Hindawi
Journal of Advanced Transportation
Volume 2019, Article ID 4971479, 11 pages
https://doi.org/10.1155/2019/4971479

Review Article

Review of Roundabout Capacity Based on Gap Acceptance

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Received 27 August 2018; Revised 12 December 2018; Accepted 15 January 2019; Published 4 February 2019

Academic Editor: Peter Furth

Circulating vehicles have priority at modern roundabouts. Entrance vehicles can enter the roundabout when there is a time gap larger than the critical gap; otherwise, the vehicles need to wait until there is a large enough gap. The gap acceptance theory was used to analyze the entrance capacity of roundabouts, which can be derived by queuing theory involving two vehicle streams. The paper introduces the main styles of headway distribution, which are named as bunched exponential distribution or M3 distribution. The calculation model of free stream ratio is also introduced. The entrance capacity models can be classified by different entrance vehicle types, which are piecewise function or linear function, or by different critical gap types, which are constant or stochastic function. For each form, the typical capacity expressions are given. The calculation values show a very small difference between these kinds of models. The capacity value based on the critical gap of stochastic function is more realistic and more complex in function style. Some conclusions were derived that drivers' nonhomogeneous and inconsistent character is more realistic than the fixed critical gap and following gap. The calculation results of capacity are similar to the field capacity under the assumption of homogeneity and continuance, with only a minor percent deviation. Finally, the paper points out additional problems and the suggested research in capacity of roundabouts.

1. Introduction

Modern roundabouts have some obvious characteristics such as small diameter of the central island, entrance vehicles yield to circulating vehicles, deviation of entrance vehicles, and split island between entrance and exit. The bottleneck appears at the entrance of the roundabout because vehicles need time to judge whether to enter the roundabout or to wait for another lager gap. Therefore, the research emphasis of modern roundabouts is always the entrance capacity.

Vehicles on the entrance lanes should yield to vehicles on the circulating lanes because of the entrance yield rule. The entrance lanes are regarded as the minor road and the circulating lanes as the major road. The roundabout can be regarded as the typical priority-controlled intersection. The entrance capacity, delay and queue length can be calculated by using the gap acceptance theory. The gap acceptance theory was well developed in Germany [1, 2]. The base theory was proposed by Major, Buckley and Tanner et al. (refer to [3, 4]). The capacity model had been developed based on different signal timing, different lane numbers, and different vehicle traffic characteristics.

Brilon [5] defined the full capacity as the sum of arrival flow rates when the saturated degree is 1 on the lanes. It is too difficult to find the field status because the arrival flow rate of every entrance is equal to entrance capacity based on the definition. Some research [6, 7] regarded the sum of entrance capacities as the full capacity when an entrance is saturated with the increase of flow rate of all entries by a certain proportion.

Some methods advised 3000 vehicles per hour as the capacity value of a single lane roundabout. Weaving theory was used to describe the traffic characteristics of roundabouts, such as Clayton's method, the equation of DOE (Department of Environment in UK) and Wardrop's method, which was proposed by Transport and Road Research Laboratory (TRRL) in UK.

After the 1970s, Wardrop's method could not be used to calculate the weaving capacity of roundabouts because the entrance yield rule was applied in the field. The regression

method and gap acceptance method were since developed (Ashworth, 1989); [8, 9].

Philbrick proposed the linear regression method (refer to [10]), which is mainly used in England. The exponential regression methods (Brilon and Stuwe, 1993) were based on numerous survey data among the saturated entrance flow rate and conflicting flow rate, geometry, etc.

The linear regression models in England were developed from traditional roundabouts, which had the large-diameter island design and entrance priority. The capacity, such as in the models proposed by Kimber [11], McNell and Smith [12], and Hollis and Semmens et al. [13], was regressed with geometry parameters of roundabouts and the total circulating traffic volume [14] regardless of traffic status of every lane.

In gap acceptance theory, some parameters should be determined including headway distribution of circulating vehicles, the critical gap, and following gap although they are variable with different geometry and traffic conditions of roundabouts. Vehicles enter the roundabout by use of acceptable gaps of circulating flow and the capacity is mainly determined by circulating flow rate and headway distribution.

Some countries such as USA, Germany, Australia, UK, Japan, France, and Russia have built complete capacity methods which are suitable for their own traffic conditions, including Highway Capacity Manual in USA, aaSIDRA, AUSTROADS and NAASRA in Australia, Swedish CAPCAL, SETRA method, and CETUR method in Germany.

The gap acceptance can be regarded as the signal timing process, which was proposed by Achelik and Chung (1994). Both the gap acceptance theory and regression method can be used to calculate the delay and queue length. The gap acceptance models are identical among roundabouts, two-way stop, and all-way stop intersections.

The capacity models and traffic control methods of roundabouts were studied in many countries. Al-Madani [15] collected the data of 13 large roundabouts including circulating and exiting flows, the number of lanes, and lateral position of vehicles in Bahrain. The developed model matched the field data reasonably well and fell well in other methods. Khoo and Tang [16] proposed a control strategy which was effective in reducing system travel time and increasing volume especially during medium to high levels of demand. A case study of a two-lane roundabout in Malaysia was developed in a microscopic simulation environment to study the roundabout system properties and to test the effectiveness of the proposed control strategy. Macioszek (2016) applied the capacity method of HCM 2010 in the field roundabouts in Poland. The calculated values were similar as the experience values.

