Hindawi
Journal of Advanced Transportation
Volume 2019, Article ID 2143042, 11 pages
https://doi.org/10.1155/2019/2143042

Research Article

A Capacitated Location-Inventory Model with Demand Selection

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Received 4 September 2018; Revised 6 January 2019; Accepted 9 January 2019; Published 23 January 2019

Academic Editor: Zhi-Chun Li

In this paper, we consider an integrated supply chain network design problem, which incorporates inventory and pricing decisions into the capacitated facility location model. We assume that each warehouse has a capacity limitation that limits the average demand flowing through the warehouse and that the supplier can choose whether to satisfy each potential retailer's demand. We formulate the problem as a nonlinear integer programming model and solve the model via a Lagrangian relaxation based approach. We develop an efficient algorithm to solve the subproblem that arises from the Lagrangian relaxation procedure. Finally, we conduct extensive computational experiments to test the performance of the algorithms proposed in this paper and provide the managerial insights based on the computational results.

1. Introduction

Network design is among the most important supply chain decisions, as their implications are significant and long lasting. Thus, in designing a supply chain distribution network, the supply chain drivers, such as facilities, transportation, inventory, and pricing, should be considered together to support the competitive strategy of a company and maximize the supply chain profits [1]. Traditionally, decisions associated with these drivers are made separately. In this paper, we study an integrated supply chain distribution network design problem, which incorporates inventory and pricing decisions into the capacitated facility location model. We consider that a supplier (company) owns the production facility and produces one type of product to serve a set of potential retailers, each facing an uncertain demand. The supplier replenishes the retailers via the warehouses that serve as intermediate facilities for the shipment of the product. The objective of the supplier is to determine how many warehouses to locate and where to locate them, which retailers to assign to each warehouse, when and how many to reorder and what level of the safety stock to maintain at warehouses, and how much the sale price of the product at each region associated with a particular warehouse to maximize the total profits, while ensuring that the capacity constraints for warehouses are not exceeded and maintaining a specific service level. The total profits for the supplier are equal to the total revenues subtracting the transportation cost, the working inventory and safety stock holding cost at the warehouses, and the locating and operating cost of the warehouses.

In the supply chain distribution network design problems, most papers assume that all the demands should be served by the supplier, and then the objective is to minimize the system-wide costs. In practice, it is advisable that the supplier may lose some potential retailers to competitors in order to increase the profits because either the costs of serving those retailers can be very costly [2, 3] or the warehouses serving those retailers have capacity limitations, possibly both. Therefore, in this paper, we consider that the supplier can choose whether to satisfy each potential retailer's demand and then explore the benefits that the supplier will achieve by conducting this market strategy. The supplier determines which retailers to serve via the pricing decisions. In the pricing decisions, each retailer holds a reserve price for the product, and if the per unit cost charged for a retailer by the supplier is higher than the retailer's reserve price, the supplier will lose the retailer. The per unit cost charged for a retailer contains the wholesale price of the product and the per unit transportation cost from a specific warehouse to the retailer. Thus, it is important for the supplier to determine the

wholesale price of the product at each region associated with a particular warehouse, which directly affects the supplier in terms of the level of profit achieved as well as the demand profile that the supplier attempts to serve.

Furthermore, in this paper, we assume that each warehouse is subject to a capacity limitation, such as in the capacitated fixed charged facility location problem, which limits the average demand that flows through the warehouse. This situation might occur, for example, in contexts where the plant has a finite annual production capacity, or the warehouse has only a fixed number of square feet of storage space. The warehouse capacity constraints considered in this paper are different with those considered in Ozsen et al. [4], which limit the maximum possible inventory accumulation at the warehouses.

In summary, the distribution network design problem studied in this paper has the following features. (i) Each warehouse has a capacity limitation that limits the average demand flowing through the warehouse; (ii) each retailer faces an uncertain demand; (iii) the supplier can choose which potential retailers to serve; (iv) the objective of the supplier is to maximize the total profits. The main contribution of this paper is that, in designing the supply chain distribution network, we consider these features simultaneously and introduce an algorithm to solve the problem effectively.

The rest of this paper is organized as follows. In Section 2, we provide a review of the literature. In Section 3, we present a nonlinear integer programming model for the capacitated distribution network design problem. In Section 4, we introduce a Lagrangian relaxation based approach to solve the capacitated distribution network design problem and an efficient algorithm to solve the subproblem that arises from the Lagrangian relaxation procedure. In Section 5, we present the computational results. Finally, we conclude our discussions in Section 6.

2. Literature Review

Recently, there is an extensive body of research on the supply chain network design, such as supply chain network design with multiechelon inventory [5, 6], multicommodity supply chain network design [7, 8], multisourcing supply chain network design [9–11], multiobjective supply chain network design [12, 13], reliable supply chain network design [14–16], closed-loop supply chain network design [17, 18], and green supply chain network design [19–22]. We refer the readers to Shen [23], Melo et al. [24], Farahani et al. [25], and Govindan et al. [26] for reviews of related research.

In this paper, we only review literature on the location-inventory models that aim to minimize the sum of facility location costs, transportation costs, and inventory-related costs by choosing the optimal warehouse locations among given candidate sites to serve a set of retailers and at the same time determining the optimal inventory replenishment policies for the warehouses. Nozick and Turnquist [27] study a distribution network design problem, in which the inventory policy adopted at the distribution centers is an (s-1, s) continuous review policy, and the safety stock holding costs are approximated by a linear function of the number of distribution centers. They formulate the problem as a fixed-charge facility location model with inventory costs included in the fixed location cost as a constant. Shen et al. [28] study an uncapacitated location-inventory problem, in which the mean-to-variance ratio of the demand at each retailer is identical for all retailers, and the risk-pooling benefits are considered. They formulate the problem as a set-covering model and then make use of a column generation based approach to solve the set-covering model. Daskin et al. [29] formulate the same problem as a nonlinear integer programming model, and they develop a Lagrangian relaxation procedure to solve their model. Shu et al. [30] extend the works of Shen et al. [28] by proposing an effective algorithm for the pricing problem that arises from the column generation for all demand patterns. Sourirajan et al. [31] study a distribution network design problem that focuses on the trade-off between the safety stocks and the leading times at the DCs, in which they assume the leading time at a DC is directly related to the total demands assigned to the DC. They consider the facility location costs, the pipeline inventory costs between the production facility and the DCs, and the safety stock holding costs at the DCs, but they ignore the transportation costs between the DCs and the retailers in their model. They solve their model by a Lagrangian relaxation based algorithm. Berman et al. [32] study a coordinated location-inventory problem in which they consider a periodic review (R, S) inventory policy at distribution centers and two types of coordination for the distribution network system, i.e., partial coordination and full coordination. They show that the full coordination increases the location and inventory costs, but it is likely to reduce the overall costs of running the distribution system.

