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Research Article

Equilibrium and Optimization in a Double-Ended Queueing System with Dynamic Control

Yuejiao Wang [1,2] and Zaiming Liu [2]

[1] *College of Mathematics and Computational Science, Hunan First Normal University, 410205 Changsha, China*
[2] *School of Mathematics and Statistics, Central South University, 410083 Changsha, China*

Correspondence should be addressed to Yuejiao Wang; yuejiao.wang@csu.edu.cn

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In this paper, we consider a double-ended queueing system which is a passenger-taxi service system. In our model, we also consider the dynamic taxi control policy which means that the manager adjusts the arrival rate of taxis according to the taxi stand congestion. Under three different information levels, we study the equilibrium strategies as well as socially optimal strategies for arriving passengers by a reward-cost structure. Furthermore, we present several numerical experiments to analyze the relationship between the equilibrium and socially optimal strategies and demonstrate the effect of different information levels as well as several parameters on social benefit.

1. Introduction

The taxi service is an important component in the comprehensive transportation hubs. However, many travelers encounter such a situation that there are no taxis in the taxi stand during peak hour and there are many taxis in the taxi stand during the nonpeak hour. In order to make more efficient use of the taxi resource, we consider optimization problems in the passenger-taxi service system under dynamic taxi control. The passenger-taxi service system can be described as a double-ended queue: a queue for passengers, a queue for taxis. Clearly, the two queues cannot exist at the same time. In this paper, we will give some efficient strategies to ensure passengers' and taxis' utilities and reduce the taxi stand congestion.

Kendall [1] first studied the double-ended queue. The passenger-taxi service system was introduced as an example. Dobbie [2] found transient behavior under the nonhomogeneous Poisson arrival of passengers and taxis. Giveen [3] showed the asymptotic behavior of the double-ended queueing system under when the mean rates of arrival of passengers and taxis vary. Kashyap [4, 5] considered a double-ended queue with limited waiting space for taxis and for passengers. He studied the expected queue lengths of taxis and passengers under the general arrival of passengers and the Poisson arrival of taxis. Wong, Wong, Bell, and Yang [6] adopted an absorbing Markov chain to model the searching process of taxi movements and proposed a useful formulation for describing the urban taxi services in a network. Conolly, Parthasarathy, and Selvaraju [7] studied double-ended queues with an impatient server or customers. Crescenzo, Giorno, Kumar, and Nobile [8] discussed a double-ended queue with catastrophes and repairs and obtained both the transient and steady-state probability distributions. Moreover, the double-ended queue can be applied to many other areas, for example, computer science, perishable inventory system, and organ allocation system. Zenios [9] illustrated a double-ended matching problem between several classes of organs and patients who would renege due to death. Wong, Szeto, and Wong [10] adopted the sequential logit approach to modeling bilevel decisions of vacant taxi drivers in customer-search. However, the above references discussed the performance measures of the double-ended queue. In this paper, we study the strategic behaviors of the passengers.

The study of queueing systems with strategic behavior of customers was first done by Naor [11], who analyzed the strategic behavior of customers under an observable queue by a linear reward-cost structure. Edelson and Hildebrand

[12] investigated the same problem following with Naor (1969). However, in this model, arriving customers do not know the queue length before his decision. Yang, Leung, Wong, and Bell [13] presented an equilibrium model for the problem of bilateral searching and meeting between taxis and passengers in a general network. Burnetas and Economou [14] discussed strategic behavior in a single server Markovian queue with setup times. Burnetas, Economou, and Vasiliadis [15] studied strategic customer behavior in a queueing system with delayed observations. Guo and Hassin [16] illustrated strategic behavior of customers and social optimization in Markovian vacation queues. Wang, Zhang, and Huang [17] considered strategic behavior of customers and social optimization in a constant retrial queue with the N-policy. The optimization problems of passenger-taxi service system under strategic behavior of passengers were first considered by Shi and Lian [18] who studied the arriving passengers' equilibrium strategies and socially optimal strategies under a limited waiting space of taxis and the same taxis' arrival rate. Shi and Lian [19] discussed a double-ended queueing system with limited waiting space for arriving taxis and arriving passengers. A passenger-taxi service system with a gated policy was considered by Wang, Wang, and Zhang [20] who studied the arriving passengers' equilibrium strategies and socially optimal strategies in fully observable, almost unobservable and fully unobservable cases, while they considered the model with same arrival rate of taxis. In order to balance the relationship between long passenger delays and high taxi's company costs, we consider a passenger-taxi service system with dynamic taxi control. The taxi control is to improve the arrival rate of taxis when the queue length of passengers is large so as to reduce delays and decrease it at times of increased queue length of taxis so as to reduce the costs of taxis' derivers. In this model, we study the (Nash) equilibrium strategies and socially optimal strategies of arriving passengers. Our model will improve the social benefit in the observable case and unobservable case if the waiting space of taxis is large enough, compared with the results in [18].

In the passenger-taxi service system with dynamic taxi control, arriving passengers decide whether to join the taxi stand or balk based on a linear reward-cost structure. We discuss the equilibrium strategies and socially optimal strategies under three different information levels: (1) fully observable case: the arriving passengers are noticed the number of passengers and taxis in the taxi stand; (2) almost unobservable case: the arriving passengers are only informed of the state of taxis; (3) fully unobservable case: the arriving passengers are not informed of the number of passengers or taxis in the taxi stand. The passenger's strategic behavior is under two different types: "selfishly optimal" and "socially optimal". "Selfishly optimal" is the strategy under (Nash) equilibrium conditions. "Socially optimal" is the strategy to maximize the social benefit. The contribution of the present paper is as follows: (1) study the passenger's selfishly optimal threshold and socially optimal threshold in fully observable case; (2) obtain the selfishly optimal joining probabilities and socially optimal joining probabilities in the almost unobservable case; (3) investigate the selfishly optimal joining probabilities and social benefit function in the fully unobservable case; (4) present several numerical experiments to analyze the relationship between the equilibrium and socially optimal strategies and demonstrate the effect of different information levels as well as several parameters on social benefit.

The rest of the paper is organized as follows. In Section 2, we describe precisely the passenger-taxi service system. Sections 3, 4, and 5 discuss the equilibrium strategies and socially optimal strategies of passengers in three different information levels. In Section 6, we present some numerical examples to show how different information levels and parameters impact passenger's strategic behavior and social benefit. Section 7 concludes the paper with a summary.

2. Model Description

In this paper, we consider a passenger-taxi service system which is a double-ended queueing system. Now we give a precise description of the model. Passengers (one to four passengers traveling together and will arrive at the same destination can be seen as one passenger) arrive according to a Poisson process with rate λ_1. Taxis arrive according to a Poisson process. The arrive rate sets to λ_0 whenever the number of passengers in the system equals to 0 and sets to λ_2 otherwise, where $\lambda_0 < \lambda_2$. Passengers and taxis are served according to first-in-first-out discipline and leave the taxi stand at once if a taxi takes one passenger. In the taxi stand, the taxis' capacity is N which means that a taxi cannot join the taxi stand if there are N taxis waiting for passengers. The passengers can join the taxi stand without any limit. Let $N(t)$ represent the queue length of passengers or taxis in taxi stand in time t. If $N(t) > 0$, it shows that passengers are waiting for taxis. If $N(t) = 0$, it shows that the system is empty. If $N(t) < 0$, it shows that taxis waiting for new passengers. Obviously, we know that $\{N(t), t \geq 0\}$ is a one-dimensional continuous time Markov chain with state space $\mathscr{F} = \{-N, -N+1, \cdots, -1, 0, 1, \cdots\}$. The state transition diagram is shown in Figure 1.

We assume that every joining passenger incurs a waiting cost C_1 per unit time of waiting in the passenger queue, pays a taxi fare of p_1, and obtains a reward of R after arriving at his definition. Let C_2 be the waiting cost of a taxi per unit time. Finally, we assume that joining passengers are not allowed to retrial and renege.

3. Almost Unobservable Case

In this section, we consider the almost unobservable case where an arriving passenger is only informed the state of taxis. If the number of taxis is more than zero, an arriving passenger will take a taxi immediately, to join the taxi stand without a doubt. But if the number of taxis equals 0, the passenger is not informed the number of passengers. The joining probability of an arriving passenger is q_{au} and the balking probability is $1 - q_{au}$. The state transition diagram is shown in Figure 2. For stability, let $\rho_0 = \lambda_1/\lambda_0$, $\rho_1 = \lambda_1/\lambda_2$, and $\rho_{1,2} = \lambda_1 q_{au}/\lambda_2 < 1$.

