Hindawi
Journal of Advanced Transportation
Volume 2017, Article ID 5649823, 13 pages
https://doi.org/10.1155/2017/5649823

WILEY | Hindawi

Research Article

Optimal Signal Design for Mixed Equilibrium Networks with Autonomous and Regular Vehicles

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Received 30 April 2017; Accepted 13 June 2017; Published 16 August 2017

Academic Editor: Xiaobo Qu

A signal design problem is studied for efficiently managing autonomous vehicles (AVs) and regular vehicles (RVs) simultaneously in transportation networks. AVs and RVs move on separate lanes and two types of vehicles share the green times at the same intersections. The signal design problem is formulated as a bilevel program. The lower-level model describes a mixed equilibrium where autonomous vehicles follow the Cournot-Nash (CN) principle and RVs follow the user equilibrium (UE) principle. In the upper-level model, signal timings are optimized at signalized intersections to allocate appropriate green times to both autonomous and RVs to minimize system travel cost. The sensitivity analysis based method is used to solve the bilevel optimization model. Various signal control strategies are evaluated through numerical examples and some insightful findings are obtained. It was found that the number of phases at intersections should be reduced for the optimal control of the AVs and RVs in the mixed networks. More importantly, incorporating AVs into the transportation network would improve the system performance due to the value of AV technologies in reducing random delays at intersections. Meanwhile, travelers prefer to choose AVs when the networks turn to be congested.

1. Introduction

In the past decade, the automobile industries have made significant technological development by bringing computerization into driving. Such development is constantly accelerated under the circumstance where quite a few companies such as Google, Volvo, and BMW have been advocating autonomous vehicles (AVs) that navigate without direct human operations. Once AVs enter into the market, there is a mixed traffic in the transportation networks. Moreover, the traffic pattern and related management in a transportation system change accordingly when AVs are involved.

With no doubt, AVs could alleviate vehicle crashes dramatically. According to the study of NHTSA, more than 40% of fatal crashes are attributed to human fails, such as alcohol, distraction, and fatigue [1]. In this regard, the AVs have potential to reduce quite a number of fatal accidents in that the related human failings can be overcome by AVs essentially. In addition, to improve driving safety, AVs are introduced in the transportation system to mitigate traffic congestion. Baking and acceleration actions of leading vehicles can be well detected and anticipated by the advanced automated driving technology. Such technology could reduce the traffic-destabilizing shockwave propagation and consequently enhance the capacity of road links. Traffic speed could be increased by 8% to 13% in freeway, depending on the communication method and traffic smoothing approaches [2]. In fact, other techniques such as vehicle-to-vehicle (V2V) and vehicle-to-infrastructure (V2I) communication can be applied to alleviate traffic congestion as well as automated driving technology.

AVs can pass intersections more efficiently by shortening start-up times and headways among vehicles at signal controlled intersections [3]. Moreover, by utilizing the V2V and V2I communication, random delays at signal controlled intersections can be decreased substantially. Therefore, AVs can more effectively use green times at signals and eventually better use intersection capacities. In the presence of V2V communication, all involved AVs could be programmed to choose their own routes to maximize the group interest by following the principle of Cournot-Nash (CN) equilibrium. In other words, AVs are under the control of a central manager and behave cooperatively in travel routing decisions. In contrast, RVs behave fully competitively and follow user equilibrium to find the path with the shortest travel cost. Overall, the AVs have the potential and inherent advantage in reducing traffic congestion in the network.

In order to keep up with the technical development of AVs and make full use of the related merits, it is critical for the government to design tangible policies and control strategies to adapt the deployment of the technology. In this paper, we focus on designing optimal signal control strategies of AVs.

Despite the infusive technical developments, there is still a long time, perhaps several decades, to go for the AV vehicles to reach a high market share or dominate the full market. In this case, a heterogeneous traffic stream consisting of both RVs and AVs is more reasonable in the foreseeable future. In fact, it is believed that the government can identify critical locations for managing and/or separating AVs from the heterogeneous traffic steam. Meanwhile, in spirit of applying exclusive bus lanes, an exclusive AV lane would be an attractive and effective choice to implement AVs in urban traffic system. For example, Tientrakool et al. reported that the efficiency of the AVs lane is three times the common lane with mixed traffic flows [4]. Therefore, the use of AV lanes can save travel time substantially, which can in turn enhance the market share of AVs.

So far, much of the research in AV filed from micro perspective focused on analyzing AVs' on-road vehicle-following behaviors [5–10]. Different from the comparatively simple on-road moving condition, the driving actions of AVs at intersections turn to be more complicated because several conflict points resulted from crossing, moving in different directions. To characterize such problem, Dresner and Stone proposed an intersection management algorithm for AVs by utilizing a cell-based intersection reservation system [3].

The research of AVs from the macro perspective of travel mode split, network design, and parking behavior analysis has attracted an increasing attention in recent years. For example, quite a few studies suggested that a shared taxi fleet composed of AVs may replace the traditional taxi pattern. More importantly, incorporating AVs into the transportation system could alleviate traffic congestion in the networks [11]. Fagnant and Kockelman developed an agent-based model to investigate the impact of AVs on urban transport system, and they found that one AV would be able to replace eleven RVs but would only lead to 10% more travel cost [12].

Some researchers studied the traffic improvement methods based on transportation networks, considering the behaviors of drivers or travelers. Network optimization problems for the mixed AV traffic flows include AV lane configuration problem [13] and AV zone design problem [14]. For example, Chen et al. [13] proposed a time-dependent model to optimally deploy AV lanes on a general network with mixed AVs and RVs. Zhou et al. found that cooperative autonomous vehicles can substantially improve the efficiency of freeway merging [15].

In this paper, we investigate the signal design problem in the transportation system consisting of AVs and RVs. The signal design problem in this paper shows some new features from previous studies. Firstly, the routing choice decisions of two types of vehicle in a mixed network give rise to a mixed equilibrium, rather than the conventional UE. Specifically, AV users follow the CN principle in routing decision-making and travelers who drive RVs behave as UE players to seek their respective shortest paths. Secondly, as mentioned above, the travel (random) delay at interactions can be reduced by applying the merits of V2V and V2I techniques. Therefore, it is attractive for the government to design an optimal signal control scheme to make better use of the green times so as to enhance network performance. However, it is a challenging task to formulate the network signal design problem by taking into the mixed routing choice behaviors of AV and RV users. We attempt to propose a general mathematical programming model to help governments design and implement tangible signal control strategies to minimize the total travel cost of a transportation network with mixed traffic flows.

In the network, AVs and RVs in the road links are managed to, respectively, use exclusive lanes (common lane and AV lane). In this case, no interference between two different types of vehicles exists in the links so that the advantage of AV technology would be fully utilized. At the same intersections, AVs and RVs share the green time. The merit of lower random delay in AV lane will attract more travelers to use AVs. The signal design problem is a Stackelberg game between the government (network management authority) and the travelers. The government is the leader of designing signal control schemes. And the travelers who use RVs and AVs behave as followers. Therefore, we can formulate the signal design problem as a bilevel programming model, where in the upper level the government designs optimal signal control schemes to manage the movement of AVs and RVs, and in the lower level a CN-UE mixed equilibrium is given to characterize the AV users' and RV users' routing behaviors. Note that the link traffic model used in the paper is a BPR static function, and some recently calibrated link model can be utilized in the future study [16].

The rest of this paper is organized as follows. Section 2 defines and formulates the signal design problem and proposes the solution algorithm. Numerical examples are provided in Section 3. The last section concludes the paper.