The models we discuss above deal with the problems in which the facilities are uncapacitated. Miranda and Garrido [33] propose a distribution network design model by incorporating inventory decisions into the capacitated facility location model, which also captures the risk-pooling effects. They develop a heuristic algorithm based on Lagrangian relaxation for solving the model by decomposing the corresponding Lagrangian dual problem into three subproblems. Ozsen et al. [4] study a capacitated logistic network design problem with risk-pooling, in which they consider the capacity constraint on the maximum possible inventory accumulation at a given warehouse instead of the capacity constraint that limits the average quantity of products flowing through the warehouse. They formulate the capacitated logistic network design problem as a nonlinear integer programming model and solve it by a Lagrangian relaxation based algorithm. Ozsen et al. [9] study a similar problem such that they consider multisourcing retailers; i.e., each retailer can be supplied by two or more warehouses. Vidyarthi et al. [34] and Park et al. [35] study the multistage distribution network design problems that need to determine the number and locations of suppliers or plants. Vidyarthi et al. [34] assume that both DCs and suppliers have finite capacity, and Park et al. [35] assume that only DCs have finite capacity.

However, the literature on supply chain distribution network design problems with integrating the pricing decisions is rather limited. Zhang [36] proposes a profit-maximizing

location model, where a firm, facing a set of potential retailers, needs to determine where to locate its center warehouse and the price for its product. He assumes that each retailer holds a reserve price for the product and if the actual price charged by the firm is higher than the reserve price, then the retailer will be lost from the firm. But he ignores the inventory cost in his model. Shen [3] studies a location-inventory problem with integrating the pricing decisions. He considers that each warehouse has no capacity limitation and each retailer faces a deterministic demand. He formulates the problem as a set-packing model and then solves it via column generation. Shu et al. [37] propose a two-echelon location-inventory model with a profit-maximizing objective. They assume that both warehouses and retailers carry inventory, and the supplier is in charge of the warehouse-retailer inventory replenishment. They formulate the problem as a set-packing model and then introduce a column generation based approach for solving the set-packing model. Ahmadi-Javid and Hoseinpour [38] study a profit-maximization multicommodity supply chain network design problem. They assume that the customer demands are price-sensitive and formulate the problem as a mixed-integer nonlinear programming model for the uncapacitated and capacitated distribution centers, respectively. They use Lagrangian relaxation based algorithm to solve their model.

In this paper, we present a capacitated location-inventory model by incorporating inventory and pricing decisions into the capacitated facility location model. The problem we studied in this paper is closely related to Miranda and Garrido [33], Shen [3], Ozsen et al. [4], Shu et al. [37], and Ahmadi-Javid and Hoseinpour [38]. Comparing with Shen [3] and Shu et al. [37], the feature of our paper is that we consider each warehouse has a capacity limitation, and each retailer faces an uncertain demand; comparing with Miranda and Garrido [33], the feature of our paper is that we integrate the pricing decisions into the supply chain network design; comparing with Ozsen et al. [4], the feature of our paper is that we consider different warehouse capacity constraints; comparing with Ahmadi-Javid and Hoseinpour [38], the feature of our paper is that we consider that the supplier can choose which retailers to serve, and each retailer faces an uncertain demand. We formulate this capacitated distribution network design problem as a nonlinear integer programming model and then introduce a Lagrangian relaxation based approach to solve this model. The subproblem that arises from the Lagrangian relaxation procedure is a nonlinear zero-one integer programming model, which maximizes a convex objective function subject to a single knapsack constraint. We also use the Lagrangian relaxation to solve the subproblem. Finally, we conduct the computational experiments to test the efficiency and effectiveness of the algorithms proposed in this paper.

3. Model Formulation

We consider a distribution network consisting of a production facility, a set of candidate warehouses, and a set of potential retailers, where the location of the production facility and the retailers is known, and the production rate of production facility is infinite. We assume that a supplier owns the production facility and produces one type of product, where we ignore the production cost. After production, the supplier transports the product to several warehouses for distribution. We also assume that the leading time from the production facility to each candidate warehouse is deterministic, and the demand at each retailer is uncorrelated across retailers, and that if the supplier chooses to serve a retailer, the supplier should satisfy all the demand of the retailer. In this paper, we consider single source strategy, which means a retailer is only served by one warehouse. The notations we used throughout the paper are defined as follows.

Sets

I: set of potential retailers;

J: set of candidate warehouse locations;

Inputs and Parameters

u_i: mean demand per year at retailer i, for each $i \in I$. We calculate u_i using the formula $u_i = \chi u_i'$, where u_i' is the mean demand per day at retailer i and χ is a constant used to convert daily demand into annual demand, e.g., $\chi = 300$;

σ_i: variance of daily demand at retailer i, for each $i \in I$;

d_{0j}: per unit transportation cost of delivering product from the supplier to warehouse j, for each $j \in J$;

d_{ij}: per unit transportation cost of delivering product from warehouse j to retailer i, for each $i \in I$;

K_j: fixed cost of placing an order from warehouse j to the supplier, for each $j \in J$;

h_j: per unit inventory holding cost rate at warehouse j per year, for each $j \in J$;

L_j: leading time in days for deliveries from the supplier to warehouse j, for each $j \in J$;

α: the desired percentage of retailers orders satisfied;

z_α: standard normal deviate such that $P(z \le z_\alpha) = \alpha$;

v_i: the reserve price of retailer i, for each $i \in I$;

$f_j(\cdot)$: the (annual) cost of locating and operating warehouse j, which is a concave and nondecreasing function of the annual demand flows through warehouse j, for each $j \in J$;

C_j: the capacity of warehouse j, for each $j \in J$;

Decision Variables

p_j: per unit wholesale price of the product at the region served by warehouse j, for each $j \in J$;

$x_j = 1$, if we locate a warehouse at candidate location j, and 0 otherwise, for each $j \in J$;

$y_{ij} = 1$, if the demand at retailer i is served by warehouse j, and 0 otherwise, for each $i \in I$, $j \in J$.