Let $\pi_i^{au} = \lim_{t\longrightarrow\infty}\mathbb{P}(N(t) = i)$, $(i \in \mathscr{F})$ be the steady-state probability of state i in the almost unobservable case. We

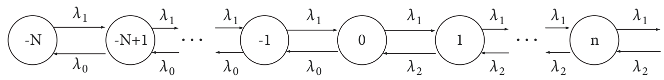

Figure 1: State transition diagram for the passenger-taxi service system.

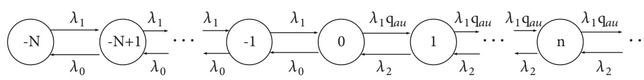

Figure 2: State transition diagram for the almost unobservable case.

can obtain the stationary distribution by solving the balance equations.

Proposition 1. *The stationary distribution in the almost unobservable case is given by*

$$\pi_{-N}^{au} = \frac{(1-\rho_0)(1-\rho_{1,2})}{1-\rho_{1,2}-\rho_0^{N+1}+\rho_0^N\rho_{1,2}}; \tag{1}$$

$$\pi_i^{au} = \frac{\rho_0^{N+i}(1-\rho_0)(1-\rho_{1,2})}{1-\rho_{1,2}-\rho_0^{N+1}+\rho_0^N\rho_{1,2}}, \quad -N+1 \le i \le 0; \tag{2}$$

$$\pi_i^{au} = \frac{\rho_0^N\rho_{1,2}^i(1-\rho_0)(1-\rho_{1,2})}{1-\rho_{1,2}-\rho_0^{N+1}+\rho_0^N\rho_{1,2}}, \quad i \ge 1. \tag{3}$$

Proof. The balance equations can be written as follows:

$$\begin{aligned}\lambda_1\pi_{-N}^{au} &= \lambda_0\pi_{-N+1}^{au};\\ (\lambda_1+\lambda_0)\pi_i^{au} &= \lambda_1\pi_{i-1}^{au}+\lambda_0\pi_{i+1}^{au}, \quad -N+1 \le i \le 1;\\ (\lambda_0+\lambda_1 q_{au})\pi_0^{au} &= \lambda_1\pi_{-1}^{au}+\lambda_2\pi_1^{au};\\ (\lambda_2+\lambda_1 q_{au})\pi_i^{au} &= \lambda_1 q\pi_{i-1}^{au}+\lambda_2\pi_{i+1}^{au}, \quad i \ge 1.\end{aligned} \tag{4}$$

Therefore, by the normalization condition, we obtain (1), (2), and (3). □

By (1), (2), and (3), we can get the expected queue length of passengers $\mathbb{E}L_{au1}$ and expected queue length of taxis $\mathbb{E}L_{au2}$, respectively,

$$\begin{aligned}\mathbb{E}L_{au1} &= \sum_{i=0}^{\infty} i\pi_i^{au}\\ &= \frac{\rho_0^N\rho_{1,2}(1-\rho_0)}{(1-\rho_{1,2})(1-\rho_{1,2}-\rho_0^{N+1}+\rho_0^N\rho_{1,2})};\end{aligned} \tag{5}$$

$$\begin{aligned}\mathbb{E}L_{au2} &= \sum_{-N}^{0}(-i)\pi_i^{au}\\ &= \frac{1-\rho_{1,2}}{1-\rho_{1,2}-\rho_0^{N+1}+\rho_0^N\rho_{1,2}}\left[N-\frac{\rho_0(1-\rho_0^N)}{1-\rho_0}\right].\end{aligned} \tag{6}$$

The effective arrival rate of passengers is

$$\begin{aligned}\lambda_c^* &= \lambda_1\sum_{j=-N}^{-1}\pi_j^{au}+\lambda_1 q_{au}\sum_{j=0}^{\infty}\pi_j^{au}\\ &= \frac{\lambda_1(1-\rho_0^N)(1-\rho_{1,2})+\lambda_1 q_{au}\rho_0^N(1-\rho_0)}{1-\rho_{1,2}-\rho_0^{N+1}+\rho_0^N\rho_{1,2}}.\end{aligned} \tag{7}$$

The effective arrival rate of taxis is

$$\begin{aligned}\lambda_T^* &= \lambda_0\sum_{j=-N+1}^{0}\pi_j^{au}+\lambda_2\sum_{j=1}^{\infty}\pi_j^{au}\\ &= \frac{\lambda_1(1-\rho_0^N)(1-\rho_{1,2})+\lambda_1 q_{au}\rho_0^N(1-\rho_0)}{1-\rho_{1,2}-\rho_0^{N+1}+\rho_0^N\rho_{1,2}}.\end{aligned} \tag{8}$$

Therefore, by Little's law, we obtain the expected waiting time of a joining passenger $\mathbb{E}W_{au1}$ and the expected waiting time of a taxi $\mathbb{E}W_{au2}$, respectively

$$\mathbb{E}W_{au1} = \frac{\mathbb{E}L_{au1}}{\lambda_c^*}; \tag{9}$$

$$\mathbb{E}W_{au2} = \frac{\mathbb{E}L_{au2}}{\lambda_T^*}. \tag{10}$$

3.1. Equilibrium Strategies of Passengers. Now we consider the equilibrium strategy of an arriving passenger in almost unobservable case. We first consider the average waiting time of a joining passenger; see the below proposition.

Proposition 2. *When the queue length of taxis in the taxi stand is zero, the expected waiting time for a joining passenger is*

$$W(q_{au} \mid N(t) \ge 0) = \frac{1}{\lambda_2(1-\rho_{1,2})}. \tag{11}$$

Proof. By Little's law and (5), we get the expected waiting time of a joining passenger

$$\begin{aligned}W(q_{au} \mid N(t) \ge 0) &= \frac{\left(\sum_{j=0}^{\infty} j\pi_j\right)/(\lambda_1 q)}{\sum_{j=0}^{\infty}\pi_j}\\ &= \frac{1}{\lambda_2(1-\rho_{1,2})}.\end{aligned} \tag{12}$$

□

Therefore, the utility of an arriving passenger is

$$U_{au}\left(q_{au} \mid N(t) \geq 0\right) = R - p_1 - C_1 \frac{1}{\lambda_2\left(1-\rho_{1,2}\right)}. \quad (13)$$

An equilibrium strategy for an arriving passenger who decides whether to join or balk is represented by q_{au}^e which is the joining probability for an arriving passenger and $1 - q_{au}^e$ is the balking probability. q_{au}^e is also called selfishly optimal joining probability.

Theorem 3. *In the almost unobservable case, the equilibrium strategy for an arriving passenger is given by*

$$q_{au}^e = \begin{cases} 0, & if\ R < p_1 + C_1 \dfrac{1}{\lambda_2}; \\ q_{au}^{e*}, & if\ p_1 + C_1 \dfrac{1}{\lambda_2} \leq R \leq p_1 + C_1 \dfrac{1}{\lambda_2\left(1-\rho_1\right)}; \\ 1, & if\ R > p_1 + C_1 \dfrac{1}{\lambda_2\left(1-\rho_1\right)}, \end{cases} \quad (14)$$

where $q_{au}^{e*} = (\lambda_2(R-p_1) - C_1)/(R-p_1)\lambda_1$.

Proof. By Proposition 2, we have

$$W_c'\left(q_{au} \mid N(t) \geq 0\right) = \left(\frac{1}{\lambda_2\left(1-\rho_{1,2}\right)}\right)' = \frac{\lambda_1}{\left(\lambda_2\left(1-\rho_{1,2}\right)\right)^2}, \quad (15)$$

so that $W_c'(q_{au} \mid N(t) \geq 0) > 0$; therefore, $W_c(q_{au} \mid N(t) \geq 0)$ is increasing for $q_{au} \in [0,1]$.

If $R < p_1 + C_1(1/\lambda_2)$, then $U_c(q_{au} \mid N(t) \geq 0) < 0$ for $q_{au} \in [0,1]$. Therefore, the best choice for an arriving passenger is balking, so that $q_{au}^e = 0$.

If $R > p_1 + C_1(1/\lambda_2(1-\rho_1))$, then $U_c(q_{au} \mid N(t) \geq 0) > 0$ for $q_{au} \in [0,1]$, so that an arriving passenger's best choice is $q_{au}^e = 1$.

