

Research Article

Macroscopic Modeling of On-Street and Garage Parking: Impact on Traffic Performance

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The short-term interactions between on-street and garage parking policies and the associated parking pricing can be highly influential to the searching-for-parking traffic and the overall traffic performance in the network. In this paper, we develop a macroscopic on-street and garage parking decision model and integrate it into a traffic system with an on-street and garage parking search model over time. We formulate an on-street and garage parking-state-based matrix that describes the system dynamics of urban traffic based on different parking-related states and the number of vehicles that transition through each state in a time slice. This macroscopic modeling approach is based on aggregated data at the network level over time. This leads to data collection savings and a reduction in computational costs compared to most of the existing parking/traffic models. This easy to implement methodology can be solved with a simple numerical solver. All parking searchers face the decision to drive to a parking garage or to search for an on-street parking space in the network. This decision is affected by several parameters including the on-street and garage parking fees. Our model provides a preliminary idea for city councils regarding the short-term impacts of on-street and garage parking policies (e.g., converting on-street parking to garage parking to garage usage information to all drivers) and parking pricing policies on: searching-for-parking traffic (cruising), the congestion in the network (traffic performance), the total driven distance (environmental impact), as well as the revenue created for the city by the hourly on-street and garage parking fee rates. This model can be used to analyze how on-street and garage parking policies can affect traffic performance; and how traffic performance can affect the decision to use on-street or garage parking. The proposed methodology is illustrated with a case study of an area within the city of Zurich, Switzerland.

1. Introduction

As the population in urban areas is increasing, more and more cars need to find parking spaces in city centers. These vehicles normally have the choice between on-street and garage parking. Both parking possibilities follow diverse policies, which can sometimes lead to rather complex interdependencies and significant changes in the performance of a transportation network. In this research, we develop a macroscopic on-street and garage parking model such that the influence of different on-street and garage parking policies on the traffic system can be studied and illustrated. Hereafter, off-street parking is referred to as garage parking. The macroscopic model is built on a traffic system with a parking search model over time. It is incorporated into the on-street parking framework from Cao and Menendez [1].

The existing literature contains a number of empirical approaches to model the interaction between on-street and garage parking. These empirical methods usually focus on collecting data for both on-street and garage parking, e.g., SFpark [2] uses its responsive pricing scheme to leave between 20 and 40 percent of on-street parking spaces open on every block, and Pierce et al. [3] introduce parking pricing to have open spaces available in public garages at all times. Other garage parking models are based on questionnaires, e.g., [4, 5]; or they use dynamic information to predict real-time garage parking availability [6]. Kobus et al. [7] estimate the effect of on-street parking fees on drivers' choice between on-street and garage parking. Gragera and Albalate [8] analyze how garage parking demand is affected by on-street parking regulations. Our macroscopic modeling approach allows us to estimate the average impact of on-street and garage parking on the traffic system without any physical devices nor large data collection efforts.

For modeling both on-street and garage parking and the associated parking fees, previous studies [9, 10] illustrate how the actual full price of parking contains both the interaction between garage operators and the cruising costs for on-street parking. They develop a spatial competition model to eliminate cruising by allocating excess cruising demand to garage parking and focus on social optimum suggestions concerning the relationship between curbside and garage fares. Shoup [11] shows that underpriced on-street parking creates an incentive for drivers to cruise. The microeconomic model explains why a driver would rather choose to cruise for free on-street parking than paying for garage parking. The decision model between on-street and garage parking in [11] is based on the garage parking fee, the driver's intended parking duration, the time spent cruising, the cost of petrol while cruising-for-parking, the number of people in the car, and the driver's and his passengers' value of time (VOT). Our parking decision, instead, is based on a macroscopic modeling approach. Arnott and Rowse [12] present a nonlinear model of parking congestion focusing on the searching-for-parking phenomenon for an available on-street parking space in a homogeneous metropolis. Arnott and Inci [13] analyze the influences of on-street parking pricing on cruising-for-parking and Arnott and Rowse [14] study the effects of on-street parking time limits on traffic performance, but do not consider garage parking. Arnott et al. [15] explore optimal location-dependent parking fees in comparison to time-varying road tolls concentrating on commuter parking and their arrival times during the morning rush hour. Arnott and Inci [16] investigate how an increasing demand affects the traffic dynamics for a uniform road network with on-street parking. Mackowski et al. [17] model variable on-street and garage pricing in real-time for effective parking access and space utilization by using a dynamic Stackelberg leader-follower game theory approach. Qian and Rajagopala [18] develop a real-time pricing approach for a parking lot based on its occupancy rate as a system optimal parking flow minimization problem. They assume a user equilibrium travel behavior and only focus on garage parking without analyzing its interdependency with on-street parking in the network. Benenson et al. [19] develop an agent-based parking model for a city by simulating the behavior of each driver in comparison to our macroscopic framework based on aggregated data. Further studies use this agent-based parking model to analyze different parking policies [20], estimate city parking patterns [21], explore cruising-for-parking [22, 23] and evaluate parking planning projects for large parking garages [24]. Wang et al. [25] study park-and-ride (P+R) networks with multiple origins and one destination and focus on an optimal parking pricing strategy. They only focus on setting optimal parking fees for P+R terminals and do not consider the interaction with on-street parking. Kladeftiras and Antoniou [26] focus on the effects of illegal parking (double parking) on traffic and environmental conditions using a microscopic simulation. Zheng and Geroliminis [27] model multimodal traffic with limited on-street and garage parking and dynamic pricing based on a congestion- and cruising-responsive feedback parking pricing scheme. The proposed framework is based on the macroscopic

fundamental diagram (MFD) reflecting the dynamics of parking flows in an urban network [28, 29]. Liu and Geroliminis [30] use an MFD approach to investigate how cruising-for-onstreet-parking influences the commuters' morning peak and develop a dynamic parking pricing model to reduce total social cost. However, they do not consider garage parking in its framework. Leclercq et al. [31] also only include on-street parking to their trip-based MFD model evaluating the on-street parking search process with respect to different vehicle parking strategies. Arnott et al. [32] study how much curbside to allocate to parking when the private sector provides garage parking. Arnott and Rowse [33] analyze parking in a spatially homogeneous downtown area where the drivers choose between on-street and garage parking. Cruising for parking contributes to congestion, such that the price of the initially cheaper on-street parking is increased until it equals the price of garage parking. Then increasing the on-street parking fee may generate an efficiency gain through the reduction of cruising. These models focus on social optimum and user equilibrium methodologies.

Compared to methodologies concentrating on long-term demand management strategies, our dynamic macroscopic modeling approach focuses on the short-term effects in the traffic network, i.e., the demand entering the network is treated as exogenous, and the on-street and garage parking capacity is taken as fixed. An on-street and garage parking-state-based matrix is used to capture the system dynamics of urban traffic. It is based on multiple parking-related traffic states and transition events to update the number of vehicles per state over time [1]. The total traffic demand entering the network is divided into two groups; through-traffic, and vehicles searching for parking. The first group of vehicles represents the proportion of traffic that is driving through this area but does not want to park or has a destination outside. Therefore, it only experiences two transition events, as seen in Figure 1(a). The second group of vehicles needs to decide between searching for on-street parking or driving towards a parking garage, as seen in Figure 1(b). During one single time slice, a vehicle may experience at most one transition event.

In case the drivers decide for on-street parking, they might need to circulate in the city to search for an available on-street parking space, which contributes to the problem of traffic congestion. In case the drivers decide for garage parking, there is no need to search-for-parking. These vehicles drive towards the closest parking garage and access it depending on its current availability. With limited data collection efforts, our macroscopic on-street and garage parking decision model shows the influence of different on-street and garage parking pricing ratios on the average searching time/distance. We analyze not only the relationship between on-street and garage parking, but also their interdependency on cruising-for-parking traffic and traffic performance with respect to different parking fees. Different pricing strategies affect the drivers' decision to park on-street or to drive towards a parking garage. Insights from this paper will help city councils or private agencies to analyze the short-term impacts on the traffic system, for example, when changing the hourly on-street and garage parking fee rates on the network, or when converting on-street to garage parking spaces (as it has been the case in cities like Zurich, Switzerland).



FIGURE 1: The transition events of urban traffic focusing on on-street and garage parking in-between different parking-related states. (a) Through-traffic. (b) Searching for parking traffic.

In summary, the existing literature approaches on-street and garage parking models either with empirical data collection efforts or with methodologies concentrating on user equilibrium or social optimum solutions that focus on long-term demand management strategies. Our on-street and garage parking decision model follows a macroscopic approach and focuses on short-term effects. The main contributions of this study are three-fold.

- (i) First, without large data collection efforts, our macroscopic decision model provides valuable insights into different on-street and garage parking fee ratios and their impacts on cruising-for-parking traffic as well as the overall traffic performance. The macroscopic model of garage parking also allows us to provide an easy to implement methodology with low computational costs based on aggregated data at the network level over time that can be easily solved with a simple numerical solver.
- (ii) Second, our framework provides the tools to study the trade-off between the parking revenue and the cruising-for-parking traffic. We analyze not only the relationship between on-street and garage parking, but also their interdependency on traffic performance with respect to different parking fees. This study can be used for city councils or private agencies to find reasonable hourly on-street and garage parking fees such that the average vehicle time/distance is not negatively affected and additionally, acceptable financial revenues are obtained.
- (iii) Third, different parking policies in city center areas, e.g., the short-term effects of converting on-street to garage parking spaces on the traffic system, or the availability of garage usage information to all drivers

can be simulated using our methodology and recommendations for city councils can be made.