Let $r_i(p_j, d_{ij})$ denote the per unit gross profits of serving retailer i using warehouse j for the supplier, and we define $r_i(p_j, d_{ij})$ in the following. We know that when the wholesale price plus the per unit transportation cost charged to a retailer is higher than the retailer's reserve price, this retailer will turn to some other suppliers; i.e., the supplier will lose this retailer. Then we have [3, 36]

$$r_i\left(p_j, d_{ij}\right) = \begin{cases} p_j + d_{ij} - d_{0j}, & p_j + d_{ij} \le v_i, \\ 0, & p_j + d_{ij} > v_i. \end{cases} \tag{1}$$

We know that when $p_j + d_{ij} \le v_i$, retailer i can be served by the supplier for the per unit cost retailer i has to pay lower than his reserve price, and thus the supplier can get revenues from serving retailer i; when $p_j + d_{ij} > v_i$, retailer i cannot be served by the supplier for the per unit cost, retailer i has to pay higher than his reserve price, and thus the supplier cannot get any revenue from retailer i. Therefore, the pricing decisions determine which retailers can be served by the supplier, which is important for maximizing the supplier's profits.

In this paper, we focus on Type I service level [39], i.e., probability of no stock out per order cycle, which is one of the most commonly used service levels in inventory management [40, 41], and for calculating the safety stocks at warehouses, we assume that the normal approximation for demands is appropriate [28]. Since retailer demands are independent and the normal approximation for demands is appropriate, the warehouse demands can be represented by the Normal distribution. Thus, for each warehouse j, the leading time demand at warehouse j is Normal distribution with mean $L_j\sqrt{\sum_{i\in I} u_i' y_{ij}}$ and variance $L_j \sum_{i\in I} \sigma_i^2 y_{ij}$, and then the safety stock required at warehouse j with the probability α of no stock out per order cycle is $z_\alpha \sqrt{L_j \sum_{i\in I} \sigma_i^2 y_{ij}}$.

Using the above notations, we formulate the capacitated distribution network design problem as follows:

$$\mathscr{P}: \max \quad \sum_{j\in J} \left\{ \sum_{i\in I} r_i\left(p_j, d_{ij}\right) u_i y_{ij} - \sqrt{2h_j K_j \sum_{i\in I} u_i y_{ij}} - h_j z_\alpha \sqrt{L_j \sum_{i\in I} \sigma_i^2 y_{ij}} - f_j\left(\sum_{i\in I} u_i y_{ij}\right) x_j \right\} \tag{2}$$

$$\text{s.t.} \quad \sum_{j\in J} y_{ij} \le 1, \quad \forall i \in I, \tag{3}$$

$$y_{ij} - x_j \le 0, \quad \forall i \in I,\ j \in J, \tag{4}$$

$$\sum_{i\in I} u_i y_{ij} \le C_j, \quad \forall j \in J, \tag{5}$$

$$y_{ij} \in \{0, 1\}, \quad \forall i \in I,\ j \in J, \tag{6}$$

$$x_j \in \{0, 1\}, \quad \forall j \in J. \tag{7}$$

The first term of the objective function (2) is the gross profits of serving retailers using warehouses; the second term describes the working inventory holding cost determined by the EOQ model with ordering cost K_j, per unit holding cost h_j, and demand $\sum_{i\in I} u_i y_{ij}$; the third term represents the safety stocks holding cost; the fourth term is the cost of locating and operating warehouses, which is associated with how many demands flow through the warehouses. Constraint (3) states that a retailer may not be assigned to any warehouse. Constraint (4) stipulates that retailers can be only assigned to opened warehouses. Constraint (5) stipulates that the total demands served by a warehouse cannot exceed the capacity limit of the warehouse. Lastly, constraints (6) and (7) are standard integrality constraints, and constraint (6) also requires that each retailer is assigned to exactly one warehouse.

Note that the above model formulated is an extension of the classic capacitated facility location problem, which is already NP-hard, and furthermore the objective function (2) of the above model contains nonlinear cost terms as well. Thus any commercial solver cannot solve the above model effectively. In what follows, we develop a Lagrangian relaxation based approach to solve the above nonlinear integer programming model.

4. Solution Approach

Lagrangian relaxation method has been used successfully to solve many integer programming problems, such as facility location problems and supply chain network design problems [4, 9, 29, 34, 42, 43]. Fisher [44] gives an excellent discussion of the Lagrangian relaxation method for solving the integer programming problems. In this section, we develop a Lagrangian relaxation procedure to solve $\mathscr{P}$. We first obtain the Lagrangian dual problem of $\mathscr{P}$ by relaxing the assignment constraint (3), and then, for fixed Lagrangian multipliers, the Lagrangian dual problem is a nonlinear zero-one integer programming model, which maximizes a convex objective function subject to a single knapsack constraint. We also use the Lagrangian relaxation method to solve the Lagrangian dual problem. We describe the details of the Lagrangian relaxation procedure as follows.

4.1. Obtaining an Upper Bound. We obtain the Lagrangian dual problem by relaxing the assignment constraint (3) and associating with a set of Lagrange multipliers, denoted by λ_i, as follows:

$$\begin{aligned}\min_{\lambda}\max_{x,y}\quad & \sum_{j\in J}\sum_{i\in I}\left(r_i\left(p_j,d_{ij}\right)u_i-\lambda_i\right)y_{ij}\\ & -\sum_{j\in J}\sqrt{2h_jK_j\sum_{i\in I}u_iy_{ij}}-\sum_{j\in J}h_jz_\alpha\sqrt{L_j\sum_{i\in I}\sigma_i^2y_{ij}}\\ & -\sum_{j\in J}f_j\left(\sum_{i\in I}u_iy_{ij}\right)x_j+\sum_{i\in I}\lambda_i\end{aligned}\tag{8}$$

$$\begin{aligned}\text{s.t.}\quad & y_{ij}-x_j\le 0,\quad \forall i\in I,\ j\in J,\\ & \sum_{i\in I}u_iy_{ij}\le C_j,\quad \forall j\in J,\\ & y_{ij}\in\{0,1\},\quad \forall i\in I,\ j\in J,\\ & x_j\in\{0,1\},\quad \forall i\in I,\end{aligned}\tag{9}$$

where $\lambda=(\lambda_1,\lambda_2,\ldots,\lambda_n)$, and $\lambda_i\ge 0, i=1,2,\ldots,n$.