Since $U_c(q_{au} \mid N(t) \geq 0)$ is a decreasing function for $q_{au} \in [0,1]$, so that there exists a unique solution of the equation $U_c(q_{au}^{e*}) = 0$ within $(0,1)$ for $p_1 + C_1(1/\lambda_2) \leq R \leq p_1 + C_1(1/\lambda_2(1-\rho_1))$. □

3.2. Socially Optimal Strategies of Passengers. Now, we consider the socially optimal strategy of an arriving passenger. By (5), (6), (7), and (8), we obtain the social benefit S_{au} in the almost unobservable case

$$\begin{aligned} S_{au}\left(q_{au}\right) &= \lambda_c^*\left(R - p_1 - C_1 \mathbb{E}W_{au1}\right) + \lambda_T^*\left(p_1 - C_2 \mathbb{E}W_{au2}\right) = R \\ &\cdot \frac{\lambda_1\left(1-\rho_0^N\right)\left(1-\rho_{1,2}\right) + \lambda_1 q_{au} \rho_0^N\left(1-\rho_0\right)}{1-\rho_{1,2}-\rho_0^{N+1}+\rho_0^N\rho_{1,2}} - C_1 \\ &\cdot \frac{\rho_0^N\rho_{1,2}\left(1-\rho_0\right)}{\left(1-\rho_{1,2}\right)\left(1-\rho_{1,2}-\rho_0^{N+1}+\rho_0^N\rho_{1,2}\right)} - C_2 \\ &\cdot \frac{1-\rho_{1,2}}{1-\rho_{1,2}-\rho_0^{N+1}+\rho_0^N\rho_{1,2}}\left[N - \frac{\rho_0\left(1-\rho_0^N\right)}{1-\rho_0}\right]. \end{aligned} \quad (16)$$

We investigate the socially optimal strategy which is represented by q_{au}^* to maximize social benefit in almost unobservable case.

Theorem 4. *$S_{au}(q_{au})$ is a concave function in $q_{au} \in [0,1]$ and reaches maximum at $q_{au}^* = \min((dS_{au}(q_{au})/q_{au})|_{q_{au}=q_{au}^*} = 0; 1)$.*

Proof. To simplify, let $C = C_2[N - \rho_0(1-\rho_0^N)/(1-\rho_0)](1/(1-\rho_0))$. Therefore,

$$\begin{aligned} S_{au}\left(q_{au}\right) &= \lambda_2 R \\ &- \left[\left(\frac{\lambda_2}{1-\rho_0} - \frac{\lambda_2\rho_0^{N+1}}{1-\rho_0} - \frac{\lambda_1\left(1-\rho_0^N\right)}{1-\rho_0}\right) R \right. \\ &\left. + C_1 \frac{\rho_0^N\rho_{1,2}\left(1-\rho_0\right)}{\left(1-\rho_0\right)\left(1-\rho_{1,2}\right)^2} + C\right] \pi_{-N}^{au}. \end{aligned} \quad (17)$$

The first derivative of $S_{au}(q_{au})$ is

$$\begin{aligned} \frac{dS_{au}\left(q_{au}\right)}{dq_{au}} &= \frac{\rho_1\rho_0^N}{\left(1-\rho_{1,2}\right)^2}\left(\pi_{-N}^{au}\right)^2\left[R\left(\frac{\lambda_2}{1-\rho_0} - \frac{\lambda_2\rho_0^{N+1}}{1-\rho_0} - \frac{\lambda_1\left(1-\rho_0^N\right)}{1-\rho_0}\right) + C_1\frac{\rho_0\rho_{1,2}}{\left(1-\rho_{1,2}\right)^2} + C \right. \\ &\left. - \frac{1+\rho_{1,2}}{1-\rho_{1,2}}\frac{C_1}{\pi_{-N}^{au}}\right] = \frac{\rho_1\rho_0^N}{\left(1-\rho_{1,2}\right)^4}\left(\pi_{-N}^{au}\right)^2\left[R\left(\frac{\lambda_2}{1-\rho_0} - \frac{\lambda_2\rho_0^{N+1}}{1-\rho_0} - \frac{\lambda_1\left(1-\rho_0^N\right)}{1-\rho_0}\right)\left(1-\rho_{1,2}\right) + C_1\rho^N\rho_{1,2} \right. \\ &\left. + C\left(1-\rho_{1,2}\right)^2 - \left(1-\rho_{1,2}^2\right)\frac{C_1}{\pi_{-N}^{au}}\right] \\ &= \frac{\rho_1\rho_0^N}{\left(1-\rho_{1,2}\right)^4}\left(\pi_{-N}^{au}\right)^2 \\ &\cdot \left[\left[R\left(\frac{\lambda_2}{1-\rho_0} - \frac{\lambda_2\rho_0^{N+1}}{1-\rho_0} - \frac{\lambda_1\left(1-\rho_0^N\right)}{1-\rho_0}\right) + C \right.\right. \\ &\left. + C_1\frac{1-\rho_0^N}{1-\rho_0}\right]\rho_{1,2}^2 \end{aligned}$$

$$
\begin{aligned}
&-2\rho_{1,2}\left(R\left(\frac{\lambda_2}{1-\rho_0}-\frac{\lambda_2\rho_0^{N+1}}{1-\rho_0}-\frac{\lambda_1\left(1-\rho_0^N\right)}{1-\rho_0}\right)\right.\\
&\left.+C\right)+\left(R\left(\frac{\lambda_2}{1-\rho_0}-\frac{\lambda_2\rho_0^{N+1}}{1-\rho_0}-\frac{\lambda_1\left(1-\rho_0^N\right)}{1-\rho_0}\right)\right.\\
&\left.\left.+C-C_1\frac{1-\rho_0^{N+1}}{1-\rho_0}\right)\right].
\end{aligned} \tag{18}
$$

Since

$$
\begin{aligned}
\Delta &= 4\left(R\left(\frac{\lambda_2}{1-\rho_0}-\frac{\lambda_2\rho_0^{N+1}}{1-\rho_0}-\frac{\lambda_1\left(1-\rho_0^N\right)}{1-\rho_0}\right)+C\right)\\
&\cdot C_1\rho_0^N+4C_1^2\frac{\left(1-\rho_0^N\right)\left(1-\rho_0^{N+1}\right)}{\left(1-\rho_0\right)^2}>4C_1^2\\
&\cdot\frac{\left(1-\rho_0^N\right)^2}{\left(1-\rho_0\right)^2},
\end{aligned} \tag{19}
$$

we have

$$
\frac{R\left(\lambda_2/\left(1-\rho_0\right)-\lambda_2\rho_0^{N+1}/\left(1-\rho_0\right)-\lambda_1\left(1-\rho_0^N\right)/\left(1-\rho_0\right)\right)+C+\sqrt{\Delta/4}}{R\left(\lambda_2/\left(1-\rho_0\right)-\lambda_2\rho_0^{N+1}/\left(1-\rho_0\right)-\lambda_1\left(1-\rho_0^N\right)/\left(1-\rho_0\right)\right)+C+C_1\left(\left(1-\rho_0^N\right)/\left(1-\rho_0\right)\right)}>1. \tag{20}
$$

Therefore, we know that $dS_{au}(q_{au})/dq_{au}$ is a decreasing function at $q_{au}\in[0,1]$. Hence, we obtain the maximum of $S_{au}(q_{au})$ in $q_{au}^*=\min((dS_{au}(q_{au})/q_{au})|_{q_{au}=q_{au}^*}=0,1)$. □

4. Fully Observable Case

We first consider the fully observable case in which arriving passengers are informed both the number of passengers and taxis in taxi stand. In this case, we consider the equilibrium strategies and socially optimal strategies for arriving passengers.

4.1. Equilibrium Strategies of Passengers. In fully observable case, the equilibrium joining strategy of an arriving passenger who decides whether to join the taxi stand or balk is represented by threshold type that is an arriving passenger will join the taxi stand if the queue length of passengers is less than threshold and balking otherwise. If there exists a threshold n^0 such that the passengers will join the taxi stand if $N(t)\le n^0$ and balk otherwise, then n^0 is called selfishly optimal threshold. The value of n^0 is given by the following Theorem 5.

Theorem 5. *In the fully observable passenger-taxi system, there exists a unique selfishly optimal threshold*

$$
n^0=\left[\frac{\lambda_2\left(R-p_1\right)}{C_1}\right] \tag{21}
$$

which is the equilibrium strategy of an arriving passenger.