The paper is organized as follows. Section 2 presents the overall framework of the macroscopic on-street and garage parking decision model. Section 3 illustrates the concept and mathematical model of the on-street and garage parking-state-based matrix. Section 4 shows a case study of an area within the city of Zurich, Switzerland. Section 5 concludes this paper.

2. On-Street and Garage Parking Decision

Several cost factors influence the on-street/garage parking decision, as seen in Figure 2. These cost variables include variables that have an impact on either the on-street parking option (e.g., the on-street parking pricing), the garage parking option (e.g., the garage parking pricing) or on both parking options (e.g., the number of parking spaces of each kind, and the desired parking duration). Drivers with desired long parking durations are more likely to choose garage parking. All drivers are assumed to be rational during their parking decision and only compare the relevant parking costs between on-street and garage parking, i.e., all drivers are treated as risk-neutral.

The parking decision is then modeled macroscopically using a logistic function based on the on-street and garage parking cost variables. The data inputs are presented in Section 2.1. All mathematical details of this modeling approach are illustrated in Section 2.2.

2.1. Data Inputs for Decision Model. The decision between on-street and garage parking is dependent on the input variables shown in Tables 1 and 2. The model parameters and



FIGURE 2: Decision model for on-street and garage parking based on several cost factors.

 TABLE 1: Independent variables for parking decision (inputs to the model): traffic network and model parameters.

Notation	Definition
L	Size (length) of the network.
b	Average length of a block in the network.
Т	Length of the simulation's time horizon.
t	Length of a time slice.
Κ	Total number of user groups for demand input of the network. Each user group has a different VOT.
VOT^k	VOT for user group $k \in K$.
υ	Free flow speed, i.e., maximum speed in the net- work, including stopped time at intersections.
w	Walking speed in the network.

all variables that are required to define the traffic network are presented in Table 1. These variables can either be directly measured, or estimated based on simulation results and/or the macroscopic fundamental diagram.

All data inputs are based on a compact urban area with a relatively homogeneous network. The total time horizon is divided into small time slices (e.g., 1 minute). All traffic and parking conditions can change over multiple time slices, but they are assumed to be steady within each time slice. As stated in [1], the urban network is abstracted as one ring road with cars driving in a single direction, which has been proven to be reasonable for small, homogeneous traffic networks [34].

It is assumed that all trips are exclusively made by car, i.e., the mode choice has been previously made. In addition, we assume that drivers do not cancel their trips while searching for parking. The VOT is assumed to be different for individual vehicles depending on their user group. Such user group can be dependent on the residents' location, income, careers, working states, etc.

Table 2 shows all independent variables associated with on-street and garage parking. This includes parking duration and parking pricing specific input parameters. These variables can be estimated based on real measurements, historical on-street and garage parking and pricing data, or defined otherwise. All variables related to the travel demand and the distances driven can be estimated based on historical data, e.g., traffic data on main roads to enter the network, etc. The distance variables that are associated with a transition into the next state can be reasonably assumed based on the length of the network, and other data collected from drivers. TABLE 2: Independent variables for parking decision (inputs to the model): on-street and garage parking parameters.

Notation	Definition
A	Total number of existing on-street parking spaces (for public use) in the area.
G	Number of parking garages in the network.
R	Total capacity of all parking garages, i.e., total number of all garage parking spaces.
t _d	Parking duration of vehicles (independently of on-street and garage parking).
$t_{d,\xi}$	Parking duration of vehicles focusing on on-street (<i>op</i>) or garage parking (<i>gp</i>), $\xi \in \{op, gp\}$.
$ au_{\xi}$	On-street (<i>op</i>) or garage parking (<i>gp</i>) time limit, $\xi \in \{op, gp\}.$
Ρξ	Hourly on-street (<i>op</i>) or garage parking (<i>gp</i>) fee rate, $\xi \in \{op, gp\}.$
$P_{\rm dist}$	Price per kilometer driven on the network (i.e., external costs as petrol, wear and tear of vehicles).
eta^i	Proportion of new arrivals during time slice <i>i</i> that corresponds to traffic that is not searching for parking.
$l_{ns/}^k$	Distance that must be driven by a vehicle from user group $k \in K$ before it starts to search for parking.
$l_{/}^{k}$	Distance that must be driven by a vehicle from user group $k \in K$ before it leaves the area without having parked.
$l_{p/}^k$	Distance that must be driven by a vehicle from user group $k \in K$ before it leaves the area after it has parked on-street or in a garage.

Given the homogeneous network, parking searchers are assumed to be homogeneously distributed within the overall driving traffic. This is reasonable, as we also assume that all on-street parking spaces (not only the available ones) are uniformly distributed on the network. Although on-street parking maneuvers have been proven to influence traffic flow [35, 36], here we do not account for them, as we focus on the influence of the number of vehicles searching for on-street parking on the overall traffic performance. Recall that we focus on small compact areas with standard parking policies (e.g., downtown areas or portions thereof). We do not need to record the location of individual cars and parking spots throughout the different time slices in the system, i.e., only average numbers of vehicles during a time slice and total/average searching times and distances are tracked.

TABLE 3: Intermediate model variables.

Notation	Definition
$C^{i,k}_{\xi}$	Total cost of on-street (<i>op</i>) or garage parking (<i>gp</i>) in time slice <i>i</i> for user group $k \in K, \xi \in \{op, gp\}$.
$\delta^{i,k}_{gp}$	Choice of drivers for garage parking (gp) in time slice <i>i</i> for user group $k \in K$.
$\gamma^{i,k}_{\xi}$	Proportion of drivers deciding for on-street (<i>op</i>) or garage parking (<i>gp</i>) in time slice <i>i</i> for user group $k \in K, \xi \in \{op, gp\}.$
ACT^i	Average cruising time for on-street parking in time slice <i>i</i> .
ADD	Average driving distance to closest garage location.
AWD_{ξ}	Average walking distance from on-street (<i>op</i>) or garage parking (<i>gp</i>) parking to destination, $\xi \in \{op, gp\}$.
v^i	Average travel speed in time slice <i>i</i> , including stopped time at intersections.
d^i	Maximum driven distance per vehicle in time slice <i>i</i> .
A^i	Number of available on-street parking spaces at the beginning of time slice <i>i</i> .
R^{i}	Garage parking availability of all parking garages in time slice <i>i</i> .
P _{tot}	Total revenue resulting from hourly on-street and garage parking fee rates for the city.

Parking garages are also assumed to be uniformly distributed within the network, and without loss of generality, all associated garage parking capacities are assumed to be equal. The distribution of desired parking durations is considered as an input to this model. Some distributions describe the parking duration better than others, see [37, 38]. In theory, however, any distribution can be used, e.g., poisson, negative binomial [1]. It is assumed that during the period of one working day, drivers do not repark their car after the on-street parking time limit has expired.

2.2. Mathematical Decision Framework. Table 3 summarizes the intermediate modeling variables that are needed to model the on-street and garage parking decision. The model outputs provide, among others, the results of the interactions between on-street and garage parking and their influence on the urban traffic system.

We model the parking decision between on-street and garage parking macroscopically in Eqs. (1)–(3). This will then be incorporated into the on-street and garage parking-state-based matrix from [1, 39] in Section 3. We assume that the on-street parking time limit τ_{op} is smaller than the garage parking time limit τ_{gp} , i.e., $\tau_{op} \leq \tau_{gp}$. All drivers with a desired parking duration of $t_d \leq \tau_{op}$ decide between on-street and garage parking, whereas drivers with $\tau_{op} < t_d \leq \tau_{gp}$ are restricted and can only park at garages. Notice that t_d is taken out of a distribution and no individual vehicles are tracked. We assume that $t_d \leq \tau_{gp}$ for all drivers, since vehicles with $t_d > \tau_{gp}$ cannot find a parking place according to their desired parking duration anywhere within this network. The choice for garage parking over on-street parking, $\delta_{gp}^{i,k}$, is modeled in Eq. (1) using a logistic function based on R (the total capacity

of all parking garages), A (the total number of existing on-street parking spaces), $C_{op}^{i,k}$ (the cost of on-street parking, modeled in Section 2.2.1), and $C_{gp}^{i,k}$ (the cost of garage parking, modeled in Section 2.2.2).

$$\delta_{gp}^{i,k} = \frac{e^{R/R + A \cdot \mathbf{C}_{op}^{i,k} - A/R + A \cdot \mathbf{C}_{gp}^{i,k} / \min\{R/R + A \cdot \mathbf{C}_{op}^{i,k} - A/R + A \cdot \mathbf{C}_{gp}^{i,k}\}}{1 + e^{R/R + A \cdot \mathbf{C}_{op}^{i,k} - A/R + A \cdot \mathbf{C}_{gp}^{i,k} / \min\{R/R + A \cdot \mathbf{C}_{op}^{i,k} - A/R + A \cdot \mathbf{C}_{gp}^{i,k}\}}.$$
 (1)

Drivers base their parking choice on expected costs. These cost variables are determined macroscopically without stochastic components, using average values and probability distributions across the whole population. Therefore, $\delta_{qp}^{i,k}$ is modeled as an average corresponding to aggregated data at the network level based on the logistic probability distribution. To make sure the parking choice takes the supply into consideration, we add the weight parameters R/(R + A) and A/(R + A) to Eq. (1). These terms are not time-dependent since there is no real-time information for on-street and garage parking availability. We assume that the drivers have only information available about the VOT for their own user group $k \in K$, about total parking capacities in the network, and about the system averages required to determine the cost variables $C_{op}^{i,k}$ and $C_{gp}^{i,k}$. It is further assumed that the drivers are unso-phisticated in their decision-making since they do not use their experienced vehicle speed values as additional information to update the available system average information required to determine $C_{op}^{i,k}$ and $C_{gp}^{i,k}$. In an alternative scenario in Section 4.5, we relax this assumption by having real-time garage parking availability information accessible to the drivers when deciding to park on-street or to drive towards a parking garage. However, no real-time on-street parking information is assumed to be available in this alternative scenario, as this is less common. In future research, other scenarios can be investigated involving a forecast about future on-street and/or garage parking availability when making their decision.

