin{aligned} U_{22}(\bar{z}) = &-\frac{T\psi(\bar{S},\theta)\lambda z}{4T - \bar{\rho}_p\bar{\omega}^2} [4F_4(\lambda\bar{z}) U_{11}(0) + k_1 F_1(\lambda\bar{z}) \phi_{11}(0) \\ &+ M_{11}(0) k_2 F_2(\lambda\bar{z}) + Q_{11}(0) k_3 F_3(\lambda z)] \\ &+ [-F_1(\lambda\bar{z}) U_{21}(0) + k_1 F_2(\lambda\bar{z}) \phi_{21}(0) \\ &+ k_2 F_2(\lambda\bar{z}) M_{21}(0) + k_3 F_4(\lambda\bar{z})] \\ &+ \frac{T\psi(\bar{S},\theta)}{4T - \bar{\rho}_p\bar{\omega}^2} [k_1 F_2(\lambda\bar{z}) \phi_{11}(0) \\ &+ 2k_2 F_3 M_{11}(0) + 3k_3 F_4(\lambda\bar{z}) Q_{11}(0)]. \end{aligned} \tag{34}$$

From the lateral displacement of active pile and passive pile, the rotation angle, shear force, and bending moment of active pile and passive pile can be obtained, and then, the relationship of the displacement, rotation angle, shear force, and bending moment of active pile and passive pile between pile top and pile bottom can be obtained as

$$\begin{Bmatrix} U_{11}(\delta) \\ \varphi_{11}(\delta) \\ Q_{11}(\delta) \\ M_{11}(\delta) \end{Bmatrix} = [T^1] \begin{Bmatrix} U_{11}(0) \\ \varphi_{11}(0) \\ Q_{11}(0) \\ M_{11}(0) \end{Bmatrix} \tag{35}$$

$$\begin{Bmatrix} U_{21}(\delta) \\ \phi_{21}(\delta) \\ Q_{21}(\delta) \\ M_{21}(\delta) \end{Bmatrix} = -\beta\lambda\delta [T^2] \begin{Bmatrix} U_{11}(0) \\ \phi_{11}(0) \\ Q_{11}(0) \\ M_{11}(0) \end{Bmatrix} + [T^1] \begin{Bmatrix} U_{21}(0) \\ \phi_{21}(0) \\ Q_{21}(0) \\ M_{21}(0) \end{Bmatrix} + \beta [T^3] \begin{Bmatrix} U_{11}(0) \\ \phi_{11}(0) \\ Q_{11}(0) \\ M_{11}(0) \end{Bmatrix}. \tag{36}$$

In which, $\delta = H/d$ is the pile length diameter ratio, and $\beta = T\psi(\bar{S},\theta)/4T - \bar{\rho}_p\bar{\omega}^2$

$$[T^1] = \begin{bmatrix} -F_1 & k_1 F_2 & k_3 F_4 & k_2 F_3 \\ \frac{4}{k_1} F_4 & F_1 & \frac{k_3}{k_1} F_3 & \frac{k_2}{k_1} F_2 \\ \frac{4}{k_2} F_3 & -4\frac{k_1}{k_2} F_4 & \frac{k_3}{k_2} F_2 & F_1 \\ \frac{4}{k_3} F_2 & -4\frac{k_1}{k_3} F_3 & F_1 & -4\frac{k_2}{k_3} F_4 \end{bmatrix}, \tag{37}$$

$$[T^2] = \begin{bmatrix} 4F_4 & k_1F_1 & k_3F_3 & k_2F_2 \\ \frac{4F_3}{k_1} & -4F_4 & \frac{k_3}{k_1}F_2 & \frac{k_2}{k_1}F_1 \\ \frac{4}{k_3}F_1 & -\frac{4k_1}{k_3}F_2 & -4F_4 & -\frac{4k_2}{k_3}F_3 \\ \frac{4}{k_2}F_2 & -4\frac{k_1}{k_2}F_3 & \frac{k_3}{k_2}F_1 & -4F_4 \end{bmatrix}, \tag{38}$$

$$[T^3] = \begin{bmatrix} 0 & k_1F_2 & 3k_3F_4 & 2k_2F_3 \\ -\frac{4F_4}{k_1} & 0 & 2\frac{k_3}{k_1}F_3 & \frac{k_2}{k_1}F_2 \\ -\frac{12}{k_3}F_2 & \frac{8k_1}{k_3}F_3 & 0 & \frac{4k_2}{k_3}F_4 \\ -\frac{8}{k_2}F_3 & 4\frac{k_1}{k_2}F_4 & \frac{k_3}{k_2}F_2 & 0 \end{bmatrix}. \tag{39}$$