Note that we solve the Lagrangian dual problem for fixed values of Lagrange multipliers, λ_i, to obtain an upper bound on optimal objective function value of the capacitated distribution network design problem. Clearly, we want to maximize the Lagrangian function (8) over decision variables, x_j, y_{ij}, and p_j, and to minimize the function over the Lagrange multipliers, λ_i. Minimizing the Lagrangian function (8) over the Lagrange multipliers, λ_i, can be done using subgradient optimization [44]. Therefore, we focus on solving the problem for fixed values of the Lagrange multipliers, λ_i.

For fixed values of Lagrange multipliers, λ_i, we solve the following subproblem for each candidate warehouse j.

$$\begin{aligned}\mathscr{P}_j:\max\quad & \sum_{i\in I}\left(r_i\left(p_j,d_{ij}\right)u_i-\lambda_i\right)y_{ij}\\ & -\sqrt{2h_jK_j\sum_{i\in I}u_iy_{ij}}-h_jz_\alpha\sqrt{L_j\sum_{i\in I}\sigma_i^2y_{ij}}\\ & -f_j\left(\sum_{i\in I}u_iy_{ij}\right)\\ \text{s.t.}\quad & \sum_{i\in I}u_iy_{ij}\le C_j,\\ & y_{i,j}\in\{0,1\}.\end{aligned}\tag{10}$$

We note that $\mathscr{P}_j$ is a 0-1 integer programming problem with a nonlinear convex objective function and a single knapsack constraint. Let O_j be the optimal objective function value of $\mathscr{P}_j$, and we easily know that $\sum_{j:O_j>0,j\in J}O_j+\sum_{i\in I}\lambda_i$ is an upper bound for the optimal objective function value of $\mathscr{P}$. Since p_j is a decision variable in the objective function of $\mathscr{P}_j$, we first show how to solve $\mathscr{P}_j$ for a fixed p_j, and then we show how to solve $\mathscr{P}_j$ when p_j is a decision variable as well.

4.1.1. Solution Approach for $\mathscr{P}_j$ with a Fixed p_j. For a fixed p_j, we solve $\mathscr{P}_j$ by the Lagrangian relaxation method as well. By relaxing the capacity constraint (5), the Lagrangian dual problem of $\mathscr{P}_j$ is then formulated as follows:

$$\begin{aligned}\min_{\overline{\lambda}_j}\max_{y}\quad & \sum_{i\in I}\left(r_i\left(p_j,d_{ij}\right)u_i-\lambda_i-\overline{\lambda}_ju_i\right)y_{ij}\\ & -\sqrt{2h_jK_j\sum_{i\in I}u_iy_{ij}}-h_jz_\alpha\sqrt{L_j\sum_{i\in I}\sigma_i^2y_{ij}}\\ & -f_j\left(\sum_{i\in I}u_iy_{ij}\right)+\overline{\lambda}_jC_j\end{aligned}\tag{11}$$

$$\text{s.t.}\quad y_{ij}\in\{0,1\},\tag{12}$$

where $\overline{\lambda}_j$ is the Lagrange multiplier associated with the capacity constraint (5), and $\overline{\lambda}_j\ge 0$.

For a fixed value of the Lagrange multiplier $\overline{\lambda}_j$, we want to maximize (11) over the assignment variables y_{ij}. Thus, for a fixed $\overline{\lambda}_j$, the Lagrangian relaxation dual problem of $\mathscr{P}_j$ reduces to the following problem:

$$\begin{aligned}\mathscr{L}_{\overline{\lambda}_j}:\max\quad & \sum_{i\in I}\left(r_i\left(p_j,d_{ij}\right)u_i-\lambda_i-\overline{\lambda}_ju_i\right)y_{ij}\\ & -\sqrt{2h_jK_j\sum_{i\in I}u_iy_{ij}}-h_jz_\alpha\sqrt{L_j\sum_{i\in I}\sigma_i^2y_{ij}}\\ & -f_j\left(\sum_{i\in I}u_iy_{ij}\right)\\ \text{s.t.}\quad & y_{ij}\in\{0,1\}.\end{aligned}\tag{13}$$

Let $\overline{O}_j$ be the optimal objective function value of $\mathscr{L}_{\overline{\lambda}_j}$, and then $\overline{O}_j+\overline{\lambda}_jC_j$ is an upper bound for the optimal objective function value of $\mathscr{P}_j$. Furthermore, we can rewrite $\mathscr{L}_{\overline{\lambda}_j}$ as a different but equivalent formulation as follows:

$$\begin{aligned}\min_{S\subseteq I}\quad & f_j\left(\sum_{i\in S}u_i\right)+\sqrt{2h_jK_j\sum_{i\in S}u_i}+h_jz_\alpha\sqrt{L_j\sum_{i\in S}\sigma_i^2}\\ & -\sum_{i\in S}\left(r_i\left(p_j,d_{ij}\right)u_i-\lambda_i-\overline{\lambda}_ju_i\right),\end{aligned}\tag{14}$$

where S is a subset of I.

Let $G_j(\sum_{i\in S}u_i)\equiv\sqrt{2h_jK_j\sum_{i\in S}u_i}+f_j(\sum_{i\in S}u_i)$ and $a_i(p_j,d_{ij})\equiv\lambda_i+\overline{\lambda}_ju_i-r_i(p_j,d_{ij})u_i$. Then, $\mathscr{L}_{\overline{\lambda}_j}$ can be reformulated as

$$\min_{S\subseteq I}\quad \sum_{i\in S}a_i\left(p_j,d_{ij}\right)+G_j\left(\sum_{i\in S}u_i\right)+h_jz_\alpha\sqrt{L_j\sum_{i\in S}\sigma_i^2}.\tag{15}$$

Let $\overline{S}_j$ denote the optimal solution of (15), and we have the following observation.