Notice that the decision of some drivers is restricted by τ_{op} and τ_{gp} . This is taken into account when calculating $\gamma_{gp}^{i,k}$ in Eq. (2), which described the proportion of vehicles deciding for garage parking, and $\gamma_{op}^{i,k}$ in Eq. (3) which describes the proportion of vehicles deciding to search for on-street parking.

$$\gamma_{gp}^{i,k} = \underbrace{\int_{0}^{\tau_{op}} f(t_d) dt_d \cdot \delta_{gp}^{i,k}}_{\text{term 1}} + \underbrace{\int_{\tau_{op}}^{\tau_{gp}} f(t_d) dt_d}_{\text{term 2}},$$
(2)

$$\gamma_{op}^{i,k} = 1 - \gamma_{gp}^{i,k}.$$
 (3)

Term 1 in Eq. (2) represents the portion of vehicles with a parking duration $t_d \leq \tau_{op}$ that have the option to decide for on-street or garage parking. Term 2 represents the portion of vehicles with $\tau_{op} < t_d \leq \tau_{gp}$ that have to park in a garage because of their desired parking duration. Notice that both $\int_0^{\tau_{op}} f(t_d) dt_d$ and $\int_{\tau_o}^{\tau_g} f(t_d) dt_d$ are assumed to be *k*-independent, i.e., the distribution of the parking durations is assumed to be independent of the drivers' VOT.

2.2.1. Cost of On-Street Parking. In Eq. (4), we derive the cost of on-street parking, $C_{op}^{i,k}$ for each user group $k \in K$ in time slice *i*.

$$C_{op}^{i,k} = \underbrace{p_{op}}_{\text{term 1}} + \underbrace{p_{\text{dist}} \cdot v^{i} \cdot \text{ACT}^{i}}_{\text{term 2}} + \underbrace{\text{VOT}^{k} \cdot \text{ACT}^{i}}_{\text{term 3}} + \underbrace{\text{VOT}^{k} \cdot \frac{\text{AWD}_{op}}{w}}_{\text{term 4}}.$$
(4)

Term 1 represents the hourly on-street parking fee rate which, in the remainder of this paper, is assumed to be constant. In theory, however, the on-street parking fee could also be modeled as a responsive parking pricing scheme [39] that takes the parking search phenomenon into consideration. Notice that the parking decision in Eq. (1) is assumed to be based on the parking fee rates per hour independently of the parking durations. Term 2 represents the average cruising distance for on-street parking (i.e., external costs as petrol, wear and tear of vehicles) converted to price units. Term 3 represents the average cruising time based on the drivers' VOT expressed in price units for $k \in K$. The average cruising time ACT' is determined as in Jakob et al. [39], and is based on a queueing diagram showing the cumulative number of vehicles going through each transition event as a function of time. Notice that the longer the drivers search for on-street parking, the higher the average cruising time ACT' is, and consequently also the $C_{op}^{i,k}$. Therefore, it is more likely that the drivers might decide for garage parking in congested areas. Term 4 represents the cost of walking from the on-street parking to the destination expressed in price units for $k \in K$. Even though our abstracted network was a ring, we may assume without loss of generality that the real network is a square grid, where the average length of a block b in the network is known. The total length of the ring network, L, is then equivalent to joining all blocks of length b together. As on-street parking spaces are uniformly distributed throughout the network, the walking costs can be determined using the average distance traveled AWD_{op} (Eq. (5)) between two random points in the square grid [40].

$$AWD_{op} = \frac{2}{3} \cdot \underbrace{b \cdot \left(-\frac{1}{2} + \sqrt{\frac{1}{4} + \frac{L}{2b}}\right)}_{\text{term 1}}.$$
 (5)

Term 1 represents the side length of the square grid.

2.2.2. Cost of Garage Parking. The cost of garage parking, $C_{gp}^{i,k}$ for each user group $k \in K$ in time slice *i* is based on multiple cost terms as shown in Eq. (6).

$$C_{gp}^{i,k} = \underbrace{p_{gp}}_{\text{term 1}} + \underbrace{p_{\text{dist}} \cdot \text{ADD}}_{\text{term 2}} + \underbrace{\text{VOT}^k \cdot \frac{\text{ADD}}{v^i}}_{\text{term 3}} + \underbrace{\text{VOT}^k \cdot \frac{\text{AWD}_{gp}}{w}}_{\text{term 4}}.$$
(6)

Term 1 represents the hourly garage parking fee rate which, in the remainder of this paper, is assumed to be constant. Terms 2 and 3 show the cost of driving from the actual vehicle's garage parking decision location to the closest garage for $k \in K$. It contains the average distance to the closest garage parking expressed in distance price units (term 2) and the average time expressed in price units for $k \in K$ (term 3). Both terms include the associated average driving distance ADD, determined in Eq. (7).

$$ADD = \frac{L}{2 \cdot G}.$$
 (7)

Remember that the actual garage locations are assumed to be uniformly distributed on the network and that we assume that traffic on the abstracted ring moves in a single direction.

Term 4 in Eq. (6) represents the cost of walking from the garage parking to the destination expressed in price units for $k \in K$. As the number of garages is limited, they are expected to require, on average, some walking distance. The walking speed *w* is assumed to be a constant input. To estimate the area served by each parking garage, we take the surface of the square grid $\left[b \cdot \left(-1/2 + \sqrt{1/4 + L/2b}\right)\right]^2$ and divide it by *G* (Figure 3). Assuming destinations are uniformly distributed in the network, we can compute the average walking distance AWD_{*gp*} in Eq. (8) as 2/3 of the radius of each of the areas served by a parking garage.

$$AWD_{gp} = \frac{2b}{3\sqrt{\pi \cdot G}} \left[-\frac{1}{2} + \sqrt{\frac{1}{4} + \frac{L}{2b}} \right].$$
 (8)

Note that we enhance $C_{gp}^{i,k}$ in Section 4.5 by including garage usage information to all drivers.

2.2.3. Total Revenue. One component of the parking decision is paying an hourly fee for on-street or garage parking. However, the drivers pay the final parking fee from on-street or garage parking depending on how long they have parked. Eq. (9) expresses the total revenue P_{tot} obtained from all user groups *K* for the time horizon *T*.

$$P_{\text{tot}} = \sum_{i=1}^{T} \sum_{k=1}^{K} \underbrace{n_{op/ns}^{i,k} \cdot p_{op} \cdot \overline{t_{d,op}}}_{\text{term 1}} + \underbrace{n_{gp/ns}^{i,k} \cdot p_{gp} \cdot \overline{t_{d,gp}}}_{\text{term 2}}.$$
 (9)

Term 1 shows the revenue from on-street parking for user group $k \in K$ during time slice *i*. Term 2 illustrates the revenue from garage parking for user group $k \in K$ during time slice *i*. $\overline{t_{d,op}}$ and $\overline{t_{d,gp}}$ illustrate the average on-street/garage parking duration obtained from all user groups *K* for the time horizon *T*. Notice that $n_{op/ns}^{i,k}$ in term 1 and $n_{gp/ns}^{i,k}$ in term 2 are both defined in Section 3.1 (Table 4).

3. On-Street and Garage Parking-State-Based Matrix

The on-street and garage parking-state-based matrix describes the system dynamics of urban traffic based on multiple parking-related states as in [1]. The matrix is used to incorporate our parking decision model into a macroscopic traffic system framework that captures the interactions over time between the on-street and garage parking systems. This section shows an overview of all on-street and garage parkingrelated traffic states (Section 3.1), and the analytical formulations for the transition events between those states (Section 3.2).



FIGURE 3: Simple example of uniformly distributed garage parking to illustrate the average walking distance in Eq. (8).

3.1. Parking-Related Traffic States. The parking-related traffic states build the foundation for the parking-state-based matrix. The matrix updates all parking-related traffic states based on the number of vehicles going through different transition events in each time slice. The matrix is updated iteratively over time until the whole period is analyzed, or a defined criterion is reached (e.g., all the cars leave the area). By integrating our on-street and garage parking decision model from Section 2, the matrix allows us to illustrate the effects of different on-street and garage parking policies on searching time and searching distance.

All transition events are modeled using a deterministic approach. However, the model is not thoroughly deterministic, as for example, the parking location of each vehicle is not fixed, nor the travel time, nor the parking duration. The model is not thoroughly stochastic either and there are no random values involved in the computation of each transition event. Having a not thoroughly stochastic model does not necessarily make the model less valuable than one under completely stochastic conditions, because the model is meant to look for average values based on some probability distributions rather than the random values themselves. Since all variables are based on average values and not on random values, every simulation run returns the same results as long as the input variables to the model are not changed. In addition, there is no need to run the model many times in order to account for its stochasticity, as it is based on probability functions (i.e., the stochasticity is already implicit within the model formulations) [1].

All vehicles searching for parking in Figure 1(b) have the option to decide for on-street or garage parking at their current location. This decision involves the on-street and garage parking decision model from Section 2. The vehicles that have decided to search for on-street parking can change their mind

and switch to garage parking later. As soon as the vehicles decide for garage parking, they will drive towards the closest parking garage and access it based on availability. For these drivers, the location of the parking garages is assumed to be known, or guidance to the garage location is available. Once the garage parking decision is made, we assume the drivers do not change their decision while driving to the garage location. If there are no available garage parking spaces, the vehicles cannot access the parking garage and might move to the searching-for-on-street-parking state. After the vehicles have accessed on-street or garage parking, they depart and move back to the non-searching state before they leave the area. All traffic states in Figure 1 are summarized in Table 5. The initial conditions of all traffic state variables are model input variables that can be measured, assumed, or simulated.

