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Research Article

Exploring the Benefits of Representing Multiplayer Game Data in a Coordinate System

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In the realm of game theory, a range of mathematical approaches exists for the representation of game data, with the extensive form (depicted as a game tree) and the normal form (illustrated as a payoff matrix) standing out as the most prevalent. However, a significant drawback associated with these approaches is their limited scalability. As the number of players or their strategic options increases, these techniques progressively lose their feasibility and become less practical for meaningful analysis. The present work proposes an alternative approach that significantly enhances the representation of data in two- or three-player games. Within this framework, the conventional payoff matrix is substituted with a payoff coordinate system, employing a coordinate plane for two-player games and a coordinate space for three-player games. This approach offers numerous advantages when compared to other methods. For instance, the Nash equilibrium can be readily identified within a game without requiring an extensive duration to exhaustively examine all strategies for its determination. By employing this approach, the representation of game data becomes more convenient and efficient, making it easier to analyze and comprehend the underlying strategies employed by players.

1. Introduction

In 1944, Von Neumann and Morgenstern published the *Theory of Games and Economic Behavior* [1], a groundbreaking book that laid the groundwork for game theory as we know it today. Since then, this theory has undergone extensive development, and it has applications in a wide range of fields such as economics, computer science, security, political science, military applications, and biology [2–11].

Various approaches exist for formalizing a game [12, 13], such as the extensive form being one of them (game tree) [14]. This approach, originally suggested by Von Neumann and Morgenstern [1] and further developed by Kuhn [15], becomes increasingly complex to illustrate as the number of strategies is expanded, making it difficult to represent. Another commonly employed and practical approach is referred to as the normal form (payoff matrix). The purpose of introducing this method was to enable the representation of a game using an array that displays the players' payoffs based on their strategies [16]. Theoretically, arrays of any dimension can be imagined, with each dimension representing a player. However, in practice, only two-dimensional arrays are easily understood. This limitation reveals that payoff matrices are primarily applicable to two-player games. Consequently, even in recent books and articles related to game theory, we predominantly encounter examples restricted to two players, each having distinct strategies. Recognizing this limitation, Özkaya et al. [17] brought the concept of 3D matrices in decision and game theories, effectively expanding the analytical scope and allowing for a more comprehensive analysis.

The Nash equilibrium is one of the essential concepts in game theory, which serves as a significant point of reference. Determining this equilibrium using the 2D or 3D matrix methods requires checking all strategies one by one, which can be quite impractical. However, by employing the payoff coordinate system, we can directly identify the Nash equilibrium, offering a more efficient way to analyze strategic relationships (see Section 4.2 for further elaboration). This paper is aimed at presenting an alternative approach that outperforms existing methods in terms of simplicity and clarity of results in both two-player and three-player games. We will use a coordinate plane in two-player games and a coordinate space in three-player games. By adopting this approach, the analysis and understanding of the strategies employed by players become significantly easier and more comprehensible. In summary, the most important contributions of this research are as follows:

(i) Our approach notably enhances data representation in two-player or three-player games

(ii) It simplifies the visualization of game data, rendering it more accessible

(iii) It facilitates a more profound comprehension of the fundamental strategies employed by players

(iv) Additionally, it substantially streamlines the analysis of these strategies

The rest of this paper is organized as follows. Section 2 reviews the most recent critical studies in the literature. In Section 3, we present some classic notions of game theory. Then, the payoff coordinate plane and the payoff coordinate space, respectively, are introduced in Sections 4 and 5. Moreover, a comparative study was carried out between the classical method (payoff matrix) and the payoff coordinate system. Section 6 concludes the paper.

2. Related Works

Over time, game theory has undergone extensive development, and many previous researchers have paid attention to this theory in their works, whether through traditional or evolutionary game paradigms.

In the realm of traditional game theory, NASA provides a noteworthy case in point where cooperative game theory with transferable utilities has been applied effectively within their PvS libraries [18]. Liu et al. [19] and Zhao et al. [20] employed cooperative game theory to construct models for offloading in their respective studies. On the contrary, some researchers have employed noncooperative game theory approaches. For example, Liu et al. [21] delved into the application of edge computing for computational offloading through a two-step Stackelberg game analysis. Furthermore, Messous et al. [22] explored the challenge of task computation offloading in MEC-based UAV networks, aiming to balance energy consumption, delay, and computational cost. In 2021, Zhou et al. [23] also employed noncooperative game theory to allocate resources, aiming to strike a balance between task owners and resource providers. A significant drawback associated with these approaches is their limited scalability. In addressing this challenge, Khoobkar et al. [5, 6] have recently turned their attention towards tackling scalability concerns in offloading by employing the evolutionary game approach and replicator dynamics. Their aim is to attain a Nash equilibrium between fog and cloud devices.