For the convenience of derivation, the matrix $[T^1]$, $[T^2]$, and $[T^3]$ are divided into four parts, i.e.,

$$\begin{aligned} [T^1] &= \begin{bmatrix} T^1_{11} & T^1_{12} \\ T^1_{21} & T^1_{22} \end{bmatrix}, \\ [T^2] &= \begin{bmatrix} T^2_{11} & T^2_{12} \\ T^2_{21} & T^2_{22} \end{bmatrix}, \\ [T^1] &= \begin{bmatrix} T^2_{11} & T^2_{12} \\ T^2_{21} & T^2_{22} \end{bmatrix}. \end{aligned} \tag{40}$$

Assuming that both the active pile and the passive pile have fixed ends, and considering the boundary conditions of the active pile, the stiffness matrix and flexibility matrix of a single pile can be obtained as

$$[K^s] = -\left[T^1_{12}\right]^{-1}\left[T^1_{11}\right], \tag{41}$$

$$[f^s] = [K^s]^{-1}. \tag{42}$$

The element $K^s(1,1)$ of the stiffness matrix $[K^s]$ is the lateral dynamic impedance of an active single pile. Considering the boundary conditions of passive piles, the pile-pile relationship matrix under lateral vibration can be obtained from Equation (36), that is

$$\begin{aligned} \left[a\left(\bar{S},\theta\right)\right] = \beta\left[T^1_{11}\right]^{-1}\{&\lambda\delta\left[T^2_{11}\right] + \lambda\delta\left[T^2_{12}\right][K^s] \\ &- \left[T^3_{11}\right] - \left[T^3_{12}\right][K^s]\}. \end{aligned} \tag{43}$$

According to the definition of interaction factor, the lateral dynamic interaction factor of pile-pile in transversely isotropic soil is

$$\alpha_{uP} = \frac{U_{21}(0)}{U_{11}(0)} = \frac{\alpha(1,1)f^s(1,1) + \alpha(1,2)f^s(2,1)}{f^s(1,1)}, \tag{44}$$

where $\alpha(1,1)$ and $\alpha(1,2)$ are the element of pile-pile interaction matrix during lateral vibration.

5. Lateral Dynamic Impedance of Pile Groups in Transversely Isotropic Soil

5.1. Solution of Lateral Vibration of Pile Groups. The lateral vibration of pile groups in transversely isotropic soil under rigid caps is investigated. The number of piles is n, the physical and geometric properties of each pile are the same, and the pile foundation and bearing platform are symmetrically distributed. Regardless of the mass of rigid cap, there is lateral harmonic load acting on the cap, and the dimensionless load amplitude is P. Since the pile top is a rigid cap, the lateral displacement at the pile groups top is equal to the lateral displacements of each pile at top, we have

$$U^G = U_i = \sum_{j=1}^{n} \frac{\alpha_{uP,ij}P_j}{K^S(1,1)}, \tag{45}$$

where U^G and U_i are the dimensionless quantities of the lateral displacement amplitude of the pile groups and the i-th pile top, and P_j is the lateral load shared by the j-th pile, $\alpha_{uP,ij}$ are lateral dynamic interaction factor between the i-th pile and the j-th pile. Taking the rigid cap as the research object, the following equilibrium equation can be obtained:

$$\sum_{j=1}^{n} P_j = P. \tag{46}$$

By solving Equations (45) and (46), the lateral displacement U^G and U_i of the pile groups and the i-th pile at top and the lateral load P_j of each pile at top can be obtained, and then, the lateral dynamic impedance of the pile groups in transversely isotropic soil can be obtained.

$$K^G(i\bar{\omega}) = \frac{P}{U^G(0)} = K_h + i\bar{\omega}C_h, \tag{47}$$

where K_h and C_h are the lateral dynamic stiffness and dynamic damping of pile groups in transversely isotropic soil.