These parking-related states are determined using the information on the transition events. We introduce the transition events in Table 4.

Eqs. (10–14) update the number of "non-searching, "searching for on-street parking", "on-street parking", "driving to garage parking," and "garage parking" vehicles, respectively. Notice that all equations need to be determined for every user group $k \in K$, where K is the total number of user groups for the demand input of the network.

$$N_{\rm ns}^{i+1} = \sum_{k=1}^{N} N_{\rm ns}^{i+1,k}, \text{ where } N_{\rm ns}^{i+1,k} = N_{\rm ns}^{i,k} + n_{/ns}^{i,k} + n_{op/ns}^{i,k} + n_{gp/ns}^{i,k} - n_{ns/dgp}^{i,k} - n_{ns/dgp}^{i,k}$$

$$N_{s}^{i+1} = \sum_{k=1}^{N} N_{s}^{i+1,k}, \text{ where } N_{s}^{i+1,k} = N_{s}^{i,k} + n_{ns/s}^{i,k} + n_{dgp/s}^{i,k} - n_{s/op}^{i,k} - n_{s/dgp}^{i,k},$$
(11)

$$N_{op}^{i+1} = \sum_{k=1}^{K} N_{op}^{i+1,k}, \text{ where } N_{op}^{i+1,k} = N_{op}^{i,k} + n_{s/op}^{i,k} - n_{op/ns}^{i,k},$$
(12)

$$N_{dgp}^{i+1} = \sum_{k=1}^{K} N_{dgp}^{i+1,k}, \text{ where } N_{dgp}^{i+1,k} = N_{dgp}^{i,k} + n_{ns/dgp}^{i,k} + n_{s/dgp}^{i,k} - n_{dgp/gp}^{i,k} - n_{dgp/s}^{i,k},$$
(13)

$$N_{gp}^{i+1} = \sum_{k=1}^{K} N_{gp}^{i+1,k}, \text{ where } N_{gp}^{i+1,k} = N_{gp}^{i,k} + n_{dgp/gp}^{i,k} - n_{gp/ns}^{i,k}.$$
(14)

Eq. (10) updates the number of "non-searching" vehicles for each $k \in K$ before aggregating them to N_{ns}^{i+1} . Vehicles entering the area (i.e., $n_{lns}^{i,k}$), and vehicles that depart from on-street or garage parking (i.e., $n_{op/ns}^{i,k}$ and $n_{gp/ns}^{i,k}$) join this state; vehicles that start searching or drive to garage parking (i.e., $n_{ns/s}^{i,k}$ and $n_{ns/dgp}^{i,k}$), and vehicles leaving the area (i.e., $n_{ns/}^{i,k}$) quit this state. Eq. (11) updates the number of "searching" vehicles for each $k \in K$ and aggregates them after to N_s^{i+1} . Vehicles starting to search for on-street parking (i.e., $n_{ns/s}^{i,k}$) and vehicles not able to access garage parking (i.e., $n_{dgp/s}^{i,k}$) join this state; vehicles accessing on-street parking (i.e., $n_{s/op}^{i,k}$) and vehicles driving to garage parking (i.e., $n_{s/dgp}^{i,k}$) leave this state. Eq. (12) updates the number of "on-street parking" vehicles for each $k \in K$ and

Notation	Name	Definition
$n^{i,k}_{/ns}$	Enter the area	Number of vehicles that enter the area and transition to "non-searching" for user group $k \in K$ during time slice <i>i</i> (i.e., travel demand per VOT user group).
$n_{ns/dgp}^{i,k}$	Go to parking (Decision to park: Driving to garage parking)	Number of vehicles that transition from "non-searching" to "driving to garage parking" (depending on their parking decision) for user group $k \in K$ during time slice <i>i</i> .
$n_{ns/s}^{i,k}$	Go to parking (Decision to park: Searching for on-street parking)	Number of vehicles that transition from "non-searching" to "searching for on-street parking" (depending on their parking decision) for user group $k \in K$ during time slice <i>i</i> .
$n_{s/dgp}^{i,k}$	Switch to garage parking	Number of vehicles that transition from "searching for on-street parking" to "driving to garage parking" for user group $k \in K$ during time slice <i>i</i> .
$n_{s/op}^{i,k}$	Find and access on-street parking	Number of vehicles that transition from "searching for on-street parking" to "on-street parking" for user group $k \in K$ during time slice <i>i</i> .
$n_{dgp/gp}^{i,k}$	Access garage parking	Number of vehicles that transition from "driving to garage parking" to "garage parking" for user group $k \in K$ during time slice <i>i</i> .
$n_{dgp/s}^{i,k}$	Not access garage parking	Number of vehicles that transition from "driving to garage parking" to "searching for on-street parking" for user group $k \in K$ during time slice <i>i</i> .
$n_{op/ns}^{i,k}$	Depart on-street parking	Number of vehicles that transition from "on-street parking" to "non-searching" for user group $k \in K$ during time slice <i>i</i> .
$n_{gp/ns}^{i,k}$	Depart garage parking	Number of vehicles that transition from "garage parking" to "non-search- ing" for user group $k \in K$ during time slice <i>i</i> .
$n_{ns/}^{i,k}$	Leave the area	Number of vehicles that leave the area and transition from "non-search- ing" for user group $k \in K$ during time slice <i>i</i> .

TABLE 4: All transition event variables for the on-street and garage parking-state-based matrix per time slice.

TABLE 5: All traffic state variables for the on-street and garage parking-state-based matrix per time slice.

Notation	Name	Definition
$N_{ m ns}^{i,k}$	Non-searching	Number of vehicles in the state "non-searching" for user group $k \in K$ at the beginning of time slice <i>i</i> .
$N_s^{i,k}$	Searching for on-street parking	Number of vehicles in the state "searching for on-street parking" for user group $k \in K$ at the beginning of time slice <i>i</i> .
$N_{op}^{i,k}$	On-street parking	Number of vehicles in the state "on-street parking" for user group $k \in K$ at the beginning of time slice <i>i</i> .
$N_{dgp}^{i,k}$	Driving to garage parking	Number of vehicles in the state "driving to garage parking" for user group $k \in K$ at the beginning of time slice <i>i</i> .
$N_{gp}^{i,k}$	Garage parking	Number of vehicles in the state "garage parking" for user group $k \in K$ at the beginning of time slice <i>i</i> .

then aggregates them to N_{op}^{i+1} . Vehicles accessing an on-street parking space (i.e., $n_{s/op}^{i,k}$) join this state; vehicles departing from on-street parking (i.e., $n_{op/ns}^{i,k}$) leave this state. Eq. (13) updates the number of vehicles that "drive to garage parking" for each user group $k \in K$ and then aggregates them to N_{dgp}^{i+1} . Vehicles that drive to a parking garage during time slice i (i.e., $n_{ns/dgp}^{i,k}$ and $n_{s/dgp}^{i,k}$) join this state; vehicles that actually access garage parking (i.e., $n_{dgp/gp}^{i,k}$) and vehicles that cannot access garage parking (i.e., $n_{dgp/g}^{i,k}$) quit this state. Eq. (14) updates the number of "garage parking" vehicles for each $k \in K$ before aggregating them to N_{gp}^{i+1} . Vehicles that access a garage during time slice i(i.e., $n_{dgp/gp}^{i,k}$) join this state; vehicles that depart garage parking (i.e., $n_{gp/ns}^{i,k}$) join this state. The total number of vehicles driving in the network at the beginning of time slice i is $N_{ns}^{i} + N_{s}^{i} + N_{dgp}^{i}$. The total number of vehicles parked at the beginning of time slice i is $N_{op}^{i} + N_{gp}^{i}$.

3.2. Transition Events. We model the transition events introduced in Table 4 in Sections 3.2.1–3.2.9.

3.2.1. Enter the Area. The traffic demand $n_{/ns}^{i,k}$ is an input to the model. It can be based on a probability distribution or it can be extracted from an agent-based model (e.g., MATSim). However, similar to all other transition events, $n_{/ns}^{i,k}$ is deterministic and represents average values, i.e., there are no random values involved in their computation. A portion β^i of all vehicles entering the area is considered as through traffic, i.e., these vehicles will drive through the area without needing to park.