Lemma 1. *For each $i\in I$, we have $a_i(p_j,d_{ij})<0$ if $i\in\overline{S}_j$.*

Proof. We prove it by contradiction. Suppose that there exists $i \in \overline{S}_j$ such that $a_i(p_j, d_{ij}) \geq 0$. Then we let $\widehat{S}_j = \overline{S}_j \setminus \{i\}$, and note that the objective function value of (15) for $\widehat{S}_j$ is less than that for $\overline{S}_j$, which contradicts the optimality of $\overline{S}_j$. We therefore establish that if $i \in \overline{S}_j$, then $a_i(p_j, d_{ij}) < 0$. □

We define $I_j^- = \{i \in I \mid a_i(p_j, d_{ij}) < 0\}$. For a fixed p_j, from Lemma (2), we know that when we solve $\mathscr{L}_{\overline{\lambda}_j}$, we only focus on the retailers within the set I_j^- instead of the set I. Then we let $a_i \equiv a_i(p_j, d_{ij})$, $b_i \equiv u_i$, and $c_i \equiv \sigma_i^2$. As shown in Shu et al. [30], we know that the optimal solution of (15) is $\{i \mid \beta(-b_i/c_i) + \gamma(-c_i/a_i) < 1, i \in I_j^-\}$, where both β and γ are arbitrary positive real numbers, i.e., $\beta > 0, \gamma > 0$.

For a fixed p_j, (15) can be solved in $O(n^2 \log n)$ time [23, 30], where n is the number of retailers. In this paper, we solve (15) by a dual algorithm [23], and we depict the steps of the dual algorithm as follows.

Step 1. For each pair of (i, i'), $i \in I_j^-, i' \in I_j^-$, we solve the following linear equations:

$$\begin{aligned} \beta\left(-\frac{b_i}{a_i}\right) + \gamma\left(-\frac{c_i}{a_i}\right) &= 1 \\ \beta\left(-\frac{b_{i'}}{a_{i'}}\right) + \gamma\left(-\frac{c_{i'}}{a_{i'}}\right) &= 1. \end{aligned} \tag{16}$$

Let $(\beta_{ii'}, \gamma_{ii'})$ denote the solution of (16), and we choose the solution if both $\beta_{ii'}$ and $\gamma_{ii'}$ are positive.

Step 2. For the ease of exposition, we relabel the $\beta_{ii'}$ as β_k so that $\beta_k \leq \beta_{k+1}$, $k = 1, 2, \ldots, m$ and $m \leq |I|^2$, and we also denote $\beta_0 = 0$, for $k = 0$, and $\beta_{m+1} = +\infty$, for $k = m + 1$.

Step 3. For each $k = 0, 1, \ldots, m$,

(a) Let $\Gamma_i = (1 - \beta'(-b_i/a_i))/(-c_i/a_i)$ for each $i \in I^-$, where $\beta_k \leq \beta' < \beta_{k+1}$, and we then sort the lines $\beta(-b_i/a_i) + \gamma(-c_i/a_i) = 1$ in the nondecreasing value of Γ_i for all $i \in I_j^-$.

(b) Let $k_1 \leq k_2 \leq \cdots \leq k_{|I_j^-|}$ denote the ordering of the lines when the value of Γ_i is obtained at β'. So, if $\Gamma_{k_l} \geq 0$, $l = 1, 2, \ldots, |I_j^-|$, then $\{k_l, k_{l+1}, \ldots, k_{|I_j^-|}\}, \{k_{l+1}, \ldots, k_{|I_j^-|}\}, \ldots, \{k_{|I_j^-|}\}$ are the feasible solutions of (15).

Step 4. From the feasible solutions of (15), we select the one with minimum objective function value of (15), and then this solution is the optimal solution of (15).

Having solved the Lagrangian relaxation problem $\mathscr{L}_{\overline{\lambda}_j}$, we need to minimize the Lagrangian function (11) over the Lagrange multiplier $\overline{\lambda}_j$. We do so using a subgradient optimization procedure.

We then show how to obtain a lower bound for the optimal objective function value of $\mathscr{P}_j$, and it is sufficient to find a feasible solution for $\mathscr{P}_j$. At each iteration of the Lagrangian procedure, we depict the details of the step for obtaining a feasible solution for $\mathscr{P}_j$ as follows:

Let $I_j^1 = \{i \in I_j^- \mid y_{ij} = 1\}$ and $I_j^0 = \{i \in I_j^- \mid y_{ij} = 0\}$.

Step 1. If $\sum_{i \in I_j^1} u_i y_{ij} \leq C_j$, then the current solution for the Lagrangian relaxation problem is feasible for $\mathscr{P}_j$, and for a retailer $i \in I_j^0$, we also let $y_{ij} = 1$ for which the capacity constraint is satisfied and that increases the objective function value to the maximum based on the assignments made so far.

Step 2. If $\sum_{i \in I_j^1} u_i y_{ij} \geq C_j$, for each $i \in I_j^1$, we first let $y_{ij} = 0$ for which $y_{ij} = 1$ and that decreases the objective function value to the minimum based on the assignments made so far until the capacity constraint is satisfied. Secondly, we update the set I_j^0, and then for a retailer $i \in I_j^0$, we also let $y_{ij} = 1$ for which the capacity constraint is satisfied and that increases the objective function value to the maximum based on the assignments made so far.

Step 3. Update the set $I_j^1 = \{i \in I \mid y_{ij} = 1\}$, and then I_j^1 is a feasible solution of $\mathscr{P}_j$.

4.1.2. Solution Approach for $\mathscr{P}_j$ When p_j Is a Decision Variable. In this subsection, we discuss how to solve $\mathscr{P}_j$ as well as decide the optimal sale price p_j when the price p_j charged to retailers is a decision variable. We let S_j^* denote the optimal solution of $\mathscr{P}_j$.