5.2. Examples of 3 × 3 Pile Groups in Transversely Isotropic Soil. Figures 6–9 show the variation curves of lateral dynamic stiffness and dynamic damping of pile groups in transversely isotropic soil with frequency, and the parameter value is $G_{hv} = 1.5$, $\mu_{hh} = 0.4$, $\mu_{vh} = 0.35$, $\rho_p/\rho = 2.0$, $E_p/G_v = 1000$, and $H/d = 20$. Figure 6 gives the comparison curves of lateral dynamic stiffness and dynamic damping of pile groups in homogeneous soil ($G_{hv} = 1.0$, $\mu_{hh} = \mu_{vh} = 0.35$) and transverse isotropic soil. Obviously, the lateral dynamic impedance of pile groups in transversely isotropic soil can degenerate to that in homogeneous soil, which shows the correctness of the analysis method in this paper. Poisson's ratio μ_{hh} of orthogonal lateral strain caused by lateral stress and Poisson's ratio μ_{vh} of lateral strain caused by vertical stress

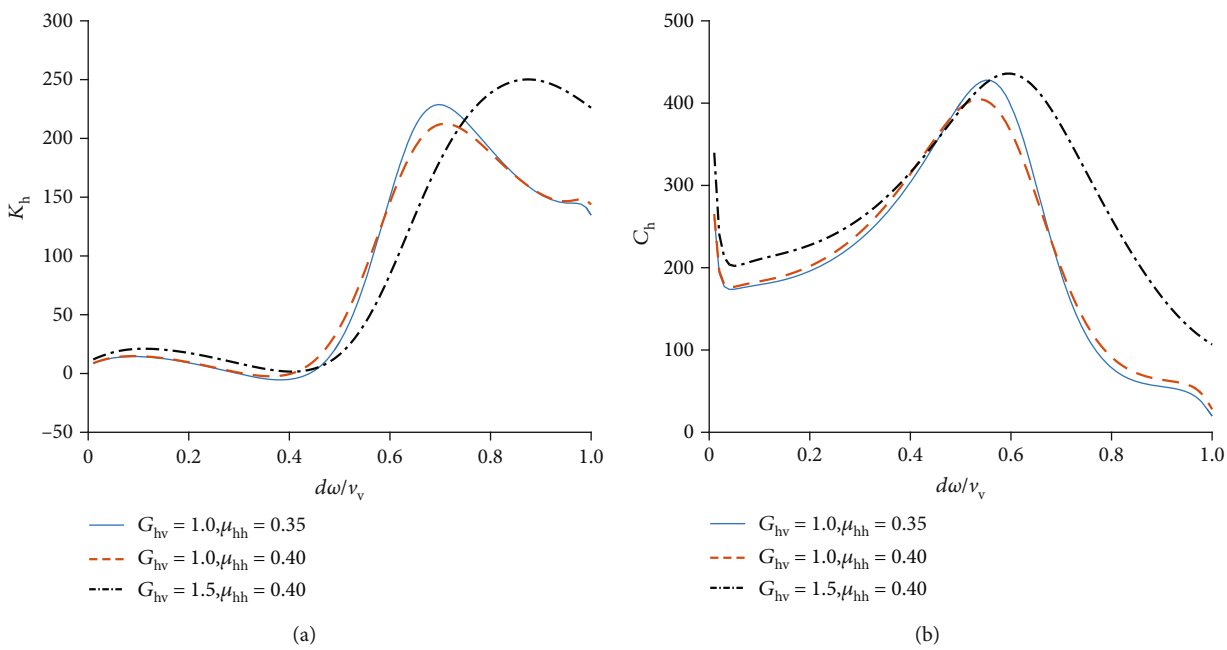

Figure 6: (a) Curves of lateral dynamic stiffness of pile groups in transversely isotropic soil varying with frequency. (b) Curves of lateral dynamic damping of pile groups in transversely isotropic soil varying with frequency.