Biel and Wong [17] proposed the entrance capacity model of circulating multilanes roundabout based on the regression model and HCM model. Under the limited priority condition, Qu et al. [18] calculated the entrance capacity based on Raff's critical gap model and the maximum likelihood method. Yap and Gibson et al. [19] proposed two regression methods and compared various methods based on the survey data of roundabouts in UK. The capacity model of negative exponential distribution showed a coincidence with the reality than the regression model under both the high flow rate and low flow rate. By using of the Germany capacity programs, Mauro and Branco [20] compared the capacity and delay model between circulating multilane roundabout and Turbo roundabout.

Weaving theory was used to describe the traffic characteristics, including weaving capacity in weaving sections. It is applicable in the traditional roundabouts where the central island is large and entry flow stream has the priority. But the regression method and gap acceptance theory can be used to calculate entrance capacity in modern roundabouts.

Linear regression method is mainly used in England. The entrance capacity is regressed with conflicting flow rate, geometry, etc. regardless of traffic status of every lane. Regression method usually can obtain satisfied results after a large number of data survey of roundabouts under the condition of saturated traffic flow.

Gap acceptance characters are related to traffic conditions of roundabouts including the headway distribution of circulating vehicles, circulating volume, critical gap, and following gap. Queuing theory was used to describe the gap acceptance process and to deduce the capacity equation. Gap acceptance theory and regression method can be used to calculate the delay and queue length too. By choosing the reasonable traffic parameters such as critical gap value or its distribution, gap acceptance theory can obtain the consistent capacity results as the field data. Gap acceptance theory was usually applied in the science research of traffic flow, whereas the regression method was more appropriate in the traffic engineering field.

Based on the gap acceptance theory, the basic assumptions were induced, the headway distributions were introduced, and the relevant parameters were summed up. Most entrance capacity models were classified according to the different critical gap types and the g(t) functions. Their applicable conditions and current application fields were concluded.

2. The Assumptions of Gap Acceptance

The headway is the time interval that elapses between the arrival of the leading vehicle and the following vehicle at the designated test point. Gap is a little different from headway in that gap is the measure of the time between the rear bumper of the first vehicle and the front bumper of the second vehicle, rather than front-to-front time.

Gap acceptance usually happens in unsignalized intersections that are controlled by priority. The entrance vehicles will enter the intersection when there are no vehicles on the major road or wait for the acceptable gap at the stop line. The entrance vehicle will enter the intersection if the gap on the major road is larger than or equal to the critical gap; otherwise the entrance vehicle will decelerate or stop so as to wait for the next gap when it is larger than the critical gap. The critical gap can be regarded as the driver's judgment threshold, which determines whether the entrance vehicle has enough time to enter the intersection safely or not. Drivers' behaviors are different from each other in reality, so the critical gaps of various drivers are different. The critical gap is usually

regarded to follow a certain distribution described by average value and variance.

Passing behavior hardly happened on the circulating lanes of a roundabout. It was supposed that the headway of circulating vehicles followed a certain distribution such as negative exponential distribution, M3 distribution, or Erlang distribution. In addition, some assumptions were given in gap acceptance theory [21–23]:

(1) Gap of vehicles is regarded as headway for ease of data collection.

(2) All gaps of circulating vehicles can be combined into single-lane traffic flow.

(3) Vehicles on the entrance arrive stochastically at the roundabout.

(4) Vehicles on the circulating lanes do not change their running behaviors when vehicles enter into the roundabout.

(5) Drivers on the entrance can recognize the vehicles going to exit roundabout before the conflict spot.

In reality, the above assumptions cannot exactly reflect all the driving operations in a roundabout. More complicated priority types, such as limited priority and priority conversation, reasonably accord with the reality. The interactions among vehicles are more complicated under these conditions.

Some influence factors are important to gap acceptance including queue length on the minor road, traffic volume on the major road, number of lanes on the major road and the minor road, the exiting vehicles, the geometry of entrance lanes, and the velocity of major road vehicles.

3. Traffic Character of Major Flow

Headway is important to analyze the gap acceptance process and calculate capacity and signal timing of intersections. The headway distribution is especially indispensable to traffic flow simulation. The regular headway distributions include negative exponential distribution also named as M1, shifted negative exponential distribution named as M2, and bunched exponential distribution named as M3 distribution. M1 and M2 distribution can be regarded as the special types of M3 distribution. We just give the probability density function and cumulative probability function of M3 distribution which is a dichotomized headway model. Erlang distribution, log-normal distribution, and mixed distribution (refer to [24, 25]) were also usually used in the field.

Critical gap t_c, gap t_f, and minimum headway t_m are also important traffic parameters. They can be constant or stochastic distribution depending on drivers' behavior character. The empirical values were given in HCM and some traffic engineering manuals. There are a lot of methods to calculate t_c.

3.1. M3 Distribution of Major Stream. Cowan [26] proposed the M3 distribution in which some vehicles run as a fleet and other vehicles run under the free flow status. The probability density function f(t) and cumulative distribution function F(t) are as follows:

$$f(t) = \begin{cases} \alpha\lambda e^{-\lambda(t-t_m)} & t \geq t_m \\ 0 & t < t_m \end{cases} \tag{1}$$

$$F(t) = \begin{cases} 1 - \alpha e^{-\lambda(t-t_m)} & t \geq t_m \\ 0 & t < t_m \end{cases} \tag{2}$$

where q is the flow rate on the major road (veh/s); α is the ratio of free flow; λ is the decay constant (1/s); t_m is the minimum headway. The estimation of λ can be found by using (3) (Troutbeck, 1997).

$$\hat{\lambda} = \frac{\alpha q}{1 - q t_m} \tag{3}$$

The estimated value of t_m is as follows:

$$\hat{t_m} = \min\{t_1, t_2, \cdots, t_n\} \tag{4}$$

Some extremely small values probably arise because individual drivers violate the priority rule. Hidden perils exist in these driving operations and such data is unreasonable. So the average headway of the vehicle fleet should be determined as t_m in the application of (4).