Table 1: A payoff matrix of a two-player game with 2 strategies.

Player 1	Player 2	
	Chooses strategy 1	Chooses strategy 2
Chooses strategy 1	(3;3)	(1;4)
Chooses strategy 2	(4;1)	(2;2)

Additional research has devoted its efforts to expanding the analytical scope of game theory. Izgi and Özkaya [24] evaluated the agricultural insurance problem through a game theory method called the matrix norm approach; they examined the issue of decision-making using real data in a zero-sum matrix game against nature. In 2021, Ulansky and Raza [25] presented a criterion for selecting the optimal decision in a game against nature as defined in [13], amidst partial a priori uncertainty. In the same year, Özkaya and Izgi [26] used game theory to analyze the impact of self-quarantine measures amid the initial surge of the COVID-19 pandemic's first wave. Their research honed in on the utilization of game theory as a tool to elucidate the consequences of quarantine across three distinct stages: the beginning, spread, and end of a pandemic. In 2022, Vdovyn et al. [27] addressed the challenges associated with evaluating, analyzing, and modeling economic systems through the lens of game theory. Their research encompassed a range of decision criteria, such as the Hurwicz, Wald, Hodges-Lehmann, Bayes-Laplace, Savage, and minimum dispersion methodologies. In 2023, Özkaya et al. [17] introduced the innovative concept of 3D matrices within the domains of decision and game theories. Their primary emphasis was on adapting and applying well-established criteria such as Laplace, Wald, Hurwicz, and Savage within a three-dimensional framework. To achieve this, they built upon the foundational definitions of 3D matrices as outlined in [28].

Researchers consistently strive to broaden and simplify concepts. Within this framework, this paper illustrates the benefits of representing multiplayer game data in a coordinate system, such as scalability, convenience, and efficiency.

3. Classic Notions of Game Theory

In this section, we start by recalling some classic notions of game theory that we will use in this work (some references used [12, 16, 29, 30, 31, 32]).

Definition 1. A dominant strategy is a strategy that provides, at least, the same utility as all the other player's strategies and strictly greater for some strategy.

A strictly dominant strategy is a strategy that always provides greater utility to the player, ensuring a strictly superior outcome, no matter what the other player's strategy is.

Definition 2. A dominated strategy is a strategy that provides, at best, the same utility as all the other player's strategies and, in some cases, results in a strictly inferior outcome.