Proof. By the passenger's utility, n^0 should satisfy the following conditions:

$$
\begin{aligned}
&R-p_1-C_1\frac{n^0}{\lambda_2}\ge 0;\\
&R-p_1-C_1\frac{n^0+1}{\lambda_2}<0.
\end{aligned} \tag{22}
$$

□

4.2. Socially Optimal Strategies of Passengers. Then we consider the socially optimal strategy for an arriving passenger. That is specified by threshold which means that there exists a unique n^* (is called socially optimal threshold) such that the social benefit reaches maximum. Clearly, we know that the system follows a one-dimensional continuous time Markov chain with state space $\mathcal{F}^0=\{-N,-N+1,\cdots,-1,0,\cdots,n\}$, where n is the passenger buffer size. The transition rate diagram is shown in Figure 3. For simplicity, let $\rho_0=\lambda_1/\lambda_0$ and $\rho_1=\lambda_1/\lambda_2$. Let π_k be the stationary distribution of state $k\in\mathcal{F}^0$ in the fully observable case. We can obtain the stationary distribution by solving the balance equations.

Proposition 6. *The stationary distribution in the fully observable case is as follows:*

$$
\begin{aligned}
\pi_{-N}&=\frac{\left(1-\rho_0\right)\left(1-\rho_1\right)}{1-\rho_1+\rho_1\rho_0^N-\rho_0^{N+1}-\rho_0^N\rho_1^{n+1}+\rho_0^{N+1}\rho_1^{n+1}};\\
\pi_i&=\frac{\rho_0^{N+1}\left(1-\rho_0\right)\left(1-\rho_1\right)}{1-\rho_1+\rho_1\rho_0^N-\rho_0^{N+1}-\rho_0^N\rho_1^{n+1}+\rho_0^{N+1}\rho_1^{n+1}},\\
&\qquad -N+1\le i\le 0;\\
\pi_i&=\frac{\rho_0^N\rho_1^i\left(1-\rho_0\right)\left(1-\rho_1\right)}{1-\rho_1+\rho_1\rho_0^N-\rho_0^{N+1}-\rho_0^N\rho_1^{n+1}+\rho_0^{N+1}\rho_1^{n+1}},\\
&\qquad 1\le i\le n.
\end{aligned} \tag{23}
$$

By the same method of (16), we obtain the social benefit function:

$$
\begin{aligned}
S_{ob}(n)&=\lambda_c\left(R-p_1-C_1\mathbb{E}W_{obc}\right)\\
&+\lambda_T\left(p_1-C_2\mathbb{E}W_{obt}\right)=\lambda_cR\\
&-C_1\mathbb{E}L_{obc}-C_2\mathbb{E}L_{obt}\\
&=\frac{1}{1-\rho_1+\rho_1\rho_0^N-\rho_0^{N+1}-\rho_0^N\rho_1^{n+1}+\rho_0^{N+1}\rho_1^{n+1}}\left[R\lambda_1\left(1\right.\right.
\end{aligned}
$$

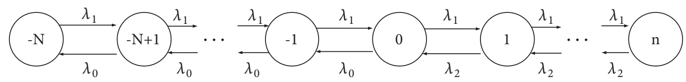

FIGURE 3: State transition diagram for the fully observable case.

$$- \rho_1 + \rho_1 \rho_0^N - \rho_0^{N+1} - \rho_0^N \rho_1^n + \rho_0^{N+1} \rho_1^n \big) - C_1 \rho_0^N (1 - \rho_0) \left[\frac{\rho_1 (1-\rho_1^n)}{1-\rho_1} - n\rho_1^{n+1} \right] - C_2 (1-\rho_1) \left(N - \frac{\rho_0 (1-\rho_0^N)}{1-\rho_0} \right) \Bigg], \tag{24}$$

where $\mathbb{E}W_{obc}$ and $\mathbb{E}W_{obt}$ represent the expected waiting time of passengers and taxis respectively, $\mathbb{E}L_{obc}$ and $\mathbb{E}L_{obt}$ represent the mean queue length of passengers and taxis respectively, and $\lambda_c = \lambda_T$ represents the effective arrival rate of passengers or taxis.

By definition of the socially optimal threshold, we know that n^* should follow the below two inequalities:

$$\begin{aligned} S_{ob}(n^*) - S_{ob}(n^*+1) &\geq 0; \\ S_{ob}(n^*) - S_{ob}(n^*-1) &\geq 0. \end{aligned} \tag{25}$$

By calculation, we have the following inequalities which are equal to the condition (25)

$$\begin{aligned} & n^* \left(1 - \rho_1 + \rho_1 \rho_0^N - \rho_0^{N+1}\right)(1-\rho_1) \\ & \quad - \rho_0^N \rho_1 (1-\rho_0) \left(1 - \rho_1^{n^*}\right) \leq \frac{M}{C_1 \rho_1} \\ & \quad \leq (n^*+1) \left(1 - \rho_1 + \rho_1 \rho_0^N - \rho_0^{N+1}\right)(1-\rho_1) \\ & \quad - \rho_0^N \rho_1 (1-\rho_0) \left(1 - \rho_1^{n^*+1}\right), \end{aligned} \tag{26}$$

where

$$\begin{aligned} M = {} & R\lambda_1 \left(1 - \rho_1 + \rho_1 \rho_0^N - \rho_0^{N+1}\right)(1-\rho_1)^2 \\ & + C_2 \rho_1 (1-\rho_1)^2 \left[N - \frac{\rho_0 (1-\rho_0^N)}{1-\rho_0} \right]. \end{aligned} \tag{27}$$

Let

$$\begin{aligned} f(x) = {} & x \left(1 - \rho_1 + \rho_1 \rho_0^N - \rho_0^{N+1}\right)(1-\rho_1) \\ & - \rho_0^N \rho_1 (1-\rho_0) (1-\rho_1^x). \end{aligned} \tag{28}$$

In the following proposition, we study the monotonicity of the function $f(x)$ for $x \geq 1$.

Proposition 7. *If $\rho_0 < 1$, $f(x)$ is an increasing function in $x \geq 1$. If $\rho_0 > 1$ and $\rho_1 \neq 1$, $f(x)$ is a decreasing function in $x \geq 1$.*

Proof. The first order derivative of $f(x)$ is

$$\begin{aligned} g(\rho_1) &:= \frac{df(x)}{dx} \\ &= \left(1 - \rho_1 + \rho_1 \rho_0^N - \rho_0^{N+1}\right)(1-\rho_1) \\ &\quad + \rho_0^N (1-\rho_0) \rho_1^{x+1} \log \rho_1. \end{aligned} \tag{29}$$

The second order derivative of $f(x)$ is

$$\frac{d^2 f(x)}{dx^2} = \rho_0^N (1-\rho_0) \rho_1^{x+1} (\log \rho_1)^2 > 0. \tag{30}$$

The first order derivative of $g(\rho_1)$ is

$$\begin{aligned} \frac{dg(\rho_1)}{d\rho_1} = {} & -2(1-\rho_1) + \rho^N (1-\rho_1) + \rho_0^N (\rho_0 - \rho_1) \\ & + (x+1) \rho_0^N (1-\rho_0) \rho_1^x \log \rho_1 \\ & + \rho_0^N (1-\rho_1) \rho_1^x. \end{aligned} \tag{31}$$

(1) If $0 < \rho_1 \leq \rho_0 < 1$ and $1 < \rho_1 \leq \rho_0$, by (31), we have

$$\begin{aligned} \frac{dg(\rho_1)}{d\rho_1} &< -2(1-\rho_1) + \rho_0^N (1-\rho_1) + \rho_0^N (1-\rho_1) \\ &\quad + \rho_0^{N+x} (1-\rho_0) - \rho_0^N (1-\rho_0) \\ &= (1-\rho_1)\left(-2 + 2\rho_0^N\right) \\ &\quad + (1-\rho_0) \rho_0^N (\rho_0^x - 1) < 0. \end{aligned} \tag{32}$$

Therefore, $g(\rho_1)$ is a decreasing function for $\rho_1 \in (0,1) \cup (1,\infty)$. Then,

$$g(1) = \left. \frac{df(x)}{dx} \right|_{\rho_1 = 1} = 0. \tag{33}$$

Hence,

$$\begin{aligned} & \frac{df(x)}{dx} > 0, \quad \forall \rho_1 \in (0,1) \\ \text{and } & \frac{df(x)}{dx} < 0, \quad \forall \rho_1 \in (1,\infty). \end{aligned} \tag{34}$$