3.2. Calculate the Ratio of Free Flow. The parameters of M3 distribution are predicted by using least square method. λ can be calculated by the following equation [27]:

$$\lambda = \frac{1}{(1/n)\sum_{i=1}^{n} t_i - \zeta} \tag{5}$$

where ζ is a headway value at which the vehicles are assumed to be free (s). ζ is accepted as 3 or 4 seconds (Troutbeck, 1997; Hagring, 1998). $(1/n)\sum_{i=1}^{n} t_i$ is the average of headways which are greater than ζ.

The ratio of free flow can be estimated to satisfy the condition that the mean headway must be equal to the reciprocal of the flow rate.

Jocabs (1980, refer to [2]) proposed the relation between traffic volume q and the ratio, as follows:

$$\alpha = e^{-kq} \tag{6}$$

A more complicated equation (7) was given in the SIDRA 2.0 and the early model (Akcelik, Chung, 1994):

$$\alpha = e^{-bqt_m} \tag{7}$$

where k and b are constants.

Tanner [28] proposed the linear relation between the ratio of free flow and traffic volume (refer to [4]; Ashworth, 1970):

$$\alpha = 1 - qt_m \tag{8}$$

The analogous relation model was proposed in the Roundabout Manual of AUSTROADS [29]:

$$\alpha = 0.75\left(1 - qt_m\right) \tag{9}$$

Table 1: Parameter estimation of free flow ratio.

Lanes numbers		1	2	>2
Non-blocking traffic flow	t_m	1.8	0.9	0.6
	$3600/t_m$	2000	4000	6000
	b	0.5	0.3	0.7
	k_d	0.2	0.3	0.3
Circulating flow in roundabout	t_m	2.0	1.0	0.8
	$3600/t_m$	1800	3600	4500
	b	2.5	2.5	2.5
	k_d	2.2	2.2	2.2

Table 2: The ratio of free flow using different equation under different flow rates on circulating vehicles.

q(veh/s)	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45
Eq. (7)	0.78	0.61	0.47	0.37	0.29	0.22	0.17	0.14	0.11
Eq. (8)	0.90	0.80	0.70	0.60	0.50	0.40	0.30	0.20	0.10
Eq. (9)	0.68	0.60	0.53	0.45	0.38	0.30	0.23	0.15	0.08
Eq. (10)	0.80	0.65	0.51	0.41	0.31	0.23	0.16	0.10	0.05

The model in SIDRA 2.0 (refer to Akcelik, 2007) can be applied in nonblocking traffic flow, which is deduced from the traffic volume model between the velocity and the flow rate and the headway distribution model:

$$\alpha = \frac{1 - qt_m}{1 - (1 - k_d)\, qt_m}, \quad \alpha \geq 0.001 \tag{10}$$

where k_d is the bunched delay constant; for convenience, 0.001 is the minimum value of α.

The constants in (7) and (10) are as listed in Table 1.

The ratio of free flow is 0 when $q = 1/t_m$ except in the exponential model.

For the circulating flow in roundabout, $t_m = 2s, b = 2.5, k_d = 2.2$, we can calculate the ratio of free flow under different circulating flow rates as Table 2 and Figure 1. Equations (8) and (9) are linear models. There are relatively small difference value among (7), (9), and (10). The values of (8) are always 0.75 times of (9).

3.3. Critical Gap and following Gap. Calculations of critical gap t_c and following gap t_f are also important in gap acceptance theory. Many methods can be used to calculate the critical gap such as Raff's method, Maximum likelihood method, Siegloch's method, and Ashworth's method [30–35].

The maximum likelihood method used log normal distribution of critical gap. It was used in measuring values for the Highway Capacity Manual. Brilon et al. [30] reached a conclusion that the maximum likelihood and Hewitts's methods provided the more consistent estimates with the least bias. The probability equilibrium method had a significant bias that was dependent on the flow in the priority stream. The modified Raff technique (Troutbeck, 2011) is an acceptable alternative. Guo [36] proposed the theoretical method which can be substituted into the capacity equations containing critical gap variable. The equation had basically identical critical gap with the maximum likelihood method.

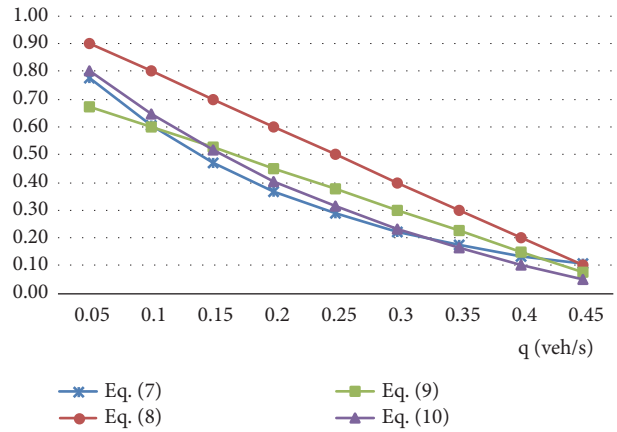

Figure 1: The ratio of free flow using different equation ($t_m = 2s, b = 2.5, k_d = 2.2$).

Table 3: Critical gap and following gap estimated in HCM 2010 (s).