2. Signal Design Problem with Mixed Traffic Flows

In this section, we firstly describe the signal design problem where both AVs and RVs share a transportation network. Then, a bilevel programming model is developed, to characterize the leader-follower behavior of the manager and the travelers. The upper-level model optimizes the total travel time in the network by determining optimal green time ratios of all phases at the signal control intersections. The lower-level problem is the CN-UE mixed equilibrium problem that determines the mixed traffic pattern. To facilitate the model formulations, the following assumptions are made. For each link, the AVs and RVs use their exclusive lanes, which means the traffic streams of AVs and RVs would not interfere with each other. The travel mode split for each commuter is determined by the discrete choice model (logit model). The capacity of AV lane is much larger than of the capacity of RV lane. The performance functions of AV and RV links are strictly increasing, as well as convex functions with respect to link flows. At the intersections, AVs and RVs share the same green times. The total delay at the intersections consists of average delay and random delay, and the AVs technologies can reduce the random delay. Thus, for AVs, the weight of random delay in the total delay is smaller than that for RVs. AVs follow the CN principle to choose the routes while the RVs follow the UE principle.

2.1. The Lower-Level Problem. The lower level of the model is a logit-based traffic equilibrium problem. RVs follow user equilibrium (UE) for traffic assignment, while AVs follow Cournot-Nash equilibrium (CN) for traffic assignment. Logit model is used to split the travel demands of the two modes.

We assume that travel time consists of road travel time and intersection delay time. The travel time of RVs and AVs can be represented by the following BPR functions, respectively:

$$
\begin{aligned}
t_a^1 &= t_{fa}^1\left[1+\beta_1\left(\frac{x_a^1}{C_a^1}\right)^{p_1}\right], \quad \forall a \in A \\
t_a^2 &= t_{fa}^2\left[1+\beta_2\left(\frac{x_a^2}{C_a^2}\right)^{p_2}\right], \quad \forall a \in A.
\end{aligned}
\tag{1}
$$

Functions (1) represent link travel time with respect to the link flow for RVs and AVs, respectively. Parameters β_1, β_2, p_1 and p_2 represent the sensitivities of link flow to the travel time. Due to the technical advantages of AVs, we assume $\beta_1 > \beta_2$ and $p_1 > p_2$.

The intersection delay times of RVs and AVs of each phase at the intersection can be expressed as

$$
\begin{aligned}
d_a^{1i} &= \frac{T_y\left(1-\varpi_{ai}^1\right)^2}{2\left(1-x_{ai}^1/C_a^{1i}\right)} + \lambda_1 \frac{x_{ai}^1/C_a^{1i}\varpi_{ai}^1}{2C_a^{1i}\left(\varpi_{ai}^1 - x_{ai}^1/C_a^{1i}\right)}, \\
&\qquad a \in A_y,\ i \in M_a^1 \\
d_a^{2i} &= \frac{T_y\left(1-\varpi_{ai}^2\right)^2}{2\left(1-x_{ai}^2/C_a^{2i}\right)} + \lambda_2 \frac{x_{ai}^2/C_a^{2i}\varpi_{ai}^2}{2C_a^{2i}\left(\varpi_{ai}^2 - x_{ai}^2/C_a^{2i}\right)}, \\
&\qquad a \in A_y,\ i \in M_a^2
\end{aligned}
\tag{2}
$$

Delay functions (2) consist of the average delay and the random delay; λ_1 and λ_2 represent the weight of the random delay in the total delay. Due to the technical improvement of AVs, we believe that AVs can effectively reduce the random delay ($\lambda_1 > \lambda_2$), thus reducing the total delay at the intersections.

Assume that the path set of the RV is K_1 and that of the AVs is K_2. We can establish a user equilibrium model as follows:

$$
\begin{aligned}
\min \quad Z &= k_1 \sum_{a\in A}\int_0^{x_a^1} t_a^1(\omega)\,\mathrm{d}\omega + k_1 \sum_{a\in\overline{A}}\sum_{i\in M_a^1}\int_0^{x_{ai}^1} d_a^{1i}(v)\,\mathrm{d}v + k_2\sum_{a\in A} t_a^2\left(x_a^2\right)x_a^2 + k_2\sum_{a\in\overline{A}}\sum_{i\in M_a^2} d_a^{2i}\left(x_{ai}^2\right)x_{ai}^2 \\
&\quad + \frac{1}{\theta}\sum q_{1,w}\left(\ln q_{1,w}-1\right) + \frac{1}{\theta}\sum q_{2,w}\left(\ln q_{2,w}-1\right)
\end{aligned}
\tag{3}
$$

$$
\text{subject to} \quad \sum_{j\in K_1} f_{1j}^w = \frac{q_{1,w}}{k_1} \tag{4}
$$

$$
\sum_{j\in K_2} f_{2j}^w = \frac{q_{2,w}}{k_2} \tag{5}
$$

$$
\begin{aligned}
&f_{1j}^w \geq 0, \\
&f_{2j}^w \geq 0
\end{aligned}
\tag{6}
$$

$$
x_a^1 = \sum_{j\in K_1} f_{1j}^w \delta_{aj}^{1w}, \quad \forall a \in A \tag{7}
$$

$$
x_a^2 = \sum_{j\in K_2} f_{2j}^w \delta_{aj}^{2w}, \quad \forall a \in A \tag{8}
$$

$$
x_{ai}^1 = \sum_{j\in K_1}\sum_{i\in M_a^1} f_{1j}^w \delta_{aj}^{1wi}, \quad \forall a \in \overline{A} \tag{9}
$$

$$
x_{ai}^2 = \sum_{j\in K_2}\sum_{i\in M_a^2} f_{2j}^w \delta_{aj}^{2wi}, \quad \forall a \in \overline{A} \tag{10}
$$

$$x_a^1 = \sum_{i\in M_a^1} x_{ai}^1, \quad \forall a \in \overline{A} \tag{11}$$

$$x_a^2 = \sum_{i\in M_a^1} x_{ai}^2, \quad \forall a \in \overline{A} \tag{12}$$

$$q_{1,w} + q_{2,w} = q_w \tag{13}$$

$$q_{1,w} = q_w \frac{1}{1 + e^{\theta(u_{1,w}-u_{2,w})}} \tag{14}$$

$$q_{2,w} = q_w \frac{1}{1 + e^{\theta(u_{2,w}-u_{1,w})}}. \tag{15}$$

For a mathematical programming problem, any local minimum solution satisfies the first-order conditions. If the first-order conditions of the model satisfy the path and mode choice principles, then the mixed equilibrium holds. We construct the following Lagrangian function:

$$\begin{aligned} L(x,q,\mu_1,\mu_2,\lambda) = Z &+ \sum_w u_{1,w}\left(\frac{q_{1,w}}{k_1} - \sum_{j\in K_1} f_{1j}^w\right) \\ &+ \sum_w u_{2,w}\left(\frac{q_{2,w}}{k_2} - \sum_{j\in K_2} f_{2j}^w\right) \\ &+ \sum_w \lambda^w \left(q_w - q_{1,w} - q_{2,w}\right). \end{aligned} \tag{16}$$

The first-order conditions are

$$\begin{aligned} &f_{1j}^w \frac{\partial L}{\partial f_{1j}^w} = 0, \quad \frac{\partial L}{\partial f_{1j}^w} \ge 0 \\ &f_{2j}^w \frac{\partial L}{\partial f_{2j}^w} = 0, \quad \frac{\partial L}{\partial f_{2j}^w} \ge 0 \\ &\frac{\partial L}{\partial q_{1,w}} = 0, \\ &\frac{\partial L}{\partial q_{2,w}} = 0 \\ &\frac{\partial L}{\partial u_{1,w}} = 0, \\ &\frac{\partial L}{\partial u_{2,w}} = 0, \\ &\frac{\partial L}{\partial \lambda^w} = 0. \end{aligned} \tag{17}$$