(2) If $\rho_0 > 1$ and $0 < \rho_1 < 1$, we have

$$\begin{aligned} \frac{dg(\rho_1)}{d\rho_1} &> 2\rho_0^N (1-\rho_1) - 2(1-\rho_1) \\ &= 2(1-\rho_1)\left(\rho_0^N - 1\right) > 0. \end{aligned} \tag{35}$$

Therefore, $g(\rho_1)$ is an increasing function for $\rho_1 \in (0,1)$ and $\rho_0 \in (1,\infty)$. Then,

$$g(1) = \left.\frac{df(x)}{dx}\right|_{\rho_1=1} = 0. \tag{36}$$

Thus,

$$\frac{df(x)}{dx} < 0, \quad \forall \rho_1 \in (0,1). \tag{37}$$

By (34) and (37), for $x \geq 1$, we obtain that $f(x)$ is an increasing function, when $\rho_0 < 1$ and $f(x)$ is a decreasing function when $\rho_0 > 1$ and $\rho_1 \neq 1$. □

Let

$$\begin{aligned} n^*\left(1-\rho_1+\rho_1\rho_0^N-\rho_0^{N+1}\right)(1-\rho_1) \\ -\rho_0^N\rho_1(1-\rho_0)\left(1-\rho_1^{n^*}\right) = \frac{M}{C_1\rho_1}. \end{aligned} \tag{38}$$

In the following theorem, we consider the socially optimal strategy which is specified by the socially optimal threshold n^* such that the social benefit reaches maximum.

Theorem 8. *In fully observable case, the socially optimal strategy is given as follows:*

(1) *If $\rho_1 = 1$ or $\rho_0 = 1$, there is no solution to (38).*

(2) *For $\rho_0 \in (0,1)$,*

(a) *if $f(1) > M/C_1\rho_1$, there is no solution to (38).*
(b) *if $f(1) < M/C_1\rho_1$, there exists a unique solution $n^* > 1$ of (38).*
(c) *if $f(1) = M/C_1\rho_1$, there exists a unique solution $n^* = 1$ of (38).*

(3) *For $\rho_0 \in (1,\infty)$,*

(a) *if $f(1) < M/C_1\rho_1$, there is no solution to (38).*
(b) *if $f(1) > M/C_1\rho_1$, there exists a unique solution $n^* > 1$ of (38).*
(c) *if $f(1) = M/C_1\rho_1$, there exists a unique solution $n^* = 1$ of (38).*

5. Fully Unobservable Case

Now we consider fully unobservable case where arriving passengers are not informed the number of passengers or taxis, but they know the arrival rates of passengers and taxis. The joining probability of an arriving passenger is q and the balking probability is $1-q$. The state transition diagram is shown in Figure 4. Let $\rho_{1,1} := \lambda_1 q/\lambda_2$ and $\rho_{0,1} := \lambda_1 q/\lambda_0$. Suppose the system is stable, so that $\rho_{1,1} = \lambda_1 q/\lambda_2 < 1$.

Let π_i^{fu} be the steady-state probability of state i in the fully unobservable case. In the following proposition, we obtain the stationary distribution by balance equations.

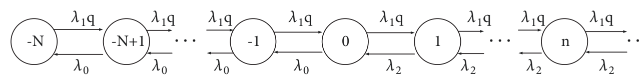

Figure 4: State transition diagram for the fully unobservable case.

Proposition 9. *The stationary distribution in fully unobservable case is given by*

$$\pi_{-N}^{fu} = \frac{(1-\rho_{0,1})(1-\rho_{1,1})}{1-\rho_{1,1}-\rho_{0,1}^{N+1}+\rho_{1,1}\rho_{0,1}^N}; \tag{39}$$

$$\pi_i^{fu} = \frac{\rho_{0,1}^{N+i}(1-\rho_{0,1})(1-\rho_{1,1})}{1-\rho_{1,1}-\rho_{0,1}^{N+1}+\rho_{1,1}\rho_{0,1}^N}, \quad -N \leq i \leq 0; \tag{40}$$

$$\pi_i^{fu} = \frac{\rho_{0,1}\rho_{1,1}^i(1-\rho_{0,1})(1-\rho_{1,1})}{1-\rho_{1,1}-\rho_{0,1}^{N+1}+\rho_{1,1}\rho_{0,1}^N}, \quad i \geq 1. \tag{41}$$

By the same method, we obtain the expected queue length of passengers $\mathbb{E}L_{fu1}$ and the expected queue length of taxis $\mathbb{E}L_{fu2}$, respectively:

$$\begin{aligned} \mathbb{E}L_{fu1} &= \sum_{i=0}^{\infty} i\pi_i^{fu} \\ &= \frac{\rho_{1,1}\rho_{0,1}^N(1-\rho_{0,1})}{(1-\rho_{1,1})\left(1-\rho_{1,1}-\rho_{0,1}^{N+1}+\rho_{1,1}\rho_{0,1}^N\right)}; \end{aligned} \tag{42}$$

$$\begin{aligned} \mathbb{E}L_{fu2} &= \sum_{i=-N}^{0}(-i)\pi_i^{fu} \\ &= \left(N - \frac{\rho_{0,1}\left(1-\rho_{0,1}^N\right)}{1-\rho_{0,1}}\right)\frac{1-\rho_{1,1}}{1-\rho_{1,1}-\rho_{0,1}^{N+1}+\rho_{1,1}\rho_{0,1}^N}. \end{aligned} \tag{43}$$

Obviously, the effective arrival rate of passengers is $\lambda_1 q$. Let the effective arrival rate of taxis be α. Then

$$\alpha = \sum_{i=-N+1}^{0}\lambda_0\pi_i^{fu} + \sum_{i=1}^{\infty}\lambda_2\pi_i^{fu} = \lambda_1 q. \tag{44}$$

The expected waiting time of a passenger $\mathbb{E}W_{fu1}(q)$ and the expected waiting time of a taxi $\mathbb{E}W_{fu2}(q)$ are, respectively,

$$\begin{aligned} \mathbb{E}W_{fu1}(q) &= \frac{\mathbb{E}L_{fu1}}{\lambda_1 q} \\ \text{and } \mathbb{E}W_{fu2}(q) &= \frac{\mathbb{E}L_{fu2}}{\lambda_1 q}. \end{aligned} \tag{45}$$

5.1. Equilibrium Strategies of Passengers. We now consider the equilibrium strategy of an arriving passenger. By (45), we get the utility of passengers

$$U_{fu}(q) = R - p_1 - C_1\mathbb{E}W_{fu1}(q). \tag{46}$$

The equilibrium strategy of an arriving passenger is specified by the joining probability, denoted by q_e such that a passenger will choose to take a taxi with probability q_e and balk with probability $1-q_e$.

Theorem 10. *In the unobservable queue case, the equilibrium strategy for each passenger is as follows:*

$$q_e = \begin{cases} 0, & if\ R < p_1; \\ q_e^*, & if\ p_1 \le R \le p_1 + C_1 \dfrac{\rho_1 \rho_0^N (1-\rho_0)}{\lambda_1 (1-\rho_1)(1-\rho_1-\rho_0^{N+1}+\rho_1\rho_0^N)}; \\ 1, & if\ R > p_1 + C_1 \dfrac{\rho_1 \rho_0^N (1-\rho_0)}{\lambda_1 (1-\rho_1)(1-\rho_1-\rho_0^{N+1}+\rho_1\rho_0^N)}, \end{cases} \quad (47)$$

where q_e^* is the unique solution of equation $U_{fu}(q_e^*) = 0$.

Proof. By (45), we know that $W_{fu}(q)$ is an increasing function for $q \in [0,1]$; thus, $U_{fu}(q)$ is a decreasing function for $q \in [0,1]$.

If $R < p_1$, $U_{fu}(q) < 0$, then the best response for an passenger is balking, so that $q_e = 0$.

If $R > p_1 + C_1(\rho_1\rho_0^N(1-\rho_0)/\lambda_1(1-\rho_1)(1-\rho_1-\rho_0^{N+1}+\rho_1\rho_0^N))$, $U_{fu}(q) > 0$; hence, his best choice is $q_e = 1$.