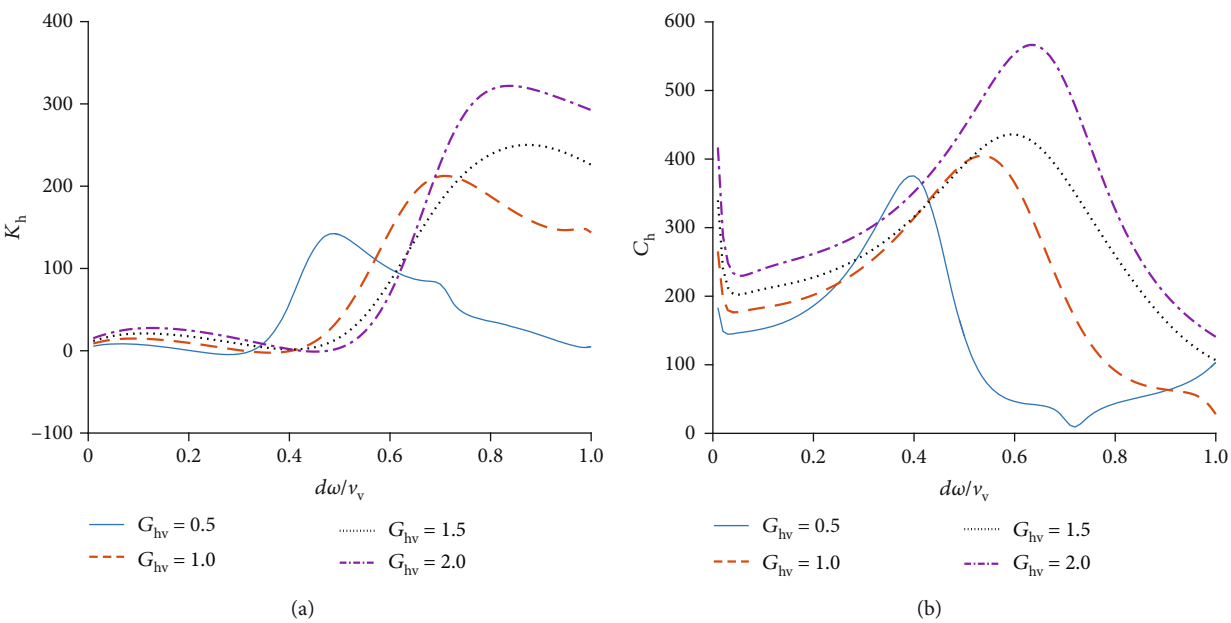

Figure 7: (a) Influence of shear modulus ratio G_{hv} on lateral dynamic stiffness of pile groups in transversely isotropic soil. (b) Influence of shear modulus ratio G_{hv} on lateral dynamic damping of pile groups in transversely isotropic soil.

have little effect on the lateral dynamic impedance of pile groups, only on the peak value of the curve of lateral dynamic stiffness and dynamic damping varying with frequency.

The ratio G_{hv} of the shear modulus in the lateral plane to the shear modulus in the vertical plane has a great impact on the lateral dynamic stiffness and dynamic damping of pile groups (Figures 6 and 7). At high frequencies, the greater the shear modulus ratio G_{hv} in both directions, the greater the lateral dynamic stiffness and dynamic damping of pile groups, while at low frequencies, it is the opposite. It can be seen that the difference of mechanical properties on different surfaces of soil around the pile has a great influence on the lateral vibration of pile groups in transversely isotropic soil, and the influence of anisotropy of soil around the pile should not be ignored when studying the lateral vibration of pile groups.

The pile spacing S/d has great influence on the lateral vibration of pile groups in transversely isotropic soil, as