By function (17), the following necessary complementary conditions can be derived:

$$\left[\sum_{a\in A} t_a^1\left(x_a^1\right) x_a^1 \delta_{aj}^{1w} + \sum_{a\in\overline{A}} \sum_{i\in M_a^1} d_a^{1i}\left(x_{ai}^1\right)\delta_{aj}^{1wi} - u_{1,w}\right] \cdot f_{1j}^w = 0, \quad j \in K_1 \tag{18}$$

$$\sum_{a\in A} t_a^1\left(x_a^1\right) x_a^1 \delta_{aj}^{1w} + \sum_{a\in\overline{A}} \sum_{i\in M_a^1} d_a^{1i}\left(x_{ai}^1\right)\delta_{aj}^{1wi} - u_{1,w} \ge 0, \quad j \in K_1 \tag{19}$$

$$\begin{aligned} &\left\{\sum_{a\in A}\left[t_a^2\left(x_a^2\right) x_a^2 \delta_{aj}^{2w} + \frac{dt_a^2\left(x_a^2\right)}{dx_a^2}\right]\right. \\ &+ \sum_{a\in\overline{A}} \sum_{i\in M_a^2}\left[d_a^{2i}\left(x_{ai}^2\right)\delta_{aj}^{2wi} + x_{ai}^2 \frac{dd_a^{2i}\left(x_{ai}^2\right)}{dx_{ai}^2}\right] \\ &\left. - u_{2,w}\right\} f_{2j}^w = 0, \quad j \in K_2 \end{aligned} \tag{20}$$

$$\begin{aligned} &\sum_{a\in A}\left[t_a^2\left(x_a^2\right) x_a^2 \delta_{aj}^{2w} + \frac{dt_a^2\left(x_a^2\right)}{dx_a^2}\right] \\ &+ \sum_{a\in\overline{A}} \sum_{i\in M_a^2}\left[d_a^{2i}\left(x_{ai}^2\right)\delta_{aj}^{2wi} + x_{ai}^2 \frac{dd_a^{2i}\left(x_{ai}^2\right)}{dx_{ai}^2}\right] - u_{2,w} \\ &\ge 0, \quad j \in K_2 \end{aligned} \tag{21}$$

$$\sum_{j\in K_1} f_{1j}^w = \frac{q_{1,w}}{k_1} \tag{22}$$

$$\sum_{j\in K_2} f_{2j}^w = \frac{q_{2,w}}{k_2} \tag{23}$$

$$\begin{aligned} &f_{1j}^w \ge 0, \\ &f_{2j}^w \ge 0 \end{aligned} \tag{24}$$

$$x_a^1 = \sum_{j\in K_1} f_{1j}^w \delta_{aj}^{1w}, \quad \forall a \in A \tag{25}$$

$$x_a^2 = \sum_{j\in K_2} f_{2j}^w \delta_{aj}^{2w}, \quad \forall a \in A \tag{26}$$

$$x_{ai}^1 = \sum_{j\in K_1} \sum_{i\in M_a^1} f_{1j}^w \delta_{aj}^{1wi}, \quad \forall a \in \overline{A} \tag{27}$$

$$x_{ai}^2 = \sum_{j\in K_2} \sum_{i\in M_a^2} f_{2j}^w \delta_{aj}^{2wi}, \quad \forall a \in \overline{A} \tag{28}$$

$$x_a^1 = \sum_{i \in M_a^1} x_{ai}^1, \quad \forall a \in \overline{A} \tag{29}$$

$$x_a^2 = \sum_{i \in M_a^1} x_{ai}^2, \quad \forall a \in \overline{A} \tag{30}$$

$$q_{1,w} + q_{2,w} = q_w \tag{31}$$

$$q_{1,w} = q_w \frac{1}{1 + e^{\theta(u_{1,w} - u_{2,w})}} \tag{32}$$

$$q_{2,w} = q_w \frac{1}{1 + e^{\theta(u_{2,w} - u_{1,w})}}. \tag{33}$$

Functions (18) and (19) satisfy the UE principle of RVs. Functions (18) and (19) satisfy the CN principle of AVs. Functions (31)–(33) satisfy the logit-based mode choice. It is obvious that the solution of the minimization problem (3)–(11) satisfies the logit-based mode choice, namely, the mixed equilibrium condition of RVs and AVs.

And the local optimal solution of a convex program is also the global optimal solution. If the objective function is a strictly convex function, there is only one optimal solution of the model. Obviously $(1/\theta)\sum q_{1,w}(\ln q_{1,w} - 1)$ and $(1/\theta)\sum q_{2,w}(\ln q_{2,w} - 1)$ are both strictly convex functions. Furthermore, the link performance functions are strictly increasing and convex. Thus, objective function (3) is strictly convex. So the lower-level model has a unique solution. Therefore, the existence and uniqueness of the solution of the lower-level model are guaranteed.

2.2. The Upper-Level Model. The upper-level model introduces the optimal signal control scheme for AVs and RVs, with the aim of minimizing system travel time:

$$\begin{aligned} \min \quad & W \\ & = k_1 \left(\sum_{a \in A} t_a^1 \left(x_a^1\right) x_a^1 + \sum_{a \in \overline{A}} \sum_{i \in M_a^1} d_a^{1i} \left(x_{ai}^1\right) x_{ai}^1 \right) \\ & \quad + k_2 \left(\sum_{a \in A} t_a^2 \left(x_a^2\right) x_a^2 + \sum_{a \in \overline{A}} \sum_{i \in M_a^2} d_a^{2i} \left(x_{ai}^2\right) x_{ai}^2 \right). \end{aligned} \tag{34}$$

The decisional variables are the green time ratios of all phases at the signal controlled intersections. Meanwhile, different phases have to meet certain requirements at the same signal controlled intersections; namely, the following constraint needs to be added to the upper model:

$$\mathbf{B}\mathbf{I}_y = \mathbf{D}, \quad \forall y \in Y. \tag{35}$$

$\mathbf{B}$ and $\mathbf{D}$ are the matrixes to represent the relationship among different phases at the intersection y, so that function (35) implies that the phases at the intersection y should satisfy certain linear constraints.

2.3. Solution Algorithm. This problem belongs to the second-best network design problem; the sensitivity analysis based method is used to solve this model. Thus, we must evaluate the changes in equilibrium link flows caused by the changes in the green time ratios. It is difficult to evaluate the changes in equilibrium link flows directly because of the implicit, nonlinear function form of equilibrium link flows. The linear function can be used to approximate the nonlinear function of equilibrium link flows. Relative algorithm was proposed by Yang and Yagar [17]. The procedure of the algorithm is outlined below.

Step 0. Determine an initial set of green time ratio $\mathbf{I}_y^{(0)}$ and let $n = 0$.

Step 1. By using the given $\mathbf{I}_y^{(n)}$, solve the lower-level mixed equilibrium problem which yields the initial vector of traffic volume $\mathbf{x}^{(n)}$, intersection delay $\mathbf{d}^{(n)}$, and link travel time $\mathbf{t}^{(n)}$.

Step 2. Calculate the derivatives, using the sensitive analysis, and derive $\partial\mathbf{x}^{(n)}/\partial\mathbf{I}_y^{(n)}$, $\partial\mathbf{t}^{(n)}/\partial\mathbf{I}_y^{(n)}$, and $\partial\mathbf{d}^{(n)}/\partial\mathbf{I}_y^{(n)}$.