3.2.2. Go to Parking (Decision to Park). We assume that the vehicles from user group $k \in K$ make their parking decision (searching for on-street parking or driving to garage parking) after driving a distance $l_{ns/}^k$ since they enter the area. $l_{ns/}^k$ can be

fixed or taken out of any given probability density function. The vehicles have the option to drive to garage parking as modeled in Eq. (15), or search for an on-street parking space as shown in Eq. (16). Both $n_{ns/dgp}^{i,k}$ and $n_{ns/s}^{i,k}$ may consist of vehicles from user group $k \in K$ entering the network in any former time slice $i' \in [1, i-1]$.

$$n_{ns/dgp}^{i,k} = \left[\sum_{i'=1}^{i-1} \underbrace{\left(1 - \beta^{i'}\right) \cdot n_{/ns}^{i',k}}_{\text{term 1}} \cdot \underbrace{\gamma_{ns/}^{i',k}}_{\text{term 2}} \cdot \underbrace{\gamma_{gp}^{i,k}}_{\text{term 3}}, \quad (15)\right]$$

$$n_{ns/s}^{i,k} = \left[\sum_{i'=1}^{i-1} \underbrace{\left(1-\beta^{i'}\right)\cdot n_{ns'}^{i',k}}_{\text{term 1}} \cdot \underbrace{\gamma_{ns'}^{i',k}}_{\text{term 2}}\right] \cdot \underbrace{\gamma_{op}^{i,k}}_{\text{term 4}}, \quad (16)$$

where

$$\gamma_{ns/}^{i',k} = \begin{cases} 1, & \text{if } l_{ns/}^k \le \sum_{j=i'}^{j=i-1} d^j \text{ and } \sum_{j=i'}^{j=i-1} d^j \le l_{ns/}^k + d^{i-1}, \\ 0, & \text{otherwise.} \end{cases}$$
(17)

Term 1 in Eqs. (15) and (16) shows the portion of the total demand $n_{jns}^{i',k}$ that needs to park, i.e., all vehicles excluding through traffic. The proportion of through-traffic, $\beta^{i'}$, is assumed to be independent of the individual user group $k \in K$. Term 2 indicates whether these vehicles can decide for parking in time slice *i* or need to continue driving until they cover a distance $l_{ns/}^k$ (Eq. (17)). Term 3 in Eq. (15) expresses the proportion of drivers deciding to drive towards a parking garage (from Eq. (2)) in time slice *i* depending on user group *k*. Term 4 in Eq. (16) expresses the proportion of drivers deciding to search for an on-street parking space (from Eq. (3)) in time slice *i* depending on user group *k*.

3.2.3. Switch to Garage Parking. In this section, the transition event $n_{s/dgp}^{i,k}$ is modeled in Eq. (18) to determine the number of vehicles switching to garage parking after being in the searching-for-on-street-parking state for at least one time slice. This represents the drivers who change their mind regarding where to park.

$$n_{s/dgp}^{i,k} = \left[\underbrace{N_s^{i,k} - n_{s/op}^i \cdot \frac{N_s^{i,k}}{N_s^i}}_{\text{term 1}}\right] \cdot \underbrace{\delta_{gp}^{i,k}}_{\text{term 2}} \cdot \underbrace{\min\left\{\left(N_s^i\right)^{-\alpha}; 1\right\}}_{\text{term 3}}.$$
(18)

Term 1 represents all searching vehicles of user group k that have not parked on-street in this time slice i. Further details on the computation of $n_{s/op}^i$ can be found in Cao and Menendez [1] based on probability theory. These details are omitted in this paper for brevity and because they are not part of the main contributions here. Term 2 shows the proportion of searching vehicles deciding to drive towards a parking garage (Eq. (1)). Notice that the same vehicles have to go over the same decision at multiple time slices in the transition events "Go to parking" and "Switch to garage parking" (potentially revising their previous decision). Term 3 represents a penalty term that prevents drivers from flipping between $n_{s/dgp}^{i,k}$ and $n_{dgp/s}^{i,k}$. It is dependent on N_s^i since the likelihood of flipping is high when there are a lot of searching vehicles on the network. The level of the penalty for the simulation is characterized by α . This parameter is defined as $\alpha > 1$ such that $(N_s^i)^{-\alpha} < 1$, if $N_s^i > 1$. We introduce a minimum function in term 3 to keep term 3 as a probability value, i.e., between 0 and 1. It can be shown in a sensitivity analysis that as long as $\alpha > 1$, changes to its value only have a marginal influence on the average searching time/ distance, the average time/distance of drivers driving to garage parking, and on the revenue collected by on-street and garage parking fees in the network, but the details are omitted in this paper for brevity. In the remainder of this study, we assume a square root dependency and set $\alpha = 2$.

3.2.4. Find and Access On-Street Parking. The vehicles $n_{s/op}^{\iota k}$ from user group k searching for on-street parking that find and access a parking space are determined in Eq. (19).

$$n_{s/op}^{i,k} = n_{s/op}^{i} \cdot \frac{N_s^{i,k}}{N_s^{i}}.$$
 (19)

Notice that all drivers decide to access the first available on-street parking space in the network, as all parking spaces have the same price. As previously stated, details on $n_{s/op}^{i}$ can be found in [1].

3.2.5. Access Garage Parking. The transition event $n_{dgp/gp}^{t,k}$ in Eq. (20) describes the process of accessing a parking garage. After the vehicles have decided to use garage parking, they drive towards the parking garage where they realize whether it is possible for them to access it depending on the garage parking availability.

$$n_{dgp/gp}^{i,k} = \underbrace{\frac{N_{dgp}^{i,k}}{N_{dgp}^{i}}}_{\text{term 1}} \cdot \min\left\{\sum_{k=1}^{K} \sum_{i'=1}^{i-1} \underbrace{\left(n_{ns/dgp}^{i',k} + n_{s/dgp}^{i',k}\right)}_{\text{term 2}} \cdot \underbrace{\gamma_{ADD}^{i',k}}_{\text{term 3}}; R^{i}\right\},\tag{20}$$

where

$$\gamma_{\text{ADD}}^{i',k} = \begin{cases} 1, & \text{if ADD} - l_{ns/}^k \leq \sum_{j=i'}^{j=i-1} d^j \text{ and } \sum_{j=i'}^{j=i-1} d^j \\ & \leq \text{ADD} - l_{ns/}^k + d^{i-1}, \\ 0, & \text{otherwise.} \end{cases}$$
(21)

Term 1 in Eq. (20) represents the portion of vehicles trying to access garage parking that belong to user group k. Term 2 shows the sum of all vehicles (from Sections 3.2.2 and 3.2.3) that have decided to use garage parking in any former time slice $i' \in [1, i-1]$. Term 3 (computed in Eq. (21)) indicates whether these vehicles have arrived at the garage after reaching ADD – $l_{ns/}^k$ (Section 2.2.2). Note that the drivers driving to garage parking are assumed to drive directly towards their garage as soon as they enter the area. Thus, the distance $l_{ns/}^{k}$ is deducted from ADD. Two conditions must be satisfied: the vehicles have driven enough distance to arrive at a parking garage after having decided for it, and they have not accessed a garage in a former time slice. Finally, the number of vehicles that can actually access garage parking is the minimum of the available garage parking spaces and the number of vehicles that want to park.

3.2.6. Not Access Garage Parking. This transition event includes all vehicles that do not access garage parking due to limited availability. In this situation, some of these vehicles $n_{dgp/s}^{i,k}$ in Eq. (22) return back to searching-for-on-street-parking state. However, depending on A and R, some drivers might prefer to stay in the "drive to garage parking" state as a result of a low total number of existing on-street parking spaces compared to the total existing garage capacity.

$$n_{dgp/s}^{i,k} = \frac{N_{dgp}^{i,k}}{N_{dgp}^{i}} \cdot \max\left\{\sum_{k=1}^{K}\sum_{i'=1}^{i-1} \underbrace{\left(n_{ns/dgp}^{i',k} + n_{s/dgp}^{i',k}\right)}_{\text{term 2}} \cdot \underbrace{\gamma_{ADD}^{i',k}}_{\text{term 3}} - R^{i}; 0\right\} \cdot \underbrace{\frac{A}{R+A}}_{\text{term 4}}.$$
(22)

Terms 1, 2, and 3 are already determined as in Eq. (20). In case the garage parking availability limit is reached and the vehicles that would like to enter a parking garage, i.e., $\sum_{k=1}^{K} \sum_{i'=1}^{i-1} \left(n_{ns/dgp}^{i',k} + n_{s/dgp}^{i',k} \right) \cdot \gamma_{ADD}^{i',k}$, surpass R^i , the remaining vehicles need to return to searching-for-parking state; otherwise all vehicles can successfully enter a garage. This portion of vehicles returning back to searching-for-on-street-parking state is reduced by term 4 that represents the drivers' decision to stay in the "drive to garage parking" state due to a low A in comparison to R + A. This term is not time-dependent since there is no real-time usage information available. This constraint is relaxed later (Section 4.5) when real-time information is available. Notice that for more realistic applications, the capacity of garage parking will not be an active constraint. It is included here, however, for the sake of completeness.

3.2.7. Depart On-Street Parking. The number of vehicles that depart from on-street parking is based on the distribution of on-street parking durations $f(t_{d,op})$ and on the number of vehicles having accessed on-street parking, $n_{s/op}^{i',k}$, in a former time slice $i' \in [1, i-1]$. The probability that these vehicles depart from on-street parking in time slice *i* equals to the probability of the on-street parking duration being between $(i - i') \cdot t$ and $(i + 1 - i') \cdot t$, i.e., $\int_{(i-i') \cdot t}^{(i+1-i') \cdot t} f(t_{d,op}) dt_{d,op}$. The transition event is formulated as $n_{op/ns}^{i,k}$ in Eq. (23) as in [1].

$$n_{op/ns}^{i,k} = \sum_{i'=1}^{i-1} n_{s/op}^{i',k} \cdot \int_{(i-i')\cdot t}^{(i+1-i')\cdot t} f(t_{d,op}) dt_{d,op}.$$
 (23)

The on-street parking availability A^{i} is updated in Eq. (24) after vehicles access or depart from on-street parking. A^{*i*} cannot surpass the total number of existing on-street parking spaces, i.e., $A^i \leq A$ for all time slices *i*.

$$A^{i+1} = A^{i} + \sum_{k=1}^{K} n^{i,k}_{op/ns} - \sum_{k=1}^{K} n^{i,k}_{s/op}.$$
 (24)

3.2.8. Depart Garage Parking. The transition event $n_{gp/ns}^{i,k}$ in Eq. (25) is modeled analogously to $n_{op/ns}^{i,k}$. As we know the number of vehicles having decided to use garage parking in all former time slices, we can find $n_{gp/ns}^{i,k}$ based on the distribution of garage parking durations $f(t_{d,qp})$.