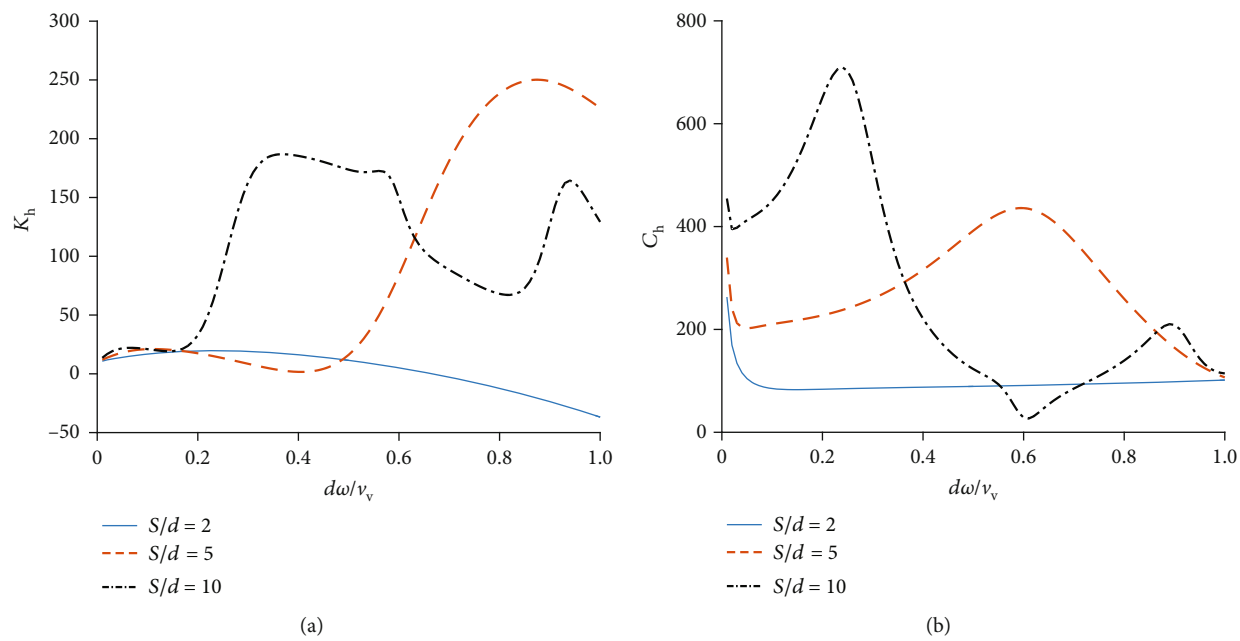

Figure 8: (a) Influence of pile spacing S/d on lateral dynamic stiffness of pile groups in transversely isotropic soil. (b) Influence of pile spacing S/d on lateral dynamic damping of pile groups in transversely isotropic soil.