Since $U_{fu}(q)$ is a decreasing function for q, so that there exists a unique solution to the equation $U_{fu}(q_e^*) = 0$ within $(0,1)$ if $p_1 \le R \le p_1 + C_1(\rho_1\rho_0^N(1-\rho_0)/\lambda_1(1-\rho_1)(1-\rho_1-\rho_0^{N+1}+\rho_1\rho_0^N))$. □

5.2. Social Benefit Function. In this section, we study the social benefit function in fully unobservable case. By (42), (43), and (44), we obtain the social benefit function

$$\begin{aligned} S_{fu} &= \lambda_1 q R - C_1 \\ &\quad \cdot \frac{\rho_{1,1}\rho_{0,1}^N (1-\rho_{0,1})}{(1-\rho_{1,1})(1-\rho_{1,1}-\rho_{0,1}^{N+1}+\rho_{1,1}\rho_{0,1}^N)} \\ &\quad - C_2 \left(N - \frac{\rho_{0,1}(1-\rho_{0,1}^N)}{1-\rho_{0,1}} \right) \\ &\quad \cdot \frac{1-\rho_{1,1}}{1-\rho_{1,1}-\rho_{0,1}^{N+1}+\rho_{1,1}\rho_{0,1}^N}. \end{aligned} \quad (48)$$

Since the social benefit function is complicated, the first and second order derivatives are too difficult to analyze. So, we study the socially optimal strategies which is represented by a joining probability (is called socially optimal joining probability) such that the social benefit reaches maximum by numerical experiments.

6. Numerical Experiments

In this section, we will show some tables to find the relationship between the equilibrium strategies and socially optimal strategies of arriving passengers under three different information levels (fully observable case, almost unobservable case, and fully unobservable case). Moreover, we will present figures to compare the socially optimal joining probabilities in the almost unobservable case with those in the fully unobservable case. We also find the effect of three different information levels as well as several parameters: taxi buffer size N, arrival rate of passengers λ_1, the low arrival rate of taxis λ_0, and the high arrival rate of taxis λ_2 on social benefit.

We first study the equilibrium joining probabilities in almost unobservable and fully unobservable cases, respectively. These results are shown in Figure 5. From the left of Figure 5, we can see that the equilibrium joining probabilities in the fully unobservable case is increasing as N and λ_0 increase, respectively. It is obvious that the equilibrium joining probabilities in the fully unobservable case is always larger than that in the almost unobservable case. From the right of Figure 5, we know that the equilibrium joining probability in the almost unobservable case decreases with respect to λ_1. Moreover, the equilibrium joining probability in the almost unobservable case is increasing as λ_2 increases.

We second consider the socially optimal strategies in three different information levels. These results are shown in Figures 6 and 7. In the left of Figure 6, we find that socially optimal joining probabilities in the almost unobservable case are decreasing as N increases. When $\lambda_0 < \lambda_2$, the socially optimal joining probabilities is larger than that in the case $\lambda_0 = \lambda_2$. From the right of Figure 6, we can observe that socially optimal joining probabilities in the fully unobservable case is increasing with respect to N. Furthermore, the socially optimal joining probabilities in the case $\lambda_0 < \lambda_2$ is less than that in the case $\lambda_0 = \lambda_2$. In Figure 7, we study the socially optimal threshold in two cases. When $\lambda_2 > \lambda_1$, the socially optimal threshold is increasing as N increases. The socially optimal threshold in the case $\lambda_0 = \lambda_2$ is larger than that in the case $\lambda_0 < \lambda_2$. When $\lambda_2 < \lambda_1$, the relationship is opposite.

We then consider the optimal social benefit in the fully unobservable and fully observable cases, respectively. These results are shown in Figure 8. In the left of Figure 8, we find that the optimal social benefit in the observable case is decreasing with respect to N. If N is less than 5, the best choice is $\lambda_0 = \lambda_2$ which is the case in [18]. If N is larger than 5, the social benefit in the case $\lambda_0 < \lambda_2$ is larger than that in the case $\lambda_0 = \lambda_2$. From this behavior, we know that our model can be used to improve the optimal social benefit. The right of Figure 8 shows the relationship between the social benefit and the arrival rates of taxis.

In the last numerical example, we investigate the effect of three different information levels as well as several parameters

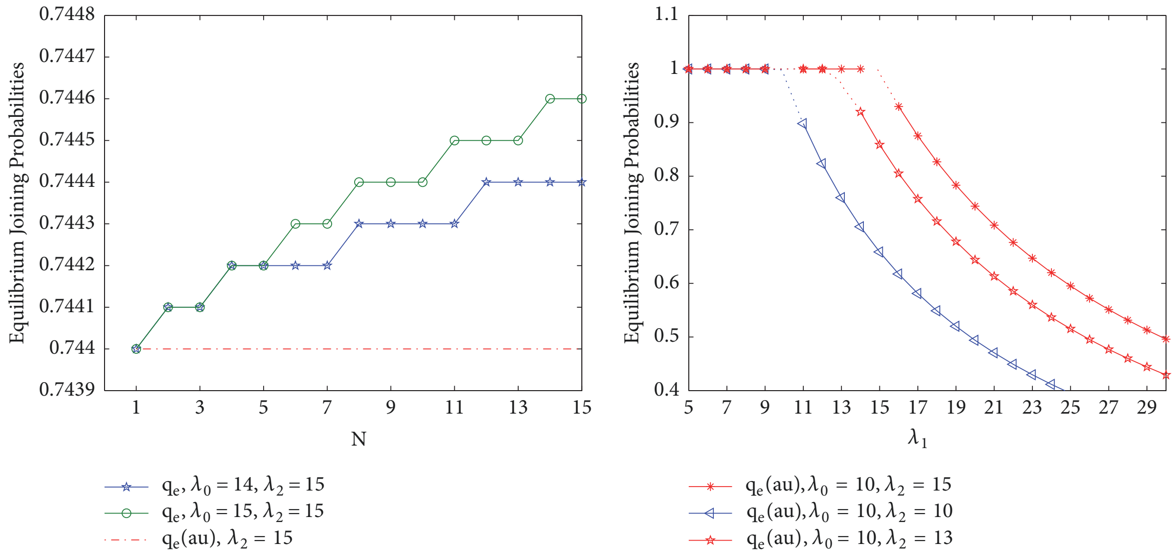

Figure 5: Left: equilibrium joining probabilities for the almost and fully unobservable cases vs. N for $R = 30$; $p_1 = 5$; $C_1 = 3$; $\lambda_1 = 20$. Right: rquilibrium joining probabilities for the almost and fully unobservable cases vs. λ_1 for $R = 30$, $p_1 = 5$, $C_1 = 3$, $N = 5$.

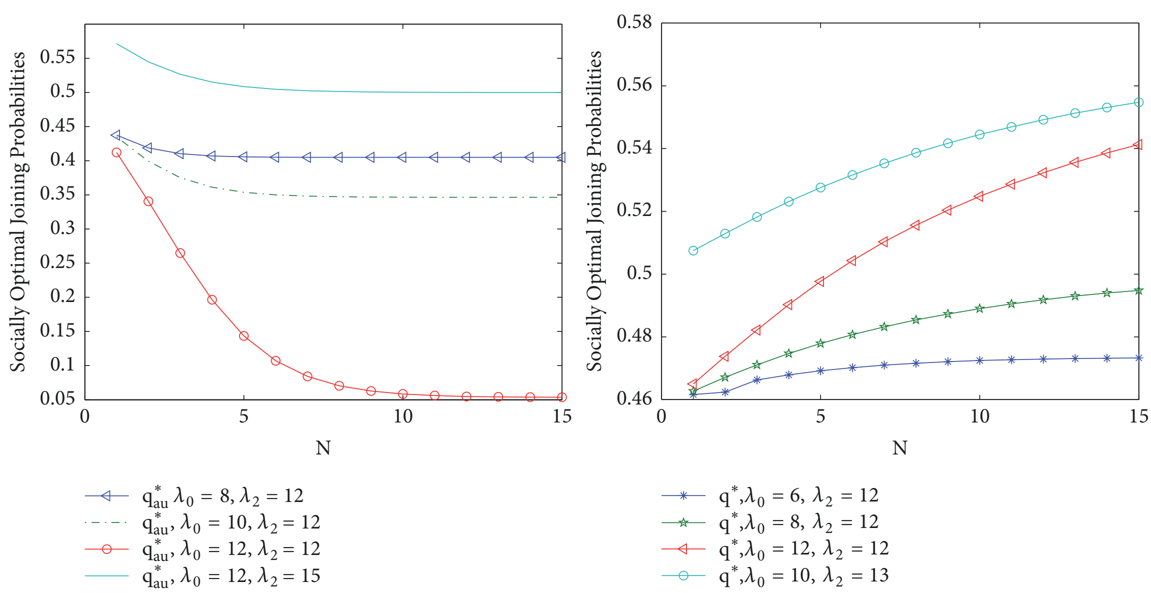

Figure 6: Left: socially optimal joining probabilities for the almost unobservable case vs. N for $R = 30$, $C_1 = 15$, $C_2 = 10$, $\lambda_1 = 20$. Right: socially optimal joining probabilities for the unobservable case vs. N for $R = 30$, $C_1 = 20$, $C_2 = 15$, $\lambda_1 = 20$.