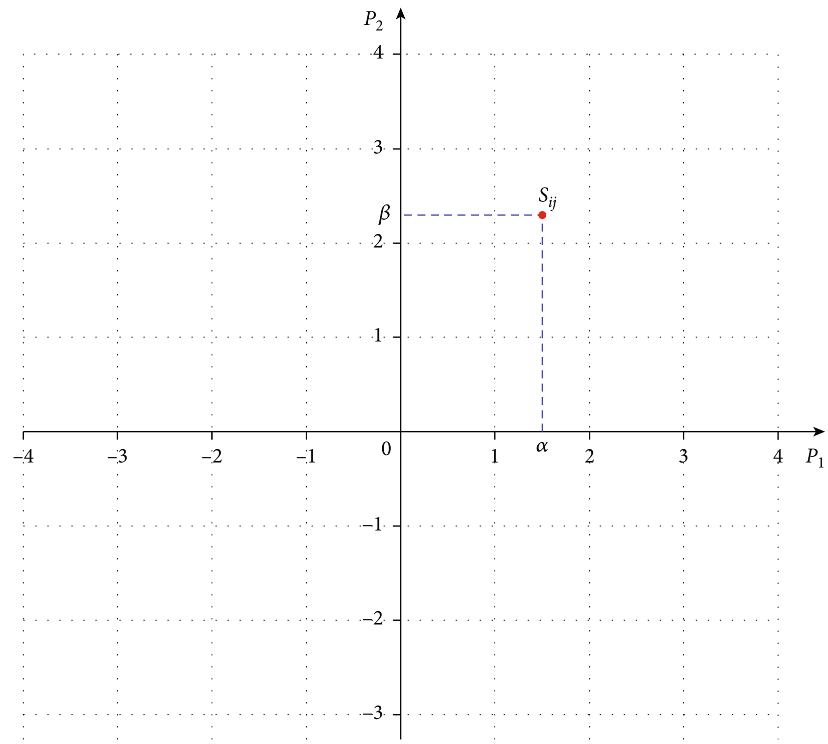

FIGURE 1: A payoff coordinate plane of a two-player game with n strategies.

A strictly dominated strategy is a strategy that consistently results in lower utility for the player, ensuring a strictly inferior outcome, no matter what the other player's strategy is.

Definition 3. A Nash equilibrium is an outcome that no player wants to unilaterally deviate from their strategy, regardless of the choices of others.

Example 1. Imagine a situation where player 1 participates in a game with an undisclosed partner, referred to as player 2, the payoff matrix is shown in Table 1.

Player 1's first strategy is strictly dominated because, regardless of player 2's choice, player 1 achieves a higher gain by playing the second strategy. Consequently, the first strategy is strictly dominated, while the second strategy is strictly dominant for player 1.

Likewise, it can be confirmed that the first strategy constitutes a strictly dominated strategy for player 2, while the second strategy emerges as a strictly dominant strategy for the same player.

Within the game, the only Nash equilibrium emerges at (2;2), with both players opting for strategy 2. In this equilibrium, neither player finds it advantageous to alter their chosen strategy.

4. Payoff Coordinate Plane

In this section, we will introduce the concept of a payoff point. Following that, we will establish a coordinate plane dedicated to payoff analysis. Subsequently, by plotting the payoff points on this coordinate plane, we will conduct a comparative analysis between our method and the other method (payoff matrix). The main results are supported by a practical example.

4.1. Definitions

Definition 4. Let n and m be two integers greater than 2. Consider a two-player game with $n \times m$ strategies.

We define the payoff points $S_{ij}(\alpha\,;\beta)$ with $i \in \{1\,;2\,;\cdots\,;n\}$ and $j \in \{1\,;2\,;\cdots\,;m\}$ by the following:

i is the strategy chosen by the first player

j is the strategy chosen by the second player

α is the payoff for player 1 by choosing i^{th} strategy

β is the payoff for player 2 by choosing j^{th} strategy

S_{ij} is the payoff for both players when the first player chooses the i^{th} strategy and the second chooses the j^{th} strategy

Definition 5. A payoff coordinate plane is a way of representing a two-player game as a coordinate plane showing the players' payoffs (in the form of payoff points) according to their respective strategies.

The x-axis will be reserved for the gain of the first player, and the y-axis will be reserved for the gain of the second player. The payoff coordinate plane is depicted in Figure 1.

Example 2. Consider a two-player game with three strategies. The number of points to build is $3^2 = 9$.