	Single lane	Left lane in Multi lanes	Right lane in Multi-lanes
t_c	4.8	4.7	4.4
t_f	2.5	2.2	2.2

Brilon [37] found the traditional log normal distribution is not an essential part of the maximum likelihood method. The logistic distribution is easier to handle.

The constant critical gap in HCM 2010 is listed in Table 3 based on the field data of roundabouts in California.

4. Entrance Capacity of a Roundabout

4.1. The Theoretical Model of Entrance Capacity in Priority-Controlled Intersections. Based on gap acceptance theory, the equation of theoretical capacity C of the minor road in unsignalized intersections is as follows (Siegloch, 1973; refer to [2]):

$$C = q \int_0^{\infty} f(t)\, g(t)\, dt \tag{11}$$

where $g(t)$ is the number of vehicles entering the intersection when headways of vehicles on the major road are equal to t. It can be divided into continuous function and piecewise function because the number of vehicles can be expressed as an integer or decimal number.

Siegloch (1973; refer to [2]) proposed the continuous linear function of $g(t)$ as follows:

$$g(t) = \begin{cases} \dfrac{t - t_0}{t_f}, & t > t_0 \\ 0, & t \leq t_0 \end{cases} \tag{12}$$

$$t_0 = t_c - \frac{t_f}{2}$$

where t_f is the following headway; t_0 is the minimum acceptable headway, the intercept in horizontal coordinate of headway. The integration interval is $[t_c, \infty)$ in (11).

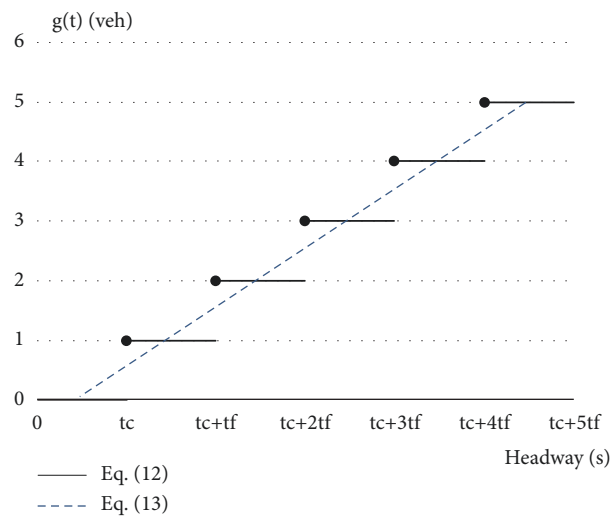

Figure 2: The g(t) of continuous function and piecewise function.

When $g(t)$ is a piecewise function, it has an equation as follows (Harders, 1976; refer to [2]):

$$g(t) = nP_n(t), \quad n = 1, 2, \cdots, \infty$$
$$P_n(t) = \begin{cases} 1, & t_c + (n-1)t_f \le t < t_c + nt_f \\ 0, & t < t_c \end{cases} \tag{13}$$

where t_c is the critical gap. The integration interval is $[t_c, \infty)$ in (11).

From Figure 2, the interval of circulating headway which can be accepted is $[t_0, +\infty)$ in continuous $g(t)$ function. The interval of circulating headway which can be accepted is $[t_c, +\infty)$ in piecewise $g(t)$ function.

Equation (11) shows that the capacity is related to the traffic volume on the major road, headway distribution, and $g(t)$ function. The lower limit of integral and $g(t)$ function is dependent on the critical gap.

Capacity model can be divided into two basic types according to critical gap, which can be a constant or stochastic distribution function. On the other side, the capacity model can be divided into two basic types including the continuous function and the piecewise function of $g(t)$ when the critical gap is constant. The basic equations are all based on a single-lane roundabout.

4.2. Entrance Capacity Model When Critical Gap Is Constant

4.2.1. When g(t) Is Piecewise Function. When the critical gap is constant, the model varies with the headway distribution. As mentioned before, M1, M2, M3, Erlang, log-normal distribution, etc. usually were used to describe the arrival of circulating vehicles. M3 model was mainly recommended because it can be regarded as a general equation of M1, M2, and M3T.

Troutbeck [38] proposed (14) of entrance capacity where the headway on the major road followed M3 distribution and g(t) is piecewise function. Equation (14) is used in AUSTROADS [29].

$$C = \begin{cases} \dfrac{\alpha q e^{-\lambda(t_c - t_m)}}{1 - e^{-\lambda t_f}}, & q > 0 \\ \dfrac{1}{t_f}, & q = 0 \end{cases} \tag{14}$$

The following are the capacity equations in which the distribution parameters are specially determined in M3 distribution. When headway follows M1 distribution, vehicles are under the free flow status and $t_m = 0$ where multiple circulating lanes were usually regarded as single lane. When headway follows M2 distribution, vehicles are under the free flow status and $t_m \neq 0$ in one lane. When headway follows M3T distribution, a dichotomized headway model assumes that a proportion, α, of all vehicles are free and the 1-α bunched vehicles have the same t_m.