(taxi buffer size N, the arrival rate of passengers λ_1, and the low arrival rate of taxis λ_0) on social benefit. These results are shown in Figures 9, 10, and 11. From the left of Figure 9, when $\lambda_1 > \lambda_2$, we can see that the social benefit in fully observable case and almost unobservable case increase as λ_0 increases. In the fully unobservable case, we can see that the social benefit function is unimodal; then, we get the optimal low taxi arrival rate. In the right of Figure 9, when $\lambda_1 < \lambda_2$, the social benefits in three cases are convex functions for λ_0. However, social benefits in three cases are basically the same. Moreover, from Figure 9 we know that the dynamic taxi control policy can improve the social benefit in several cases compared with the same taxi arrival rate case. In the left of Figure 10, when $\lambda_1 < \lambda_2$, we know that the social benefit in three cases is not much difference. When $\lambda_1 > \lambda_2$, the social benefits in fully observable case and almost unobservable case are larger than that in fully unobservable case. From this behavior we know that providing the taxi stand information is an efficiency policy to improve the social benefit. In the right of Figures 10 and 11, the social benefit functions are convex in three cases. In other words, we obtain an optimal taxi buffer size which maximizes the social benefit. From the left of Figure 11, it can be seen that the social benefit in the fully observable case is more than that in the fully unobservable case. Moreover, the gap between the social benefit in the almost unobservable case and fully unobservable case becomes smaller as N

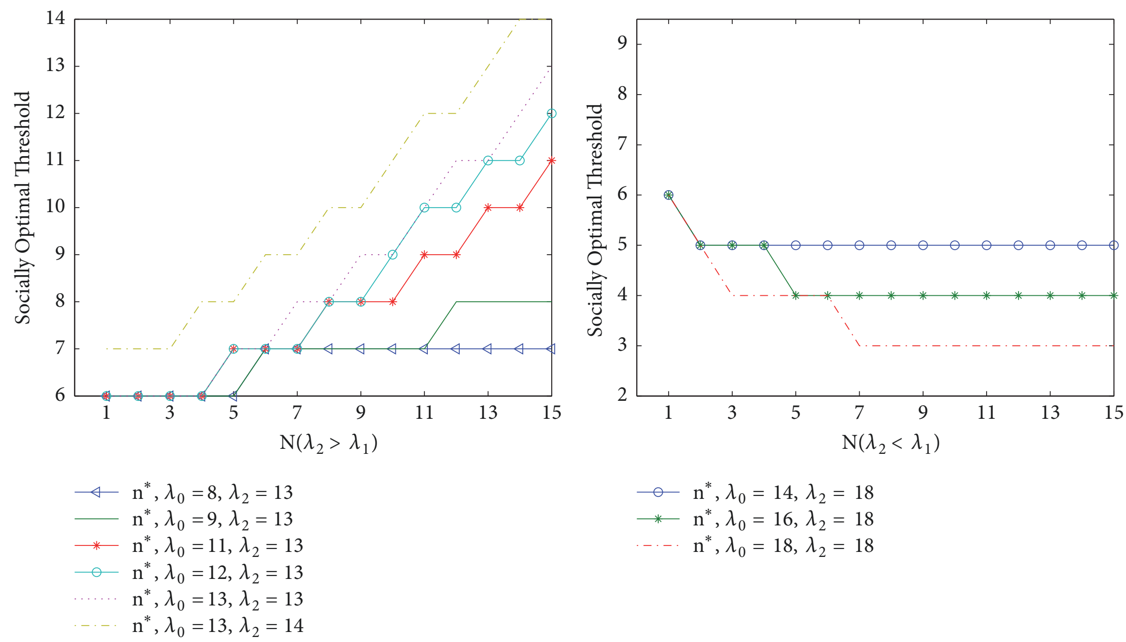

FIGURE 7: Left: socially optimal threshold in the observable case vs. $N(\lambda_2 > \lambda_1)$ for $R = 30$, $C_1 = 20$, $C_2 = 15$, $\lambda_1 = 10$. Right: socially optimal threshold in the observable case vs. $N(\lambda_2 < \lambda_1)$ for $R = 30$; $C_1 = 15$; $C_2 = 10$; $\lambda_1 = 20$.

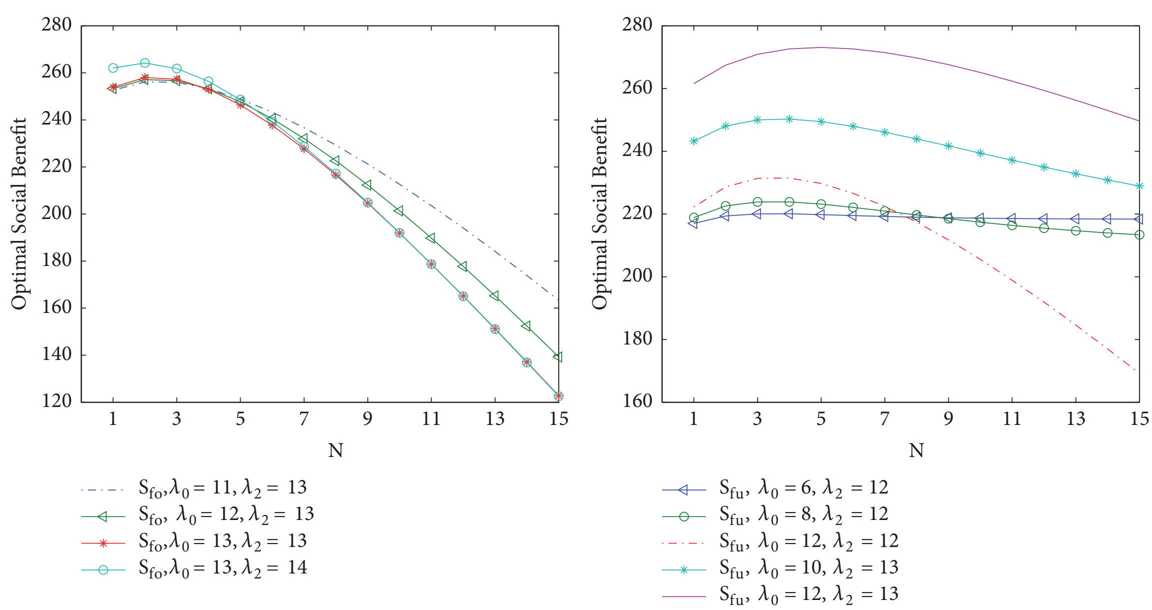

FIGURE 8: Left: optimal social benefit in observable case vs. N for $R = 30$, $C_1 = 10$, $C_2 = 10$, $\lambda_1 = 15$. Right: optimal social benefit in the unobservable case vs. N for $R = 30$, $C_1 = 10$, $C_2 = 10$, $\lambda_1 = 15$.

increases. This behavior indicates that providing the queue length of passengers and taxis can improve the social benefit. From the right of Figure 11, when $\lambda_0 < \lambda_2 < \lambda_1$, we know that the social benefits in the fully observable case and almost unobservable case are larger than that in the fully unobservable case. However, the gap between the social benefit in the fully observable case and almost unobservable case becomes smaller as N increases. This phenomenon indicates that when $\lambda_0 < \lambda_2 < \lambda_1$, if the cost of providing the fully information of the system is large, announcing the state of taxis can also greatly improve the social benefit.