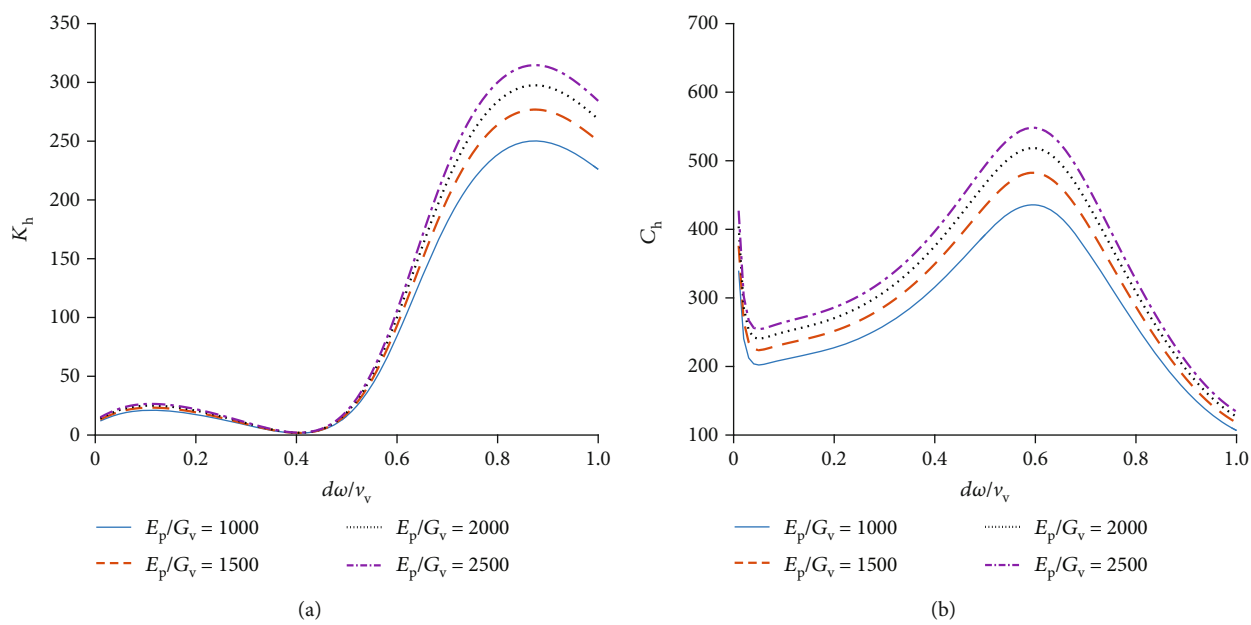

Figure 9: (a) Influence of pile-soil modulus ratio E_v/G_v on lateral dynamic stiffness of pile groups in transversely isotropic soil. (b) Influence of pile-soil modulus ratio E_v/G_v on lateral dynamic damping of pile groups in transversely isotropic soil.

shown in Figure 8. When the pile spacing is small ($S/d = 2.0$), the change of lateral dynamic impedance of pile groups in transversely isotropic soil with frequency is relatively small, and the dynamic stiffness gradually decreases with the increase of frequency and tends to negative value, while the change of dynamic damping with frequency is small. At this time, the lateral vibration of pile groups is similar to that of embedded foundation. The curves of lateral dynamic stiffness and dynamic damping of pile groups in transversely isotropic soil varying with frequency will appear peaks and valleys when the pile spacing increases. The larger the pile spacing is, the more severe and complex the variation curves of lateral dynamic stiffness and dynamic damping with frequency are. The influence of the ratio E_p/G_v of the elastic modulus of the pile to the vertical plane shear modulus of the soil around the pile on the lateral vibration of pile groups in transversely isotropic soil is shown in Figure 9. At low frequency, the influence of modulus ratio on lateral dynamic

stiffness is relatively small, while at high frequency, the influence is larger, while the influence of modulus ratio on dynamic damping is larger and is not affected by frequency. The greater the elastic modulus of the pile, the greater the stiffness of the pile, the smaller the lateral displacement of the pile, and the greater the lateral dynamic stiffness and dynamic damping at the pile top.

6. Conclusions

Based on Novak's plane assumption and considering the anisotropy of the soil around the pile, the concept of pile-pile dynamic interaction factor is extended to the transversely isotropic soil, and the lateral vibration of pile groups is studied. The following conclusions are obtained through numerical analysis: (1) the anisotropy of the soil around the pile has a great influence on the propagation of the radiation wave, and the influence of the anisotropy cannot be ignored. The attenuation function of the lateral displacement obtained by the simplified solution of Dobry and Gazetas is different from the solution of the plane model; (2) when the pile spacing is small, the real and imaginary parts of the lateral displacement attenuation function of the soil change slowly and stably with frequency, and when the pile spacing is large, the curve fluctuates greatly with frequency; (3) Poisson's ratio μ_{hh} of orthogonal lateral strain caused by lateral stress and Poisson's ratio μ_{vh} of lateral strain caused by vertical stress only have certain influence on the peak value of the curve of lateral dynamic stiffness and dynamic damping varying with frequency; (4) the ratio of G_{hv} the shear modulus in the lateral plane to the shear modulus in the vertical plane and the pile spacing S/d have a great impact on the lateral vibration of pile groups in transversely isotropic soil. The impact of the difference of mechanical properties of the soil around the pile on the lateral vibration of pile groups in transversely isotropic soil should not be ignored. (5) The ratio E_p/G_v of elastic modulus of pile to vertical plane shear modulus of soil has an effect on the lateral dynamic stiffness of pile groups in transversely isotropic soil, and the influence is related to frequency, but the influence of E_p/G_v on dynamic damping is not affected by frequency. (6) In this paper, Novak's plane model is used to study the lateral vibration of pile groups in transversely isotropic soil. Some simplification and assumptions are made to the problem, and the vertical displacement of transversely isotropic soil is ignored. Although it can meet the needs of engineering, its accuracy is a little lower than that of three-dimensional model. Three-dimensional model can be used to study the lateral vibration of pile groups in transversely isotropic soil by finite element method or boundary element method and to carry out comparative study with engineering examples.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest to report regarding the present study.

Acknowledgments

This research was supported by the National Natural Science Foundation of China (no. U1504505) and the Key Scientific Research Projects of Colleges and Universities in Henan Province (23B560001).

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