$$n_{gp/ns}^{i,k} = \sum_{i'=1}^{i-1} n_{dgp/gp}^{i',k} \cdot \int_{(i-i')\cdot t}^{(i+1-i')\cdot t} f(t_{d,gp}) dt_{d,gp}.$$
 (25)

After vehicles access or depart from garage parking, the availability R^i is updated in Eq. (26). R^i cannot surpass the total capacity, i.e., $R^i \leq R$ for all time slices *i*.

$$R^{i+1} = R^{i} + \sum_{k=1}^{K} n_{gp/ns}^{i,k} - \sum_{k=1}^{K} n_{dgp/gp}^{i,k}.$$
 (26)

3.2.9. Leave the Area. The vehicles leave the area after having driven for a given distance l_j^k or $l_{p_j}^k$ depending on whether they have parked or not. Notice that the distances l_j^k and $l_{p_j}^k$ analogous to $l_{ns_j}^k$ can be fixed or taken out of any given modeled as $n_{ns/}^{i,k}$ in Eq. (27)–Eq. (29) and include through-traffic vehicles, $\beta^{i'} \cdot n_{/ns'}^{i',k}$ and vehicles from the transition events $n_{op/ns}^{i',k}$ and $n_{gp/ns}^{i',k}$.

$$n_{ns/}^{i,k} = \sum_{i'=1}^{i-1} \left(\beta^{i'} \cdot n_{/ns}^{i',k} \cdot \gamma_{/}^{i',k} + \left(n_{op/ns}^{i',k} + n_{gp/ns}^{i',k} \right) \cdot \gamma_{p/}^{i',k} \right), \quad (27)$$

where

$$\gamma_{l}^{i',k} = \begin{cases} 1, & \text{if } l_{l}^{k} \leq \sum_{j=i}^{j=i-1} d^{j} \text{ and } \sum_{j=i}^{j=i-1} d^{j} \leq l_{l}^{k} + d^{i-1}; \\ 0, & \text{otherwise.} \end{cases}$$

$$\gamma_{pl}^{i',k} = \begin{cases} 1, & \text{if } l_{pl}^{k} \leq \sum_{j=i}^{j=i-1} d^{j} \text{ and } \sum_{j=i}^{j=i-1} d^{j} \leq l_{pl}^{k} + d^{i-1}; \\ 0, & \text{otherwise.} \end{cases}$$

$$(29)$$

otherwise.

Further details can be found in [1]. These details are omitted in this paper for brevity and because they are not part of the main contributions here.

4. Applications

In this section, a case study of an area within the city of Zurich, Switzerland, is provided to illustrate the influences of on-street and garage parking on the traffic system. We use real data obtained by Cao et al. [34]. The results are obtained with the aid of a simple numerical solver such as Matlab. We discuss the findings regarding on-street and garage parking pricing, the related parking decision, and the impacts on the average searching time/distance. We analyze the short-term effects of including garage usage information to all drivers, as well as the influences of converting on-street to garage parking spaces on the traffic system.

4.1. Case Study of an Area within the City of Zurich, Switzerland. Our study area (0.28km²) in Figure 4(a) is located around the shopping area Jelmoli in the city center of Zurich [34]. There is a significant amount of retail space and offices from the financial sector in this area. The total length of all roads in the area is L = 7.7 km with an associated area radius of 0.3 km and b = 76 m. Most streets in this area have two lanes (one per direction or two one-way lanes).

There are A = 207 on-street parking spaces and G = 2 parking garages (Jelmoli and Talgarten garage) with a total capacity of R = 332 spaces. The on-street parking price is on average $p_{op} = 1.5 CHF/h$ and the garage parking price is on average $p_{gp} = 3 CHF/h$ [34]. We consider time slices of 1 min during a working day, i.e., t = 1min for a time horizon of T = 1440 min. The MFD of the city of Zurich was used for the traffic properties (i.e., v = 12.5 km/h), based on [41, 42, 43].

The parking demand (Figure 4(b)), parking durations, and initial conditions are extracted from an agent-based model in MATSim that is based on previous measurements. This has been validated and proven reasonable for the city of Zurich in Waraich and Axhausen [44]. Note that the parking demand (Figure 4(b)) is a deterministic demand that changes throughout the day. There is a total travel demand of 2687 trips spread between four different user groups (892/956/838/956 trips) in the network associated with different VOTs ($VOT^1 = 29.9$ CHF/h; $VOT^2 = 25.4$ CHF/h; $VOT^3 = 25.8$ CHF/h; $VOT^4 = 17.2 \text{ CHF/h}$). All VOT values are based on the estimated mean values for the VOT for car journeys in Switzerland [45]. Based on the parking demand and parking usage 23% (618 trips) of the daily demand (i.e., $\beta^i = 0.23, \forall i$) does not search for parking and can be considered as through-traffic, while 77% (2069 trips) of the daily traffic searches for parking [34]. At the beginning of every working day 183 vehicles are already in the area, where $N_{op}^0 = 70$ are parked on-street and $N_{gp}^0 = 113$ are in a garage. All other initial conditions are considered as zero, i.e., $N_{ns}^0 = N_s^0 = N_{dgp}^0 = 0$. Taking the network properties into account, the travel distances $l_{ns/}^k$, $l_{/}^k$, and $l_{p/}^k$ are all uniformly distributed between 0.1 and 0.7 km for all $k \in \{1, ..., 4\}$.

The parking durations of vehicles are differentiated by their parking destination. Figure 5(a) displays the distribution of on-street parking durations and Figure 5(b), the distribution of garage parking durations. The histogram in Figure 5(a) is comparable to a gamma distribution with a shape parameter of $a_1 = 3.5$ and a scale parameter of $a_2 = 28.5$. The histogram in Figure 5(b) represents the two types of drivers, those that have to park in a garage because $t_d > \tau_{op}$ and those that chose to do it as $t_d \leq \tau_{op}$. It is modeled using the gamma distribution with a shape parameter of $a_1 = 2.1$ and a scale parameter of $a_2 = 137.4$. Depending on the frequency, a bi-modal gamma distribution might be suitable for other case studies. All on-street parking spaces have a parking time limit of τ_{op} = 180 min and the garages have no limit within 24 hours, i.e., $\tau_{gp} = 1440 \text{ min}$ [34]. The price per distance driven is assumed as $p_{dist} = 0.3$ CHF/km and the walking speed is set to w = 5 km/h [46].

4.2. Validation. Given that the original framework [1] focusing only on on-street parking has already been validated—in terms of parking usage and cruising time – in former studies [34] and has been used in [47], here we focus on the validation of the garage parking usage. We validate the garage parking occupancy rate using empirical data collected by the city of Zurich. The real garage occupancy data in Figure 6 is generated through a local monitoring system (PLS Zurich) based on 15-minute intervals between the 1st and the 22nd of April, 2016. Only data from

Tuesdays, Wednesday, and Thursdays from the Jelmoli and Talgarten garages are included in the study to represent a working day demand. Compared to Cao et al. [34], the garage parking occupancy obtained in this paper is already close to 100% after the 9.5th hour. This happens because the garage parking duration used here (gamma distribution with mean $\mu = 293 \text{ min}$ in Figure 5(b)) is on average longer than that used in Cao et al. [34] (gamma distribution with mean $\mu = 230.2 \text{ min}$) due to our differentiation of parking durations based on parking destinations. Hence, the turnover rate of the garage parking spaces is reduced, and the 100% garage occupancy is reached at an earlier hour of the day.

The curve reflecting the estimated garage parking occupancy rate shows a rather similar pattern to that of the real data. The approximation is more accurate compared to the validation in Cao et al. [34], where no differentiation between on-street and garage parking is modeled. The mean absolute error of our estimation is 0.046, less than in [34].

4.3. Model Results. In this section, we present some valuable insights with respect to on-street and garage parking. Table 6 illustrates the average/total time and driven distance for the vehicles in the states "Searching for on-street parking," "Drive to garage parking" and "Non-searching" during a typical working day.

On average, each vehicle spends 9.7 minutes in the network (excluding the time spent parked). Not surprisingly, vehicles spend on average longer in the "Searching for on-street parking"- state (3.7 minutes) than in the "Drive to garage parking"- state (3 minutes). A similar behavior can be detected when looking at the average driven distance in the network (Table 6). What is interesting, however, is that the absolute difference in average travel time between the two parking options is less than a minute. This happens because of two reasons. First, the area itself is rather small. Second, based on our decision framework in Eq. (2), on average, only 48.8% of the parking vehicles are able to make a decision between on-street and garage parking. The remaining 51.2% must drive toward a parking garage, given that the on-street parking duration limit is set to $\tau_{op} = 180$ min.

Following the parking demand (Figure 4(b)), the number of vehicles searching for on-street parking increases drastically between the 9th and the 13th hour, and the number of available on-street parking spaces goes down (Figure 7(a)). After the 9.5th hour, the number of available garage parking spaces gets close to zero (Figure 7(b)). The vehicles that cannot access garage parking then return back to the searching-for-onstreet-parking state. This leads to more searching vehicles and less available on-street parking spaces at an earlier hour compared to Cao et al. [34] (Figure 7(a)). The number of vehicles driving to garage parking behaves analogously to the parking demand (Figure 4(b)) and increases between the 5th and the 20th hour (Figure 7(b)). Given the distribution of garage parking durations and the resulting turnover, the number of available garage parking spaces decreases drastically between the 9.5th and the 14th hour (see also Figure 6).

Once there are no available on-street parking spaces anymore (Figure 7(a)), the average cruising time increases



FIGURE 4: Case study area and parking demand per minute computed as a moving average over 10 min (Source: [34]). (a) Case study area in the city center of Zurich. (b) Number of vehicles entering the area over one day.