7. Conclusions

In this paper, we study the passenger-taxi service system with dynamic taxi control by a double-ended Markovian queueing system. The taxi control is to improve the arrival rate of taxis

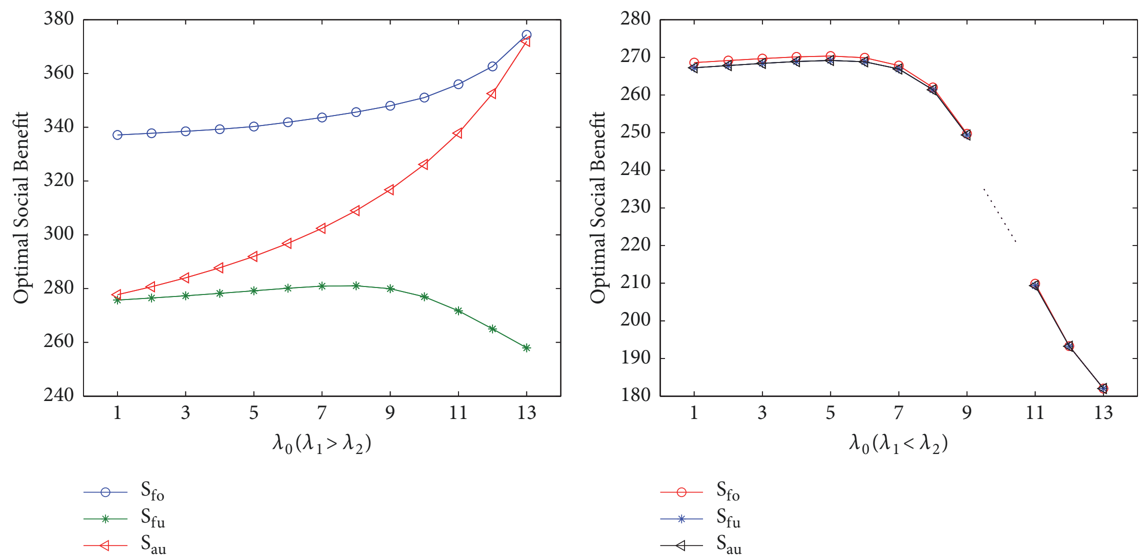

FIGURE 9: Left: Optimal social benefit under three different information levels vs. $\lambda_0(\lambda_1 > \lambda_2)$ for $R = 30$, $C_1 = 10$, $C_2 = 10$, $\lambda_1 = 20$, $\lambda_2 = 13$, $N = 15$; Right: Optimal social benefit under three different information levels vs. $\lambda_0(\lambda_1 < \lambda_2)$ for $R = 30$, $C_1 = 10$, $C_2 = 10$, $\lambda_1 = 10$, $\lambda_2 = 13$, $N = 15$.

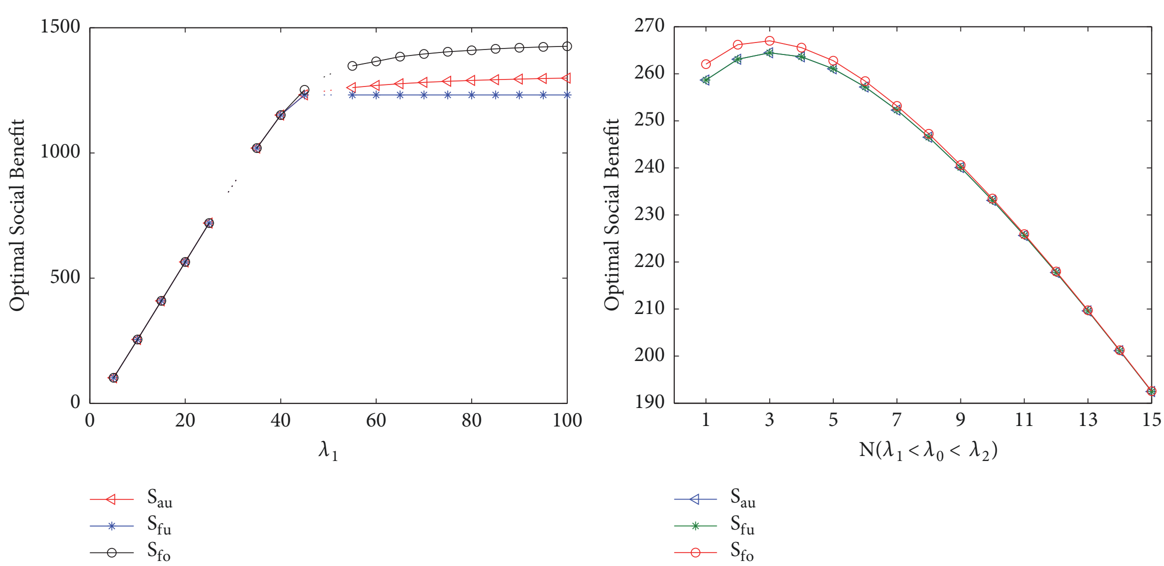

FIGURE 10: Left: optimal social benefit under three different information levels vs. λ_1 for $R = 30$, $C_1 = 15$, $C_2 = 10$, $\lambda_0 = 30$, $\lambda_2 = 50$, $N = 5$. Right: optimal social benefit under three different information levels vs. $N(\lambda_1 < \lambda_0 < \lambda_2)$ for $R = 30$, $C_1 = 15$, $C_2 = 10$, $\lambda_1 = 10$, $\lambda_0 = 12$, $\lambda_2 = 13$.

when the queue length of passengers is large so as to reduce delays and decrease it at times of increased queue length of taxis so as to reduce the costs of taxis. We derive the passenger's and taxi's expected waiting times in three different information levels. By the reward-cost structure, we analyze the strategic behavior of arriving passengers from their individual utility and social benefit under three different information levels.

We study the (Nash) equilibrium strategies and socially optimal strategies in three different information levels, respectively. We obtain the selfishly and socially optimal thresholds of passengers in the fully observable case. In the almost unobservable case and the fully unobservable case, we consider the selfishly and socially optimal joining probabilities for arriving passengers. Furthermore, the numerical results showed that dynamic taxi control policy can greatly improve the social benefit compared with the model with the same arrival rate of taxis.

In order to reduce the waiting time of passengers, a possible extension to this work can be to consider a model with priority. In another direction, the extension of the study to non-Markovian models with general interarrival times seems also important.

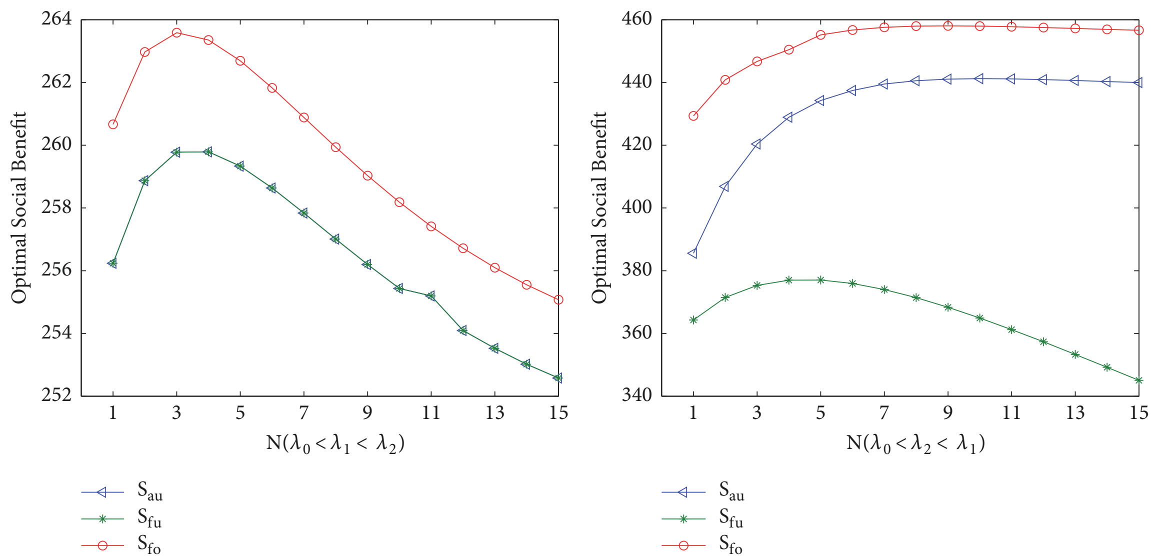

Figure 11: Left: optimal social benefit under three different information levels vs. $N(\lambda_0 < \lambda_1 < \lambda_2)$ for $R = 30$, $C_1 = 15$, $C_2 = 10$, $\lambda_1 = 10$, $\lambda_0 = 8$, $\lambda_2 = 13$. Right: optimal social benefit under three different information levels vs. $N(\lambda_0 < \lambda_2 < \lambda_1)$ for $R = 30$, $C_1 = 20$, $C_2 = 15$, $\lambda_1 = 20$, $\lambda_0 = 16$, $\lambda_2 = 18$.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

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