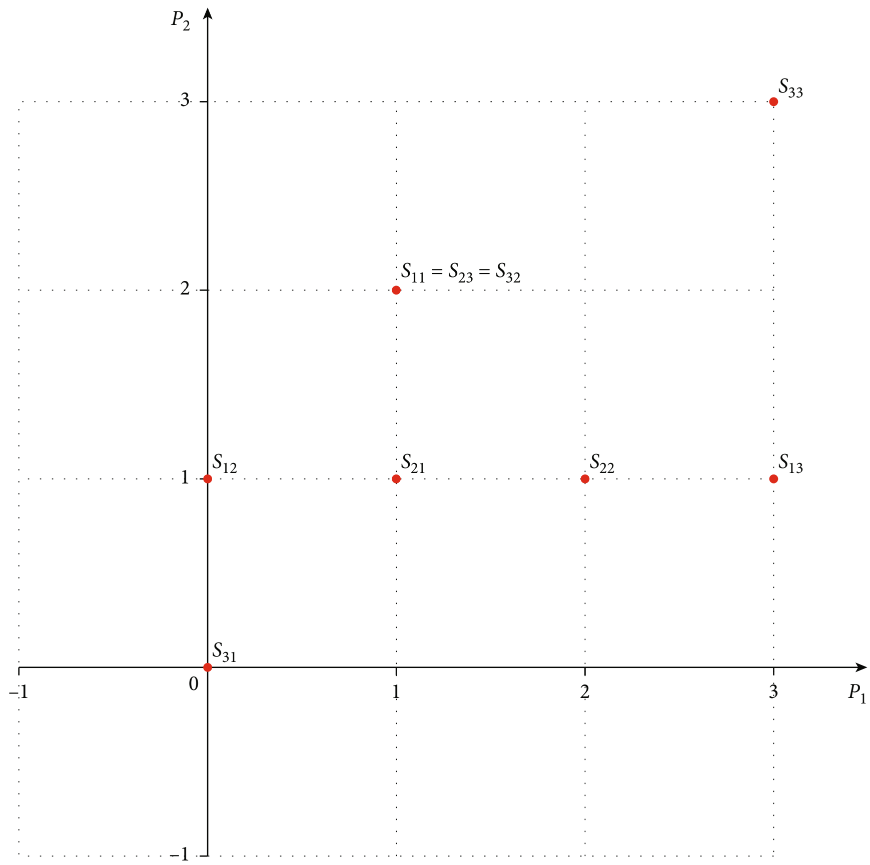

Figure 2: A payoff coordinate plane of a two-player game with 3 strategies.

Table 2: A payoff matrix of a two-player game with 3 strategies.

Player 1	Player 2 Chooses strategy 1	 Chooses strategy 2	 Chooses strategy 3
Chooses strategy 1	(1;2)	(0;1)	(3;1)
Chooses strategy 2	(1;1)	(2;1)	(1;2)
Chooses strategy 3	(0;0)	(1;2)	(3;3)

Suppose we have the following points: $S_{11}(1\,;2)$, $S_{12}(0\,;1)$, $S_{13}(3\,;1)$, $S_{21}(1\,;1)$, $S_{22}(2\,;1)$, $S_{23}(1\,;2)$, $S_{31}(0\,;0)$, $S_{32}(1\,;2)$, and $S_{33}(3\,;3)$.

This means that if both players play strategy 1, then the first player wins 1 and the second wins 2. Similarly, if player 1 plays strategy 1 and player 2 plays strategy 2, then player 1 wins 0 and player 2 wins 1 and so on. The payoff coordinate plane is depicted in Figure 2.

If we use the normal form (payoff matrix), we get the data presented in Table 2.

4.2. Advantages. According to the example above, it is clear that the couple $(0\,;0)$ is not favorable, neither to the first player nor to the second. That is, if the first player chooses the third strategy, then the second player must rationally choose either the second strategy or the third strategy. Similarly, we can notice that the couple $(3\,;3)$ is a Nash equilibrium, and if we compare the two methods (payoff matrix and payoff coordinate plane), we can conclude the following:

(1) The representation of the payoff points makes it much easier to read the benefits of each player. Indeed, it suffices to make the orthogonal projection of x-axis to know the gain of the first player, and similarly, we make the orthogonal projection of y -axis to know the gain of the second player

(2) The Nash equilibrium in pure strategies can be easily determined in a game. Indeed, if we use the payoff matrix, we need to check all the couples to determine the Nash equilibrium; on the other hand, if we use the payoff coordinate plane, we can determine it from the first observation, which will be the point at the top right (S_{33} in the previous example)

(3) The strategies with the same power can be easily deduced. Indeed, in our coordinate plane, we can clearly see that the points S_{11}, S_{23}, and S_{32} are merged, and this means that, if both players play strategy 1, or player 1 plays strategy 2 and player 2 plays strategy 3, or player 1 plays strategy 3 and player 2 plays strategy 2, then each player's gain will not change; on the other hand, it is difficult to make this kind of remark if we use the payoff matrix, and it