FIGURE 5: Parking durations for on-street and garage parking. (a) Histogram of the on-street parking durations. (b) Histogram of the garage parking durations.

(Figure 8). This leads to an increase in the costs associated with cruising-for-on-street-parking.

Figure 9(a) shows the share of vehicles searching for on-street parking, driving to garage parking, or non-searching over time. This traffic composition is related only to the vehicles circulating on the network, and not those that are parked. Between the 10th and the 13th hour, the network has the highest percentage of vehicles searching for on-street parking. Figure 9(b) shows the number of vehicles $n_{ns/dgp}^{i,k}$, $n_{s/dgp}^{i,k}$ and $n_{dgp/s}^{i,k}$ summed over all user groups $k \in K$ over a typical working day. $n_{ns/dgp}^{i,k}$ behaves analogously to the parking demand (Figure 4(b)). It increases between the 5th and the 20th hour. $n_{s/dgp}^{i,k}$ is negligibly small. $n_{dgp/s}^{i,k}$ increases from approximately the 9.5th hour since the garage parking occupancy rate is close to 100% (Figure 6). Thus, not enough available garage parking spaces are left (Figure 7(b)) and vehicles are not able to access the parking garages.

4.4. Impacts of On-Street and Garage Parking Pricing. We now use our model to capitalize on the interactions between

on-street and garage parking pricing to improve traffic performance in the short term (i.e., minimize the average searching time and distance). This might be accomplished by increasing the attractiveness of garage parking such that fewer vehicles insist on searching for an on-street parking space, or vice versa, once the garages become full. Remember that this is only possible for drivers who actually have a choice and not for drivers who can only use a garage (Eq. (2)) due to the on-street parking time limit restrictions. What is the ideal ratio between on-street and garage parking fees to attract drivers such that they avoid cruising for on-street parking? We study the impacts of a limited on-street and garage capacity in combination with different on-street and garage parking pricing parameters, i.e., due to the limited number of garage parking spaces and different related pricing schemes, congestion might occur and affect the traffic performance in the network.

Remember that both the hourly on-street and garage parking fee rates, p_{op} and p_{gp} , are part of the decision-related cost variables for on-street and garage parking. Based on these cost variables, the drivers decide for on-street or garage parking, affecting the



FIGURE 6: Comparison between the empirical garage and the estimated garage parking occupancy. (Empirical data were collected and averaged over 12 working days from 3 weeks during 1st-22nd April, 2016).

average travel time in each parking-related state as illustrated in Figure 10. Increasing the ratio p_{op}/p_{gp} leads to a higher cost variable $C_{op}^{i,k}$ (Section 2.2.1) and the drivers are more likely to drive to garage parking. Thus, the average time for vehicles driving to garage parking increases, while the average searching time decreases (Figure 10). Both times are equal for $p_{op}/p_{gp} = 1.75$. At the same time, the average vehicle time for both searching and driving to garage parking vehicles increases in the network after some initial drop (green dotted line in Figure 10). The results for the average distance driven follow a similar pattern as Figure 10, but the details are omitted in this paper for brevity.

Table 7 highlights these findings by comparing the reference scenario in Section 4.3 with an average $p_{op} = 1.5$ CHF/ hour and an average $p_{gp} = 3$ CHF/hour, i.e., $p_{op}/p_{gp} = 1/2$, with the following scenarios:

- (i) $p_{op} = 0.75 CHF$ /hour and $p_{gp} = 3 CHF$ /hour, i.e., $p_{op}/p_{gp} = 1/4$,
- (ii) $p_{op} = 3 CHF/hour$ and $p_{gp} = 3 CHF/hour$, i.e., $p_{op}/p_{gp} = 1$,
- (iii) $p_{op} = 6 CHF/hour$ and $p_{gp} = 3 CHF/hour$, i.e., $p_{op}/p_{gp} = 2$.

As one would expect, the short-term financial benefits for the city, i.e., the total revenue from both on-street and garage parking pricing, increase as either p_{op} and p_{gp} increase. As a matter of fact, increasing only the price of on-street parking (e.g., to 3 *CHF* or 6 *CHF*) increases the revenues for both the on-street and the garage parking. The former is intuitive; the latter comes from the fact that more vehicles move into garage parking in order to avoid the expensive on-street parking fees. Only doubling p_{op} compared to the reference scenario (scenario $p_{op}/p_{gp} = 1$) leads to 23.8% more parking revenue. The total revenue would increase slightly faster if p_{ap} were to increase,

since more than 60% of the parking spaces in the network are garage parking spaces. Every city can estimate their best on-street and garage parking fees according to its plans on improving the traffic performance, the congestion and the environmental conditions and on collecting parking fee revenue (see the different on-street and garage parking pricing ratios in Table 7). It is also possible to use more advanced sensitivity analysis techniques (e.g., as in [48, 49]) to further understand the impact of different inputs (dependent or independent) on these metrics.

Different on-street and garage parking fee rates can lead not only to more vehicle time/distance on the network, worse traffic performance, and worse environmental conditions, but also to various financial revenue outputs. Based on these results, cities can find reasonable hourly on-street and garage parking fees such that the average time driving to garage parking and searching for on-street parking are not negatively affected and additionally, acceptable financial revenues are obtained. Our methodology provides the tools to do a cost-benefit analysis and to study the trade-off between revenues and the average travel time of vehicles trying to park.

4.5. Availability of Garage Usage Information to All Drivers. In reality, the actual garage parking availability also influences the drivers' decision to park on-street or to drive towards a parking garage. This garage usage information can be made available to the drivers by providing real-time smartphone applications or garage information signs in the traffic network.

In this section, we include full real-time information of garage parking availability into our on-street and garage parking model, i.e., the drivers have access to real-time usage information but no forecast of future garage availability. Since this garage parking availability influences the driver's parking decision, we replace all R by R^i in Eq. (1) and Eq. (22). Table 7 illustrates the average time and driven distance

State	Average time per vehicle (min/veh)	Total time (min)	Average driven distance (km/veh)	Total driven distance (km)
Searching for on-street parking state	3.72	4323	0.77	901
Driving to garage parking state	2.96	7458	0.69	1554
Non-searching state	4.46	10047	0.93	2093
Total	9.69	21827	2.02	4547

TABLE 6: Average/total time and driven distance in the network during a typical working day.



FIGURE 7: On-street and garage parking demand and supply over a typical working day. (a) Searching vehicles and available on-street parking. (b) Vehicles driving to garage and available garage parking.



FIGURE 8: Average cruising time for on-street parking over a typical working day.

for the scenario with garage availability information given to all drivers in the network during a typical working day. This additional information helps to reduce the average searching time by 21.2% and the average time driving to garage parking by 27.7% compared to the scenario without garage information available (Section 4.3). The average driven distance in the network reduces similarly and the revenue from on-street and garage parking fees stays constant (Table 7). Allowing drivers to make their on-street or garage parking decision based on real-time occupancy data leads to a better traffic performance on the network, and on average, a faster journey for drivers searching for parking, without affecting much the parking revenues.

The parking choice for garage over on-street parking decreases drastically for drivers with available garage usage information between the 9.5th and the 14th hour compared to drivers who have no garage information available (Figure 11). Since the increase in the average cruising time (Figure 8) has an impact on the drivers' decision, more drivers without any available garage usage information drive to garage parking between the 9.5th and the 13th hour. Due to the lack of garage information, this parking choice is made even if the garage occupancy rate is low. Note that this parking choice only affects the portion of the parking demand that can make a decision between on-street and garage parking due to the on-street parking duration limit. By including the garage usage information into the decision framework, the drivers react towards the garage occupancy rate. The garage occupancy rate (Figure 6) is then reflected in Figure 11 and the parking choice for garage parking increases from the 14th hour analogously to the decrease in the garage occupancy rate in Figure 6.

4.6. Impacts of Converting On-Street Parking to Garage Parking Spaces. It has been one of the policies in Zurich, Switzerland to convert on-street to garage parking spaces. In this section, we evaluate the effects of this policy on traffic performance and the city's revenue. Converting on-street to garage parking spaces is not a trivial task, since real estate in downtown areas is normally expensive to be dedicated to parking garages. However, many cities around the world have indeed done it in order to remove on-street parking spaces without necessarily downsizing the overall parking supply. For example, since the



FIGURE 9: Traffic composition and garage parking-related transition events as a moving average over 10min. (a) Traffic composition. (b) Vehicles in garage parking related transition events.



FIGURE 10: The impact of on-street and garage parking pricing schemas on average time searching/driving to garage parking.

1990's, Zurich has introduced a parking supply cap system in the inner-city [50], i.e., in case a new parking space is created in a parking garage, an existing on-street parking space must be removed such that the parking supply is kept the same [51]. Since the introduction of this policy a few parking garages (e.g., City Parkhaus Zurich and Globus Parkhaus Zurich) and office parking lots have been built, and as a result, on-street parking space has been recovered for other activities. In this paper, we assume a new parking garage is built and the number of parking garages increases to G = 3. At the same time, valuable on-street parking space becomes available. We assume that the recovered road space while converting on-street parking to garage parking spaces has no influence on the traffic flow, and can be used for other activities as in the case of Zurich (e.g., to create pedestrian zones or bicycle lanes). It is further assumed that the conversion of on-street parking does not influence the on-street and garage parking accessibility in the network. The total garage capacity starts at R = 332 (as in Section 4.3) and increases dependent on the number of converted on-street

parking spaces. Note that the initial conditions for N_{op}^0 and N_{qp}^0 are adapted accordingly.

The outputs in Figure 12(a)-12(b) show the impacts of the on-street parking conversion on the average time searching/ driving to garage parking and the parking fee revenue. The impacts on the average/total driven distance follow a similar pattern, but the details are omitted in this paper for brevity.