Step 3. Formulate local linear approximations of upper-level objective function (36) by using derivative information and solve the linear programming to obtain an auxiliary solution.

$$\begin{aligned} W(\mathbf{x}, \mathbf{d}, \mathbf{t}) = {} & W\left(\mathbf{x}^{(n)}, \mathbf{d}^{(n)}, \mathbf{t}^{(n)}\right) + \left(\frac{dW\left(\mathbf{x}, \mathbf{d}^{(n)}, \mathbf{t}^{(n)}\right)}{d\mathbf{I}_y} \right. \\ & \left. + \frac{dW\left(\mathbf{x}^{(n)}, \mathbf{d}, \mathbf{t}^{(n)}\right)}{d\mathbf{I}_y} + \frac{dW\left(\mathbf{x}^{(n)}, \mathbf{d}^{(n)}, \mathbf{t}\right)}{d\mathbf{I}_y} \right) \left(\mathbf{I}_y \right. \\ & \left. - \mathbf{I}_y^{(n)}\right). \end{aligned} \tag{36}$$

Step 4. Update the toll vector $\mathbf{I}_y^{(n+1)} = \mathbf{I}_y^{(n)} + (1/(n+1))(\overline{\mathbf{I}}_y^{(n)} - \mathbf{I}_y^{(n)})$.

Step 5. Terminate the algorithm when $\|\mathbf{I}_y^{(n+1)} - \mathbf{I}_y^{(n)}\| \leq \varepsilon$, where ε is the convergence criterion; otherwise $n = n + 1$ and go to Step 1.

For detailed equations of calculating the derivatives, please refer to [18, 19].

3. Numerical Examples

The road network with 14 links and 6 nodes in the numerical examples is provided in Figure 1. The road network has 12 OD pairs: 1-3, 1-5, 1-6, 3-1, 5-1, 6-1, 5-6, 6-5, 3-5, 5-3, 3-6, and 6-3. The solid lines in the figure represent the lanes for RVs. The dash lines indicate the lanes for AVs. Relevant parameters of the road network are shown in Table 1. Nodes 2 and 4 represent signal controlled intersections. For the sake of simplicity, we assume that, at intersections 2 and 4, for RVs the capacity proportion of left turn lane, through lane, and the right turn lane is 1 : 2 : 1. At intersection 2, for AVs, the capacity proportion of left turn lane, through lane, and right turn lane is 1 : 2 : 1. At intersection 4, the AVs lane has only through lane. In this study, intersection 4 takes one control strategy, while there are four different signal control strategies for intersection 2 as candidates. In our numerical examples, we examine the optimal green time ratios and some

Table 1: Relevant parameters of the road network.

Link	t_{fa}^1	C_a^1	t_{fa}^2	C_a^2
1	18	2000	18	2000
2	18	2000	18	2000
3	26	2000	26	2000
4	26	2000	26	2000
5	21	2000	21	2000
6	21	2000	21	2000
7	14	2000	14	2000
8	14	2000	14	2000
9	35	2000	—	—
10	35	2000	—	—
11	40	2000	—	—
12	40	2000	—	—
13	25	2000	25	2000
14	25	2000	25	2000

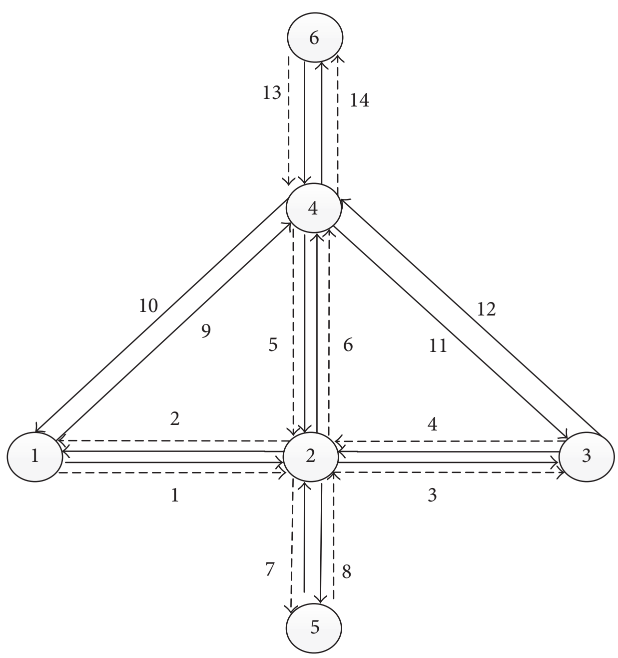

Figure 1: Topology of the transportation network.

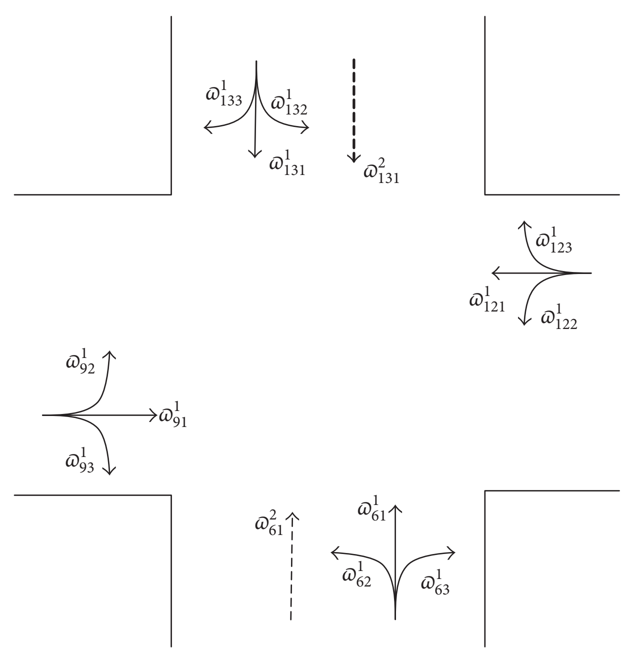

Figure 2: Illustration of green time ratios in intersection 4.

sensitive analyses under different signal control strategies. In the numerical examples, we have $k_1 = 2$, $k_2 = 3$, $T_y = 2$, $\lambda_1 = 0.9$, $\beta_1 = 0.3$, $p_1 = 4$, and $p_2 = 1$.

3.1. Different Signal Control Strategies. Based on the experience and theories of traffic engineering, we make the following assumptions for the traffic organization and signal control at the intersections as follows:

(i) Conflict points between RVs and AVs are not allowed in the same phase, because it is difficult to establish the communication between RVs and AVs. So these conflict points should be eliminated by signal control strategies for the sake of safety.

(ii) The conflicts among AVs are allowed in one phase, but the number should be limited in one signal phase, because the V2V technologies could help the AVs to avoid crash if there is enough space. But if too many conflict points exist in the same phase, the space is not insufficient for AVs to prevent crashes. So we assume, in the numerical examples, that there are at most four conflict points in the same phase.

For intersection 4, we propose only one control strategy. Illustration of green time ratios of different movements is shown in Figure 2.

A five-phase control strategy is used at intersection 4; in the strategy, 4 phases are utilized by RVs; one phase is

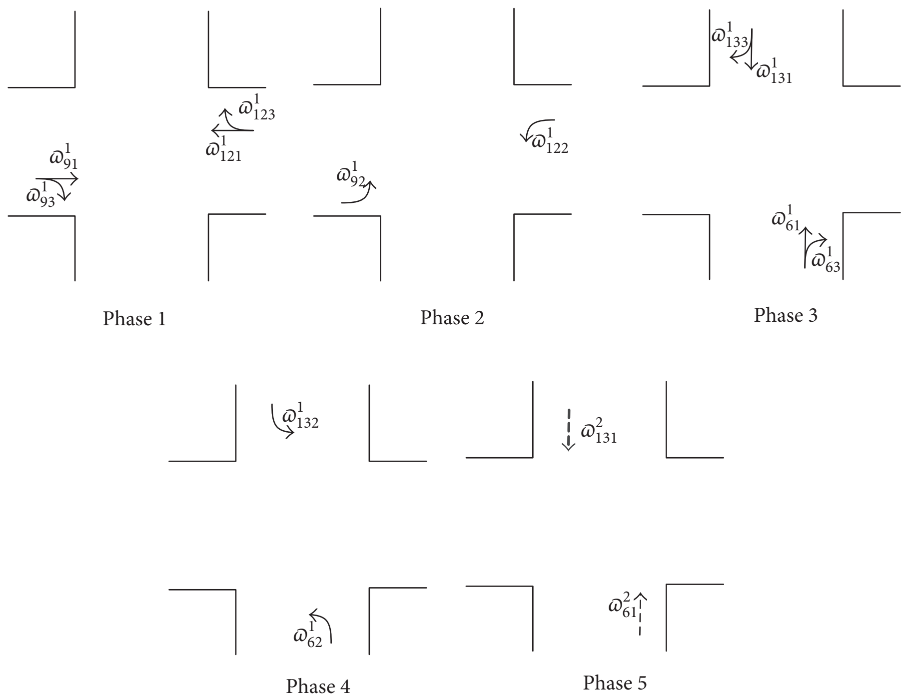

Figure 3: Five-phase signal control strategy in intersection 4.