Let $I_j = \{i \mid d_{ij} < v_i, i \in I\}$. Note that I_j is a set of potential retailers that can be served by warehouse j when the per unit sale price of the product is zero. To solve $\mathscr{P}_j$ and decide the optimal sale price p_j, we first give the following theorem.

Theorem 2. *Let p_j^* be the optimal price of $\mathscr{P}_j$ and $p_j^i = v_i - d_{ij}, i \in I_j$. When the optimal solution S_j^* of $\mathscr{P}_j$ is a nonempty set, the optimal price p_j^* is in the set $\{p_j^i, i \in I_j\}$.*

Proof. For the notational convenience, we relabel the retailers in set I_j as $\{1, 2, \ldots, |I_j|\}$ so that $p_j^{i+1} \geq p_j^i, i = 1, 2, \ldots, |I_j|$.

If $p_j^* > p_j^{|I_j|}$, $p_j^* + d_{ij} \geq v_i, \forall i \in I$. Then we have $r_i(p_j^*, d_{ij})u_i - \lambda_i \leq 0, \forall i \in I$, and it is obvious that the optimal solution S_j^* of $\mathscr{P}_j$ is an empty set. This is to say, no retailers can be served by warehouse j because of the high price, and at this time the optimal price has no practical significance. Thus we have $0 < p_j^* \leq p_j^{|I_j|}$.

Since $p_j^* \leq p_j^{|I_j|}$, there exists an $i_0 \in \{1, 2, \ldots, |I_j|\}$ such that $p_j^{i_0} \geq p_j^*$ and $p_j^{i_0} = \arg\min_{p_j^i, i \in I_j}(p_j^i - p_j^*)$. If $p_j^* = p_j^{i_0}$, then $p_j^* \in \{p_j^i, i \in I_j\}$. If $p_j^* < p_j^{i_0}$, then $r_i(p_j^*, d_{ij})u_i - \lambda_i \leq 0, i \in \{1, 2, \ldots, i_0 - 1\}$ and $r_i(p_j^*, d_{ij})u_i - \lambda_i > 0, i \in \{i_0, i_0 + 1, \ldots, |I_j|\}$, which implies that the optimal solution $S_j^* \subseteq \{i_0, i_0+1, \ldots, |I_j|\}$. Notice that $r_i(p_j^*, d_{ij}) < r_i(p_j^{i_0}, d_{ij}), i \in \{i_0, i_0+1, \ldots, |I_j|\}$. Then, we have $r_i(p_j^*, d_{ij}) - \lambda_i < r_i(p_j^{i_0}, d_{ij}) - \lambda_i, i \in \{i_0, i_0 + 1, \ldots, |I_j|\}$, and we also have

$$\sum_{i \in S_j^*} \left(r_i\left(p_j^*, d_{ij}\right) u_i - \lambda_i\right) - \sqrt{2h_j K_j \sum_{i \in S_j^*} u_i}$$

Table 1: Parameters for problem instances.

Parameter	u_i'	σ_i	v_i	K_j	h_j	L_j	z_α	χ
Value	$[50, 100]$	$[10, 50]$	$[10, 50]$	1000	5	10	1.96	300

$$\begin{aligned} &- h_j z_\alpha \sqrt{L_j \sum_{i \in S_j^*} \sigma_i^2} - f_j \left(\sum_{i \in S_j^*} u_i \right) \\ &< \sum_{i \in S_j^*} \left(r_i \left(p_j^{i_0}, d_{ij} \right) u_i - \lambda_i \right) - \sqrt{2 h_j K_j \sum_{i \in S_j^*} u_i} \\ &- h_j z_\alpha \sqrt{L_j \sum_{i \in S_j^*} \sigma_i^2} - f_j \left(\sum_{i \in S_j^*} u_i \right). \end{aligned} \tag{17}$$

This would imply that p_j^* is not the optimal price of $\mathscr{P}_j$ and, therefore, a contradiction exists. We establish that $p_j^* \in \{p_j^i, i \in I_j\}$. □

Theorem 2 shows that we can attain the optimal price p_j^* for $\mathscr{P}_j$ within the set $\{p_j^i, i \in I_j\}$. Based on Theorem 2, we describe how to solve $\mathscr{P}_j$ when p_j is a decision variable as follows:

Step 1. For each $i \in I_j$, we first solve $\mathscr{P}_j$ at $p_j = p_j^i$ by the algorithm proposed in Section 4.1.1.

Step 2. Then, we select the one with maximum objective value, and the corresponding solution and price are the optimal solution and price of $\mathscr{P}_j$.

4.2. Obtaining a Lower Bound. In this subsection, we show how to obtain a feasible solution of $\mathscr{P}$, and, therefore, we obtain a lower bound on optimal objective function value for $\mathscr{P}$. We depict the details of obtaining a feasible solution of $\mathscr{P}$ as follows. At each iteration of the Lagrangian procedure, we first fix the warehouse locations at those locations for which $x_j = 1$ in the current Lagrangian solution. Then we assign the retailers to the fixed warehouses in a two-step heuristic process.

Step 1. For retailer i, if $\sum_{j \in J} y_{ij} \geq 1$, then we assign retailer i to warehouse j for which $y_{ij} = 1$ and that increases the profit to the maximum based on the assignments made so far.

Step 2. For retailer i, if $\sum_{j \in J} y_{ij} = 0$, then we assign retailer i to warehouse j for which the capacity constraint is satisfied and that increases the total profit to the maximum based on the assignments made so far.

5. Computational Results

In this section, we summarize our computational experience with the algorithms outlined in the above section. We conduct two sets of experiments as follows. The first set of experiments is designed to solve $\mathscr{P}_j$ that arises from the Lagrangian relaxation procedure, and the second set of experiments is designed to solve $\mathscr{P}$. We solve $\mathscr{P}_j$ via the algorithm proposed in Section 4.1 and solve $\mathscr{P}$ by the Lagrangian relaxation procedure. All the algorithms are coded in C++. We conduct all the experiments on a PC with a dual-core CPU of 2.27 GHz and 4G RAM running the Windows 7 operating system. We randomly generate the potential retailer locations and the candidate warehouse locations on the plane $[0, 1000] \times [0, 1000]$, and we denote the point $(0, 0)$ as the supplier or plant. Without loss of generality, we let $\chi = 300$.