The more on-street parking spaces that are converted to garage parking spaces, the less drivers chose to go to on-street parking in the first place. This leads to a decreasing average searching time and an increasing average time driving to garage parking in the short term (Figure 12(a)). Table 7 highlights these findings by comparing the reference scenario in Section 4.3 with the scenarios of having a 10%, 30%, and 50% conversion rate from on-street to garage parking spaces. The average time/ distance for vehicles that wish to park decreases as more on-street parking spaces are converted to garage parking. When converting on-street parking, for simplicity, we assume that the distribution for the garage parking durations becomes the same for all levels (based on the combination of Figure 5(a) and Figure 5(b)) as in Cao et al. [34]. Figure 12(b) and Table 7 show the impact of the on-street parking conversion on the total revenue created by on-street/garage parking. While a decreasing number of on-street parking spaces leads to a decreasing total on-street parking revenue, it leads to an increasing total revenue from both on-street and garage parking fees. A conversion of on-street parking to garage parking spaces leads to a reduced travel time and distance for vehicles wishing to park and an increase in the total parking revenue for the city.

5. Conclusions

In this study, we develop a dynamic macroscopic on-street and garage parking model such that the short-term influences of different on-street and garage parking policies on the traffic system can be studied and illustrated. The macroscopic model is built on a traffic system with a parking search model over time. It is incorporated into the on-street parking framework from Cao and Menendez [1]. We validate this model based on

verting on-street to garage parking) to) the reference sc	enario in Section 4		arentnesis represent	is the percentage change	e with respect to t	he reference scena	110.
Policies	Section 4.3: Reference scenario	Section 4.4: Diff	ferent on-street an pricing	d garage parking	Section 4.5: Garage usage information available	Section 4.6: C	conversion rate from garage parking	n on-street to
Scenario	$\frac{p_{op}}{p_{gp}} = \frac{1}{2}$	$\frac{p_{op}}{p_{gp}} = \frac{1}{4}$	$rac{p_{op}}{p_{gp}}=1$	$rac{p_{op}}{p_{op}}=2$	Known to all drivers	10%	30%	50%
Average time for vehicles searching for on-street parking (min/veh)	3.72	4.32 (+16.1%)	3.61 (-3%)	3.53 (-5.1%)	2.93 (-21.2%)	2.61 (-29.8%)	0.89 (-76.1%)	0.18 (-95.2%)
Average time for vehicles driving to garage parking (min/veh)	2.96	2.29 (-22.6%)	3.31 (+11.8%)	3.83 (+29.4%)	2.14 (-27.7%)	4.04 (+36.5%)	5.38 (+81.8%)	5.93 (+100%)
Average time for vehicles searching for on-street parking and driving to	6.68	6.61 (-1.1%)	6.92 (+3.6%)	7.36 (+10.2%)	5.07 (-24.1%)	6.65 (-0.4%)	6.27 (-6.1%)	6.11 (-8.5%)
garage parking (min/ven) Average time for vehicles non-searching (min/veh)	4.46	4.46(+0%)	4.45 (-0.2%)	4.44 (-0.5%)	4.48 (+0.5%)	4.41 (-1.1%)	4.4 (-1.3%)	4.4 (-1.3%)
Average driven distance for vehicles searching for on-street parking (km/veh)	0.77	0.9 (+16.9%)	0.75 (-2.6%)	0.73 (-5.2%)	0.61 (-20.8%)	0.53 (-31.1%)	0.18 (-76.6%)	0.1 (-87%)
Average driven distance for vehicles driving to garage parking (km/veh)	0.69	0.54 (-21.7%)	0.77 (+11.6%)	0.89 (+29%)	0.5 (-27.5%)	0.92 (+33.3%)	1.19 (+72.5%)	1.23 (+78.3%)
Average driven distance for vehicles searching for on-street parking and	1.46	1.44(-1.4%)	1.52 (+4.1%)	1.62 (+11%)	1.11 (-24%)	1.45 (-0.7%)	1.37 (-6.2%)	1.33 (-8.9%)
driving to garage parking (km/veh) Average driven distance for vehicles non-searching (km/veh)	0.93	0.93 (+0%)	0.93 (+0%)	0.93 (+0%)	0.93 (+0%)	0.93 (+0%)	0.92 (-1.1%)	0.92 (-1.1%)
Total on-street parking revenue	3968	2040 (-48.6%)	7764 (+95.7%)	13845 (+248.9%)	3974 (+0.2%)	3720 (-6.3%)	3192 (-19.6%)	2634 (-33.6%)
Total garage parking revenue	13163	12714 (-3.4%)	13443 (+2.1%)	13634 (+3.6%)	13131 (-0.2%)	14314 (+8.7%)	16606 (+26.2%)	19008(+44.4%)
Total revenue created by both on-street and garage parking	17131	14754 (-13.9%)	21207 (+23.8%)	27479 (+60.4%)	17105 (-0.2%)	18034 (+5.3%)	19798 (+15.6%)	21642 (+26.3%)

TABLE 7: Comparison of different policies presented in Section 4.4 (Different on-street and garage parking fees), Section 4.5 (Availability of garage usage information), and Section 4.6 (Con-



FIGURE 11: Parking choice for garage over on-street parking over a typical working day. The parking choice is illustrated for the scenarios without (Section 4.3) and with (Section 4.5) available garage usage information for all drivers. This choice is only possible for drivers with desired parking duration $t_d \leq \tau_{ov}$.



FIGURE 12: The influence of converting on-street to garage parking on the average/total time searching and driving to garage parking, and on the parking fee revenue in the network. (a) Impact on average time. (b) Impact on on-street and garage parking fee revenue.

real data for a case study of an area within the city of Zurich, Switzerland.

The main contributions of this paper are three-fold.

First, we model garage parking macroscopically, including the parking searchers' decision to drive to a parking garage or search for an on-street parking space in the network. This includes the influences on the searching-for-parking traffic (cruising), the congestion in the network (traffic performance), the total driven distance (environmental impact), and the revenue created by on-street and garage parking fees for the city.

Second, we analyze not only the relationship between on-street and garage parking but also their interdependency on cruising-for-parking traffic and traffic performance with respect to different parking fees. Different hourly on-street and garage parking fee ratios can lead not only to more vehicle time/distance in the network, but also to various financial revenue outputs. Thus, this analysis can be used for city councils or private agencies to find reasonable hourly on-street and garage parking fee ratios such that the average vehicle time/ distance is not negatively affected and additionally, acceptable financial revenues are obtained. Our methodology provides the tools to do a cost-benefit analysis and to study the trade-off between the revenue and the average travel time. In the longterm, drivers might avoid paying high on-street or garage parking fees and quit their journeys. This could affect the demand, but long-term effects are beyond the scope of this paper. Third, our model allows us to analyze parking policies in city center areas, e.g., the short-term effects of converting on-street to garage parking spaces on the traffic system can be simulated, and recommendations for city councils can be made. In the city of Zurich, a conversion of on-street parking to garage parking spaces might lead to a higher average time driving to garage parking and a lower average searching time in the short-term with an increase in the total parking revenue. Additionally, the impact of the availability of garage usage information on all drivers can be analyzed. This might lead to a better traffic performance on the network and an on average faster and shorter journey for each driver searching for parking.

The general framework provides an easy to implement methodology to macroscopically model on-street and garage parking. All methods are based on very limited data inputs, including travel demand, VOT, number of garages with their capacity, the traffic network, and initial parking specifications. Only aggregated data at the network level over time are required such that there is no need for individual on-street and garage parking data. This macroscopic approach saves on data collection efforts and reduces the computational costs significantly compared to existing literature. Additionally, there is no requirement of complex simulation software, and the model can be easily solved with a simple numerical solver.

So far, the model only accounts for cars, so any changes in the demand and how those could affect other transportation modes have not been studied. However, recent advances in the MFD and its multimodal extension, the 3D-MFD [52, 53, 42], could open new opportunities to enhance the model by including public transport. If public transport was introduced, the effect of using parking revenue to improve or subsidize mass transit and increase service frequency could potentially be analyzed. Also, here we assume that double parking is not an issue, and we do not account explicitly for delivery parking. Notice, however, that there are already some studies on the development of dynamic delivery parking spots [54], which could also be integrated into the proposed framework in the future.

Overall, the usage of the model is far beyond the illustration in the case study presented here. Here we have assumed that the driving time/distance within the parking garages is negligible. We have done so because that travel does not affect traffic performance at the city level. However, it would be relatively easy to integrate the cost of travel within the parking garages into our decision model, and account for the fact that drivers prefer to park on lower floors close to the exits. A further consideration is tiered parking pricing, which could also be included into the model. Certain cities have tiered pricing for both on-street and garage parking such that the driver may pay a low rate for the first hours, and then the rate jumps up significantly to increase turnover and promote higher parking availability. The model could then be extended to study responsive parking pricing schemes. Time-dependent parking fees can be used to move the parking demand away from the daily peak. Future research can investigate parking fees that are not only dependent on the parking demand of each user group over time, but also on the available parking supply in the network. Additionally, this could include a traffic

demand split with a fixed (low subsidized) parking fee for all on-street and/or garage parking spaces. All remaining portions of demand could be treated responsively, reflecting the external costs for parking. This approach can be motivated by, e.g., the subsidy by a company or a city for their residents.

In summary, the model can be used to efficiently analyze the influence of different on-street and garage parking policies on the traffic system for a smaller geographic scale network, despite its simplicity in data requirements. Based on scarce aggregated data, this model can be used to analyze how on-street and garage parking policies can affect the traffic performance; and how the traffic performance can affect the decision to use on-street or garage parking.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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