(1) M1 Distribution. $\lambda = q$ when $t_m = 0$ and $\alpha = 1$. Buckley et al. ([39], refer to [40]) proposed (15).

$$C = \begin{cases} \dfrac{q e^{-q t_c}}{1 - e^{-q t_f}}, & q > 0 \\ \dfrac{1}{t_f}, & q = 0 \end{cases} \tag{15}$$

(2) M2 Distribution. $\lambda = q/(1 - qt_m)$ when $t_m \neq 0$, t_m is a constant, and $\alpha = 1$. Equation (16) can be referred to Luttinen [14] as follows:

$$C = \begin{cases} \dfrac{q e^{-\lambda(t_c - t_m)}}{1 - e^{-\lambda t_f}}, & q > 0 \\ \dfrac{1}{t_f}, & q = 0 \end{cases} \tag{16}$$

(3) M3T Distribution. $\lambda = q$ when $t_m \neq 0$, t_m is a constant, and $\alpha = 1 - qt_m$. The equation can be referred to Tanner [28] and Luttein [14].

$$C = \begin{cases} \dfrac{(1 - qt_m) q e^{-\lambda(t_c - t_m)}}{1 - e^{-\lambda t_f}}, & q > 0 \\ \dfrac{1}{t_f}, & q = 0 \end{cases} \tag{17}$$

(4) M3 Distribution and Critical Gap Model. When the headway on the major road follows M3 distribution, (18) can be deduced when the function of critical gap [34, 36] is substituted into (14).

$$C = \frac{\beta_a q}{1 - e^{-\lambda t_f}} \tag{18}$$

where β_a is total acceptance coefficient, the proportion of the number of accepted gaps to the number of total gaps.

4.2.2. When g(t) Is Continuous Function. The entrance capacity can be expressed as the product of the saturated flow rate s of entrance and the probability larger than the minimum acceptable gap t_0, $P\{t > t_0\}$ (Akcelik, 1998). That is to say, $C = sP\{t > t_0\}$, where $P\{t > t_0\}$ is the probability larger than the minimum acceptable gap t_0.

(1) M1 Model. Siegloch (1973; refer to Wu 1999) proposed (19) in which the headway on the major road followed negative exponent distribution as in (15).

$$C = \frac{e^{-qt_0}}{t_f} \tag{19}$$

Equation (19) was used in HCM 2010, and parameters were set as $t_c = 4.8s$ and $t_f = 2.5s$.

(2) M2 Model. Jacobs (1980, refer to [2]) proposed (20) in which the headway on the major road followed shifted negative exponent distribution as (16).

$$C = \frac{q}{\lambda t_f} e^{-\lambda(t_0 - t_m)} = \frac{1 - qt_m}{t_f} e^{-\lambda(t_0 - t_m)}, \quad t_0 > t_m \tag{20}$$

The equation also can be expressed in similar form as

$$C_p = \frac{e^{-\lambda(t_0 - t_m)}}{t_f(1 + \lambda t_m)}, \quad t_0 > t_m \tag{21}$$

McDonald and Armitage [41] used the above equations as the capacity of roundabouts. They can be compared to signalized intersections where t_f^{-1} is regarded as the saturated flow rate and t_0 is regarded as lost time.

(3) M3 Model. In Cowan's M3 distribution, the minimum acceptable gap is larger than minimum headway, $t_0 > t_m$. The difference between M3 distribution and M2 distribution is the ratio of free flow, α. The potential capacity is as

$$\begin{aligned} C_p &= \frac{q}{t_f}\left[\alpha\lambda \int_{t_0}^{\infty} t e^{-\lambda(t - t_m)} dt - t_0 R(t_0)\right] \\ &= \frac{\alpha q}{\lambda t_f} e^{-\lambda(t_0 - t_m)}, \quad t_0 > t_m \end{aligned} \tag{22}$$

The M3 model is used in AUSTROADS [29] and SIDRA (Akcelik, 1998). Multilane traffic flow can be regarded as a single-lane traffic flow, and the traffic volume is equal to the sum of all circulating traffic volume.

4.3. Entrance Capacity Model When Critical Gap Follows a Distribution. Different drivers have various critical gaps because the drivers' behaviors or the types of vehicles are different. Ideally, driver behaviors are supposed to be homogeneous and consistent [21, 42]. When a driver is consistent, every driving behavior is identical making the critical gap of the driver a constant. When different drivers are homogeneous, their drive behaviors are similar so they have the same critical gap whether constant or a distribution.

Catchpole and Plank [21] proposed the classification of entrance capacity models, including the constant critical gap and critical gap distribution. The models in Section 4.2 are the first part of Section 4.3.1. Heidemann and Wegmann [4] proposed the models based on a certain distribution of t_c, t_m, t_f and M3 distribution of the circulating stream. Wu's equations [2] were based on Erlang distribution of the circulating headway, whereas Guo's equations [34, 36] were based on M3 distribution of the circulating headway and the exponent function of rejected proportion.

4.3.1. Equations of Catchpole and Plank. Catchpole and Plank [21] proposed the capacity model based on different driver behaviors. When g(t) is piecewise function, the capacity can be expressed as follows.

(1) When the driver behaviors are homogeneous or consistent, that is to say, t_c is a constant, the capacity models are as (14) to (22) mentioned before.

(2) When the driver behaviors are consistent but nonhomogeneous, the capacity is as follows.

$$C = \frac{q(1 - qt_m) e^{qt_m} \mathsf{L}(t_c(q))}{1 - \mathsf{L}(t_f(q))} \tag{23}$$

where $\mathsf{L}(t_c(q))$ is the Laplace transformation of t_c distribution and $\mathsf{L}(t_f(q))$ is the Laplace transformation of t_f.

(3) It can be regarded as a special style of the following number (4) situation when the driver behaviors are homogeneous but inconsistent.

(4) When the driver behaviors are inconsistent and nonhomogeneous, the capacity is as follows.

$$C = \frac{1}{\sum_j (\theta_j / q_j)} = \frac{\alpha q e^{\lambda t_m}}{1 - e^{-\lambda t_f}} \cdot \frac{1}{\sum_j \theta_j / \mathsf{L}(\theta_j(q))} \tag{24}$$

where θ_j is the proportion of the jth kind of entrance vehicle; $\mathsf{L}(\theta_j(q))$ is the Laplace transformation of θ_j.