utilized by AVs. The illustration is shown in Figure 3. And the relationship among the green time ratios in the strategy can be represented by the following linear functions:

$$\begin{gathered}
\omega_{91}^{1}=\omega_{93}^{1}=\omega_{121}^{1}=\omega_{123}^{1} \\
\omega_{92}^{1}=\omega_{122}^{1} \\
\omega_{61}^{1}=\omega_{63}^{1}=\omega_{131}^{1}=\omega_{133}^{1} \\
\omega_{62}^{1}=\omega_{132}^{1} \\
\omega_{61}^{2}=\omega_{131}^{2} \\
\omega_{91}^{1}+\omega_{92}^{1}+\omega_{61}^{1}+\omega_{62}^{1}+\omega_{61}^{2}=1.
\end{gathered} \tag{37}$$

For intersection 2, illustration of green time ratios of different directions is shown in Figure 4.

And then we propose four signal control strategies for intersection 2.

Control Strategy 1. There are five phases in Strategy 1; the illustration of the five phases is shown in Figure 5.

And the relationship among the green time ratios in the strategy can be represented by the following linear functions:

$$\begin{gathered}
\omega_{82}^{1}=\omega_{83}^{1}=\omega_{52}^{1}=\omega_{53}^{1}=\omega_{83}^{2}=\omega_{53}^{2} \\
\omega_{81}^{1}=\omega_{51}^{1}=\omega_{81}^{2}=\omega_{51}^{2} \\
\omega_{12}^{1}=\omega_{13}^{1}=\omega_{42}^{1}=\omega_{43}^{1}=\omega_{13}^{2}=\omega_{43}^{2} \\
\omega_{11}^{1}=\omega_{41}^{1}=\omega_{11}^{2}=\omega_{41}^{2} \\
\omega_{13}^{2}=\omega_{43}^{2}=\omega_{53}^{2}=\omega_{83}^{2} \\
\omega_{81}^{2}+\omega_{82}^{2}+\omega_{12}^{1}+\omega_{11}^{1}+\omega_{13}^{2}=1.
\end{gathered} \tag{38}$$

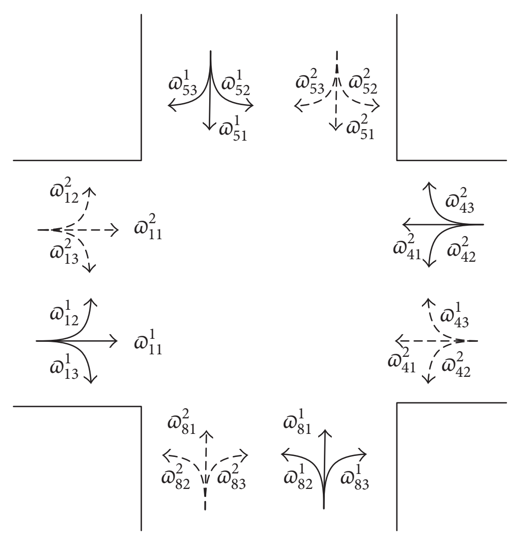

Figure 4: Illustration of green time ratios in intersection 2.

Control Strategy 2. There are five phases in Strategy 2. The illustration of the five phases is shown in Figure 6.

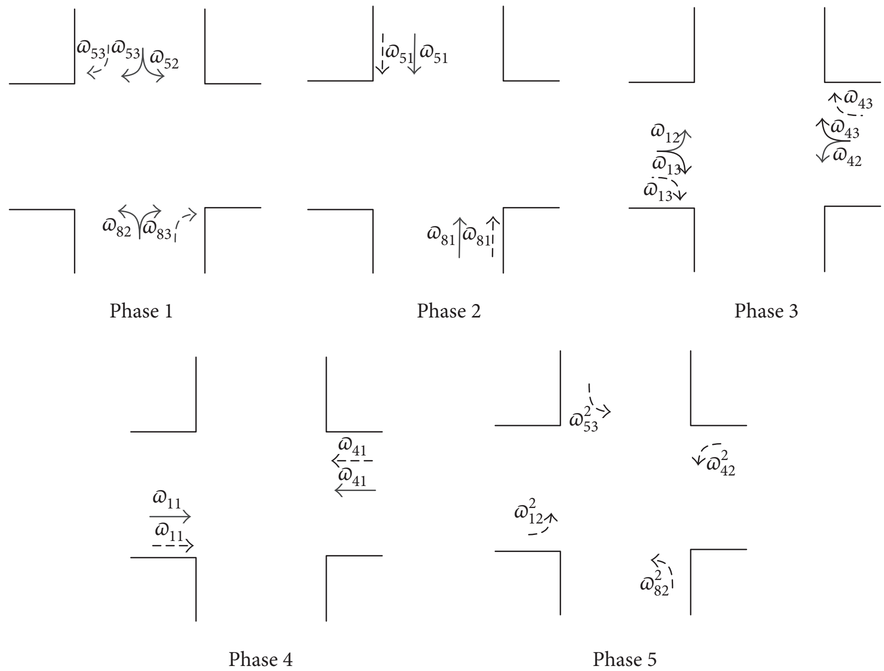

Figure 5: Illustration of control strategy 1 in intersection 2.