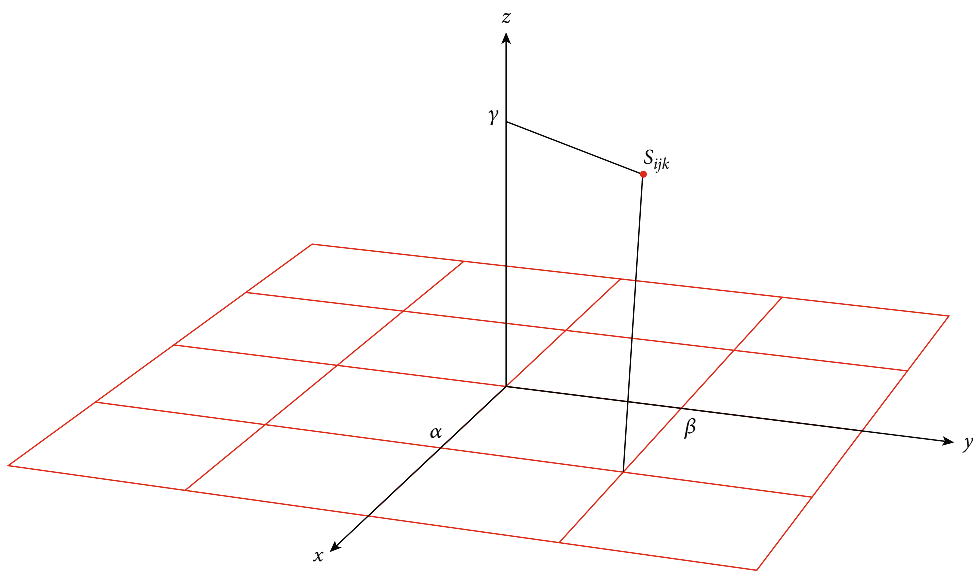

FIGURE 3: A payoff coordinate space of a three-player game with n strategies.

will be very difficult if the number of strategies increases

(4) The most beneficial strategy for the first player can easily be determined. Indeed, in the payoff coordinate plane, it suffices to choose the point on the extreme right. For example, in Figure 2, if player 1 chooses strategy 2, then the second player will also prefer to play strategy 2

(5) The most beneficial strategy for the second player can easily be determined. Indeed, in the payoff coordinate plane, it suffices to choose the highest point

(6) Dominant and dominated strategies can be easily identified

(7) This method can be extended to three (3) dimensions (i.e., a game with three players). Indeed, instead of using a coordinate plane, we will use a coordinate space

5. Payoff Coordinate Space

In this section, we will introduce the concept of a payoff point in 3D rather than 2D. Following that, we will establish a coordinate space dedicated to payoff analysis. Subsequently, by plotting the payoff points on this coordinate space, we will conduct a comparative analysis between our method and the other method (payoff matrix). The main results are supported by a practical example.

5.1. Definitions

Definition 6. Let n_1, n_2, and n_3 be three integers greater than 2. Consider a three-player game with $n_1 \times n_2 \times n_3$ strategies.

We define the payoff points $S_{ijk}(\alpha\,;\beta\,;\gamma)$ with $i \in \{1\,;2\,;\cdots;n_1\}$, $j \in \{1\,;2\,;\cdots;n_2\}$, and $k \in \{1\,;2\,;\cdots;n_3\}$ by the following:

i is the strategy chosen by the first player
j is the strategy chosen by the second player
k is the strategy chosen by the third player
α is the payoff for player 1 by choosing the i^{th} strategy
β is the payoff for player 2 by choosing j^{th} strategy
γ is the payoff for player 3 by choosing k^{th} strategy
S_{ijk} is the payoff of the three players when the first player chooses the i^{th} strategy, the second chooses the j^{th} strategy, and the third chooses the k^{th} strategy

Definition 7. A payoff coordinate space is a way of representing a three-player game as a coordinate space showing the players' payoffs (in the form of payoff points) according to their respective strategies.

The x-axis will be reserved for the gain of the first player, the y-axis will be reserved for the gain of the second player, and the z-axis will be reserved for the gain of the third player. The payoff coordinate space is depicted in Figure 3.

Example 3. Consider a three-player game with two strategies. The number of points to be constructed is $2^3 = 8$.

Suppose we have the following points: $S_{111}(0\,;3\,;1)$, $S_{112}(2\,;1\,;1)$, $S_{121}(4\,;2\,;3)$, $S_{122}(1\,;0\,;0)$, $S_{211}(1\,;0\,;0)$, $S_{212}(3\,;4\,;2)$, $S_{221}(0\,;0\,;1)$, and $S_{222}(3\,;2\,;4)$.

We will compare the classical method (payoff matrix) and the payoff coordinate space. The payoff coordinate space related to this example is depicted in Figure 4.