4.3.2. Equations of Heidemann and Wegmann. Heidemann and Wegmann [4] proposed that the capacity equation of unsignalized intersections follow a certain distribution for t_c, t_m, t_f. Based on M3 distribution of the headway on a major road, the capacity is (25) when g(t) is the piecewise function, and it is (26) when g(t) is the continuous function.

$$\begin{aligned} C &= \frac{\lambda}{1 + \lambda \overline{B}} \frac{\mathsf{L}(t_c'(\lambda))}{1 - \mathsf{L}(t_f(\lambda))} \\ &= \frac{\lambda}{1 + \lambda \overline{B}} \frac{\mathsf{L}(t_c(\lambda)) \mathsf{L}(t_m(-\lambda))}{1 - \mathsf{L}(t_f(\lambda))} \end{aligned} \tag{25}$$

$$C = \frac{\lambda}{1+\lambda\overline{B}} \frac{1}{\left(1-\mathsf{L}\left(t_f(\lambda)\right)\right)\mathsf{L}\left(t_c'(-\lambda)\right)} = \frac{\lambda}{1+\lambda\overline{B}} \frac{1}{\left(1-\mathsf{L}\left(t_f(\lambda)\right)\right)\mathsf{L}\left(t_c(-\lambda)\right)\mathsf{L}\left(t_m(\lambda)\right)} \tag{26}$$

where $\overline{B} = \bar{t}_m/\alpha$, $t_c' = t_c - t_m$; $\mathsf{L}(t_c'(\lambda))$ is the Laplace transformation of t_c' at λ, $\mathsf{L}(t_c'(\lambda)) = \mathsf{L}(t_c(\lambda))\mathsf{L}(t_m(-\lambda))$; $\mathsf{L}(t_f(\lambda))$ is the Laplace transformation of t_f at λ; $\mathsf{L}(t_m(-\lambda))$ is the Laplace transformation of t_m at $-\lambda$.

4.3.3. Wu Equation. Wu [2] proposed the probability density function when the headway on a major road follows Erlang distribution as follows.

$$f(t_x) = \frac{\gamma}{\left(\alpha_{t_x}-1\right)!}\left(\gamma t_x\right)^{\alpha_{t_x}-1} e^{-\gamma t_x} \tag{27}$$

where $\gamma = \alpha_{t_x}/\bar{t}_x$; $\bar{t}_x$ is the average value of t_x; α_{t_x} is the constant in Erlang distribution of t_x.

(1) When g(t) Is a Piecewise Function. It is supposed that the minimum value of t_c is τ_{t_c} and $\tau_{t_c} > \tau_{t_m}$, the minimum value of t_f is $\tau_{t_f} > 0$ and $\tau_{t_f} > 0$, the minimum value of t_m is τ_{t_c} and $\tau_{t_m} > 0$, and t_c, t_m, t_f all follow negative exponent distribution. The capacity is as follows when the drivers are nonhomogeneous.

$$C_{bunch} = \alpha q \frac{\left(\lambda\left(\bar{t}_c-\tau_{t_c}\right)/k_{t_c}+1\right)^{-\alpha_{t_c}} e^{-\lambda\tau_{t_c}} \left(-\lambda\left(\bar{t}_m-\tau_{t_m}\right)/k_{t_m}+1\right)^{-\alpha_{t_m}} e^{\lambda\tau_{t_m}}}{1-\left(\lambda\left(\bar{t}_f-\tau_{t_f}\right)/k_{t_f}+1\right)^{-\alpha_{t_f}} e^{-\lambda\tau_{t_f}}} \tag{28}$$

The capacity is as follows when the drivers are homogeneous.

$$C_{bunch} = \alpha q \frac{\left(-\lambda\left(\bar{t}_c-\tau_{t_c}\right)/k_{t_c}+1\right)^{\alpha_{t_c}} e^{-\lambda\tau_{t_c}} \left(\lambda\left(\bar{t}_m-\tau_{t_m}\right)/k_{t_m}+1\right)^{\alpha_{t_m}} e^{\lambda\tau_{t_m}}}{1-\left(\lambda\left(\bar{t}_f-\tau_{t_f}\right)/k_{t_f}+1\right)^{-\alpha_{t_f}} e^{-\lambda\tau_{t_f}}} \tag{29}$$

(2) When g(t) Is Continuous Function. Some assumptions have been given that the headway on a major road follows shifted Erlang distribution and t_f follows random distribution. The capacity is as follows when the drivers are nonhomogeneous.

$$C_{bunch} = \left(1-q\bar{t}_m\right)\frac{1}{\bar{t}_f}\left(\frac{\lambda\left(\bar{t}_0-\tau_{t_0}\right)}{k_{t_0}}+1\right)^{-\alpha_{t_0}} \cdot e^{-\lambda\tau_{t_0}}\left(\frac{-\lambda\left(\bar{t}_m-\tau_{t_m}\right)}{k_{t_m}}+1\right)^{-\alpha_{t_m}} e^{\lambda\tau_{t_m}} \tag{30}$$

The capacity is as follows when the drivers are homogeneous.

$$C_{bunch} = \left(1-q\bar{t}_m\right)\frac{1}{\bar{t}_f}\left(\frac{-\lambda\left(\bar{t}_0-\tau_{t_0}\right)}{k_{t_0}}+1\right)^{\alpha_{t_0}} \cdot e^{-\lambda\tau_{t_0}}\left(\frac{\lambda\left(\bar{t}_m-\tau_{t_m}\right)}{k_{t_m}}+1\right)^{\alpha_{t_m}} e^{\lambda\tau_{t_m}} \tag{31}$$

4.3.4. Guo Equation. The equation of Guo [34, 36] is based on M3 distribution. All the headways of the free flow on the major road have rejected proportions; that is to say, every headway in the interval of (t_m, ∞) on the major road is probably to be accepted. The rejected proportion function is the exponent function and the accepted proportion increases when the headway on the major road increases.