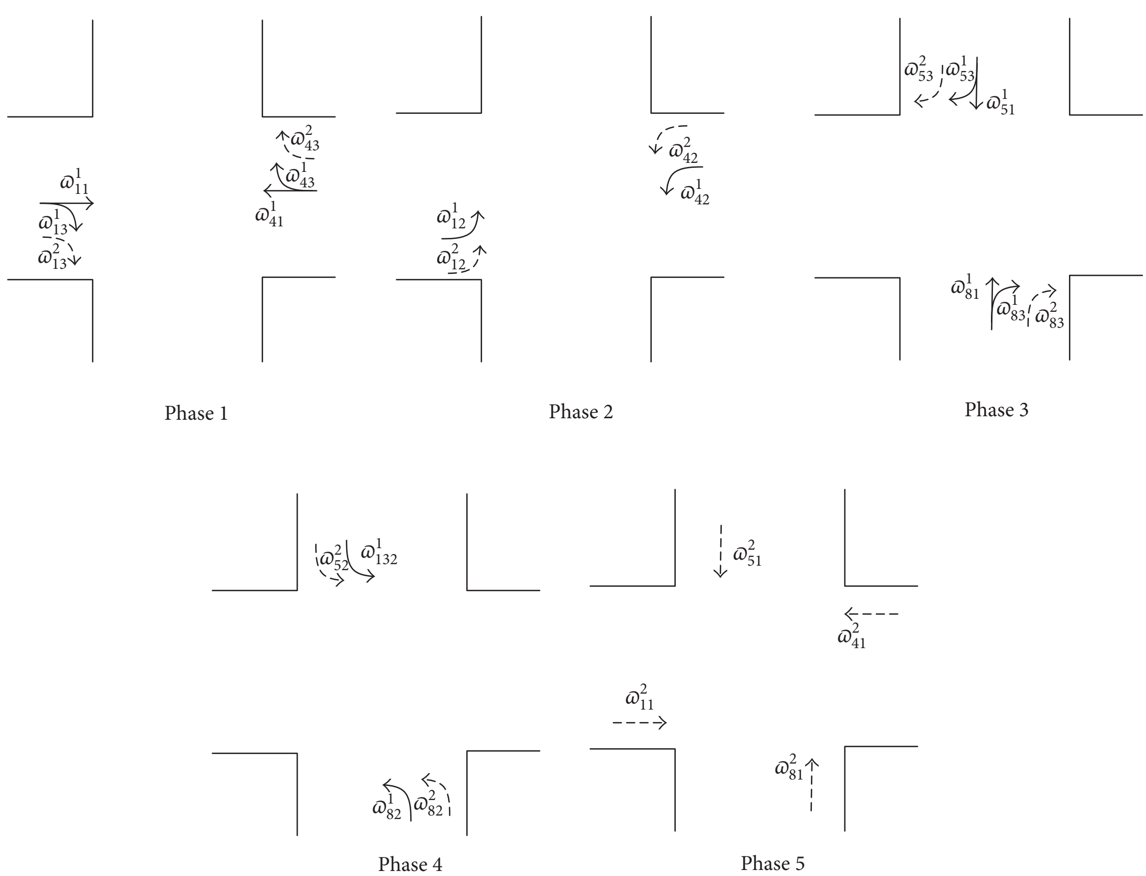

Figure 6: Illustration of control strategy 2 in intersection 2.

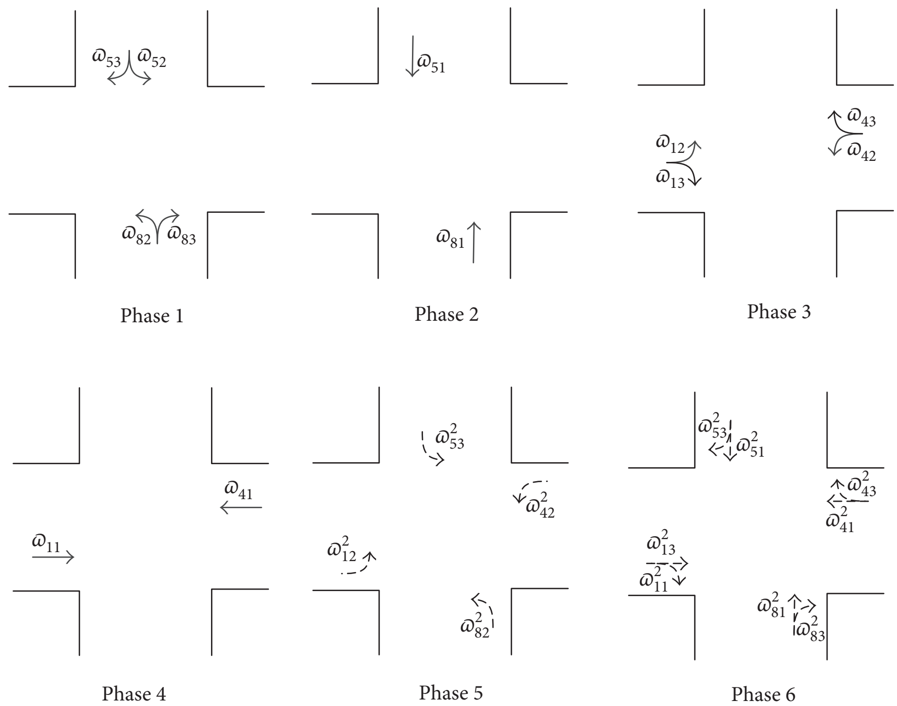

Figure 7: Illustration of control strategy 3 in intersection 2.

And the relationship among the green time ratios in the strategy can be represented by the following linear functions:

$$\begin{aligned}
&\varpi_{82}^{1}=\varpi_{83}^{1}=\varpi_{52}^{1}=\varpi_{53}^{1}=\varpi_{83}^{2}=\varpi_{53}^{2}\\
&\varpi_{81}^{1}=\varpi_{51}^{1}=\varpi_{81}^{2}=\varpi_{51}^{2}\\
&\varpi_{12}^{1}=\varpi_{13}^{1}=\varpi_{42}^{1}=\varpi_{43}^{1}=\varpi_{13}^{2}=\varpi_{43}^{2}\\
&\varpi_{11}^{1}=\varpi_{41}^{1}=\varpi_{11}^{2}=\varpi_{41}^{2}\\
&\varpi_{13}^{2}=\varpi_{43}^{2}=\varpi_{53}^{2}=\varpi_{83}^{2}\\
&\varpi_{81}^{2}+\varpi_{82}^{2}+\varpi_{12}^{1}+\varpi_{11}^{1}+\varpi_{13}^{2}=1.
\end{aligned}\tag{39}$$

Control Strategy 3. There are six phases in strategy 3; the illustration of the six phases is shown in Figure 7. The relationship among the green time ratios in the strategy can be represented by the following linear functions:

$$\begin{aligned}
&\varpi_{82}^{1}=\varpi_{83}^{1}=\varpi_{52}^{1}=\varpi_{53}^{1}=\varpi_{83}^{2}=\varpi_{53}^{2}\\
&\varpi_{81}^{1}=\varpi_{51}^{1}=\varpi_{81}^{2}=\varpi_{51}^{2}\\
&\varpi_{12}^{1}=\varpi_{13}^{1}=\varpi_{42}^{1}=\varpi_{43}^{1}=\varpi_{13}^{2}=\varpi_{43}^{2}\\
&\varpi_{11}^{1}=\varpi_{41}^{1}=\varpi_{11}^{2}=\varpi_{41}^{2}\\
&\varpi_{13}^{2}=\varpi_{43}^{2}=\varpi_{53}^{2}=\varpi_{83}^{2}\\
&\varpi_{81}^{2}+\varpi_{82}^{2}+\varpi_{12}^{1}+\varpi_{11}^{1}+\varpi_{13}^{2}=1.
\end{aligned}\tag{40}$$

Control Strategy 4. There are five phases in strategy 6; the illustration of the six phases is shown in Figure 8.

The relationship among the green time ratios in the strategy can be represented by the following linear functions:

$$\begin{aligned}
&\varpi_{82}^{1}=\varpi_{83}^{1}=\varpi_{52}^{1}=\varpi_{53}^{1}=\varpi_{83}^{2}=\varpi_{53}^{2}\\
&\varpi_{81}^{1}=\varpi_{51}^{1}=\varpi_{81}^{2}=\varpi_{51}^{2}\\
&\varpi_{12}^{1}=\varpi_{13}^{1}=\varpi_{42}^{1}=\varpi_{43}^{1}=\varpi_{13}^{2}=\varpi_{43}^{2}\\
&\varpi_{11}^{1}=\varpi_{41}^{1}=\varpi_{11}^{2}=\varpi_{41}^{2}\\
&\varpi_{13}^{2}=\varpi_{43}^{2}=\varpi_{53}^{2}=\varpi_{83}^{2}\\
&\varpi_{81}^{2}+\varpi_{82}^{2}+\varpi_{12}^{1}+\varpi_{11}^{1}+\varpi_{13}^{2}=1.
\end{aligned}\tag{41}$$

3.2. Four Numerical Examples under Different Strategies. The first example is simulated to examine the performance of signal control strategies. In the example, there are 3000 travelers for each OD pair. For AVs, we set the weight of the random delay in the total delay $\lambda_2 = 0.1$, and $\beta_2 = 0$ in the case. The results are shown in Table 2. From these results, the green time ratios for AVs are relatively small. For example, in strategy 1, the green time ratios for phase 5 of intersection 4 is just 0.073, which implies that the exclusive phase for the AVs occupies the least green time. One can conclude that in order to minimize system travel time by setting traffic signals at intersections, AVs always need to occupy less green time; more green time should be allocated to the RVs. For strategy 4, total system travel cost is 2995322, increased by 3.38%, 19.40%, and 23.65% compared with the results of strategy 3, strategy 2, and strategy 1, respectively. Remarkably, strategies 3 and 4 are the 6 phases' control strategies, and

TABLE 2: Optimal green time ratios under different strategies.