All the parameters for the problem instances are generated as follows. For each retailer i, we randomly generate the daily mean demand u_i' in $[50, 100]$, the variance of daily demand σ_i in $[10, 50]$, and the reserve price v_i in $[10, 50]$. The fixed ordering cost K_j is set to 1000, the per unit holding cost rate h_j is set to 5, the lead time L_j is set to 10, and the standard normal deviate z_α is set to 1.96 corresponding to a 97.5% service level. Table 1 summarizes the values for the parameters.

Let D be the total potential retailer demands, i.e., $D = \sum_{i \in I} u_i$. In the first experiment, we fix a potential warehouse, say j, and solve the corresponding subproblem $\mathscr{P}_j$. We denote the capacity of the fixed warehouse j as $0.5D$, $0.7D$, and $0.9D$, respectively, which allows for the test problem instances with small capacity, medium capacity, and large capacity relative to the total potential retailer demands. In the second experiment, for each candidate warehouse j, we randomly generate the capacity limit C_j in $[0.3D, 0.5D]$, $[0.5D, 0.7D]$, and $[0.7D, D]$, respectively, which allows for the test problem instances with small capacity, medium capacity, and large capacity relative to the total potential retailer demands. We let the locating and operating cost for warehouse j be $1000 \times \sqrt{D_j}$, where D_j is the total demands that flow through warehouse j, $j \in J$.

Let t_{0j} be the Euclidean distance between the supplier and warehouse j and t_{ij} be the Euclidean distance between warehouse j and retailer i. Then, we define the per unit transportation cost from the supplier to warehouse j, d_{0j}, and the per unit transportation cost from warehouse j to retailer i, d_{ij}, in the following:

$$d_{0j} = \begin{cases} 2, & \text{if } 0 < t_{0j} \leq 100, \\ 4, & \text{if } 100 < t_{0j} \leq 300, \\ 6, & \text{if } 300 < t_{0j} \leq 600, \\ 8, & \text{if } 600 < t_{0j} \leq 1000, \\ 10, & \text{if } 1000 < t_{0j} \leq 1500, \end{cases}$$

Table 2: Parameters for Lagrangian procedures.

Parameter	Value
Maximum number of iterations	500
Minimum θ value allowed	0.00001
Number of no progress iterations before halving θ	5
Initial θ value	2
Initial Lagrange multipliers	0

Table 3: Performance of the algorithm for solving $\mathscr{P}_j$.

# Retailers			10	30	50	80	100
SC	CPU Time(s)	Ave	0.057	4.594	41.886	288.915	844.497
		Max	0.109	7.753	55.364	343.967	1105.920
	Gap(%)	Ave	0.00	0.41	1.12	0.32	0.06
		Max	0.00	2.17	5.87	2.57	0.67
MC	CPU Time(s)	Ave	0.043	4.017	33.824	217.616	726.392
		Max	0.109	6.957	54.241	310.329	950.070
	Gap(%)	Ave	0.00	0.00	0.00	0.00	0.00
		Max	0.00	0.00	0.00	0.00	0.00
LC	CPU Time(s)	Ave	0.014	1.903	9.626	99.271	309.888
		Max	0.046	3.853	46.909	370.034	648.387
	Gap(%)	Ave	0.00	0.00	0.00	0.00	0.00
		Max	0.00	0.00	0.00	0.00	0.00

$$d_{ij} = \begin{cases} 3, & \text{if } 0 < t_{ij} \leq 100, \\ 6, & \text{if } 100 < t_{ij} \leq 300, \\ 9, & \text{if } 300 < t_{ij} \leq 600, \\ 12, & \text{if } 600 < t_{ij} \leq 1000, \\ 15, & \text{if } 1000 < t_{ij} \leq 1500, \end{cases} \tag{18}$$

Note that the above transportation cost structures reflect different quantity discount schemes [3] and also display the economics of scale.

For all problem instances, the parameters for the Lagrangian relaxation procedure are shown in Table 2, where θ is the stepsize. In practice, the stopping criterion that the subgradient is zero for the Lagrangian procedure is rarely met, and then we terminate the Lagrangian procedure based on the maximum number of iterations allowed, or the minimum value of θ, whichever occurs first.

5.1. The Computational Results for $\mathscr{P}_j$. In this subsection, we report the computational results for $\mathscr{P}_j$ and explore how the capacity limit of warehouse j affect the computational results for $\mathscr{P}_j$. Note that when we solve $\mathscr{P}_j$, without loss of generality, we let $\lambda_j = 0$.

For each size of problem, we randomly generate 20 problem instances, and the number of the problem instances tested in this computational experiment is $20 \times 5 \times 3 = 300$. We solve all the problem instances and report the average and worst-case results. Table 3 gives the performance of the algorithm for solving $\mathscr{P}_j$ with small capacity, medium capacity, and large capacity. The row titled "# Retailers" represents the number of retailers in the test problem instances; the rows titled "SC", "MC", and "LC" represent small capacity problem instances, medium capacity problem instances, and large capacity problem instance, respectively. The row titled "CPU Time(s)" represents the CPU time taken by the algorithm for solving $\mathscr{P}_j$, where "Ave" represents the average CPU time and "Max" represents the maximum CPU time. The row titled "Gap(%)" represents the gap between the upper bound and lower bound obtained in Section 4.1, where Gap(%) = 100 × (upper bound - lower bound)/lower bound, "Ave" represents the average gap, and "Max" represents the maximum gap. From Table 3, we observe that the algorithm proposed in Section 4.1 can quickly solve $\mathscr{P}_j$ for medium size problem instances, and the CPU time taken by the algorithm for solving $\mathscr{P}_j$ decreases as the capacity constraint of the warehouse becomes loose. We also observe that for all the problem instances, the average gap is 0.47% and the maximum gap is no more than 6%, and furthermore, for most of problem instances with medium and large capacity, we can obtain the optimal solution of $\mathscr{P}_j$ via the algorithm proposed in Section 4.1. Then we conclude that the algorithm proposed in Section 4.1 for solving the subproblem $\mathscr{P}_j$ is very efficient.