5.2. Advantages. According to the example above, player 3's optimal strategy is to choose the second option when both player 2 and player 1 also select the second strategy. When both player 1 and player 3 opt for the second strategy, player 2's optimal move is to select the first option. Similarly, if player 2 chooses the second strategy and player 3 opts for

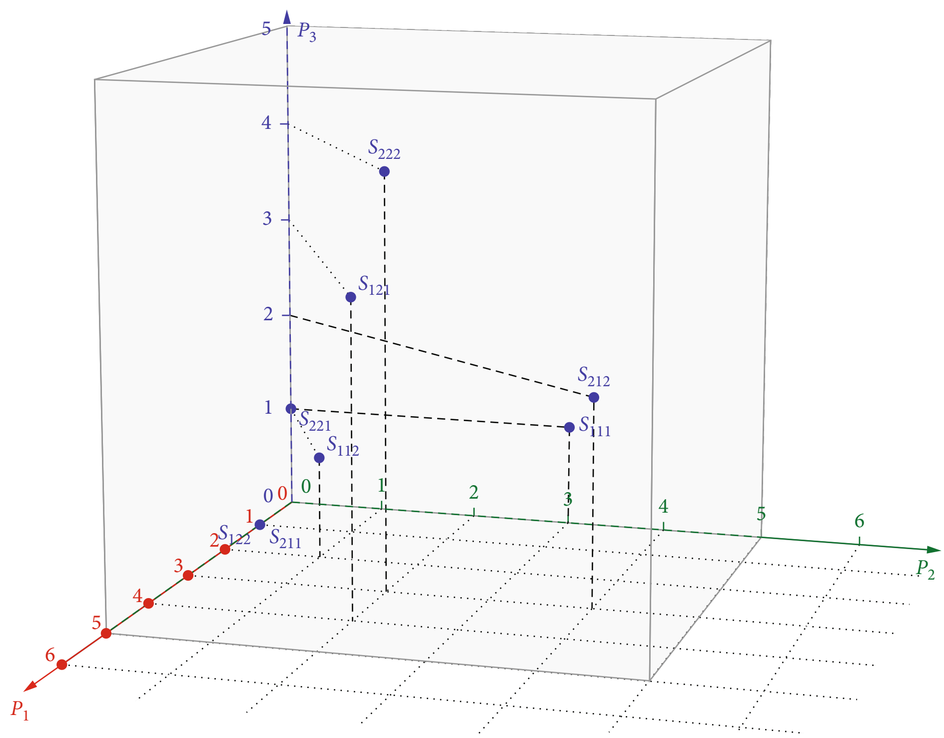

Figure 4: A payoff coordinate space of a three-player game with two strategies.

the first strategy, then player 1's optimal move is to select the first option.

In analogy with the previous section, it becomes evident that the representation of the payoff points greatly improves the comprehensibility of the benefits attributed to each player in both 2D and 3D.

The most beneficial strategy for player 1 is at the extreme point of the (Ox) axis.

The most beneficial strategy for player 2 is at the extreme point of the (Oy) axis.

The most beneficial strategy for player 3 is at the extreme point of the (Oz) axis.

(1) Nash equilibrium is easy to determine in a game

(2) Strategies of the same power are easy to deduce

(3) Dominant and dominated strategies are easy to deduce

6. Conclusion

In this paper, we have sought to address and overcome one of the biggest problems with traditional game theory methods: scalability. Within this context, we presented an alternative approach that outperforms existing methods in terms of scalability, convenience, and efficiency. Subsequently, a comparative analysis was conducted between the classical method (payoff matrix) and the payoff coordinate system, employing practical examples across both two-player and three-player game scenarios.

In conclusion, based on the aforementioned results, it can be confidently stated that the payoff coordinate system approach demonstrates remarkable efficacy in terms of its simplicity and the clarity of its results, regardless of whether it is applied to two-player or three-player games.

We believe that the extensions and contributions presented in this paper have broad applicability across numerous problems. They have the potential to simplify processes and may indeed play a pivotal role in effectively modeling complex situations. This is particularly evident in cases where the number of strategies increases in both two-player and three-player games. Hence, this paper could prove to be a valuable point of reference for future investigations and studies.

Data Availability

The data used are included within the article.

Disclosure

This study has no affiliation with or involvement in any organization or entity with any financial interest.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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