		Strategy 1	Strategy 2	Strategy 3	Strategy 4
Optimal green time ratios of intersection 4	Phase 1	0.288	0.265	0.276	0.225
	Phase 2	0.212	0.158	0.208	0.156
	Phase 3	0.225	0.164	0.254	0.278
	Phase 4	0.202	0.285	0.162	0.228
	Phase 5	0.073	0.128	0.1	0.113
Optimal green time ratios of intersection 2	Phase 1	0.252	0.229	0.201	0.189
	Phase 2	0.164	0.182	0.223	0.301
	Phase 3	0.284	0.256	0.189	0.156
	Phase 4	0.192	0.165	0.172	0.123
	Phase 5	0.108	0.168	0.119	0.165
	Phase 6	—	—	0.096	0.066
Total system cost		2422422	2508682	2897253	2995322

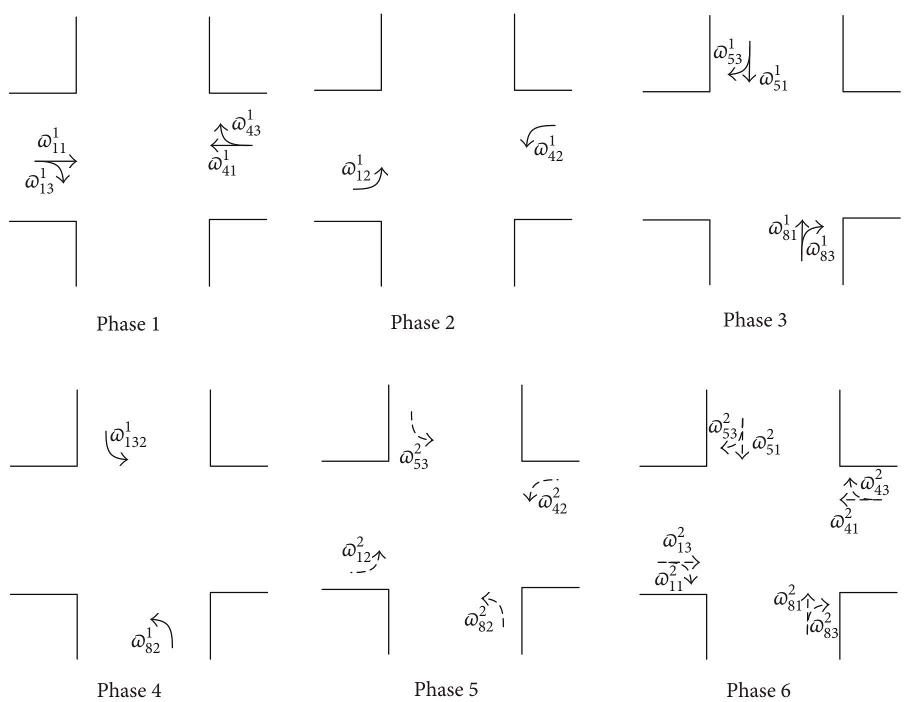

FIGURE 8: Illustration of control strategy 4 in intersection 2.

the gap of total travel cost between them is relatively small. Meanwhile, by using 5-phase control strategies, the social cost can be decreased greatly. The results indicate that the total system cost is more sensitive to the number of phases. For the optimal control of the AVs and RVs mixed network, government should reduce the number of phases at the intersections by arranging the AVs and RVs into the same phase as much as possible.

The second example is proposed to examine the sensitivity of the parameter λ_2 to the total travel time in the four strategies. Parameter λ_2 is the weight of the random delay in the total intersection delay for AVs. It reflects the technical development of AVs. With the V2V technology developing, the random delay at the intersection would be smaller; that is to say, λ_2 decreases. In the example travel demands of all OD pairs are 3000, and $\beta_2 = 0$. And results are shown in Figure 9. Above all, in all the four strategies, total travel time is positively related to λ_2. And the total travel time is more sensitive to λ_2 when λ_2 is smaller than 0.2. Otherwise when λ_2 is greater than 0.2, it is less sensitive to the total travel time. That is to say, the initial technical development of AVs for reducing λ_2 improves the network performance slowly. However, the subsequent development of technical development for reducing λ_2 (approaching 0) could improve the network performance significantly. Thus, we can find that the marginal benefit of technologies for reducing intersection delay increases as the technical development. The results imply that the system cost can be reduced due to the AVs technologies of reducing the random intersection delay, and the technology accumulation can accelerate the improvement.

Table 3: Total travel time and travel ratio of AVs under different travel demand.

	Travel demand	500	1000	1500	2000	2500	3000	3500	4000
Strategy 1	Travel time	242242	581380.8	944743.8	1356555	1816815	2422422	3391388	5038634
	Travel ratio of AVs	0.58	0.62	0.66	0.7	0.74	0.82	0.8	0.78
Strategy 2	Travel time	250868	602083.2	978385.2	1404861	1881510	2508682	3512152	5218054
	Travel ratio of AVs	0.58	0.62	0.65	0.71	0.74	0.82	0.82	0.76
Strategy 3	Travel time	289725	695340	1129928	1622460	2172938	2897253	4056150	6026280
	Travel ratio of AVs	0.47	0.51	0.56	0.6	0.63	0.75	0.83	0.78
Strategy 4	Travel time	299532	718876.8	1168175	1677379	2246490	2995322	4193448	6230266
	Travel ratio of AVs	0.48	0.52	0.56	0.6	0.63	0.75	0.83	0.78

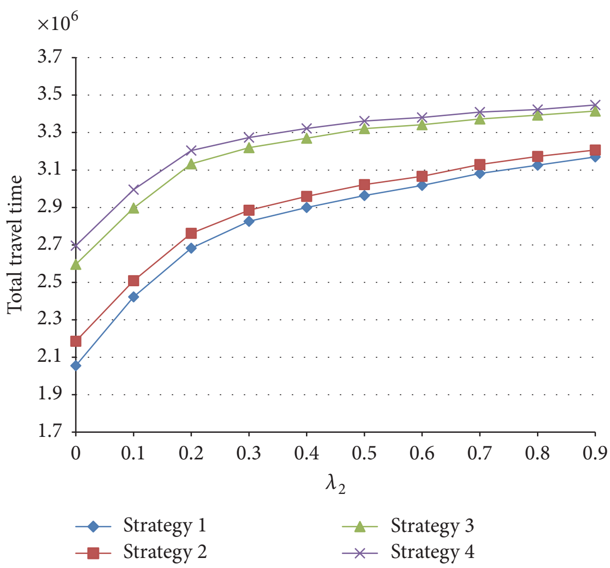

Figure 9: Relation between λ_2 and total travel time.

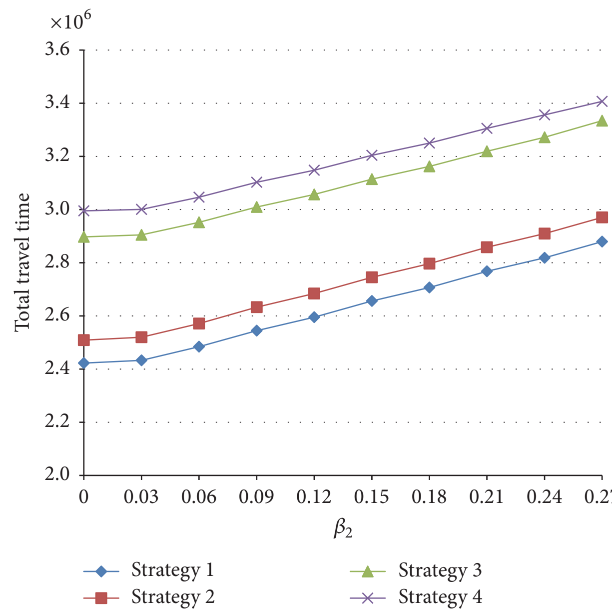

Figure 10: Relation between β_2 and total travel time.