5.2. The Computational Results for $\mathscr{P}$. In this subsection, we report the computational results of the second set of experiments and explore how the capacity limits of the warehouses affect the computational results for $\mathscr{P}$.

Tables 4–6 give the computational results for $\mathscr{P}$ with small capacity, medium capacity, and large capacity, respectively. The columns titled "#WHs" and "#Ret" represent the

Table 4: The computational results for $\mathscr{P}$ with small capacity.

Input		Output			
#WHs	#Ret	#WHs Open	#Ret Served	CPU (s)	Gap (%)
5	5	2	2	18.43	1.86
5	10	3	4	185.63	5.22
10	10	5	5	410.08	3.75
10	20	6	9	2764.90	2.46
20	30	10	14	6535.57	6.33
20	40	13	20	19059.60	2.12

Table 5: The computational results for $\mathscr{P}$ with medium capacity.

Input		Output				
#WHs	#Ret	#WHs Open	#Ret Served	CPU(s)	Gap(%)	Δ (%)
5	5	2	4	10.67	2.57	6.60
5	10	3	8	106.71	4.88	9.81
10	10	3	8	313.42	2.69	7.61
10	20	4	17	1929.37	1.91	10.26
20	30	7	23	3374.58	7.16	12.56
20	40	9	33	8425.08	3.80	14.70

Table 6: The computational results for $\mathscr{P}$ with large capacity.

Input		Output				
#WHs	#Ret	#WHs Open	#Ret Served	CPU(s)	Gap (%)	Δ (%)
5	5	2	4	10.67	2.57	6.60
5	10	3	8	76.71	4.88	9.81
10	10	3	9	258.44	1.66	6.03
10	20	4	17	829.37	1.91	10.26
20	30	7	26	2667.79	5.99	11.05
20	40	9	33	4231.18	3.80	14.70

number of warehouses and the number of potential retailers in the test problem instances, respectively; the columns titled "#WHs Opened", "#Ret Served", "CPU(s)", and "Gap(%)" represent the number of warehouses opened, the number of the retailers served by the opened warehouses, the number of CPU seconds taken by the Lagrangian procedure, and the percentage gap between upper bound and lower bound solutions, where Gap (%) = 100 × (Upper Bound - Lower Bound)/Lower Bound; the column titled "Δ(%)" in Tables 5 and 6 represents the percentage gap between the flexible model (with the constraints $\sum_{j\in J} y_{ij} \leq 1, \forall i \in I$) and the corresponding serve-all model (with the constraints $\sum_{j\in J} y_{ij} = 1, \forall i \in I$), where $\Delta(\%) = 100 \times (Z_1 - Z_2)/Z_2$, Z_1 is the best objective value of the flexible model we obtained, and Z_2 is the best objective value of the corresponding serve-all model we obtained. We solve the serve-all model via the Lagrangian relaxation algorithm as well. Note that the serve-all model means that all the potential retailers must be served by the opened warehouses, which requires that $\sum_{j\in J} C_j \geq D$, i.e., the problem instances with medium and large capacity constraints.

In Tables 4–6, we observe that the average gap is 3.64% and the largest gap is no more than 8%, which shows that the algorithm proposed in this paper for solving $\mathscr{P}$ is practical and efficient. As shown in Tables 5 and 6, the percentage of extra profits gained from the flexible model is 10% on average, and the largest percentage of extra profits gained from the flexible model is 14.7%, which shows that the flexible model is more profitable than the serve-all model. From Tables 5 and 6, we also obtain that the percentage of extra profits gained from the flexible model decreases with increasing of the capacities of the warehouses and increases with increasing of the number of potential retailers. These results show that the flexible model proposed in this paper can gain more profits when the problem instances are large-scale and when the capacities of the warehouses are small relative to the retailer's demands. Based on the above analyses, we conclude that the demand selection strategy is more profitable for the supplier in practice.

6. Conclusions

In this paper, we study a supply chain distribution network design problem in which each warehouse has a capacity limitation that limits the average demand flowing through the warehouse and each retailer faces an uncertain demand.

We consider that the supplier can choose whether to satisfy each potential retailer's demand for the warehouse capacity limitation, or the high cost of serving the retailer, and the supplier determines which retailers to serve via the pricing decisions. The one contribution of this paper to the literature is that in designing the supply chain distribution network, we consider warehouses have capacity limits, retailers face uncertain demands, and the supplier has flexibility in determining which retailers to serve simultaneously. We formulate this problem as a nonlinear integer programming model and solve the nonlinear integer programming model via a Lagrangian relaxation based approach. The subproblem that arises from the Lagrangian relaxation procedure is a 0-1 integer programming problem, which maximizes a nonlinear convex objective function subject to a single knapsack constraint. The other contribution of this paper to the literature is that we introduce an efficient algorithm to solve the subproblem. Finally, we conduct two sets of computational experiments to test the efficiency of the algorithms proposed in this paper. From the computational experiments, we obtain the following results: (1) the algorithm proposed in this paper for solving the subproblem $\mathscr{P}_j$ is efficient, and, in most cases, the algorithm can solve the $\mathscr{P}_j$ to optimality; (2) the Lagrangian relaxation based algorithm proposed in this paper for solving the master problem $\mathscr{P}$ is efficient, where the average gap is 3.64%, and the largest gap is no more than 8%; (3) the flexible model is more profitable than the serve-all model, which means the demand selection strategy is more profitable for the supplier; (4) when the problem instances are large-scale, and when the capacities of the warehouses are tight, the demand selection strategy can make more profits for the supplier.

The problem studied in this paper can be extended in many ways. First, in this paper, we consider the capacity of each warehouse is exogenous; i.e., the capacity of each warehouse is regarded as a fixed input, and, in practice, we want to determine the best capacity level to install at each warehouse. Thus, an extension of the problem studied in this paper is to allow the capacity of each warehouse to be a decision variable [45, 46]. Second, we only deal with medium size problem instances, and, in the future, we want to explore how to design the efficient approximation algorithms for the problem [6, 47–50].

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This research is supported by Humanity and Social Science Youth Foundation of the Ministry of Education of China (No. 14YJC630078).

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