The rejected proportion function $f_{ro}(t)$ is the exponent function as follows.

$$f_{r0}(t) = \begin{cases} e^{-r(t-t_m)}, & t \geq t_m \\ 0, & t < t_m \end{cases} \tag{32}$$

where r is the rejected proportion coefficient.

The capacity can be deduced when g(t) is continuous function as follows.

$$C = \alpha q\left[\frac{r}{(\lambda+r)} + \frac{e^{-\lambda t_f}}{t_f\lambda} - \frac{\lambda e^{-(\lambda+r)t_f}}{t_f(\lambda+r)^2}\right] \tag{33}$$

The capacity can be deduced when g(t) is piecewise function as follows.

$$C = \frac{\alpha q}{(\lambda+r)}\left[\frac{r}{\left(1-e^{-\lambda t_f}\right)} - \frac{\lambda e^{-(\lambda+r)t_f}}{\left[1-e^{-(\lambda+r)t_f}\right]}\left(\frac{r}{\lambda+r}\right)^{(\lambda+r)/\lambda}\right] \tag{34}$$

4.4. The Capacity of Limited Priority Merge. Under some special situations, gap acceptance theory can be based on different assumptions such as limited priority merge or priority conversion. The driving behavior under limited priority merge [43–46] means that vehicles on the circulating lane need to adjust the gaps for the vehicles entering the intersection.

Another priority style, priority conversion means that vehicles on the entrance are forced to enter into the intersection and the circulating vehicles have to yield for the entrance vehicles. These driving behaviors are more feasible in the field and their equations are more complicated to be applied for calculation.

Troutbeck [45] proposed the following equation of limited priority merge:

$$C = \frac{\alpha f_L q e^{-\lambda(t_c - t_m)}}{1 - e^{-\lambda t_f}}, \quad t_m < t_c < t_f + t_m$$
$$f_L = \frac{1 - e^{-\lambda t_f}}{1 - e^{-\lambda(t_c - t_m)} - \lambda\left(t_c - t_f - t_m\right) e^{-\lambda(t_c - t_m)}} \tag{35}$$

When $t_c \geq t_f + t_m, f_L = 1$, the vehicles on the major road are not delayed after merging and the system can be modeled as an absolute priority system. It can be referred to Bunker and Troutbeck [46], Troutbeck [44, 45], Troutbeck and Kako [43].

If both t_c and t_f are set up as t_m, all the gaps are t_m after merging. The capacity is as follows:

$$C = \frac{1}{t_m} - q \tag{36}$$

where $f_L = (1 - e^{-\lambda t_m})/\lambda t_m$ and $C = \alpha q/\lambda t_m$.

5. Comparison of Different Models

When critical gap follows a stochastic distribution, that is to say, a driver might reject a gap which he accepted before or drivers have different critical gaps, this effect results in an increase of capacity comparing with the condition of constant critical gap.

The limited priority merge can have a significant effect on the entry capacity at roundabouts. The limited priority capacity is very close to the empirical results from the UK linear regression method. The capacity prediction based on the limited priority and the UK method are reasonable in general.

It is difficult to compare different models in the field because each model adapts in different road geometrical and traffic conditions. The headway distribution of the circulating vehicles, the ratio of free flow, the type of g(t) function, the critical gap t_c, and other parameters, such as t_f, t_m, vary with different geometrical and traffic conditions, and they affect the calculation value of entrance capacity in roundabouts.

Suppose that the headway of circulating vehicles follow the M3 distribution as (1), the decay constant λ is (3), and the ratio of free flow is (7). Equations (14), (18), (22), (34), and (35) can be compared using the same parameter values

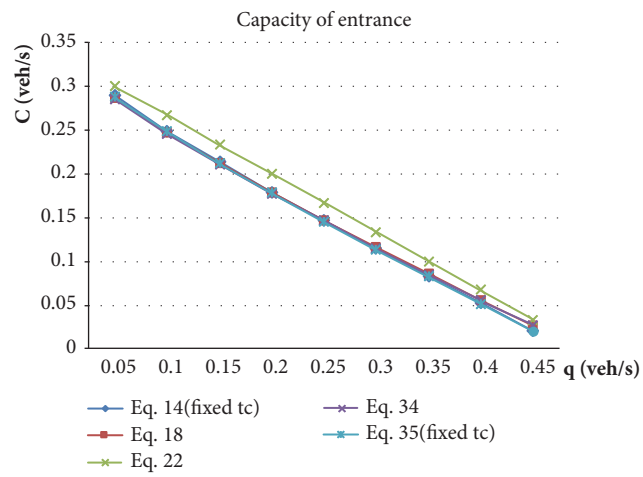

Figure 3: Calculation of entrance capacity under different flow rates on circulating vehicles.

where $t_m = 2s$ and $t_f = 3s$. Due to different g(t) functions and calculation models of t_c, some different parameters need to be set in some Equations. In (14) and (35), $t_c = 4.35s$. In (22) and (34), $t_0 = t_m = 2s$. In (18) and (35), $r = 0.365$. The entrance capacity C can be calculated using the five equations under a different flow rate q. The results can be shown in Table 4 and Figure 2.