The third example is proposed to exam the sensitivity of the parameter β_2 in BPR function to the total travel time in the four strategies. It also reflects the technical development of AVs. With the technology development, β_2 decreases so that the delay in the road link is reduced. In the example, numbers of travelers in all OD pairs are 3000, and $\lambda_2 = 0.1$. And results are shown in Figure 10. In all of the four strategies, total travel time is positively related to β_2. And the total travel time is less sensitive to β_2 if β_2 is smaller than 0.03; otherwise it is more sensitive to β_2. That is to say, the initial technical development of AVs for reducing β_2 improves the network performance substantially. However, the subsequent technical development for reducing β_2 close to 0 could not improve the network performance significantly. The results imply that network system cost can be further reduced due to the application of AV technologies in reducing the road link delay, but the technology accumulation could not accelerate the improvement.

The fourth example is proposed to examine the effect of travel demand on the total travel time in the four strategies. In the case of $\lambda_2 = 0.1$ and $\beta_2 = 0$, we set eight sets of travel demand and stimulate the optimal travel time and the travel ratio of AVs. The results are shown in Table 3. When the travel demand increases, total travel time increases more significantly in all the four strategies. And travel ratio of AVs is not positively related to the total travel demand. For the first two strategies, the travel ratio of AVs reaches the maximum when travel demand is 3000, and if travel demand is greater than 3000, the ratio decreases. For example, in strategy 1, when travel demand is 3000, the ratio of AVs is 0.82, but if the travel demand increases to 3500 and 4000, the ratio decreases to 0.8 and 0.78, respectively. For the last two strategies, the ratio gets its peak value when travel demand reaches 3500. For example, in strategy 3, when travel demand is 3500, the ratio of AVs is 0.83. But when the travel demand increases to 4000, the ratio decreases to 0.78. It implies that travelers tend to choose the AVs as the travel demand increases until it exceeds a threshold.

4. Conclusions

In order to keep up with the technical development of AVs and make full use advantages of AVs technology, it is critical for government to adopt suitable policies and control strategies to adapt the deployment of the technology. In the paper, we investigate the optimal signal control of AVs in the road network. In the network, AVs and RVs in the road link use exclusive lanes, respectively. So they do not interfere with each other in the links. But at intersections AVs and RVs share the green times. Since information and communication technologies are beneficial to reducing the random delay of AVs, it is important to design a reasonable signal system to improve the efficiency of AV movements.

A bilevel model is proposed to describe the problem of signal design for mixed networks with both AVs and RVs. The lower-level model is described as a mixed equilibrium of both types of vehicles. The AV vehicles follow the CN principle for route choice, and the RVs follow the UE principle. In the upper-level model, system managers use optimal signal control schemes to allocate appropriate green times to AVs and RVs to optimize the performance of the network. The sensitivity analysis based method is used to solve the model. Numerical examples are presented based on a road network with 14 links and 6 nodes. Four numerical examples are used to demonstrate the optimal green time ratios under different signal control strategies. Some observations have been obtained from the examples. First, for the optimal signal control of the mixed network, the number of phases at the intersections should be reduced, to improve the efficiency. Second, the system cost can be reduced due to the AVs technologies of reducing the random intersection delay, and the technology accumulation could accelerate the performance improvement. Third, system cost can be further reduced due to the AVs technologies in reducing the road link delay, but the technology accumulation could not accelerate the improvement. Finally, travelers would tend to choose the AVs as the total travel demand increases until it exceeds a threshold.

Symbols

N:	The set of intersections in the road network
Y:	The set of signal control intersections in the road network
y:	Signal controlled intersection, $y \in Y$
A_y:	The set of road links where the signal control intersection y is downstream
T_y:	The total time period of signal control intersection y
$\overline{A}$:	A set of upstream links at all intersections
$M_a^1 = \{1, 2, \ldots, \Delta_a^1\}$:	A set of phases of RVs at the intersection downstream of the road link a
$M_a^2 = \{1, 2, \ldots, \Delta_a^2\}$:	A set of phases of AVs at the intersection downstream of the road link a
$\Delta_a^1 = \lvert M_a^1 \rvert$:	The number of phases of RVs at the intersection downstream of the road link a
$\Delta_a^2 = \lvert M_a^2 \rvert$:	The number of phases of AVs at the intersection downstream of the road link a
$I_a^1 = \{\varpi_{a1}^1, \varpi_{a2}^1, \ldots, \varpi_{\Delta_a}^1\}$:	A set of green time ratios of each phase of RVs at the intersection downstream of the road link a
$I_a^2 = \{\varpi_{a1}^2, \varpi_{a2}^2, \ldots, \varpi_{\Delta_a^2}^2\}$:	A set of green time ratios of each phase of AVs at the intersection downstream of the road link a
$\mathbf{I_y} = \{I_a^1, I_a^2, \forall a \in A_y\}$:	A set of green time ratios of each phase of intersection y
$X_a^1 = \{x_{a1}^1, x_{a2}^1, \ldots, x_{a\Delta_a^1}^1\}$:	A set of RVs flows based on phases at the road link a
$X_a^2 = \{x_{a1}^2, x_{a2}^2, \ldots, x_{a\Delta_a^2}^2\}$:	A set of AVs flows based on phases at the road link a
δ_{aj}^{1w}:	$\delta_{aj}^{1w} = 1$ if the path j of RVs within the O–D pair w uses link a otherwise $\delta_{aj}^{1w} = 0$
δ_{aj}^{2w}:	$\delta_{aj}^{2w} = 1$ if path j of AVs within the O–D pair w uses link a otherwise $\delta_{aj}^{2w} = 0$
δ_{aj}^{1wi}:	if the path j of RVs within the O–D pair w uses link a and belongs to the phase $i \in M_a^1$, $\delta_{aj}^{1wi} = 1$; otherwise $\delta_{aj}^{1wi} = 0$
δ_{aj}^{2wi}:	$\delta_{aj}^{2wi} = 1$, if the path j of RVs within the O–D pair w uses link a and belongs to the phase $i \in M_a^2$; otherwise $\delta_{aj}^{2wi} = 0$
t_a^1:	The travel time of RVs at the link a
t_a^2:	The travel time of AVs at the link a
t_{fa}^1:	The free flow travel time of RVs at the link a
t_{fa}^2:	The free flow travel time of AVs at the link a
C_a^1:	The traffic capacity of RVs at the link a
C_a^2:	The traffic capacity of AVs at the link a
x_{ai}^1:	The traffic volume of RVs of the phase $i \in M_a^1$ at the link a
C_a^{1i}:	The traffic capacity of RVs of the phase $i \in M_a^1$ at the link a
x_{ai}^2:	The traffic volume of AVs of the phase $i \in M_a^2$ at the link a
C_a^{2i}:	The traffic capacity of AVs of the phase $i \in M_a^2$ at the link a
d_a^{1i}:	The average of RVs of the phase $i \in M_a^1$ at the link a
d_a^{2i}:	The average of AVs of the phase $i \in M_a^2$ at the link a
k_1:	The average passengers of a RV
k_2:	The average passengers of an AV vehicle.

Conflicts of Interest

The author declares that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

This study has been substantially supported by two grants from the National Natural Science Foundation Council of China (71531011, 71601142), a grant from Shanghai Shuguang Program (Project no. 13SG23), and a grant from Shanghai Pujiang Program (16PJC090).

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