From Figure 3, it can be concluded that the values of (22) are larger than values of other equations. The values of (14), (18), (34), and (35) are so similar that different curves cannot be clearly recognized. Because the number of vehicles entering the intersection can be considered as a decimal if continuous g(t), (22) with continuous g(t) function will result in a larger value of entrance capacity than the other equations based on the piecewise g(t) function. The influence of different critical gap models on capacity value is limited. The capacity values of (18) and (34) are slightly larger than (14) and (35) when the circulating vehicles are close to the saturated flow rate, where q=0.4 and q=0.45. The capacity values of the four equations are almost equal with each other.

The above capacity models have very small differences of each other regardless of the critical gap type and the g(t) type. Therefore, the simple calculating models based on the constant critical gap and piecewise g(t) function should be recommended in the field.

6. Application Conclusions of Different Models

The capacity models based on gap acceptance theory are analysis methods in which traffic parameters have a certain physical meaning. When the critical gap is constant, the deviation of the capacity model is conveniently used to obtain the equations and calculate the accurate capacity values. Drivers' behavior characteristics are different in different countries. For example, the critical gap of the single-lane roundabout is 4.8s in America; however, it is about 3.5s in China. Drivers' nonhomogeneous and inconsistent character

Table 4: Calculation of entrance capacity under different flow rates on circulating vehicles ($t_m = 2s$, $t_f = 3s$).

No.	q(veh/s)	alpha	λ(1/s)	βa	tc(s)	Eq. (14)(fixed tc) (veh/s)	Eq. (18)(veh/s)	Eq. (22)(veh/s)	Eq. (34)(veh/s)	FL in Equ. (35)	Eq. (35)(fixed tc)(veh/s)
1	0.05	0.779	0.043	0.696	4.589	0.289	0.286	0.300	0.285	0.997	0.288
2	0.1	0.607	0.076	0.502	4.489	0.249	0.247	0.267	0.246	0.995	0.248
3	0.15	0.472	0.101	0.370	4.418	0.213	0.212	0.233	0.210	0.994	0.212
4	0.2	0.368	0.123	0.275	4.362	0.179	0.179	0.200	0.177	0.993	0.178
5	0.25	0.287	0.143	0.206	4.311	0.146	0.147	0.167	0.146	0.991	0.145
6	0.3	0.223	0.167	0.153	4.255	0.114	0.116	0.133	0.115	0.990	0.113
7	0.35	0.174	0.203	0.112	4.179	0.083	0.086	0.100	0.084	0.989	0.082
8	0.4	0.135	0.271	0.078	4.050	0.052	0.056	0.067	0.055	0.986	0.051
9	0.45	0.105	0.474	0.046	3.756	0.021	0.027	0.033	0.027	0.982	0.020

is more realistic than the fixed critical gap and following gap. Although the calculation results of capacity are similar as the field capacity under the assumption of homogeneity and continuance, there is only a little percent deviation.

Several widely used models and the parameters can be compared as follows.

(1) The headway was usually regarded as negative exponent distribution. M3 distribution was also paid enough attention [4].

(2) Based on a field traffic survey in Australia [22], the capacity Equation (22) of M3 distribution and continuous g(t) was applied in aaSIDRA (Akcelik et al., 1994, 1997, 1998) in which the following gap and critical gap are varied from the geometry of the roundabout, the flow rate of entrance and circulating lanes. The capacity is related to the circulating flow rate, lane use, OD metric of flow rate, queue on entrance lanes and the ratio of free flow. All the lanes on entrance can be modeled at the same time.

(3) Based on the survey of a limited number of roundabouts in America and in comparison with the experience values of other countries, the capacity equation (15) of M1 distribution and piecewise function g(t) was used in HCM (2000). In the model, the constant parameters of gap acceptance were used which did not vary from the geometry of the roundabout and traffic volume. The calculation results have limitations when the circulating volume increased up to 1200pcu/h for a single-lane roundabout.

(4) The capacity equation (19) of M1 distribution and continuous function g(t) was used in HCM (2010), in which both t_c and t_f were determined and some equations were built based on a different number of entrance lanes and different number of circulating lanes. The calculating process of traffic characteristics was proposed including capacity, delay, level of service, and queue length.

(5) The capacity equation (18) was deduced by directly plugging the critical gap model into the capacity equation (14). Equation (18) has reasonable results in accordance with some classical methods and it has more accurate results under some conditions; for example, the low capacity is closer to the reality than linear regression under low flow rate conditions.

(6) The method based on stochastic critical gap is more realistic with operation in roundabouts, but most of the equations are too complicated to calculate quickly. Simplification is needed to apply in the field.

With the increase of traffic volume, the application of traditional roundabouts becomes more and more difficult. Many roundabouts, especially in China, were dismantled and few roundabouts were newly built. The field capacity becomes very small because many drivers do not obey the priority rule. Traffic jams or even traffic accidents occurred at roundabouts. In reality, signal timing was applied in some roundabouts, in which the inner lanes of the weaving area were designed as left-turn waiting areas. Left-turn vehicles need to watch the signal light two times when passing through the roundabout. The validity needs to be verified further.

Drivers are sensitive to some problems of roundabouts at peak hours, but ignore the advantages at off-peak time, such as safe, environment friendly, suitable to large-volume left turns. To the researchers and engineers in the traffic field, traffic organization and optimization should be the key focus to increase capacity and reliability at peak hours.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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